

ELECTRIC CHARGE AND ELECTRIC FIELD

- 21.1. (a) IDENTIFY and SET UP:** Use the charge of one electron (-1.602×10^{-19} C) to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e} = \frac{-3.20 \times 10^{-9} \text{ C}}{-1.602 \times 10^{-19} \text{ C/electron}} = 2.00 \times 10^{10} \text{ electrons.}$$

(b) IDENTIFY and SET UP: Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is 207×10^{-3} kg/mol, so the number of moles in 8.00×10^{-3} kg is

$$n = \frac{m_{\text{tot}}}{M} = \frac{8.00 \times 10^{-3} \text{ kg}}{207 \times 10^{-3} \text{ kg/mol}} = 0.03865 \text{ mol. } N_A \text{ (Avogadro's number) is the number of atoms in 1 mole, so the}$$

number of lead atoms is $N = nN_A = (0.03865 \text{ mol})(6.022 \times 10^{23} \text{ atoms/mol}) = 2.328 \times 10^{22}$ atoms. The number of excess electrons per lead atom is $\frac{2.00 \times 10^{10} \text{ electrons}}{2.328 \times 10^{22} \text{ atoms}} = 8.59 \times 10^{-13}$.

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

- 21.2. IDENTIFY:** The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e = 1.60 \times 10^{-19}$ C.

EXECUTE: The rate of charge flow is 20,000 C/s and $t = 100 \mu\text{s} = 1.00 \times 10^{-4}$ s.

$$Q = (20,000 \text{ C/s})(1.00 \times 10^{-4} \text{ s}) = 2.00 \text{ C. The number of electrons is } n_e = \frac{Q}{1.60 \times 10^{-19} \text{ C}} = 1.25 \times 10^{19}.$$

EVALUATE: This is a very large amount of charge and a large number of electrons.

- 21.3. IDENTIFY:** From your mass estimate the number of protons in your body. You have an equal number of electrons.

SET UP: Assume a body mass of 70 kg. The charge of one electron is -1.60×10^{-19} C.

EXECUTE: The mass is primarily protons and neutrons of $m = 1.67 \times 10^{-27}$ kg. The total number of protons and neutrons is $n_{\text{p and n}} = \frac{70 \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 4.2 \times 10^{28}$. About one-half are protons, so $n_p = 2.1 \times 10^{28} = n_e$. The number of

electrons is about 2.1×10^{28} . The total charge of these electrons is

$$Q = (-1.60 \times 10^{-19} \text{ C/electron})(2.1 \times 10^{28} \text{ electrons}) = -3.35 \times 10^9 \text{ C.}$$

EVALUATE: This is a huge amount of negative charge. But your body contains an equal number of protons and your net charge is zero. If you carry a net charge, the number of excess or missing electrons is a very small fraction of the total number of electrons in your body.

- 21.4. IDENTIFY:** Use the mass m of the ring and the atomic mass M of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_A = 6.02 \times 10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms

$$\text{is } N_A n = (6.02 \times 10^{23} \text{ atoms/mol}) \left(\frac{17.7 \text{ g}}{197 \text{ g/mol}} \right) = 5.41 \times 10^{22} \text{ atoms. The number of protons is}$$

$$n_p = (79 \text{ protons/atom})(5.41 \times 10^{22} \text{ atoms}) = 4.27 \times 10^{24} \text{ protons. } Q = (n_p)(1.60 \times 10^{-19} \text{ C/proton}) = 6.83 \times 10^5 \text{ C.}$$

(b) The number of electrons is $n_e = n_p = 4.27 \times 10^{24}$.

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

- 21.5. IDENTIFY:** Apply $F = \frac{k|q_1q_2|}{r^2}$ and solve for r .

SET UP: $F = 650 \text{ N}$.

EXECUTE: $r = \sqrt{\frac{k|q_1q_2|}{F}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.0 \text{ C})^2}{650 \text{ N}}} = 3.7 \times 10^3 \text{ m} = 3.7 \text{ km}$

EVALUATE: Charged objects typically have net charges much less than 1 C.

- 21.6. IDENTIFY:** Apply Coulomb's law and calculate the net charge q on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$. This gives $|q| = \sqrt{4\pi\epsilon_0 Fr^2} = \sqrt{4\pi\epsilon_0 (4.57 \times 10^{-21} \text{ N})(0.200 \text{ m})^2} = 1.43 \times 10^{-16} \text{ C}$. And

therefore, the total number of electrons required is $n = |q|/e = (1.43 \times 10^{-16} \text{ C})/(1.60 \times 10^{-19} \text{ C/electron}) = 890 \text{ electrons}$.

EVALUATE: Each sphere has 890 excess electrons and each sphere has a net negative charge. The two like charges repel.

- 21.7. IDENTIFY:** Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

(a) EXECUTE: $q_1 = q_2 = q$

$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2}$ so $q = r \sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150 \text{ m} \sqrt{\frac{0.220 \text{ N}}{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 7.42 \times 10^{-7} \text{ C (on each)}$

(b) $q_2 = 4q_1$

$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2}$ so $q_1 = r \sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \frac{1}{2} r \sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \frac{1}{2} (7.42 \times 10^{-7} \text{ C}) = 3.71 \times 10^{-7} \text{ C}$.

And then $q_2 = 4q_1 = 1.48 \times 10^{-6} \text{ C}$.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the sphere have equal charges or not.

- 21.8. IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_A = 6.02 \times 10^{23} \text{ atoms/mol}$. $|q| = n'_e e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$n_e = n_p = (13)(N_A) \left(\frac{0.0250 \text{ kg}}{0.026982 \text{ kg/mol}} \right) = 7.25 \times 10^{24} \text{ electrons}$.

(b) For a force of $1.00 \times 10^4 \text{ N}$ to act between the spheres, $F = 1.00 \times 10^4 \text{ N} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$. This gives

$|q| = \sqrt{4\pi\epsilon_0 (1.00 \times 10^4 \text{ N})(0.0800 \text{ m})^2} = 8.43 \times 10^{-4} \text{ C}$. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27 \times 10^{15} \text{ electrons}$.

(c) $n'_e/n_e = 7.27 \times 10^{-10}$.

EVALUATE: When ordinary objects receive a net charge the fractional change in the total number of electrons in the object is very small.

- 21.9. IDENTIFY:** Apply $F = ma$, with $F = k \frac{|q_1q_2|}{r^2}$.

SET UP: $a = 25.0g = 245 \text{ m/s}^2$. An electron has charge $-e = -1.60 \times 10^{-19} \text{ C}$.

EXECUTE: $F = ma = (8.55 \times 10^{-3} \text{ kg})(245 \text{ m/s}^2) = 2.09 \text{ N}$. The spheres have equal charges q , so $F = k \frac{q^2}{r^2}$ and

$|q| = r \sqrt{\frac{F}{k}} = (0.150 \text{ m}) \sqrt{\frac{2.09 \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 2.29 \times 10^{-6} \text{ C}$. $N = \frac{|q|}{e} = \frac{2.29 \times 10^{-6} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 1.43 \times 10^{13} \text{ electrons}$. The

charges on the spheres have the same sign so the electrical force is repulsive and the spheres accelerate away from each other.

EVALUATE: As the spheres move apart the repulsive force they exert on each other decreases and their acceleration decreases.

- 21.10. (a) IDENTIFY:** The electrical attraction of the proton gives the electron an acceleration equal to the acceleration due to gravity on earth.

SET UP: Coulomb's law gives the force and Newton's second law gives the acceleration this force produces.

$$ma = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \quad \text{and} \quad r = \sqrt{\frac{e^2}{4\pi\epsilon_0 ma}}.$$

$$\text{EXECUTE: } r = \sqrt{\frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(9.80 \text{ m/s}^2)}} = 5.08 \text{ m}$$

EVALUATE: The electron needs to be about 5 m from a single proton to have the same acceleration as it receives from the gravity of the entire earth.

(b) IDENTIFY: The force on the electron comes from the electrical attraction of all the protons in the earth.

SET UP: First find the number n of protons in the earth, and then find the acceleration of the electron using Newton's second law, as in part (a).

$$n = m_E/m_p = (5.97 \times 10^{24} \text{ kg})/(1.67 \times 10^{-27} \text{ kg}) = 3.57 \times 10^{51} \text{ protons.}$$

$$a = F/m = \frac{\frac{1}{4\pi\epsilon_0} \frac{|q_p q_e|}{R_E^2}}{m_e} = \frac{\frac{1}{4\pi\epsilon_0} n e^2}{m_e R_E^2}.$$

EXECUTE: $a = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.57 \times 10^{51})(1.60 \times 10^{-19} \text{ C})^2 / [(9.11 \times 10^{-31} \text{ kg})(6.38 \times 10^6 \text{ m})^2] = 2.22 \times 10^{40} \text{ m/s}^2$. One can ignore the gravitation force since it produces an acceleration of only 9.8 m/s^2 and hence is much much less than the electrical force.

EVALUATE: With the electrical force, the acceleration of the electron would nearly 10^{40} times greater than with gravity, which shows how strong the electrical force is.

- 21.11. IDENTIFY:** In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration: $a = F/m = (1/4\pi\epsilon_0)(e^2/r^2)/m$.

EXECUTE: (a) $a = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2 / [(0.00250 \text{ m})^2(1.67 \times 10^{-27} \text{ kg})] = 2.21 \times 10^4 \text{ m/s}^2$.

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

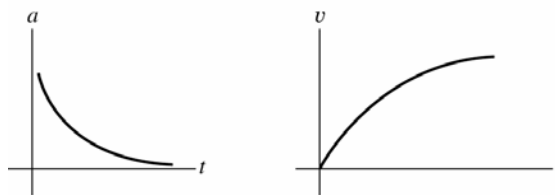


Figure 21.11

- 21.12. IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: (a) $F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$. This gives $0.200 \text{ N} = \frac{1}{4\pi\epsilon_0} \frac{(0.550 \times 10^{-6} \text{ C})|q_2|}{(0.30 \text{ m})^2}$ and $|q_2| = +3.64 \times 10^{-6} \text{ C}$. The

force is attractive and $q_1 < 0$, so $q_2 = +3.64 \times 10^{-6} \text{ C}$.

(b) $F = 0.200 \text{ N}$. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. IDENTIFY: Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k \frac{|q_2 q_3|}{r_2^2}$ and is in the $+x$ direction. $\vec{F}_1$ must be in

the $-x$ direction, so q_1 must be positive. $F_1 = F_2$ gives $k \frac{|q_1||q_3|}{r_1^2} = k \frac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2| \left(\frac{r_1}{r_2} \right)^2 = (3.00 \text{ nC}) \left(\frac{2.00 \text{ cm}}{4.00 \text{ cm}} \right)^2 = 0.750 \text{ nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_2 .

21.14. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on Q .

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x -components cancel. We only need the y -components, and each charge contributes equally.

$$F_{1y} = F_{2y} = -\frac{1}{4\pi\epsilon_0} \frac{(2.0 \times 10^{-6} \text{ C})(4.0 \times 10^{-6} \text{ C})}{(0.500 \text{ m})^2} \sin \alpha = -0.173 \text{ N (since } \sin \alpha = 0.600). \text{ Therefore, the total force is}$$

$$2F = 0.35 \text{ N, in the } -y\text{-direction}.$$

EVALUATE: If q_1 is $-2.0 \mu\text{C}$ and q_2 is $+2.0 \mu\text{C}$, then the net force is in the $+y$ -direction.

21.15. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on q_1 .

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the $+x$ -direction.

$$\text{EXECUTE: } F_2 = k \frac{|q_1 q_2|}{r_{12}^2} = 6.749 \times 10^{-5} \text{ N, } F_3 = k \frac{|q_1 q_3|}{r_{13}^2} = 1.124 \times 10^{-4} \text{ N. } F = F_2 + F_3 = 1.8 \times 10^{-4} \text{ N.}$$

$$F = 1.8 \times 10^{-4} \text{ N and is in the } +x\text{-direction.}$$

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1 \text{ on } 3} = -\vec{F}_{3 \text{ on } 1}$, as required by Newton's third law.

21.16. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on q_2 .

SET UP: $\vec{F}_{2 \text{ on } 1}$ is in the $+y$ -direction.

$$\text{EXECUTE: } F_{2 \text{ on } 1} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.0 \times 10^{-6} \text{ C})(2.0 \times 10^{-6} \text{ C})}{(0.60 \text{ m})^2} = 0.100 \text{ N. } (F_{2 \text{ on } 1})_x = 0 \text{ and}$$

$$(F_{2 \text{ on } 1})_y = +0.100 \text{ N. } F_{Q \text{ on } 1} \text{ is equal and opposite to } F_{1 \text{ on } Q} \text{ (Example 21.4), so } (F_{Q \text{ on } 1})_x = -0.23 \text{ N and}$$

$$(F_{Q \text{ on } 1})_y = 0.17 \text{ N. } F_x = (F_{2 \text{ on } 1})_x + (F_{Q \text{ on } 1})_x = -0.23 \text{ N. } F_y = (F_{2 \text{ on } 1})_y + (F_{Q \text{ on } 1})_y = 0.100 \text{ N} + 0.17 \text{ N} = 0.27 \text{ N.}$$

$$\text{The magnitude of the total force is } F = \sqrt{(0.23 \text{ N})^2 + (0.27 \text{ N})^2} = 0.35 \text{ N. } \tan^{-1} \frac{0.23}{0.27} = 40^\circ, \text{ so } \vec{F} \text{ is}$$

40° counterclockwise from the $+y$ axis, or 130° counterclockwise from the $+x$ axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.17. IDENTIFY and SET UP: Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1 . Add these forces as vectors to get the net force. The target variable is the x -coordinate of q_3 .

EXECUTE: $\vec{F}_2$ is in the x -direction.

$$F_2 = k \frac{|q_1 q_2|}{r_{12}^2} = 3.37 \text{ N, so } F_{2x} = +3.37 \text{ N}$$

$$F_x = F_{2x} + F_{3x} \text{ and } F_x = -7.00 \text{ N}$$

$$F_{3x} = F_x - F_{2x} = -7.00 \text{ N} - 3.37 \text{ N} = -10.37 \text{ N}$$

For F_{3x} to be negative, q_3 must be on the $-x$ -axis.

$$F_3 = k \frac{|q_1 q_3|}{x^2}, \text{ so } |x| = \sqrt{\frac{k|q_1 q_3|}{F_3}} = 0.144 \text{ m, so } x = -0.144 \text{ m}$$

EVALUATE: q_2 attracts q_1 in the $+x$ -direction so q_3 must attract q_1 in the $-x$ -direction, and q_3 is at negative x .

21.18. IDENTIFY: Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract. Let $\vec{F}_{21}$ be the force that q_2 exerts on q_1 and let $\vec{F}_{31}$ be the force that q_3 exerts on q_1 .

EXECUTE: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$ direction. Calculating the two forces:

$$F_{21} = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r_{12}^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})}{(0.200 \text{ m})^2} = 3.375 \text{ N}, \text{ in the } +x \text{ direction.}$$

$$F_{31} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.00 \times 10^{-6} \text{ C})(8.00 \times 10^{-6} \text{ C})}{r_{13}^2} = \frac{0.216 \text{ N} \cdot \text{m}^2}{r_{13}^2}, \text{ in the } -x \text{ direction.}$$

$$\text{We need } F_x = F_{21} - F_{31} = -7.00 \text{ N}, \text{ so } 3.375 \text{ N} - \frac{0.216 \text{ N} \cdot \text{m}^2}{r_{13}^2} = -7.00 \text{ N}. \quad r_{13} = \sqrt{\frac{0.216 \text{ N} \cdot \text{m}^2}{3.375 \text{ N} + 7.00 \text{ N}}} = 0.144 \text{ m}. \quad q_3$$

is at $x = 0.144 \text{ m}$.

EVALUATE: $F_{31} = 10.4 \text{ N}$. F_{31} is larger than F_{21} , because $|q_3|$ is larger than $|q_2|$ and also because r_{13} is less than r_{12} .

21.19. IDENTIFY: Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.19a.

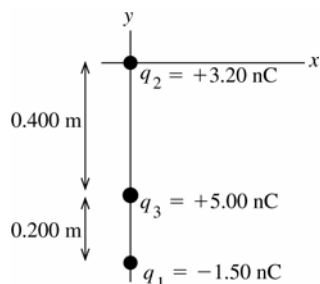


Figure 21.19a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.19b.

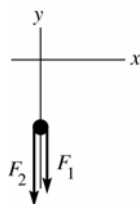


Figure 21.19b

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r_{23}^2}$$

$$F_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.50 \times 10^{-9} \text{ C})(5.00 \times 10^{-9} \text{ C})}{(0.200 \text{ m})^2} = 1.685 \times 10^{-6} \text{ N}$$

$$F_2 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(3.20 \times 10^{-9} \text{ C})(5.00 \times 10^{-9} \text{ C})}{(0.400 \text{ m})^2} = 8.988 \times 10^{-7} \text{ N}$$

The resultant force is $\vec{R} = \vec{F}_1 + \vec{F}_2$.

$$R_x = 0.$$

$$R_y = F_1 + F_2 = 1.685 \times 10^{-6} \text{ N} + 8.988 \times 10^{-7} \text{ N} = 2.58 \times 10^{-6} \text{ N}.$$

The resultant force has magnitude $2.58 \times 10^{-6} \text{ N}$ and is in the $-y$ -direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is repulsive.

21.20. IDENTIFY: Apply $F = k \frac{qq'}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2 .

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.20.

EXECUTE: $F_1 = k \frac{|q_1 q_3|}{r_1^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(4.00 \times 10^{-9} \text{ C})(0.600 \times 10^{-9} \text{ C})}{(0.200 \text{ m})^2} = 5.394 \times 10^{-7} \text{ N}.$

$F_2 = k \frac{|q_2 q_3|}{r_2^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(5.00 \times 10^{-9} \text{ C})(0.600 \times 10^{-9} \text{ C})}{(0.300 \text{ m})^2} = 2.997 \times 10^{-7} \text{ N}.$

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40 \times 10^{-7} \text{ N}.$ The net force has magnitude $2.40 \times 10^{-7} \text{ N}$ and is in the $+x$ direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

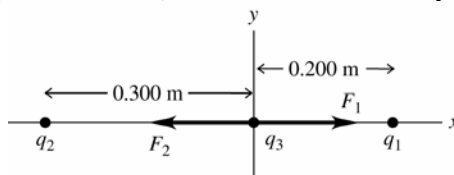


Figure 21.20

21.21. IDENTIFY: Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$.

EXECUTE: (a) The two forces on $-Q$ are shown in Figure 21.21a. $\sin \theta = \frac{a}{(a^2 + x^2)^{1/2}}$ and $r = (a^2 + x^2)^{1/2}$ is the distance between q and $-Q$ and between $-q$ and $-Q$.

(b) $F_x = F_{1x} + F_{2x} = 0$. $F_y = F_{1y} + F_{2y} = 2 \frac{1}{4\pi\epsilon_0} \frac{qQ}{(a^2 + x^2)} \sin \theta = \frac{1}{4\pi\epsilon_0} \frac{2qQa}{(a^2 + x^2)^{3/2}}.$

(c) At $x = 0$, $F_y = \frac{1}{4\pi\epsilon_0} \frac{2qQ}{a^2}$, in the $+y$ direction.

(d) The graph of F_y versus x is given in Figure 21.21b.

EVALUATE: $F_x = 0$ for all values of x and $F_y > 0$ for all x .

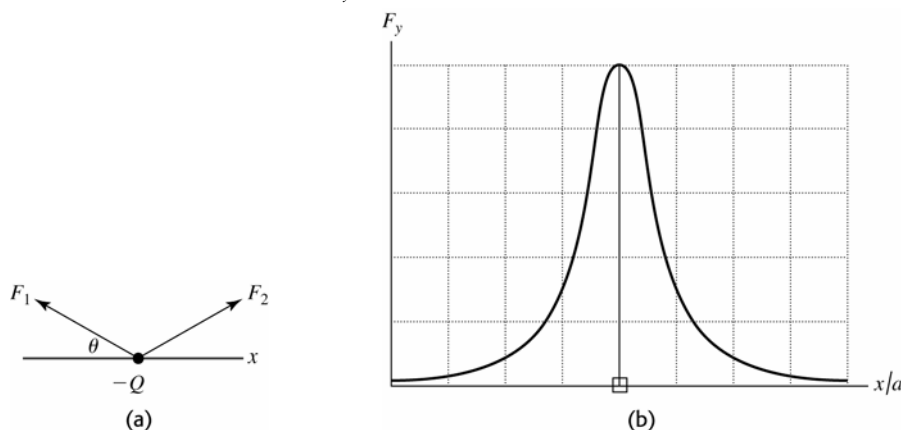


Figure 21.21

21.22. IDENTIFY: Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$. The distance between each charge q and Q is $r = (a^2 + x^2)^{1/2}$. $\cos \theta = \frac{|x|}{(a^2 + x^2)^{1/2}}.$

EXECUTE: (a) The two forces on $-Q$ are shown in Figure 21.22a.

(b) When $x > 0$, F_{1x} and F_{2x} are negative. $F_x = F_{1x} + F_{2x} = -2 \frac{1}{4\pi\epsilon_0} \frac{qQ}{(a^2 + x^2)} \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{-2qQx}{(a^2 + x^2)^{3/2}}.$ When

$x < 0$, F_{1x} and F_{2x} are positive and the same expression for F_x applies. $F_y = F_{1y} + F_{2y} = 0.$

(c) At $x = 0$, $F_x = 0.$

(d) The graph of F_x versus x is sketched in Figure 21.22b.

EVALUATE: The direction of the net force on $-Q$ is always toward the origin.

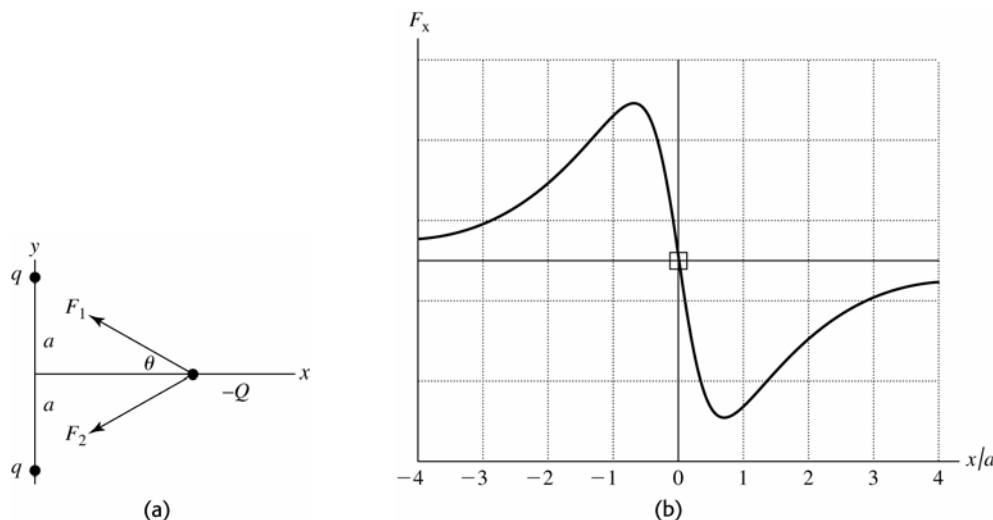


Figure 21.22

21.23. IDENTIFY: Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

(a) SET UP: The charges are placed as shown in Figure 21.23a.

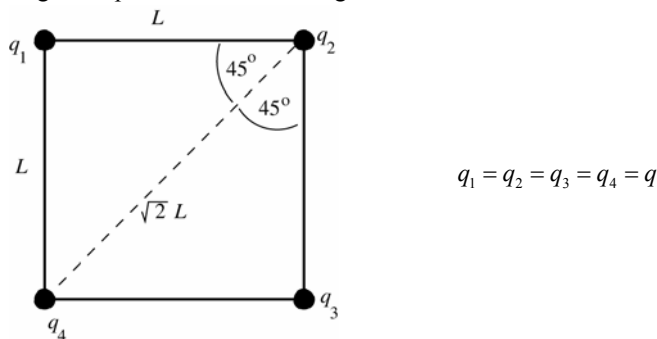


Figure 21.23a

Consider forces on q_4 . The free-body diagram is given in Figure 21.23b. Take the y -axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2 . Then $\vec{F}_2$ is in the $+y$ -direction.

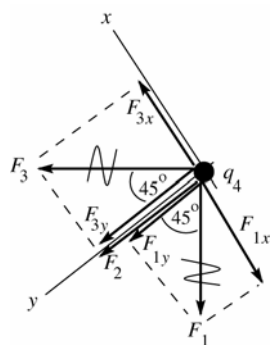


Figure 21.23b

EXECUTE: $F_3 = F_1 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{L^2}$

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{2L^2}$$

$$F_{1x} = -F_1 \sin 45^\circ = -F_1/\sqrt{2}$$

$$F_{1y} = +F_1 \cos 45^\circ = +F_1/\sqrt{2}$$

$$F_{3x} = +F_3 \sin 45^\circ = +F_3/\sqrt{2}$$

$$F_{3y} = +F_3 \cos 45^\circ = +F_3/\sqrt{2}$$

$$F_{2x} = 0, F_{2y} = F_2$$

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2}) \frac{1}{4\pi\epsilon_0} \frac{q^2}{L^2} + \frac{1}{4\pi\epsilon_0} \frac{q^2}{2L^2} = \frac{q^2}{8\pi\epsilon_0 L^2} (1 + 2\sqrt{2})$$

$$R = \frac{q^2}{8\pi\epsilon_0 L^2} (1 + 2\sqrt{2}). \text{ Same for all four charges.}$$

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

- 21.24. IDENTIFY:** Apply $F = \frac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50 \mu\text{C}$ and $q_2 = -3.50 \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1 , since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1 . Let $+q$ be a distance d to the left of q_1 , so it is a distance $d + 0.600 \text{ m}$ from q_2 .

EXECUTE: $F_1 = F_2$ gives $\frac{kq|q_1|}{d^2} = \frac{kq|q_2|}{(d + 0.600 \text{ m})^2}$. $d = \pm \sqrt{\frac{|q_1|}{|q_2|}}(d + 0.600 \text{ m}) = \pm(0.8452)(d + 0.600 \text{ m})$. d must

be positive, so $d = \frac{(0.8452)(0.600 \text{ m})}{1 - 0.8452} = 3.27 \text{ m}$. The net force would be zero when $+q$ is at $x = -3.27 \text{ m}$.

EVALUATE: When $+q$ is at $x = -3.27 \text{ m}$, $\vec{F}_1$ is in the $-x$ direction and $\vec{F}_2$ is in the $+x$ direction.

- 21.25. IDENTIFY:** $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass $1.67 \times 10^{-27} \text{ kg}$.

EXECUTE: (a) $F = (1.60 \times 10^{-19} \text{ C})(2.75 \times 10^3 \text{ N/C}) = 4.40 \times 10^{-16} \text{ N}$

(b) $a = \frac{F}{m} = \frac{4.40 \times 10^{-16} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 2.63 \times 10^{11} \text{ m/s}^2$

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63 \times 10^{11} \text{ m/s}^2)(1.00 \times 10^{-6} \text{ s}) = 2.63 \times 10^5 \text{ m/s}$

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

- 21.26. IDENTIFY:** For a point charge, $E = k \frac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: (a) The field is toward the negative charge so is downward.

$E = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{3.00 \times 10^{-9} \text{ C}}{(0.250 \text{ m})^2} = 432 \text{ N/C}$.

(b) $r = \sqrt{\frac{k|q|}{E}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.00 \times 10^{-9} \text{ C})}{12.0 \text{ N/C}}} = 1.50 \text{ m}$

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

- 21.27. IDENTIFY:** The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) **SET UP:** First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50 \times 10^6 \text{ m/s})^2 + 2a(0.0320 \text{ m})$ and $a = 3.16 \times 10^{14} \text{ m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and $E = ma/e = (1.67 \times 10^{-27} \text{ kg})(3.16 \times 10^{14} \text{ m/s}^2)/(1.60 \times 10^{-19} \text{ C}) = 3.30 \times 10^6 \text{ N/C}$, to the left.

(b) **SET UP:** Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50 \times 10^6 \text{ m/s})/(3.16 \times 10^{14} \text{ m/s}^2) = 1.42 \times 10^{-8} \text{ s} = 14.2 \text{ ns}$

(c) **SET UP:** In part (a) we saw that the electric field is proportional to m , so we can use the ratio of the electric fields. $E_e/E_p = m_e/m_p$ and $E_e = (m_e/m_p)E_p$.

EXECUTE: $E_e = [(9.11 \times 10^{-31} \text{ kg})/(1.67 \times 10^{-27} \text{ kg})](3.30 \times 10^6 \text{ N/C}) = 1.80 \times 10^3 \text{ N/C}$, to the right

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.28. IDENTIFY: Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: (a) $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50 \text{ m})}{(3.00 \times 10^{-6} \text{ s})^2} = 1.00 \times 10^{12} \text{ m/s}^2. \quad E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^{12} \text{ m/s}^2)}{1.60 \times 10^{-19} \text{ C}} = 5.69 \text{ N/C}$$

The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11} g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.29. (a) IDENTIFY: Eq. (21.4) relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.29. The weight is mg , downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.

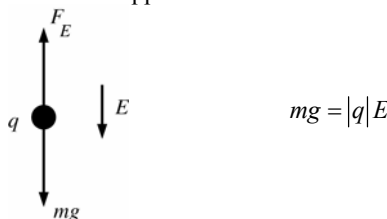


Figure 21.29

EXECUTE: $|q| = \frac{mg}{E} = \frac{(1.45 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{650 \text{ N/C}} = 2.19 \times 10^{-5} \text{ C}$ and $q = -21.9 \mu\text{C}$

(b) **SET UP:** The electrical force has magnitude $F_E = |q|E = eE$. The weight of a proton is $w = mg$. $F_E = w$ so $eE = mg$

EXECUTE: $E = \frac{mg}{e} = \frac{(1.673 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2)}{1.602 \times 10^{-19} \text{ C}} = 1.02 \times 10^{-7} \text{ N/C}$.

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g$. In part (b) the $m/|q|$ ratio is much smaller ($\sim 10^{-8}$) than in part (a) ($\sim 10^{-2}$) so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.30. IDENTIFY: Apply $E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$.

SET UP: The iron nucleus has charge $+26e$. A proton has charge $+e$.

EXECUTE: (a) $E = \frac{1}{4\pi\epsilon_0} \frac{(26)(1.60 \times 10^{-19} \text{ C})}{(6.00 \times 10^{-10} \text{ m})^2} = 1.04 \times 10^{11} \text{ N/C}$.

(b) $E_{\text{proton}} = \frac{1}{4\pi\epsilon_0} \frac{(1.60 \times 10^{-19} \text{ C})}{(5.29 \times 10^{-11} \text{ m})^2} = 5.15 \times 10^{11} \text{ N/C}$.

EVALUATE: These electric fields are very large. In each case the charge is positive and the electric fields are directed away from the nucleus or proton.

21.31. IDENTIFY: For a point charge, $E = k \frac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge q in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1 . Point B is 0.100 m from q_1 and 0.350 m from q_2 .

EXECUTE: (a) The electric fields due to the charges at point A are shown in Figure 21.31a.

$$E_1 = k \frac{|q_1|}{r_{A1}^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.25 \times 10^{-9} \text{ C}}{(0.150 \text{ m})^2} = 2.50 \times 10^3 \text{ N/C}$$

$$E_2 = k \frac{|q_2|}{r_{A2}^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{12.5 \times 10^{-9} \text{ C}}{(0.100 \text{ m})^2} = 1.124 \times 10^4 \text{ N/C}$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.

$$E = E_2 - E_1 = 8.74 \times 10^3 \text{ N/C, to the right.}$$

(b) The electric fields at points B are shown in Figure 21.31b.

$$E_1 = k \frac{|q_1|}{r_{B1}^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.25 \times 10^{-9} \text{ C}}{(0.100 \text{ m})^2} = 5.619 \times 10^3 \text{ N/C}$$

$$E_2 = k \frac{|q_2|}{r_{B2}^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{12.5 \times 10^{-9} \text{ C}}{(0.350 \text{ m})^2} = 9.17 \times 10^2 \text{ N/C}$$

Since the fields are in the same direction, we add their magnitudes to find the net field. $E = E_1 + E_2 = 6.54 \times 10^3 \text{ N/C}$, to the right.

(c) At A , $E = 8.74 \times 10^3 \text{ N/C}$, to the right. The force on a proton placed at this point would be

$$F = qE = (1.60 \times 10^{-19} \text{ C})(8.74 \times 10^3 \text{ N/C}) = 1.40 \times 10^{-15} \text{ N, to the right.}$$

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

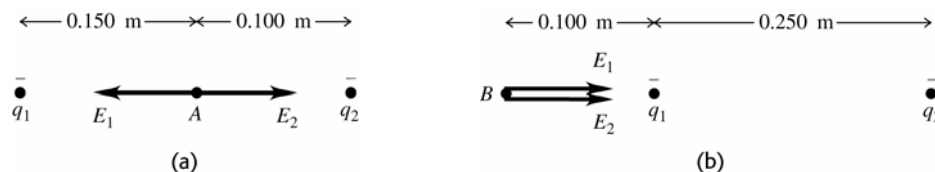


Figure 21.31

21.32. IDENTIFY: The electric force is $\vec{F} = q\vec{E}$.

SET UP: The gravity force (weight) has magnitude $w = mg$ and is downward.

EXECUTE: (a) To balance the weight the electric force must be upward. The electric field is downward, so for an upward force the charge q of the person must be negative. $w = F$ gives $mg = |q|E$ and

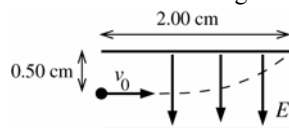
$$|q| = \frac{mg}{E} = \frac{(60 \text{ kg})(9.80 \text{ m/s}^2)}{150 \text{ N/C}} = 3.9 \text{ C}.$$

(b) $F = k \frac{|qq'|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(3.9 \text{ C})^2}{(100 \text{ m})^2} = 1.4 \times 10^7 \text{ N}$. The repulsive force is immense and this is not a feasible means of flight.

EVALUATE: The net charge of charged objects is typically much less than 1 C.

21.33. IDENTIFY: Eq. (21.3) gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

(a) **SET UP:** The motion is sketched in Figure 21.33a.



For an electron $q = -e$.

Figure 21.33a

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.33b.

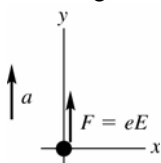


Figure 21.33b

$$\text{EXECUTE: } \sum F_y = ma_y$$

$$eE = ma$$

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00 \text{ cm}$ when $y - y_0 = +0.500 \text{ cm}$.

x-component

$$v_{0x} = v_0 = 1.60 \times 10^6 \text{ m/s}, a_x = 0, x - x_0 = 0.0200 \text{ m}, t = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.0200 \text{ m}}{1.60 \times 10^6 \text{ m/s}} = 1.25 \times 10^{-8} \text{ s}$$

In this same time t the electron travels 0.0050 m vertically:

y-component

$$t = 1.25 \times 10^{-8} \text{ s}, v_{0y} = 0, y - y_0 = +0.0050 \text{ m}, a_y = ?$$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050 \text{ m})}{(1.25 \times 10^{-8} \text{ s})^2} = 6.40 \times 10^{13} \text{ m/s}^2$$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$

$$E = \frac{ma}{e} = \frac{(9.109 \times 10^{-31} \text{ kg})(6.40 \times 10^{13} \text{ m/s}^2)}{1.602 \times 10^{-19} \text{ C}} = 364 \text{ N/C}$$

Note that the acceleration produced by the electric field is much larger than g , the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the electron in this problem.

$$(b) a = \frac{eE}{m_p} = \frac{(1.602 \times 10^{-19} \text{ C})(364 \text{ N/C})}{1.673 \times 10^{-27} \text{ kg}} = 3.49 \times 10^{10} \text{ m/s}^2$$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25 \times 10^{-8} \text{ s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49 \times 10^{10} \text{ m/s}^2$.

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.49 \times 10^{10} \text{ m/s}^2)(1.25 \times 10^{-8} \text{ s})^2 = -2.73 \times 10^{-6} \text{ m}. \text{ The displacement is } 2.73 \times 10^{-6} \text{ m, downward.}$$

(c) EVALUATE: The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

21.34. IDENTIFY: Apply Eq.(21.7) to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1 , $\hat{r} = \hat{j}$. For q_2 , $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$, where θ is the angle between $\vec{E}_2$ and the $+x$ -axis.

$$\text{EXECUTE: (a) } \vec{E}_1 = \frac{q_1}{4\pi\epsilon_0 r_1^2} \hat{j} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-5.00 \times 10^{-9} \text{ C})}{(0.0400 \text{ m})^2} = (-2.813 \times 10^4 \text{ N/C}) \hat{j}.$$

$$|\vec{E}_2| = \frac{q_2}{4\pi\epsilon_0 r_2^2} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.00 \times 10^{-9} \text{ C})}{(0.0300 \text{ m})^2 + (0.0400 \text{ m})^2} = 1.080 \times 10^4 \text{ N/C}. \text{ The angle of } \vec{E}_2, \text{ measured from the}$$

$$x\text{-axis, is } 180^\circ - \tan^{-1}\left(\frac{4.00 \text{ cm}}{3.00 \text{ cm}}\right) = 126.9^\circ. \text{ Thus}$$

$$\vec{E}_2 = (1.080 \times 10^4 \text{ N/C})(\hat{i} \cos 126.9^\circ + \hat{j} \sin 126.9^\circ) = (-6.485 \times 10^3 \text{ N/C})\hat{i} + (8.64 \times 10^3 \text{ N/C})\hat{j}$$

$$(b) \text{ The resultant field is } \vec{E}_1 + \vec{E}_2 = (-6.485 \times 10^3 \text{ N/C})\hat{i} + (-2.813 \times 10^4 \text{ N/C} + 8.64 \times 10^3 \text{ N/C})\hat{j}.$$

$$\vec{E}_1 + \vec{E}_2 = (-6.485 \times 10^3 \text{ N/C})\hat{i} - (1.95 \times 10^4 \text{ N/C})\hat{j}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2 , since q_2 is positive.

21.35. IDENTIFY: Apply constant acceleration equations to the motion of the electron.

SET UP: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

EXECUTE: Use the horizontal motion to find the time when the electron emerges from the field.

$$x - x_0 = 0.0200 \text{ m}, a_x = 0, v_{0x} = 1.60 \times 10^6 \text{ m/s}. x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ gives } t = 1.25 \times 10^{-8} \text{ s}. \text{ Since } a_x = 0,$$

$$v_x = 1.60 \times 10^6 \text{ m/s}. y - y_0 = 0.0050 \text{ m}, v_{0y} = 0, t = 1.25 \times 10^{-8} \text{ s}. y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t \text{ gives } v_y = 8.00 \times 10^5 \text{ m/s}.$$

$$\text{Then } v = \sqrt{v_x^2 + v_y^2} = 1.79 \times 10^6 \text{ m/s}.$$

EVALUATE: $v_y = v_{0y} + a_y t$ gives $a_y = 6.4 \times 10^{13} \text{ m/s}^2$. The electric field between the plates is

$$E = \frac{ma_y}{e} = \frac{(9.11 \times 10^{-31} \text{ kg})(6.4 \times 10^{13} \text{ m/s}^2)}{1.60 \times 10^{-19} \text{ C}} = 364 \text{ V/m}. \text{ This is not a very large field.}$$

- 21.36. IDENTIFY:** Use the components of $\vec{E}$ from Example 21.6 to calculate the magnitude and direction of $\vec{E}$. Use $\vec{F} = q\vec{E}$ to calculate the force on the -2.5 nC charge and use Newton's third law for the force on the -8.0 nC charge.

SET UP: From Example 21.6, $\vec{E} = (-11 \text{ N/C})\hat{i} + (14 \text{ N/C})\hat{j}$.

EXECUTE: (a) $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-11 \text{ N/C})^2 + (14 \text{ N/C})^2} = 17.8 \text{ N/C}$. $\tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}(14/11) = 51.8^\circ$, so

$\theta = 128^\circ$ counterclockwise from the $+x$ -axis.

(b) (i) $\vec{F} = \vec{E}q$ so $F = (17.8 \text{ N/C})(2.5 \times 10^{-9} \text{ C}) = 4.45 \times 10^{-8} \text{ N}$, at 52° below the $+x$ -axis.

(ii) $4.45 \times 10^{-8} \text{ N}$ at 128° counterclockwise from the $+x$ -axis.

EVALUATE: The forces in part (b) are repulsive so they are along the line connecting the two charges and in each case the force is directed away from the charge that exerts it.

- 21.37. IDENTIFY and SET UP:** The electric force is given by Eq. (21.3). The gravitational force is $w_e = m_e g$. Compare these forces.

(a) **EXECUTE:** $w_e = (9.109 \times 10^{-31} \text{ kg})(9.80 \text{ m/s}^2) = 8.93 \times 10^{-30} \text{ N}$

In Examples 21.7 and 21.8, $E = 1.00 \times 10^4 \text{ N/C}$, so the electric force on the electron has magnitude

$$F_E = |q|E = eE = (1.602 \times 10^{-19} \text{ C})(1.00 \times 10^4 \text{ N/C}) = 1.602 \times 10^{-15} \text{ N}.$$

$$\frac{w_e}{F_E} = \frac{8.93 \times 10^{-30} \text{ N}}{1.602 \times 10^{-15} \text{ N}} = 5.57 \times 10^{-15}$$

The gravitational force is much smaller than the electric force and can be neglected.

(b) $mg = |q|E$

$$m = |q|E/g = (1.602 \times 10^{-19} \text{ C})(1.00 \times 10^4 \text{ N/C})/(9.80 \text{ m/s}^2) = 1.63 \times 10^{-16} \text{ kg}$$

$$\frac{m}{m_e} = \frac{1.63 \times 10^{-16} \text{ kg}}{9.109 \times 10^{-31} \text{ kg}} = 1.79 \times 10^{14}; \quad m = 1.79 \times 10^{14} m_e.$$

EVALUATE: m is much larger than m_e . We found in part (a) that if $m = m_e$ the gravitational force is much smaller than the electric force. $|q|$ is the same so the electric force remains the same. To get w large enough to equal F_E , the mass must be made much larger.

(c) The electric field in the region between the plates is uniform so the force it exerts on the charged object is independent of where between the plates the object is placed.

- 21.38. IDENTIFY:** Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

SET UP: A proton has mass $m_p = 1.67 \times 10^{-27} \text{ kg}$ and charge $+e$. Let $+x$ be in the direction of motion of the proton.

EXECUTE: (a) $v_{0x} = 0$. $a = \frac{eE}{m_p}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = \frac{1}{2}a_x t^2 = \frac{1}{2}\frac{eE}{m_p}t^2$. Solving for E gives

$$E = \frac{2(0.0160 \text{ m})(1.67 \times 10^{-27} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(1.50 \times 10^{-6} \text{ s})^2} = 148 \text{ N/C}.$$

(b) $v_x = v_{0x} + a_x t = \frac{eE}{m_p}t = 2.13 \times 10^4 \text{ m/s}$.

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

- 21.39. IDENTIFY:** Find the angle θ that $\hat{r}$ makes with the $+x$ -axis. Then $\hat{r} = (\cos\theta)\hat{i} + (\sin\theta)\hat{j}$.

SET UP: $\tan\theta = y/x$

EXECUTE: (a) $\tan^{-1}\left(\frac{-1.35}{0}\right) = -\frac{\pi}{2} \text{ rad}$. $\hat{r} = -\hat{j}$.

(b) $\tan^{-1}\left(\frac{12}{12}\right) = \frac{\pi}{4} \text{ rad}$. $\hat{r} = \frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$.

(c) $\tan^{-1}\left(\frac{2.6}{+1.10}\right) = 1.97 \text{ rad} = 112.9^\circ$. $\hat{r} = -0.39\hat{i} + 0.92\hat{j}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{r}$ is a unit vector, because $\hat{r} \cdot \hat{r} = 1$.

21.40. IDENTIFY: The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50 \mu\text{C}$ charge is given in Figure 21.40. F_E is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the $+x$ axis to be to the right, as shown in the figure.

EXECUTE: (a) $F = |q|E = (6.50 \times 10^{-6} \text{ C})(1.85 \times 10^8 \text{ N/C}) = 1.20 \times 10^3 \text{ N}$

$$F_q = k \frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(6.50 \times 10^{-6} \text{ C})(8.75 \times 10^{-6} \text{ C})}{(0.0250 \text{ m})^2} = 8.18 \times 10^2 \text{ N}$$

$$\sum F_x = 0 \text{ gives } T + F_q - F_E = 0 \text{ and } T = F_E - F_q = 382 \text{ N}.$$

(b) Now F_q is to the left, since like charges repel.

$$\sum F_x = 0 \text{ gives } T - F_q - F_E = 0 \text{ and } T = F_E + F_q = 2.02 \times 10^3 \text{ N}.$$

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

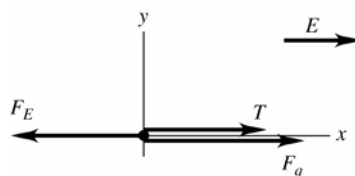


Figure 21.40

21.41. IDENTIFY and SET UP: Use $\vec{E}$ in Eq. (21.3) to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let $+x$ be east.

(a) **EXECUTE:** $F_x = |q|E = (1.602 \times 10^{-19} \text{ C})(1.50 \text{ N/C}) = 2.403 \times 10^{-19} \text{ N}$

$$a_x = F_x/m = (2.403 \times 10^{-19} \text{ N})/(9.109 \times 10^{-31} \text{ kg}) = +2.638 \times 10^{11} \text{ m/s}^2$$

$$v_{0x} = +4.50 \times 10^5 \text{ m/s}, a_x = +2.638 \times 10^{11} \text{ m/s}^2, x - x_0 = 0.375 \text{ m}, v_x = ?$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \text{ gives } v_x = 6.33 \times 10^5 \text{ m/s}$$

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) **EXECUTE:** $F_x = -|q|E = -(1.602 \times 10^{-19} \text{ C})(1.50 \text{ N/C}) = -2.403 \times 10^{-19} \text{ N}$

$$a_x = F_x/m = (-2.403 \times 10^{-19} \text{ N})/(1.673 \times 10^{-27} \text{ kg}) = -1.436 \times 10^8 \text{ m/s}^2$$

$$v_{0x} = +1.90 \times 10^4 \text{ m/s}, a_x = -1.436 \times 10^8 \text{ m/s}^2, x - x_0 = 0.375 \text{ m}, v_x = ?$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \text{ gives } v_x = 1.59 \times 10^4 \text{ m/s}$$

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.42. IDENTIFY: Coulomb's law for a single point-charge gives the electric field.

(a) **SET UP:** Coulomb's law for a point-charge is $E = (1/4\pi\epsilon_0)q/r^2$.

$$\text{EXECUTE: } E = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})/(1.50 \times 10^{-15} \text{ m})^2 = 6.40 \times 10^{20} \text{ N/C}$$

(b) Taking the ratio of the electric fields gives

$$E/E_{\text{plates}} = (6.40 \times 10^{20} \text{ N/C})/(1.00 \times 10^4 \text{ N/C}) = 6.40 \times 10^{16} \text{ times as strong}$$

EVALUATE: The electric field within the nucleus is huge compared to typical laboratory fields!

21.43. IDENTIFY: Calculate the electric field due to each charge and find the vector sum of these two fields.

SET UP: At points on the x -axis only the x component of each field is nonzero. The electric field of a point charge points away from the charge if it is positive and toward it if it is negative.

EXECUTE: (a) Halfway between the two charges, $E = 0$.

$$(b) \text{ For } |x| < a, E_x = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{(a+x)^2} - \frac{q}{(a-x)^2} \right) = -\frac{4q}{4\pi\epsilon_0} \frac{ax}{(x^2 - a^2)^2}.$$

$$\text{For } x > a, E_x = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{(a+x)^2} + \frac{q}{(a-x)^2} \right) = \frac{2q}{4\pi\epsilon_0} \frac{x^2 + a^2}{(x^2 - a^2)^2}.$$

$$\text{For } x < -a, E_x = \frac{-1}{4\pi\epsilon_0} \left(\frac{q}{(a+x)^2} + \frac{q}{(a-x)^2} \right) = -\frac{2q}{4\pi\epsilon_0} \frac{x^2 + a^2}{(x^2 - a^2)^2}.$$

The graph of E_x versus x is sketched in Figure 21.43.

EVALUATE: The magnitude of the field approaches infinity at the location of one of the point charges.

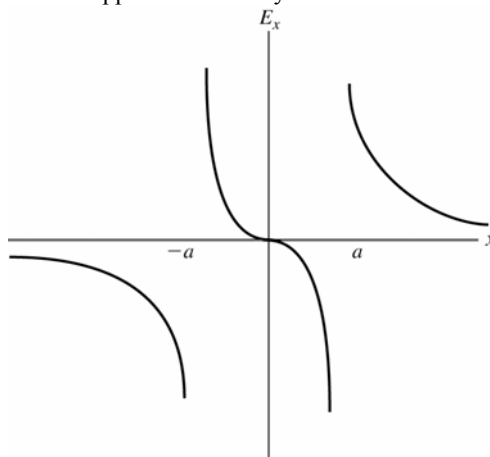


Figure 21.43

- 21.44. IDENTIFY:** For a point charge, $E = k \frac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500 \text{ nC}$ and $q_2 = +8.00 \text{ nC}$. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.44. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a

distance $1.20 \text{ m} - x$ from q_2 . $E_1 = E_2$ gives $k \frac{0.500 \text{ nC}}{x^2} = k \frac{8.00 \text{ nC}}{(1.20 - x)^2}$. $16x^2 = (1.20 \text{ m} - x)^2$. $4x = \pm(1.20 \text{ m} - x)$

and $x = 0.24 \text{ m}$ is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

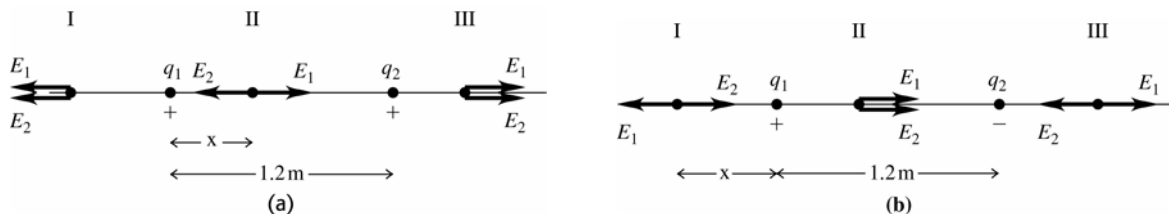


Figure 21.44

- 21.45. IDENTIFY:** Eq.(21.7) gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use Eq.(21.3) to calculate the force, using the electric field calculated in part (a).

(a) SET UP: The placement of charges is sketched in Figure 21.45a.

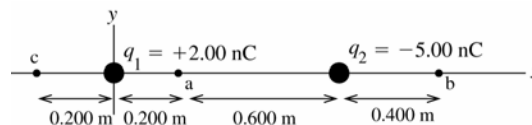


Figure 21.45a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$, where r is the distance

between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45b.

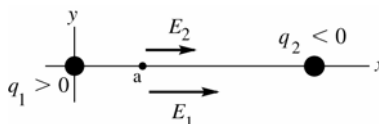


Figure 21.45b

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2.00 \times 10^{-9} \text{ C}}{(0.200 \text{ m})^2} = 449.4 \text{ N/C}$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.00 \times 10^{-9} \text{ C}}{(0.600 \text{ m})^2} = 124.8 \text{ N/C}$$

$$E_{1x} = 449.4 \text{ N/C}, E_{1y} = 0$$

$$E_{2x} = 124.8 \text{ N/C}, E_{2y} = 0$$

$$E_x = E_{1x} + E_{2x} = +449.4 \text{ N/C} + 124.8 \text{ N/C} = +574.2 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = 0$$

The resultant field at point a has magnitude 574 N/C and is in the $+x$ -direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45c.

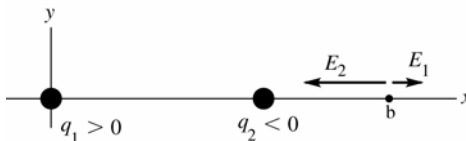


Figure 21.45c

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2.00 \times 10^{-9} \text{ C}}{(1.20 \text{ m})^2} = 12.5 \text{ N/C}$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.00 \times 10^{-9} \text{ C}}{(0.400 \text{ m})^2} = 280.9 \text{ N/C}$$

$$E_{1x} = 12.5 \text{ N/C}, E_{1y} = 0$$

$$E_{2x} = -280.9 \text{ N/C}, E_{2y} = 0$$

$$E_x = E_{1x} + E_{2x} = +12.5 \text{ N/C} - 280.9 \text{ N/C} = -268.4 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = 0$$

The resultant field at point b has magnitude 268 N/C and is in the $-x$ -direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45d.

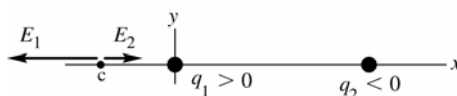


Figure 21.45d

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2.00 \times 10^{-9} \text{ C}}{(0.200 \text{ m})^2} = 449.4 \text{ N/C}$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.00 \times 10^{-9} \text{ C}}{(1.00 \text{ m})^2} = 44.9 \text{ N/C}$$

$$E_{1x} = -449.4 \text{ N/C}, E_{1y} = 0$$

$$E_{2x} = +44.9 \text{ N/C}, E_{2y} = 0$$

$$E_x = E_{1x} + E_{2x} = -449.4 \text{ N/C} + 44.9 \text{ N/C} = -404.5 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = 0$$

The resultant field at point b has magnitude 404 N/C and is in the $-x$ -direction.

(b) **SET UP:** Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602 \times 10^{-19} \text{ C})(574.2 \text{ N/C}) = 9.20 \times 10^{-17} \text{ N}$, $-x$ -direction

(ii) $F = (1.602 \times 10^{-19} \text{ C})(268.4 \text{ N/C}) = 4.30 \times 10^{-17} \text{ N}$, $+x$ -direction

(iii) $F = (1.602 \times 10^{-19} \text{ C})(404.5 \text{ N/C}) = 6.48 \times 10^{-17} \text{ N}$, $+x$ -direction

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$ - or $-x$ -direction depends on where the field point is relative to the charge that produces the field. In part (a) the field magnitudes were added because the fields were in the same direction and in (b) and (c) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.46. IDENTIFY: Apply Eq.(21.7) to calculate the field due to each charge and then require that the vector sum of the two fields to be zero.

SET UP: The field of each charge is directed toward the charge if it is negative and away from the charge if it is positive.

EXECUTE: The point where the two fields cancel each other will have to be closer to the negative charge, because it is smaller. Also, it can't be between the two charges, since the two fields would then act in the same direction. We could use Coulomb's law to calculate the actual values, but a simpler way is to note that the 8.00 nC charge is twice as large as the -4.00 nC charge. The zero point will therefore have to be a factor of $\sqrt{2}$ farther from the 8.00 nC charge for the two fields to have equal magnitude. Calling x the distance from the -4.00 nC charge: $1.20 + x = \sqrt{2}x$ and $x = 2.90$ m.

EVALUATE: This point is 4.10 m from the 8.00 nC charge. The two fields at this point are in opposite directions and have equal magnitudes.

21.47. IDENTIFY: $E = k \frac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges q_1 , q_2 and q_3 , as shown in Figure 21.47a. This figure also shows additional distances and angles. The electric fields at point P are shown in Figure 21.47b. This figure also shows the xy coordinates we will use and the x and y components of the fields $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.00 \times 10^{-6} \text{ C}}{(0.100 \text{ m})^2} = 4.49 \times 10^6 \text{ N/C}$

$$E_2 = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2.00 \times 10^{-6} \text{ C}}{(0.0600 \text{ m})^2} = 4.99 \times 10^6 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} = 0 \text{ and } E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1 \cos 53.1^\circ = 1.04 \times 10^7 \text{ N/C}$$

$E = 1.04 \times 10^7 \text{ N/C}$, toward the $-2.00 \mu\text{C}$ charge.

EVALUATE: The x -components of the fields of all three charges are in the same direction.

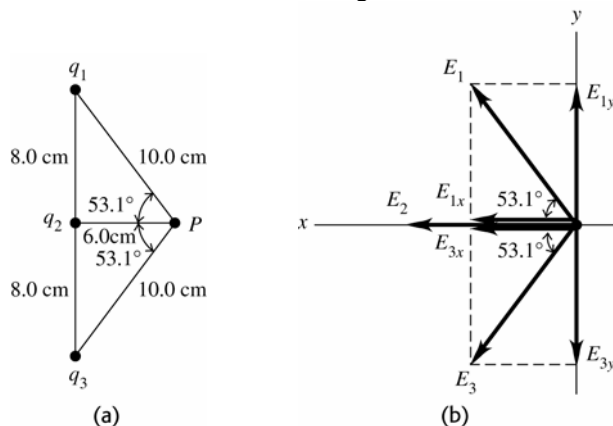


Figure 21.47

21.48. IDENTIFY: A positive and negative charge, of equal magnitude q , are on the x -axis, a distance a from the origin. Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of these fields.

SET UP: $\vec{E}$ due to a point charge is directed away from the charge if it is positive and directed toward the charge if it is negative.

EXECUTE: (a) Halfway between the charges, both fields are in the $-x$ -direction and $E = \frac{1}{4\pi\epsilon_0} \frac{2q}{a^2}$, in the $-x$ -direction.

(b) $E_x = \frac{1}{4\pi\epsilon_0} \left(\frac{-q}{(a+x)^2} - \frac{q}{(a-x)^2} \right)$ for $|x| < a$. $E_x = \frac{1}{4\pi\epsilon_0} \left(\frac{-q}{(a+x)^2} + \frac{q}{(a-x)^2} \right)$ for $x > a$.

$E_x = \frac{1}{4\pi\epsilon_0} \left(\frac{-q}{(a+x)^2} - \frac{q}{(a-x)^2} \right)$ for $x < -a$. E_x is graphed in Figure 21.48.

EVALUATE: At points on the x axis and between the charges, E_x is in the $-x$ -direction because the fields from both charges are in this direction. For $x < -a$ and $x > +a$, the fields from the two charges are in opposite directions and the field from the closer charge is larger in magnitude.

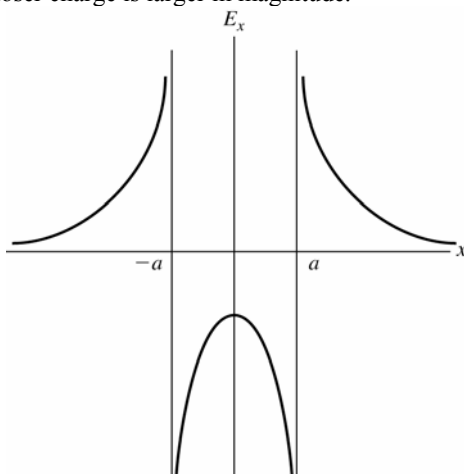


Figure 21.48

- 21.49. IDENTIFY:** The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.49a.

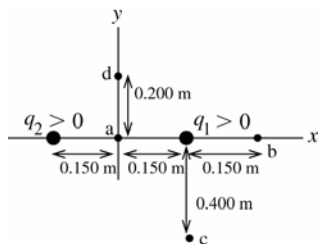


Figure 21.49a

EXECUTE: (a) The directions of the two fields are shown in Figure 21.49b.

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \text{ with } r = 0.150 \text{ m.}$$

$$E = E_2 - E_1 = 0; E_x = 0, E_y = 0$$

Figure 21.49b

(b) The two fields have the directions shown in Figure 21.49c.

$$E = E_1 + E_2, \text{ in the } +x\text{-direction}$$

Figure 21.49c

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.150 \text{ m})^2} = 2396.8 \text{ N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.450 \text{ m})^2} = 266.3 \text{ N/C}$$

$$E = E_1 + E_2 = 2396.8 \text{ N/C} + 266.3 \text{ N/C} = 2660 \text{ N/C}; E_x = +2660 \text{ N/C}, E_y = 0$$

(c) The two fields have the directions shown in Figure 21.49d.

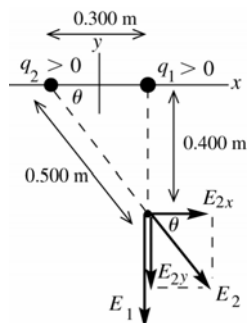


Figure 21.49d

$$\sin \theta = \frac{0.400 \text{ m}}{0.500 \text{ m}} = 0.800$$

$$\cos \theta = \frac{0.300 \text{ m}}{0.500 \text{ m}} = 0.600$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.400 \text{ m})^2} = 337.1 \text{ N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2}$$

$$E_2 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.500 \text{ m})^2} = 215.7 \text{ N/C}$$

$$E_{1x} = 0, E_{1y} = -E_1 = -337.1 \text{ N/C}$$

$$E_{2x} = +E_2 \cos \theta = +(215.7 \text{ N/C})(0.600) = +129.4 \text{ N/C}$$

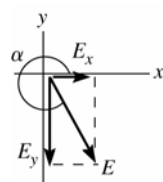
$$E_{2y} = -E_2 \sin \theta = -(215.7 \text{ N/C})(0.800) = -172.6 \text{ N/C}$$

$$E_x = E_{1x} + E_{2x} = +129 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = -337.1 \text{ N/C} - 172.6 \text{ N/C} = -510 \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129 \text{ N/C})^2 + (-510 \text{ N/C})^2} = 526 \text{ N/C}$$

$\vec{E}$ and its components are shown in Figure 21.49e.



$$\tan \alpha = \frac{E_y}{E_x}$$

$$\tan \alpha = \frac{-510 \text{ N/C}}{+129 \text{ N/C}} = -3.953$$

$$\alpha = 284^\circ \text{C, counterclockwise from } +x\text{-axis}$$

Figure 21.49e

(d) The two fields have the directions shown in Figure 21.49f.

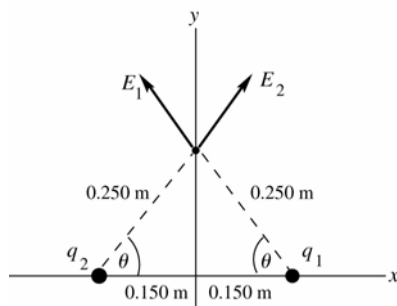


Figure 21.49f

$$\sin \theta = \frac{0.200 \text{ m}}{0.250 \text{ m}} = 0.800$$

The components of the two fields are shown in Figure 21.49g.

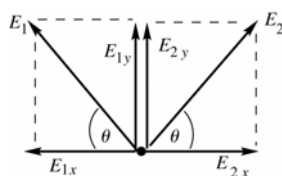


Figure 21.49g

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$$

$$E_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.250 \text{ m})^2}$$

$$E_1 = E_2 = 862.8 \text{ N/C}$$

$$E_{1x} = -E_1 \cos \theta, E_{2x} = +E_2 \cos \theta$$

$$E_x = E_{1x} + E_{2x} = 0$$

$$E_{1y} = +E_1 \sin \theta, E_{2y} = +E_2 \sin \theta$$

$$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1 \sin \theta = 2(862.8 \text{ N/C})(0.800) = 1380 \text{ N/C}$$

$$E = 1380 \text{ N/C, in the } +y\text{-direction.}$$

EVALUATE: Point *a* is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point *b* is to the right of both charges and both electric fields are in the $+x$ -direction and the resultant field is in this direction. At point *c* both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the 4th quadrant. At point *d* both fields have an upward component but by symmetry they have equal and opposite x -components so the net field is in the $+y$ -direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.50. IDENTIFY: Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.50.

$$\text{EXECUTE: } \vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{(6.00 \times 10^{-9} \text{ C})}{(0.6 \text{ m})^2} (-\hat{i}) = -150 \hat{i} \text{ N/C.}$$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} (4.00 \times 10^{-9} \text{ C}) \left(\frac{1}{(1.00 \text{ m})^2} (0.600) \hat{i} + \frac{1}{(1.00 \text{ m})^2} (0.800) \hat{j} \right) = (21.6 \hat{i} + 28.8 \hat{j}) \text{ N/C.}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4 \text{ N/C}) \hat{i} + (28.8 \text{ N/C}) \hat{j}. E = \sqrt{(128.4 \text{ N/C})^2 + (28.8 \text{ N/C})^2} = 131.6 \text{ N/C at}$$

$$\theta = \tan^{-1} \left(\frac{28.8}{128.4} \right) = 12.6^\circ \text{ above the } -x \text{ axis and therefore } 196.2^\circ \text{ counterclockwise from the } +x \text{ axis.}$$

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

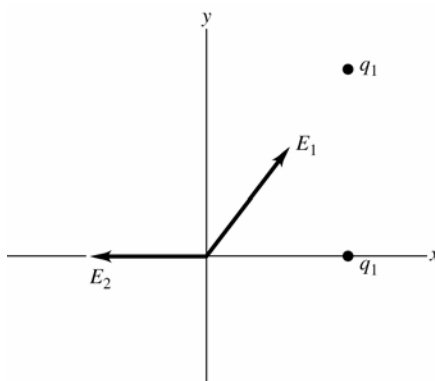


Figure 21.50

21.51. IDENTIFY: The resultant electric field is the vector sum of the field $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 .

SET UP: The placement of the charges is shown in Figure 21.51a.

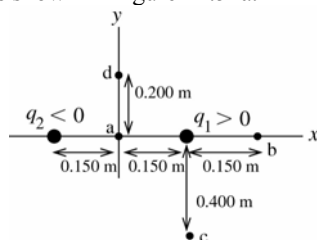


Figure 21.51a

EXECUTE: (a) The directions of the two fields are shown in Figure 21.51b.

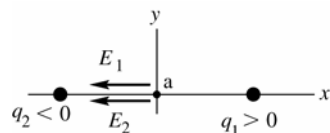


Figure 21.51b

$$E_{1x} = -2397 \text{ N/C}, E_{1y} = 0 \quad E_{2x} = -2397 \text{ N/C}, E_{2y} = 0$$

$$E_x = E_{1x} + E_{2x} = 2(-2397 \text{ N/C}) = -4790 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = 0$$

The resultant electric field at point a in the sketch has magnitude 4790 N/C and is in the $-x$ -direction.

(b) The directions of the two fields are shown in Figure 21.51c.

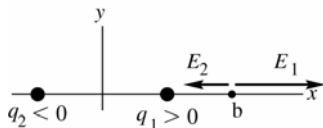


Figure 21.51c

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.150 \text{ m})^2} = 2397 \text{ N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.450 \text{ m})^2} = 266 \text{ N/C}$$

$$E_{1x} = +2397 \text{ N/C}, E_{1y} = 0 \quad E_{2x} = -266 \text{ N/C}, E_{2y} = 0$$

$$E_x = E_{1x} + E_{2x} = +2397 \text{ N/C} - 266 \text{ N/C} = +2130 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = 0$$

The resultant electric field at point b in the sketch has magnitude 2130 N/C and is in the $+x$ -direction.

(c) The placement of the charges is shown in Figure 21.51d.

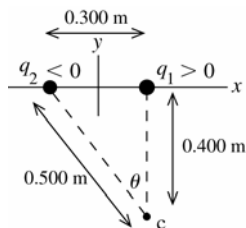


Figure 21.51d

$$\sin \theta = \frac{0.300 \text{ m}}{0.500 \text{ m}} = 0.600$$

$$\cos \theta = \frac{0.400 \text{ m}}{0.500 \text{ m}} = 0.800$$

The directions of the two fields are shown in Figure 21.51e.

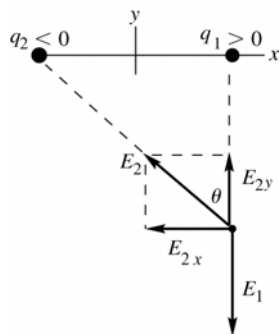


Figure 21.51e

$$E_{1x} = 0, E_{1y} = -E_1 = -337.0 \text{ N/C}$$

$$E_{2x} = -E_2 \sin \theta = -(215.7 \text{ N/C})(0.600) = -129.4 \text{ N/C}$$

$$E_{2y} = +E_2 \cos \theta = +(215.7 \text{ N/C})(0.800) = +172.6 \text{ N/C}$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.400 \text{ m})^2}$$

$$E_1 = 337.0 \text{ N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r_2^2}$$

$$E_2 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.500 \text{ m})^2}$$

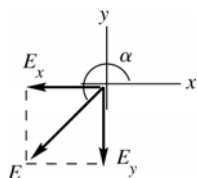
$$E_2 = 215.7 \text{ N/C}$$

$$E_x = E_{1x} + E_{2x} = -129 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = -337.0 \text{ N/C} + 172.6 \text{ N/C} = -164 \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} = 209 \text{ N/C}$$

The field $\vec{E}$ and its components are shown in Figure 21.51f.



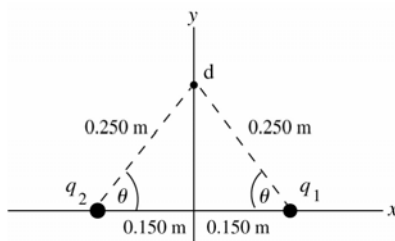
$$\tan \alpha = \frac{E_y}{E_x}$$

$$\tan \alpha = \frac{-164 \text{ N/C}}{-129 \text{ N/C}} = +1.271$$

$$\alpha = 232^\circ, \text{ counterclockwise from } +x\text{-axis}$$

Figure 21.51f

(d) The placement of the charges is shown in Figure 21.51g.

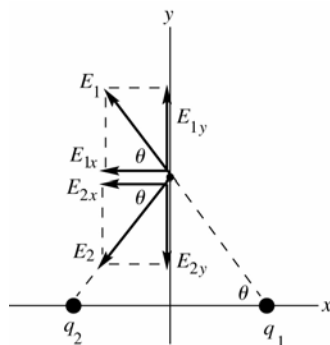


$$\sin \theta = \frac{0.200 \text{ m}}{0.250 \text{ m}} = 0.800$$

$$\cos \theta = \frac{0.150 \text{ m}}{0.250 \text{ m}} = 0.600$$

Figure 21.51g

The directions of the two fields are shown in Figure 21.51h.



$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$$

$$E_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{6.00 \times 10^{-9} \text{ C}}{(0.250 \text{ m})^2}$$

$$E_1 = 862.8 \text{ N/C}$$

$$E_2 = E_1 = 862.8 \text{ N/C}$$

Figure 21.51h

$$E_{1x} = -E_1 \cos \theta, E_{2x} = -E_2 \cos \theta$$

$$E_x = E_{1x} + E_{2x} = -2(862.8 \text{ N/C})(0.600) = -1040 \text{ N/C}$$

$$E_{1y} = +E_1 \sin \theta, E_{2y} = -E_2 \sin \theta$$

$$E_y = E_{1y} + E_{2y} = 0$$

$$E = 1040 \text{ N/C, in the } -x\text{-direction.}$$

EVALUATE: The electric field produced by a charge is toward a negative charge and away from a positive charge. As in Exercise 21.45, we can use this rule to deduce the direction of the resultant field at each point before doing any calculations.

21.52. IDENTIFY: For a long straight wire, $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\frac{1}{2\pi\epsilon_0} = 4.49 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

EXECUTE: $r = \frac{1.5 \times 10^{-10} \text{ C/m}}{2\pi\epsilon_0 (2.50 \text{ N/C})} = 1.08 \text{ m}$

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

- 21.53. IDENTIFY:** Apply Eq.(21.10) for the finite line of charge and $E = \frac{\lambda}{2\pi\epsilon_0}$ for the infinite line of charge.

SET UP: For the infinite line of positive charge, $\vec{E}$ is in the $+x$ direction.

EXECUTE: (a) For a line of charge of length $2a$ centered at the origin and lying along the y -axis, the electric field is given by Eq.(21.10): $\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x\sqrt{x^2/a^2 + 1}} \hat{i}$.

(b) For an infinite line of charge: $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x} \hat{i}$. Graphs of electric field versus position for both distributions of charge are shown in Figure 21.53.

EVALUATE: For small x , close to the line of charge, the field due to the finite line approaches that of the infinite line of charge. As x increases, the field due to the infinite line falls off more slowly and is larger than the field of the finite line.

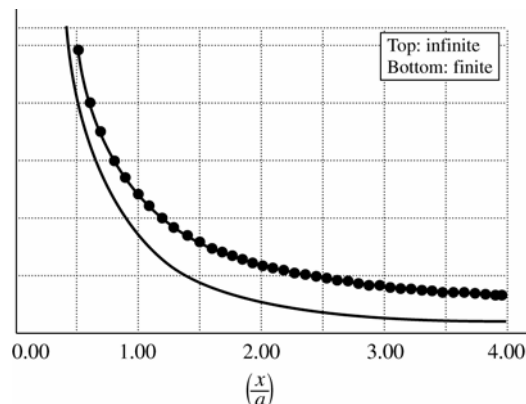


Figure 21.53

- 21.54. (a) IDENTIFY:** The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x\sqrt{(x/a)^2 + 1}} = 2 \left(\frac{1}{4\pi\epsilon_0} \right) \frac{\lambda}{x\sqrt{(x/a)^2 + 1}}.$$

EXECUTE: $E = \frac{(18.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(175 \times 10^{-9} \text{ C/m})}{(0.0600 \text{ m})\sqrt{\left(\frac{6.00 \text{ cm}}{4.25 \text{ cm}}\right)^2 + 1}} = 3.03 \times 10^4 \text{ N/C}$, directed upward.

(b) **IDENTIFY:** The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}$. We first find the radius of the circle using $2\pi r = l$.

EXECUTE: Solving for r gives $r = l/2\pi = (8.50 \text{ cm})/2\pi = 1.353 \text{ cm}$

The charge on this circle is $Q = \lambda l = (175 \text{ nC/m})(0.0850 \text{ m}) = 14.88 \text{ nC}$

The electric field is

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}} = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(14.88 \times 10^{-9} \text{ C/m})(0.0600 \text{ m})}{[(0.0600 \text{ m})^2 + (0.01353 \text{ m})^2]^{3/2}}$$

$$E = 3.45 \times 10^4 \text{ N/C, upward.}$$

EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

- 21.55. IDENTIFY:** For a ring of charge, the electric field is given by Eq. (21.8). $\vec{F} = q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q = 0.125 \times 10^{-9} \text{ C}$, $a = 0.025 \text{ m}$ and $x = 0.400 \text{ m}$.

EXECUTE: (a) $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}} \hat{i} = (7.0 \text{ N/C}) \hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(2.50 \times 10^{-6} \text{ C})(7.0 \text{ N/C}) \hat{i} = (1.75 \times 10^{-5} \text{ N}) \hat{i}$

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

- 21.56. IDENTIFY:** We must use the appropriate electric field formula: a uniform disk in (a), a ring in (b) because all the charge is along the rim of the disk, and a point-charge in (c).

(a) SET UP: First find the surface charge density (Q/A), then use the formula for the field due to a disk of charge,

$$E_x = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{(R/x)^2 + 1}} \right].$$

EXECUTE: The surface charge density is $\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{6.50 \times 10^{-9} \text{ C}}{\pi (0.0125 \text{ m})^2} = 1.324 \times 10^{-5} \text{ C/m}^2$.

The electric field is

$$E_x = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{(R/x)^2 + 1}} \right] = \frac{1.324 \times 10^{-5} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \left[1 - \frac{1}{\sqrt{\left(\frac{1.25 \text{ cm}}{2.00 \text{ cm}}\right)^2 + 1}} \right]$$

$$E_x = 1.14 \times 10^5 \text{ N/C, toward the center of the disk.}$$

(b) SET UP: For a ring of charge, the field is $E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}$.

EXECUTE: Substituting into the electric field formula gives

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}} = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.50 \times 10^{-9} \text{ C})(0.0200 \text{ m})}{[(0.0200 \text{ m})^2 + (0.0125 \text{ m})^2]^{3/2}}$$

$$E = 8.92 \times 10^4 \text{ N/C, toward the center of the disk.}$$

(c) SET UP: For a point charge, $E = (1/4\pi\epsilon_0)q/r^2$.

EXECUTE: $E = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.50 \times 10^{-9} \text{ C})/(0.0200 \text{ m})^2 = 1.46 \times 10^5 \text{ N/C}$

(d) EVALUATE: With the ring, more of the charge is farther from P than with the disk. Also with the ring the component of the electric field parallel to the plane of the ring is greater than with the disk, and this component cancels. With the point charge in (c), all the field vectors add with no cancellation, and all the charge is closer to point P than in the other two cases.

- 21.57. IDENTIFY:** By superposition we can add the electric fields from two parallel sheets of charge.

SET UP: The field due to each sheet of charge has magnitude $\sigma/2\epsilon_0$ and is directed toward a sheet of negative charge and away from a sheet of positive charge.

(a) The two fields are in opposite directions and $E = 0$.

(b) The two fields are in opposite directions and $E = 0$.

(c) The fields of both sheets are downward and $E = 2 \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$, directed downward.

EVALUATE: The field produced by an infinite sheet of charge is uniform, independent of distance from the sheet.

- 21.58. IDENTIFY and SET UP:** The electric field produced by an infinite sheet of charge with charge density σ has

magnitude $E = \frac{|\sigma|}{2\epsilon_0}$. The field is directed toward the sheet if it has negative charge and is away from the sheet if it

has positive charge.

EXECUTE: **(a)** The field lines are sketched in Figure 21.58a.

(b) The field lines are sketched in Figure 21.58b.

EVALUATE: The spacing of the field lines indicates the strength of the field. In part (a) the two fields add between the sheets and subtract in the regions to the left of A and to the right of B . In part (b) the opposite is true.

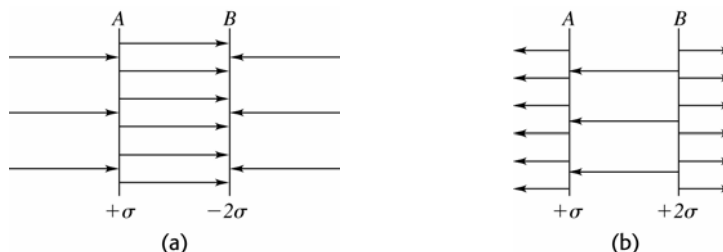


Figure 21.58

- 21.59. IDENTIFY:** The force on the particle at any point is always tangent to the electric field line at that point.
SET UP: The instantaneous velocity determines the path of the particle.
EXECUTE: In Fig. 21.29a the field lines are straight lines so the force is always in a straight line and velocity and acceleration are always in the same direction. The particle moves in a straight line along a field line, with increasing speed. In Fig. 21.29b the field lines are curved. As the particle moves its velocity and acceleration are not in the same direction and the trajectory does not follow a field line.
EVALUATE: In two-dimensional motion the velocity is always tangent to the trajectory but the velocity is not always in the direction of the net force on the particle.
- 21.60. IDENTIFY:** The field appears like that of a point charge a long way from the disk and an infinite sheet close to the disk's center. The field is symmetrical on the right and left.
SET UP: For a positive point charge, E is proportional to $1/r^2$ and is directed radially outward. For an infinite sheet of positive charge, the field is uniform and is directed away from the sheet.
EXECUTE: The field is sketched in Figure 21.60.
EVALUATE: Near the disk the field lines are parallel and equally spaced, which corresponds to a uniform field. Far from the disk the field lines are getting farther apart, corresponding to the $1/r^2$ dependence for a point charge.

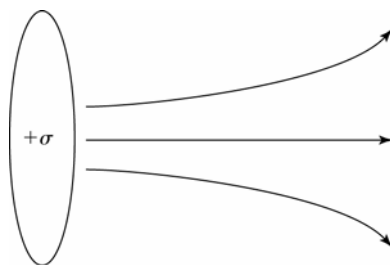


Figure 21.60

- 21.61. IDENTIFY:** Use symmetry to deduce the nature of the field lines.
(a) SET UP: The only distinguishable direction is toward the line or away from the line, so the electric field lines are perpendicular to the line of charge, as shown in Figure 21.61a.

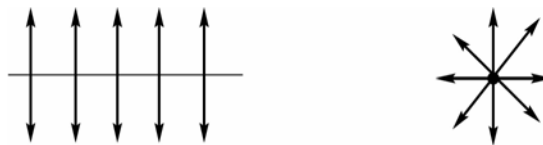


Figure 21.61a

- (b) EXECUTE and EVALUATE:** The magnitude of the electric field is inversely proportional to the spacing of the field lines. Consider a circle of radius r with the line of charge passing through the center, as shown in Figure 21.61b.

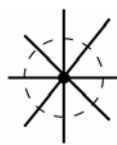


Figure 21.61b

The spacing of field lines is the same all around the circle, and in the direction perpendicular to the plane of the circle the lines are equally spaced, so E depends only on the distance r . The number of field lines passing out through the circle is independent of the radius of the circle, so the spacing of the field lines is proportional to the reciprocal of the circumference $2\pi r$ of the circle. Hence E is proportional to $1/r$.

- 21.62. IDENTIFY:** Field lines are directed away from a positive charge and toward a negative charge. The density of field lines is proportional to the magnitude of the electric field.
SET UP: The field lines represent the resultant field at each point, the net field that is the vector sum of the fields due to each of the three charges.
EXECUTE: **(a)** Since field lines pass from positive charges and toward negative charges, we can deduce that the top charge is positive, middle is negative, and bottom is positive.
(b) The electric field is the smallest on the horizontal line through the middle charge, at two positions on either side where the field lines are least dense. Here the y -components of the field are cancelled between the positive charges and the negative charge cancels the x -component of the field from the two positive charges.
EVALUATE: Far from all three charges the field is the same as the field of a point charge equal to the algebraic sum of the three charges.

- 21.63. (a) IDENTIFY and SET UP:** Use Eq.(21.14) to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.
EXECUTE: $p = qd = (4.5 \times 10^{-9} \text{ C})(3.1 \times 10^{-3} \text{ m}) = 1.4 \times 10^{-11} \text{ C} \cdot \text{m}$; The direction of $\vec{p}$ is from q_1 toward q_2 .
(b) IDENTIFY and SET UP: Use Eq. (21.15) to relate the magnitudes of the torque and field.
EXECUTE: $\tau = pE \sin \phi$, with ϕ as defined in Figure 21.63, so

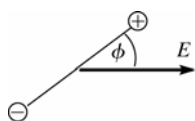


Figure 21.63

$$E = \frac{\tau}{p \sin \phi}$$

$$E = \frac{7.2 \times 10^{-9} \text{ N} \cdot \text{m}}{(1.4 \times 10^{-11} \text{ C} \cdot \text{m}) \sin 36.9^\circ} = 860 \text{ N/C}$$

EVALUATE: Eq.(21.15) gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple (Problem 11.53) and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is

$2(|q|E)(d/2) = 1.2 \times 10^{-8} \text{ N} \cdot \text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

- 21.64. (a) IDENTIFY:** The potential energy is given by Eq.(21.17).

SET UP: $U(\phi) = -\vec{p} \cdot \vec{E} = -pE \cos \phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0^\circ) = -pE$

perpendicular: $\phi = 90^\circ$ and $U(90^\circ) = 0$

$$\Delta U = U(90^\circ) - U(0^\circ) = pE = (5.0 \times 10^{-30} \text{ C} \cdot \text{m})(1.6 \times 10^6 \text{ N/C}) = 8.0 \times 10^{-24} \text{ J}.$$

$$\text{(b) } \frac{3}{2}kT = \Delta U \text{ so } T = \frac{2\Delta U}{3k} = \frac{2(8.0 \times 10^{-24} \text{ J})}{3(1.381 \times 10^{-23} \text{ J/K})} = 0.39 \text{ K}$$

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

- 21.65. IDENTIFY:** Follow the procedure specified in part (a) of the problem.

SET UP: Use that $y \gg d$.

$$\text{EXECUTE: (a) } \frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2} = \frac{(y+d/2)^2 - (y-d/2)^2}{(y^2 - d^2/4)^2} = \frac{2yd}{(y^2 - d^2/4)^2}. \text{ This gives}$$

$$E_y = \frac{q}{4\pi\epsilon_0} \frac{2yd}{(y^2 - d^2/4)^2} = \frac{qd}{2\pi\epsilon_0} \frac{y}{(y^2 - d^2/4)^2}. \text{ Since } y^2 \gg d^2/4, E_y \approx \frac{p}{2\pi\epsilon_0 y^3}.$$

(b) For points on the $-y$ -axis, $\vec{E}_-$ is in the $+y$ direction and $\vec{E}_+$ is in the $-y$ direction. The field point is closer to $-q$, so the net field is upward. A similar derivation gives $E_y \approx \frac{p}{2\pi\epsilon_0 y^3}$. E_y has the same magnitude and direction at points where $y \gg d$ as where $y \ll -d$.

EVALUATE: E falls off like $1/r^3$ for a dipole, which is faster than the $1/r^2$ for a point charge. The total charge of the dipole is zero.

- 21.66. IDENTIFY:** Calculate the electric field due to the dipole and then apply $\vec{F} = q\vec{E}$.

SET UP: From Example 21.15, $E_{\text{dipole}}(x) = \frac{p}{2\pi\epsilon_0 x^3}$.

$$\text{EXECUTE: } E_{\text{dipole}} = \frac{6.17 \times 10^{-30} \text{ C} \cdot \text{m}}{2\pi\epsilon_0 (3.0 \times 10^{-9} \text{ m})^3} = 4.11 \times 10^6 \text{ N/C}. \text{ The electric force is } F = qE =$$

$$(1.60 \times 10^{-19} \text{ C})(4.11 \times 10^6 \text{ N/C}) = 6.58 \times 10^{-13} \text{ N} \text{ and is toward the water molecule (negative } x\text{-direction).}$$

EVALUATE: $\vec{E}_{\text{dipole}}$ is in the direction of $\vec{p}$, so is in the $+x$ direction. The charge q of the ion is negative, so $\vec{F}$ is directed opposite to $\vec{E}$ and is therefore in the $-x$ direction.

- 21.67. IDENTIFY:** Like charges repel and unlike charges attract. The force increases as the distance between the charges decreases.

SET UP: The forces on the dipole that is between the slanted dipoles are sketched in Figure 21.67a.

EXECUTE: The forces are attractive because the $+$ and $-$ charges of the two dipoles are closest. The forces are toward the slanted dipoles so have a net upward component. In Figure 21.67b, adjacent dipole charges of opposite sign are closer than charges of the same sign so the attractive forces are larger than the repulsive forces and the dipoles attract.

EVALUATE: Each dipole has zero net charge, but because of the charge separation there is a non-zero force between dipoles.

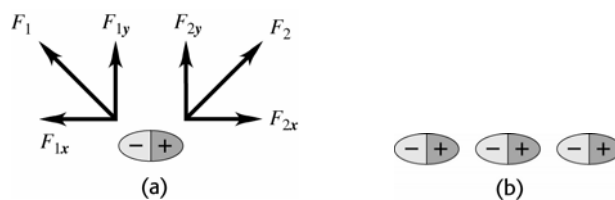


Figure 21.67

21.68. IDENTIFY: Find the vector sum of the fields due to each charge in the dipole.

SET UP: A point on the x -axis with coordinate x is a distance $r = \sqrt{(d/2)^2 + x^2}$ from each charge.

EXECUTE: (a) The magnitude of the field due to each charge is $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{(d/2)^2 + x^2} \right)$,

where d is the distance between the two charges. The x -components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y -components,

so $E = 2E_y = \frac{2q}{4\pi\epsilon_0} \left(\frac{1}{(d/2)^2 + x^2} \right) \sin\theta$, where θ is the angle below the x -axis for both fields. $\sin\theta = \frac{d/2}{\sqrt{(d/2)^2 + x^2}}$

and $E_{\text{dipole}} = \left(\frac{2q}{4\pi\epsilon_0} \right) \left(\frac{1}{(d/2)^2 + x^2} \right) \left(\frac{d/2}{\sqrt{(d/2)^2 + x^2}} \right) = \frac{qd}{4\pi\epsilon_0 ((d/2)^2 + x^2)^{3/2}}$. The field is the $-y$ direction.

(b) At large x , $x^2 \gg (d/2)^2$, so the expression in part (a) reduces to the approximation $E_{\text{dipole}} \approx \frac{qd}{4\pi\epsilon_0 x^3}$.

EVALUATE: Example 21.15 shows that at points on the $+y$ axis far from the dipole, $E_{\text{dipole}} \approx \frac{qd}{2\pi\epsilon_0 y^3}$. The expression in part (b) for points on the x axis has a similar form.

21.69. IDENTIFY: The torque on a dipole in an electric field is given by $\vec{\tau} = \vec{p} \times \vec{E}$.

SET UP: $\tau = pE \sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: (a) The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.69a.

(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.

(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.69b.

EVALUATE: The field of the dipole is directed from the $+$ charge toward the $-$ charge.

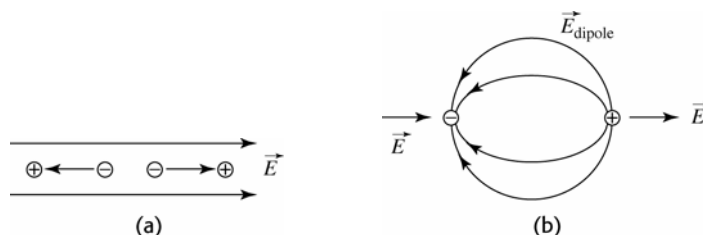


Figure 21.69

21.70. IDENTIFY: The plates produce a uniform electric field in the space between them. This field exerts torque on a dipole and gives it potential energy.

SET UP: The electric field between the plates is given by $E = \sigma/\epsilon_0$, and the dipole moment is $p = ed$. The potential energy of the dipole due to the field is $U = -\vec{p} \cdot \vec{E} = -pE \cos\phi$, and the torque the field exerts on it is $\tau = pE \sin\phi$.

EXECUTE: (a) The potential energy, $U = -\vec{p} \cdot \vec{E} = -pE \cos \phi$, is a maximum when $\phi = 180^\circ$. The field between the plates is $E = \sigma / \epsilon_0$, giving

$$U_{\max} = (1.60 \times 10^{-19} \text{ C})(220 \times 10^{-9} \text{ m})(125 \times 10^{-6} \text{ C/m}^2)/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 4.97 \times 10^{-19} \text{ J}$$

The orientation is parallel to the electric field (perpendicular to the plates) with the positive charge of the dipole toward the positive plate.

(b) The torque, $\tau = pE \sin \phi$, is a maximum when $\phi = 90^\circ$ or 270° . In this case

$$\tau_{\max} = pE = p\sigma / \epsilon_0 = ed\sigma / \epsilon_0$$

$$\tau_{\max} = (1.60 \times 10^{-19} \text{ C})(220 \times 10^{-9} \text{ m})(125 \times 10^{-6} \text{ C/m}^2)/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)$$

$$\tau_{\max} = 4.97 \times 10^{-19} \text{ N} \cdot \text{m}$$

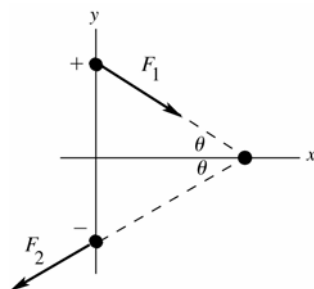
The dipole is oriented perpendicular to the electric field (parallel to the plates).

(c) $F = 0$.

EVALUATE: When the potential energy is a maximum, the torque is zero. In both cases, the net force on the dipole is zero because the forces on the charges are equal but opposite (which would not be true in a nonuniform electric field).

21.71. (a) IDENTIFY: Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.

SET UP: The two forces on each charge in the dipole are shown in Figure 21.71a.



$$\sin \theta = 1.50/2.00 \text{ so } \theta = 48.6^\circ$$

Opposite charges attract and like charges repel.

$$F_x = F_{1x} + F_{2x} = 0$$

Figure 21.71a

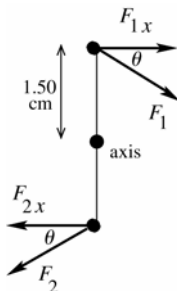
EXECUTE: $F_1 = k \frac{|qq'|}{r^2} = k \frac{(5.00 \times 10^{-6} \text{ C})(10.0 \times 10^{-6} \text{ C})}{(0.0200 \text{ m})^2} = 1.124 \times 10^3 \text{ N}$

$$F_{1y} = -F_1 \sin \theta = -842.6 \text{ N}$$

$$F_{2y} = -842.6 \text{ N so } F_y = F_{1y} + F_{2y} = -1680 \text{ N (in the direction from the } +5.00\text{-}\mu\text{C charge toward the } -5.00\text{-}\mu\text{C charge).}$$

EVALUATE: The x-components cancel and the y-components add.

(b) **SET UP:** Refer to Figure 21.71b.



The y-components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

Figure 21.71b

EXECUTE: $F_{1x} = F_1 \cos \theta = 743.1 \text{ N}$

$$\tau = 2(F_{1x})(0.0150 \text{ m}) = 22.3 \text{ N} \cdot \text{m, clockwise}$$

EVALUATE: The electric field produced by the $-10.00\mu\text{C}$ charge is not uniform so Eq. (21.15) does not apply.

21.72. IDENTIFY: Apply $F = k \frac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3 .

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.72.

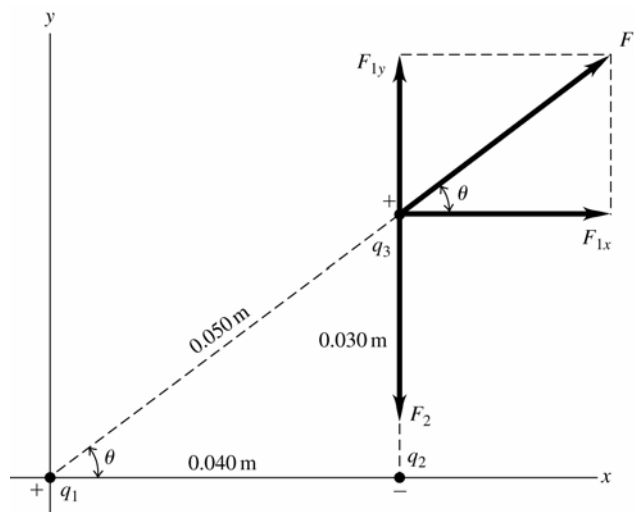


Figure 21.72

EXECUTE: (a) $F_1 = k \frac{|q_1 q_3|}{r_1^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(5.00 \times 10^{-9} \text{ C})(6.00 \times 10^{-9} \text{ C})}{(0.0500 \text{ m})^2} = 1.079 \times 10^{-4} \text{ N}$

$\theta = 36.9^\circ$. $F_{1x} = +F_1 \cos \theta = 8.63 \times 10^{-5} \text{ N}$. $F_{1y} = +F_1 \sin \theta = 6.48 \times 10^{-5} \text{ N}$.

$F_2 = k \frac{|q_2 q_3|}{r_2^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.00 \times 10^{-9} \text{ C})(6.00 \times 10^{-9} \text{ C})}{(0.0300 \text{ m})^2} = 1.20 \times 10^{-4} \text{ N}$

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20 \times 10^{-4} \text{ N}$. $F_x = F_{1x} + F_{2x} = 8.63 \times 10^{-5} \text{ N}$.

$F_y = F_{1y} + F_{2y} = 6.48 \times 10^{-5} \text{ N} + (-1.20 \times 10^{-4} \text{ N}) = -5.52 \times 10^{-5} \text{ N}$.

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02 \times 10^{-4} \text{ N}$. $\tan \phi = \left| \frac{F_y}{F_x} \right| = 0.640$. $\phi = 32.6^\circ$, below the $+x$ axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.73. (a) IDENTIFY: Use Coulomb's law to calculate the force exerted by each Q on q and add these forces as vectors to find the resultant force. Make the approximation $x \gg a$ and compare the net force to $F = -kx$ to deduce k and then $f = (1/2\pi)\sqrt{k/m}$.

SET UP: The placement of the charges is shown in Figure 21.73a.

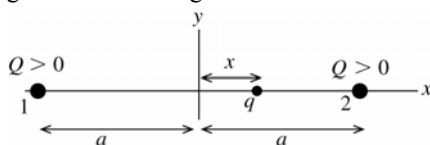


Figure 21.73a

EXECUTE: Find the net force on q .



Figure 21.73b

$F_x = F_{1x} + F_{2x}$ and $F_{1x} = +F_1$, $F_{2x} = -F_2$

$F_1 = \frac{1}{4\pi\epsilon_0} \frac{qQ}{(a+x)^2}$, $F_2 = \frac{1}{4\pi\epsilon_0} \frac{qQ}{(a-x)^2}$

$F_x = F_1 - F_2 = \frac{qQ}{4\pi\epsilon_0} \left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2} \right]$

$F_x = \frac{qQ}{4\pi\epsilon_0 a^2} \left[\left(1 + \frac{x}{a}\right)^{-2} - \left(1 - \frac{x}{a}\right)^{-2} \right]$

Since $x \ll a$ we can use the binomial expansion for $(1 - x/a)^{-2}$ and $(1 + x/a)^{-2}$ and keep only the first two terms: $(1 + z)^n \approx 1 + nz$. For $(1 - x/a)^{-2}$, $z = -x/a$ and $n = -2$ so $(1 - x/a)^{-2} \approx 1 + 2x/a$. For $(1 + x/a)^{-2}$, $z = +x/a$ and $n = -2$ so $(1 + x/a)^{-2} \approx 1 - 2x/a$. Then $F \approx \frac{qQ}{4\pi\epsilon_0 a^2} \left[\left(1 - \frac{2x}{a}\right) - \left(1 + \frac{2x}{a}\right) \right] = -\left(\frac{qQ}{\pi\epsilon_0 a^3}\right)x$. For simple harmonic motion $F = -kx$ and the frequency of oscillation is $f = (1/2\pi)\sqrt{k/m}$. The net force here is of this form, with $k = qQ/\pi\epsilon_0 a^3$. Thus $f = \frac{1}{2\pi} \sqrt{\frac{qQ}{\pi\epsilon_0 m a^3}}$.

(b) The forces and their components are shown in Figure 21.73c.

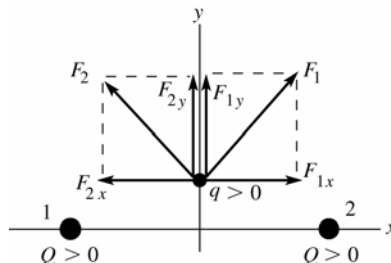


Figure 21.73c

The x-components of the forces exerted by the two charges cancel, the y-components add, and the net force is in the +y-direction when $y > 0$ and in the -y-direction when $y < 0$. The charge moves away from the origin on the y-axis and never returns.

EVALUATE: The directions of the forces and of the net force depend on where q is located relative to the other two charges. In part (a), $F = 0$ at $x = 0$ and when the charge q is displaced in the +x- or -x-direction the net force is a restoring force, directed to return q to $x = 0$. The charge oscillates back and forth, similar to a mass on a spring.

21.74. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.74. F_e is the repulsive Coulomb force between the spheres. For small θ , $\sin \theta \approx \tan \theta$.

EXECUTE: $\sum F_x = T \sin \theta - F_e = 0$ and $\sum F_y = T \cos \theta - mg = 0$. So $\frac{mg \sin \theta}{\cos \theta} = F_e = \frac{kq^2}{d^2}$. But $\tan \theta \approx \sin \theta = \frac{d}{2L}$,

so $d^3 = \frac{2kq^2L}{mg}$ and $d = \left(\frac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

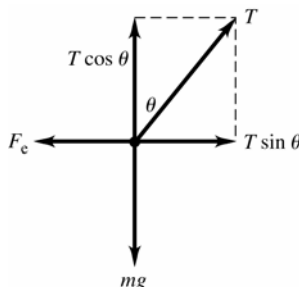


Figure 21.74

21.75. IDENTIFY: Use Coulomb's law for the force that one sphere exerts on the other and apply the 1st condition of equilibrium to one of the spheres.

(a) **SET UP:** The placement of the spheres is sketched in Figure 21.75a.

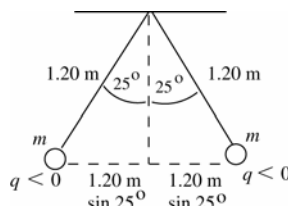


Figure 21.75a

The free-body diagrams for each sphere are given in Figure 21.75b.

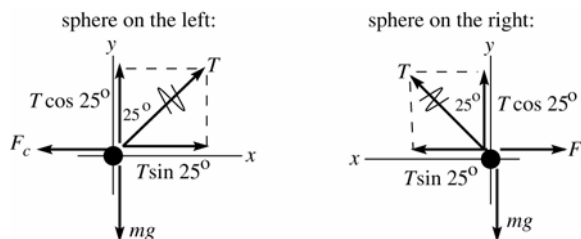


Figure 21.75b

F_c is the repulsive Coulomb force exerted by one sphere on the other.

(b) EXECUTE: From either force diagram in part (a): $\sum F_y = ma_y$

$$T \cos 25.0^\circ - mg = 0 \text{ and } T = \frac{mg}{\cos 25.0^\circ}$$

$$\sum F_x = ma_x$$

$$T \sin 25.0^\circ - F_c = 0 \text{ and } F_c = T \sin 25.0^\circ$$

Use the first equation to eliminate T in the second: $F_c = (mg / \cos 25.0^\circ)(\sin 25.0^\circ) = mg \tan 25.0^\circ$

$$F_c = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{[2(1.20 \text{ m}) \sin 25.0^\circ]^2}$$

$$\text{Combine this with } F_c = mg \tan 25.0^\circ \text{ and get } mg \tan 25.0^\circ = \frac{1}{4\pi\epsilon_0} \frac{q^2}{[2(1.20 \text{ m}) \sin 25.0^\circ]^2}$$

$$q = (2.40 \text{ m}) \sin 25.0^\circ \sqrt{\frac{mg \tan 25.0^\circ}{(1/4\pi\epsilon_0)}}$$

$$q = (2.40 \text{ m}) \sin 25.0^\circ \sqrt{\frac{(15.0 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan 25.0^\circ}{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 2.80 \times 10^{-6} \text{ C}$$

(c) The separation between the two spheres is given by $2L \sin \theta$. $q = 2.80 \mu\text{C}$ as found in part (b).

$$F_c = (1/4\pi\epsilon_0) q^2 / (2L \sin \theta)^2 \text{ and } F_c = mg \tan \theta. \text{ Thus } (1/4\pi\epsilon_0) q^2 / (2L \sin \theta)^2 = mg \tan \theta.$$

$$(\sin \theta)^2 \tan \theta = \frac{1}{4\pi\epsilon_0} \frac{q^2}{4L^2 mg} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.80 \times 10^{-6} \text{ C})^2}{4(0.600 \text{ m})^2 (15.0 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)} = 0.3328.$$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan \theta \approx \sin \theta$. With this approximation the equation becomes $\sin^3 \theta = 0.3328$ and $\sin \theta = 0.6930$, so $\theta = 43.9^\circ$. Now refine this guess:

θ	$\sin^2 \theta \tan \theta$
45.0°	0.5000
40.0°	0.3467
39.6°	0.3361
39.5°	0.3335
39.4°	0.3309

so $\theta = 39.5^\circ$

EVALUATE: The expression in part (c) says $\theta \rightarrow 0$ as $L \rightarrow \infty$ and $\theta \rightarrow 90^\circ$ as $L \rightarrow 0$. When L is decreased from the value in part (a), θ increases.

21.76. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to each sphere.

SET UP: **(a)** Free body diagrams are given in Figure 21.76. F_e is the repulsive electric force that one sphere exerts on the other.

EXECUTE: **(b)** $T = mg / \cos 20^\circ = 0.0834 \text{ N}$, so $F_e = T \sin 20^\circ = 0.0285 \text{ N} = \frac{kq_1 q_2}{r_1^2}$. (Note:

$$r_1 = 2(0.500 \text{ m}) \sin 20^\circ = 0.342 \text{ m}.)$$

(c) From part (b), $q_1 q_2 = 3.71 \times 10^{-13} \text{ C}^2$.

(d) The charges on the spheres are made equal by connecting them with a wire, but we still have

$$F_e = mg \tan \theta = 0.0453 \text{ N} = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{r_2^2}, \text{ where } Q = \frac{q_1 + q_2}{2}. \text{ But the separation } r_2 \text{ is known:}$$

$$r_2 = 2(0.500 \text{ m}) \sin 30^\circ = 0.500 \text{ m. Hence: } Q = \frac{q_1 + q_2}{2} = \sqrt{4\pi\epsilon_0 F_e r_2^2} = 1.12 \times 10^{-6} \text{ C. This equation, along}$$

with that from part (c), gives us two equations in q_1 and q_2 : $q_1 + q_2 = 2.24 \times 10^{-6} \text{ C}$ and $q_1 q_2 = 3.71 \times 10^{-13} \text{ C}^2$.

By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06 \times 10^{-6} \text{ C}$ and $q_2 = 1.80 \times 10^{-7} \text{ C}$.

EVALUATE: After the spheres are connected by the wire, the charge on sphere 1 decreases and the charge on sphere 2 increases. The product of the charges on the sphere increases and the thread makes a larger angle with the vertical.

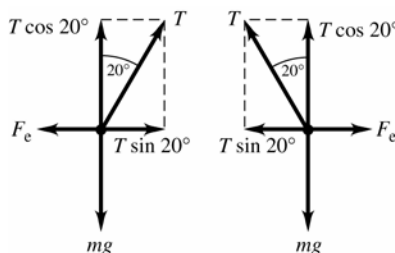


Figure 21.76

21.77. IDENTIFY and SET UP: Use Avogadro's number to find the number of Na^+ and Cl^- ions and the total positive and negative charge. Use Coulomb's law to calculate the electric force and $\vec{F} = m\vec{a}$ to calculate the acceleration.

(a) **EXECUTE:** The number of Na^+ ions in 0.100 mol of NaCl is $N = nN_A$. The charge of one ion is $+e$, so the total charge is $q_1 = nN_A e = (0.100 \text{ mol})(6.022 \times 10^{23} \text{ ions/mol})(1.602 \times 10^{-19} \text{ C/ion}) = 9.647 \times 10^3 \text{ C}$

There are the same number of Cl^- ions and each has charge $-e$, so $q_2 = -9.647 \times 10^3 \text{ C}$.

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(9.647 \times 10^3 \text{ C})^2}{(0.0200 \text{ m})^2} = 2.09 \times 10^{21} \text{ N}$$

(b) $a = F/m$. Need the mass of 0.100 mol of Cl^- ions. For Cl, $M = 35.453 \times 10^{-3} \text{ kg/mol}$, so

$$m = (0.100 \text{ mol})(35.453 \times 10^{-3} \text{ kg/mol}) = 35.45 \times 10^{-4} \text{ kg. Then } a = \frac{F}{m} = \frac{2.09 \times 10^{21} \text{ N}}{35.45 \times 10^{-4} \text{ kg}} = 5.90 \times 10^{23} \text{ m/s}^2.$$

(c) **EVALUATE:** It is not reasonable to have such a huge force. The net charges of objects are rarely larger than $1 \mu\text{C}$; a charge of 10^4 C is immense. A small amount of material contains huge amounts of positive and negative charges.

21.78. IDENTIFY: For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q , Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1} \cos \theta + F_{Qq_2} \cos \theta = 2F_{Qq_1} \cos \theta$. The electrical force on Q is

$$\text{given by Coulomb's law, } F_{Qq_1} = \frac{1}{4\pi\epsilon_0} \frac{Qq_1}{r^2} \text{ (for } q_1) \text{ and likewise for } q_2.$$

EXECUTE: First find the net force: $F = ma = (0.00500 \text{ kg})(324 \text{ m/s}^2) = 1.62 \text{ N}$. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q .

$$F = F_{Qq_1} \cos \theta + F_{Qq_2} \cos \theta = 2F_{Qq_1} \cos \theta \text{ and } F_{Qq_1} = \frac{F}{2 \cos \theta} = \frac{1.62 \text{ N}}{2 \left(\frac{2.25 \text{ cm}}{3.00 \text{ cm}} \right)} = 1.08 \text{ N. Now find the charges by}$$

solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs.

$$F_{Qq_1} = \frac{1}{4\pi\epsilon_0} \frac{Qq_1}{r^2} \text{ and } q_1 = \frac{r^2 F_{Qq_1}}{\frac{1}{4\pi\epsilon_0} Q} = \frac{(0.0300 \text{ m})^2 (1.08 \text{ N})}{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.75 \times 10^{-6} \text{ C})} = 6.17 \times 10^{-8} \text{ C.}$$

$$q_2 = -q_1 = -6.17 \times 10^{-8} \text{ C.}$$

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

- 21.79. IDENTIFY:** Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.79a.

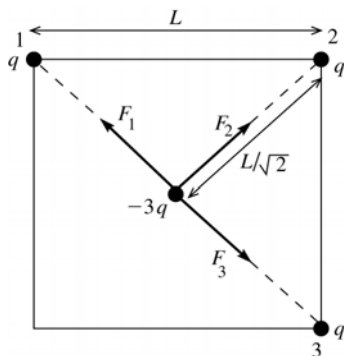


Figure 21.79a

EXECUTE: $\vec{F}_1 + \vec{F}_3 = \mathbf{0}$, so the net force is $\vec{F} = \vec{F}_2$.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q(3q)}{(L/\sqrt{2})^2} = \frac{6q^2}{4\pi\epsilon_0 L^2}, \text{ away from the vacant corner.}$$

(b) SET UP: The forces are sketched in Figure 21.79b.

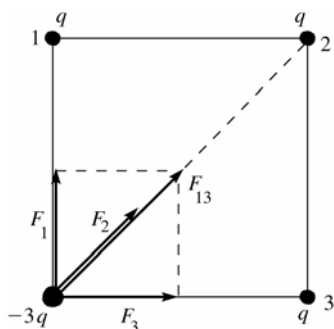


Figure 21.79b

$$\text{EXECUTE: } F_2 = \frac{1}{4\pi\epsilon_0} \frac{q(3q)}{(\sqrt{2}L)^2} = \frac{3q^2}{4\pi\epsilon_0 (2L^2)}$$

$$F_1 = F_3 = \frac{1}{4\pi\epsilon_0} \frac{q(3q)}{L^2} = \frac{3q^2}{4\pi\epsilon_0 L^2}$$

The vector sum of F_1 and F_3 is $F_{13} = \sqrt{F_1^2 + F_3^2}$.

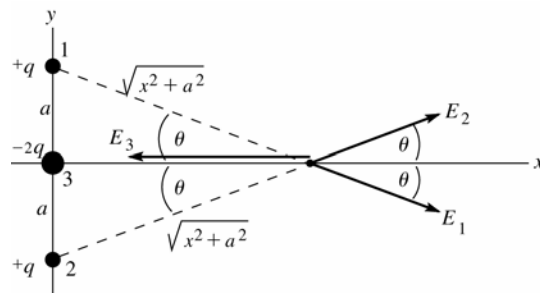
$$F_{13} = \sqrt{2}F_1 = \frac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}; \vec{F}_{13} \text{ and } \vec{F}_2 \text{ are in the same direction.}$$

$$F = F_{13} + F_2 = \frac{3q^2}{4\pi\epsilon_0 L^2} \left(\sqrt{2} + \frac{1}{2} \right), \text{ and is directed toward the center of the square.}$$

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

- 21.80. IDENTIFY:** Use Eq.(21.7) for the electric field produced by each point charge. Apply the principle of superposition and add the fields as vectors to find the net field.

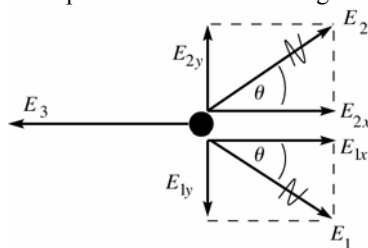
(a) SET UP: The fields due to each charge are shown in Figure 21.80a.



$$\cos \theta = \frac{x}{\sqrt{x^2 + a^2}}$$

Figure 21.80a

EXECUTE: The components of the fields are given in Figure 21.80b.



$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{a^2 + x^2} \right)$$

$$E_3 = \frac{1}{4\pi\epsilon_0} \left(\frac{2q}{x^2} \right)$$

Figure 21.80b

$$E_{1y} = -E_1 \sin \theta, E_{2y} = +E_2 \sin \theta \text{ so } E_y = E_{1y} + E_{2y} = 0.$$

$$E_{1x} = E_{2x} = +E_1 \cos \theta = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{a^2 + x^2} \right) \left(\frac{x}{\sqrt{x^2 + a^2}} \right), E_{3x} = -E_3$$

$$E_x = E_{1x} + E_{2x} + E_{3x} = 2 \left(\frac{1}{4\pi\epsilon_0} \left(\frac{q}{a^2 + x^2} \right) \left(\frac{x}{\sqrt{x^2 + a^2}} \right) \right) - \frac{2q}{4\pi\epsilon_0 x^2}$$

$$E_x = -\frac{2q}{4\pi\epsilon_0} \left(\frac{1}{x^2} - \frac{x}{(a^2 + x^2)^{3/2}} \right) = -\frac{2q}{4\pi\epsilon_0 x^2} \left(1 - \frac{1}{(1 + a^2/x^2)^{3/2}} \right)$$

$$\text{Thus } E = \frac{2q}{4\pi\epsilon_0 x^2} \left(1 - \frac{1}{(1 + a^2/x^2)^{3/2}} \right), \text{ in the } -x\text{-direction.}$$

$$\text{(b) } x \gg a \text{ implies } a^2/x^2 \ll 1 \text{ and } (1 + a^2/x^2)^{-3/2} \approx 1 - 3a^2/2x^2.$$

$$\text{Thus } E \approx \frac{2q}{4\pi\epsilon_0 x^2} \left(1 - \left(1 - \frac{3a^2}{2x^2} \right) \right) = \frac{3qa^2}{4\pi\epsilon_0 x^4}.$$

EVALUATE: $E \sim 1/x^4$. For a point charge $E \sim 1/x^2$ and for a dipole $E \sim 1/x^3$. The total charge is zero so at large distances the electric field should decrease faster with distance than for a point charge. By symmetry $\vec{E}$ must lie along the x-axis, which is the result we found in part (a).

21.81. IDENTIFY: The small bags of protons behave like point-masses and point-charges since they are extremely far apart.

SET UP: For point-particles, we use Newton's formula for universal gravitation ($F = Gm_1m_2/r^2$) and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: (a) $(0.0010 \text{ kg})/(1.67 \times 10^{-27} \text{ kg}) = 6.0 \times 10^{23}$ protons

(b) Using Coulomb's law, where the separation is twice the radius of the earth, we have

$$F_{\text{electrical}} = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.0 \times 10^{23} \times 1.60 \times 10^{-19} \text{ C})^2/(2 \times 6.38 \times 10^6 \text{ m})^2 = 5.1 \times 10^5 \text{ N}$$

$$F_{\text{grav}} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.0010 \text{ kg})^2/(2 \times 6.38 \times 10^6 \text{ m})^2 = 4.1 \times 10^{-31} \text{ N}$$

(c) **EVALUATE:** The electrical force ($\approx 200,000 \text{ lb!}$) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.82. IDENTIFY: We can treat the protons as point-charges and use Coulomb's law.

SET UP: (a) Coulomb's law is $F = (1/4\pi\epsilon_0)|q_1q_2|/r^2$.

EXECUTE: $F = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2/(2.0 \times 10^{-15} \text{ m}) = 58 \text{ N} = 13 \text{ lb}$, which is certainly large enough to feel.

(b) **EVALUATE:** Something must be holding the nucleus together by opposing this enormous repulsion. This is the strong nuclear force.

21.83. IDENTIFY: Estimate the number of protons in the textbook and from this find the net charge of the textbook.

Apply Coulomb's law to find the force and use $F_{\text{net}} = ma$ to find the acceleration.

SET UP: With the mass of the book about 1.0 kg , most of which is protons and neutrons, we find that the number of protons is $\frac{1}{2}(1.0 \text{ kg})/(1.67 \times 10^{-27} \text{ kg}) = 3.0 \times 10^{26}$.

EXECUTE: (a) The charge difference present if the electron's charge was 99.999% of the proton's is

$$\Delta q = (3.0 \times 10^{26})(0.00001)(1.6 \times 10^{-19} \text{ C}) = 480 \text{ C}.$$

(b) $F = k(\Delta q)^2/r^2 = k(480 \text{ C})^2/(5.0 \text{ m})^2 = 8.3 \times 10^{13} \text{ N}$, and is repulsive.

$$a = F/m = (8.3 \times 10^{13} \text{ N})/(1 \text{ kg}) = 8.3 \times 10^{13} \text{ m/s}^2.$$

EXECUTE: (c) Even the slightest charge imbalance in matter would lead to explosive repulsion!

- 21.84. IDENTIFY:** The electric field exerts equal and opposite forces on the two balls, causing them to swing away from each other. When the balls hang stationary, they are in equilibrium so the forces on them (electrical, gravitational, and tension in the strings) must balance.

SET UP: (a) The force on the left ball is in the direction of the electric field, so it must be positive, while the force on the right ball is opposite to the electric field, so it must be negative.

(b) Balancing horizontal and vertical forces gives $qE = T \sin \theta/2$ and $mg = T \cos \theta/2$.

EXECUTE: Solving for the angle θ gives: $\theta = 2 \arctan(qE/mg)$.

(c) As $E \rightarrow \infty$, $\theta \rightarrow 2 \arctan(\infty) = 2(\pi/2) = \pi = 180^\circ$

EVALUATE: If the field were large enough, the gravitational force would not be important, so the strings would be horizontal.

- 21.85. IDENTIFY and SET UP:** Use the density of copper to calculate the number of moles and then the number of atoms. Calculate the net charge and then use Coulomb's law to calculate the force.

EXECUTE: (a) $m = \rho V = \rho \left(\frac{4}{3} \pi r^3 \right) = (8.9 \times 10^3 \text{ kg/m}^3) \left(\frac{4}{3} \pi \right) (1.00 \times 10^{-3} \text{ m})^3 = 3.728 \times 10^{-5} \text{ kg}$

$$n = m/M = (3.728 \times 10^{-5} \text{ kg}) / (63.546 \times 10^{-3} \text{ kg/mol}) = 5.867 \times 10^{-4} \text{ mol}$$

$$N = nN_A = 3.5 \times 10^{20} \text{ atoms}$$

(b) $N_e = (29)(3.5 \times 10^{20}) = 1.015 \times 10^{22}$ electrons and protons

$$q_{\text{net}} = eN_e - (0.99900)eN_e = (0.100 \times 10^{-2})(1.602 \times 10^{-19} \text{ C})(1.015 \times 10^{22}) = 1.6 \text{ C}$$

$$F = k \frac{q^2}{r^2} = k \frac{(1.6 \text{ C})^2}{(1.00 \text{ m})^2} = 2.3 \times 10^{10} \text{ N}$$

EVALUATE: The amount of positive and negative charge in even small objects is immense. If the charge of an electron and a proton weren't exactly equal, objects would have large net charges.

- 21.86. IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0 \text{ cm}$ be the horizontal distance the drop travels and $d = 0.30 \text{ mm}$ be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and $a_y = a$.

EXECUTE: First, the mass of the drop: $m = \rho V = (1000 \text{ kg/m}^3) \left(\frac{4\pi(15.0 \times 10^{-6} \text{ m})^3}{3} \right) = 1.41 \times 10^{-11} \text{ kg}$. Next, the

$$\text{time of flight: } t = D/v = (0.020 \text{ m}) / (20 \text{ m/s}) = 0.00100 \text{ s}. \quad d = \frac{1}{2}at^2. \quad a = \frac{2d}{t^2} = \frac{2(3.00 \times 10^{-4} \text{ m})}{(0.001 \text{ s})^2} = 600 \text{ m/s}^2.$$

$$\text{Then } a = F/m = qE/m \text{ gives } q = ma/E = \frac{(1.41 \times 10^{-11} \text{ kg})(600 \text{ m/s}^2)}{8.00 \times 10^4 \text{ N/C}} = 1.06 \times 10^{-13} \text{ C}.$$

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

- 21.87. IDENTIFY:** Eq. (21.3) gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the x - and y -components of the motion, just as for projectile motion.

(a) **SET UP:** The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE$. Use coordinates where positive y is downward. Then applying $\sum \vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m$. In these coordinates the initial velocity has components $v_x = +v_0 \cos \alpha$ and $v_y = +v_0 \sin \alpha$, as shown in Figure 21.87a.

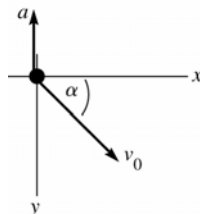


Figure 21.87a

EXECUTE: Finding $h_{\max}$: At $y = h_{\max}$ the y -component of the velocity is zero.

$$v_y = 0, v_{0y} = v_0 \sin \alpha, a_y = -eE/m, y - y_0 = h_{\max} = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

$$h_{\max} = \frac{-v_0^2 \sin^2 \alpha}{2(-eE/m)} = \frac{mv_0^2 \sin^2 \alpha}{2eE}$$

(b) Use the vertical motion to find the time t : $y - y_0 = 0$, $v_{0y} = v_0 \sin \alpha$, $a_y = -eE/m$, $t = ?$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$\text{With } y - y_0 = 0 \text{ this gives } t = -\frac{2v_{0y}}{a_y} = -\frac{2(v_0 \sin \alpha)}{-eE/m} = \frac{2mv_0 \sin \alpha}{eE}$$

Then use the x -component motion to find d : $a_x = 0$, $v_{0x} = v_0 \cos \alpha$, $t = 2mv_0 \sin \alpha / eE$, $x - x_0 = d = ?$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ gives } d = v_0 \cos \alpha \left(\frac{2mv_0 \sin \alpha}{eE} \right) = \frac{mv_0^2 2 \sin \alpha \cos \alpha}{eE} = \frac{mv_0^2 \sin 2\alpha}{eE}$$

(c) The trajectory of the proton is sketched in Figure 21.87b.

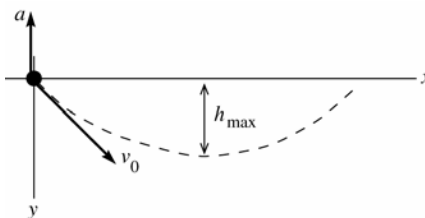


Figure 21.87b

(d) Use the expression in part (a): $h_{\max} = \frac{[(4.00 \times 10^5 \text{ m/s})(\sin 30.0^\circ)]^2 (1.673 \times 10^{-27} \text{ kg})}{2(1.602 \times 10^{-19} \text{ C})(500 \text{ N/C})} = 0.418 \text{ m}$

Use the expression in part (b): $d = \frac{(1.673 \times 10^{-27} \text{ kg})(4.00 \times 10^5 \text{ m/s})^2 \sin 60.0^\circ}{(1.602 \times 10^{-19} \text{ C})(500 \text{ N/C})} = 2.89 \text{ m}$

EVALUATE: In part (a), $a_y = -eE/m = -4.8 \times 10^{10} \text{ m/s}^2$. This is much larger in magnitude than g , the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. $h_{\max}$ and d increase when α or v_0 increase and decrease when E increases.

21.88. IDENTIFY: $E_x = E_{1x} + E_{2x}$. Use Eq.(21.7) for the electric field due to each point charge.

SET UP: $\vec{E}$ is directed away from positive charges and toward negative charges.

EXECUTE: **(a)** $E_x = +50.0 \text{ N/C}$. $E_{1x} = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{4.00 \times 10^{-9} \text{ C}}{(0.60 \text{ m})^2} = +99.9 \text{ N/C}$.

$E_x = E_{1x} + E_{2x}$, so $E_{2x} = E_x - E_{1x} = +50.0 \text{ N/C} - 99.9 \text{ N/C} = -49.9 \text{ N/C}$. Since E_{2x} is negative, q_2 must be negative. $|q_2| = \frac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \frac{(49.9 \text{ N/C})(1.20 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 7.99 \times 10^{-9} \text{ C}$. $q_2 = -7.99 \times 10^{-9} \text{ C}$

(b) $E_x = -50.0 \text{ N/C}$. $E_{1x} = +99.9 \text{ N/C}$, as in part (a). $E_{2x} = E_x - E_{1x} = -149.9 \text{ N/C}$. q_2 is negative.

$|q_2| = \frac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \frac{(149.9 \text{ N/C})(1.20 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 2.40 \times 10^{-8} \text{ C}$. $q_2 = -2.40 \times 10^{-8} \text{ C}$.

EVALUATE: q_2 would be positive if E_{2x} were positive.

21.89. IDENTIFY: Divide the charge distribution into infinitesimal segments of length dx . Calculate E_x and E_y due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx is $dQ = (Q/a)dx$. The distance between a segment at x and the charge q is $a + r - x$. $(1 - y)^{-1} \approx 1 + y$ when $|y| \ll 1$.

EXECUTE: (a) $dE_x = \frac{1}{4\pi\epsilon_0} \frac{dQ}{(a + r - x)^2}$ so $E_x = \frac{1}{4\pi\epsilon_0} \int_0^a \frac{Qdx}{a(a + r - x)^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left(\frac{1}{r} - \frac{1}{a + r} \right)$.

$a + r = x$, so $E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left(\frac{1}{x - a} - \frac{1}{x} \right)$. $E_y = 0$.

(b) $\vec{F} = q\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{a} \left(\frac{1}{x - a} - \frac{1}{x} \right) \hat{i}$.

EVALUATE: (c) For $x \gg a$, $F = \frac{kqQ}{ax} ((1 - a/x)^{-1} - 1) = \frac{kqQ}{ax} (1 + a/x + \dots - 1) \approx \frac{kqQ}{x^2} \approx \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}$. (Note that for

$x \gg a$, $r = x - a \approx x$.) The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.90. IDENTIFY: Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.11. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.

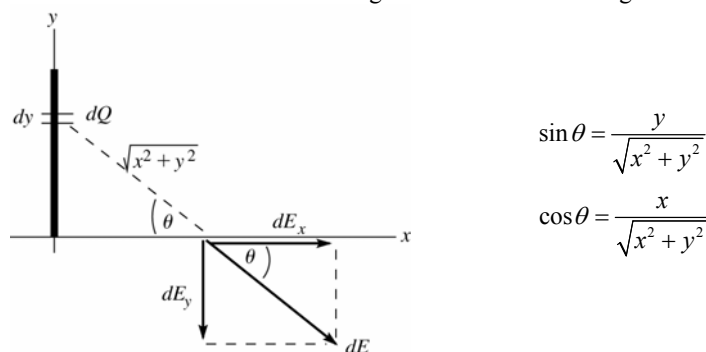


Figure 21.90

Slice the charge distribution up into small pieces of length dy . The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x -axis is dE . Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \frac{1}{4\pi\epsilon_0} \left(\frac{dQ}{x^2 + y^2} \right) = \frac{Q}{4\pi\epsilon_0 a} \left(\frac{dy}{x^2 + y^2} \right)$

$dE_x = dE \cos \theta = \frac{Qx}{4\pi\epsilon_0 a} \left(\frac{dy}{(x^2 + y^2)^{3/2}} \right)$

$dE_y = -dE \sin \theta = -\frac{Q}{4\pi\epsilon_0 a} \left(\frac{ydy}{(x^2 + y^2)^{3/2}} \right)$

$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a} \int_0^a \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a} \left[\frac{1}{x^2} \frac{y}{\sqrt{x^2 + y^2}} \right]_0^a = \frac{Q}{4\pi\epsilon_0 x} \frac{1}{\sqrt{x^2 + a^2}}$

$E_y = \int dE_y = -\frac{Q}{4\pi\epsilon_0 a} \int_0^a \frac{ydy}{(x^2 + y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a} \left[-\frac{1}{\sqrt{x^2 + y^2}} \right]_0^a = -\frac{Q}{4\pi\epsilon_0 a} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + a^2}} \right)$

(b) $\vec{F} = q_0 \vec{E}$

$F_x = -qE_x = -\frac{qQ}{4\pi\epsilon_0 x} \frac{1}{\sqrt{x^2 + a^2}}$; $F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + a^2}} \right)$

(c) For $x \gg a$, $\frac{1}{\sqrt{x^2 + a^2}} = \frac{1}{x} \left(1 + \frac{a^2}{x^2} \right)^{-1/2} = \frac{1}{x} \left(1 - \frac{a^2}{2x^2} \right) = \frac{1}{x} - \frac{a^2}{2x^3}$

$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2}$, $F_y \approx \frac{qQ}{4\pi\epsilon_0 a} \left(\frac{1}{x} - \frac{1}{x} + \frac{a^2}{2x^3} \right) = \frac{qQa}{8\pi\epsilon_0 x^3}$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \frac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$ -direction. For $x \gg a$ the charge distribution Q acts like a point charge.

21.91. IDENTIFY: Apply Eq.(21.9) from Example 21.11.

SET UP: $a = 2.50$ cm. Replace Q by $|Q|$. Since Q is negative, $\vec{E}$ is toward the line of charge and

$$\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{|Q|}{x\sqrt{x^2 + a^2}} \hat{i}.$$

EXECUTE:
$$\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{|Q|}{x\sqrt{x^2 + a^2}} \hat{i} = -\frac{1}{4\pi\epsilon_0} \frac{9.00 \times 10^{-9} \text{ C}}{(0.100 \text{ m})\sqrt{(0.100 \text{ m})^2 + (0.025 \text{ m})^2}} \hat{i} = (-7850 \text{ N/C}) \hat{i}.$$

(b) The electric field is less than that at the same distance from a point charge (8100 N/C). For large x ,

$$(x+a)^{-1/2} = \frac{1}{x} (1 + a^2/x^2)^{-1/2} \approx \frac{1}{x} \left(1 - \frac{a^2}{2x^2} \right). \quad E_{x \rightarrow \infty} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} \left(1 - \frac{a^2}{2x^2} + \dots \right).$$

The first correction term to the point charge result is negative.

(c) For a 1% difference, we need the first term in the expansion beyond the point charge result to be less than

$$0.010: \frac{a^2}{2x^2} \approx 0.010 \Rightarrow x \approx a\sqrt{1/(2(0.010))} = 0.025\sqrt{1/0.020} \Rightarrow x \approx 0.177 \text{ m}.$$

EVALUATE: At $x = 10.0$ cm (part b), the exact result for the line of charge is 3.1% smaller than for a point charge. It is sensible, therefore, that the difference is 1.0% at a somewhat larger distance, 17.7 cm.

21.92. IDENTIFY: The electrical force has magnitude $F = \frac{kQ^2}{r^2}$ and is attractive. Apply $\sum \vec{F} = m\vec{a}$ to the earth.

SET UP: For a circular orbit, $a = \frac{v^2}{r}$. The period T is $\frac{2\pi r}{v}$. The mass of the earth is $m_E = 5.97 \times 10^{24}$ kg, the orbit radius of the earth is 1.50×10^{11} m and its orbital period is 3.146×10^7 s.

EXECUTE: $F = ma$ gives $\frac{kQ^2}{r^2} = m_E \frac{v^2}{r}$. $v^2 = \frac{4\pi^2 r^2}{T^2}$, so

$$Q = \sqrt{\frac{m_E 4\pi^2 r^3}{kT^2}} = \sqrt{\frac{(5.97 \times 10^{24} \text{ kg})(4)(\pi^2)(1.50 \times 10^{11} \text{ m})^3}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.146 \times 10^7 \text{ s})^2}} = 2.99 \times 10^{17} \text{ C}.$$

EVALUATE: A very large net charge would be required.

21.93. IDENTIFY: Apply Eq.(21.11).

SET UP: $\sigma = Q/A = Q/\pi R^2$. $(1 + y^2)^{-1/2} \approx 1 - y^2/2$, when $y^2 \ll 1$.

EXECUTE: (a) $E = \frac{\sigma}{2\epsilon_0} \left[1 - \left(R^2/x^2 + 1 \right)^{-1/2} \right].$

$$E = \frac{4.00 \text{ pC}/\pi(0.025 \text{ m})^2}{2\epsilon_0} \left[1 - \left(\frac{(0.025 \text{ m})^2}{(0.200 \text{ m})^2} + 1 \right)^{-1/2} \right] = 0.89 \text{ N/C}, \text{ in the } +x \text{ direction}.$$

(b) For $x \gg R$ $E = \frac{\sigma}{2\epsilon_0} [1 - (1 - R^2/2x^2 + \dots)] \approx \frac{\sigma}{2\epsilon_0} \frac{R^2}{2x^2} = \frac{\sigma\pi R^2}{4\pi\epsilon_0 x^2} = \frac{Q}{4\pi\epsilon_0 x^2}.$

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x = 0.200$ m, the percent difference is $\frac{(0.90 - 0.89)}{0.89} = 0.01 = 1\%$. For $x = 0.100$ m,

$$E_{\text{disk}} = 3.43 \text{ N/C} \text{ and } E_{\text{point}} = 3.60 \text{ N/C}, \text{ so the percent difference is } \frac{(3.60 - 3.43)}{3.60} = 0.047 \approx 5\%.$$

EVALUATE: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x = 10.0$ cm, $R/x = 25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.94. IDENTIFY: Apply the procedure specified in the problem.

SET UP: $\int_{x_1}^{x_2} f(x) dx = -\int_{x_2}^{x_1} f(x) dx.$

EXECUTE: (a) For $f(x) = f(-x)$, $\int_{-a}^a f(x)dx = \int_{-a}^0 f(x)dx + \int_0^a f(x)dx = \int_0^{-a} f(-x)d(-x) + \int_0^a f(x)dx$. Now replace $-x$ with y . This gives $\int_{-a}^a f(x)dx = \int_0^a f(y)dy + \int_0^a f(x)dx = 2 \int_0^a f(x)dx$.

(b) For $g(x) = -g(-x)$, $\int_{-a}^a g(x)dx = \int_{-a}^0 g(x)dx + \int_0^a g(x)dx = -\int_0^{-a} -g(-x)(-d(-x)) + \int_0^a g(x)dx$. Now replace $-x$ with y . This gives $\int_{-a}^a g(x)dx = -\int_0^a g(y)dy + \int_0^a g(x)dx = 0$.

(c) The integrand in E_y for Example 21.11 is odd, so $E_y = 0$.

EVALUATE: In Example 21.11, $E_y = 0$ because for each infinitesimal segment in the upper half of the line of charge, there is a corresponding infinitesimal segment in the bottom half of the line that has E_y in the opposite direction.

21.95. IDENTIFY: Find the resultant electric field due to the two point charges. Then use $\vec{F} = q\vec{E}$ to calculate the force on the point charge.

SET UP: Use the results of Problems 21.90 and 21.89.

EXECUTE: (a) The y -components of the electric field cancel, and the x -component from both charges, as given in

Problem 21.90, is $E_x = \frac{1}{4\pi\epsilon_0} \frac{-2Q}{a} \left(\frac{1}{y} - \frac{1}{(y^2 + a^2)^{1/2}} \right)$. Therefore, $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{-2Qq}{a} \left(\frac{1}{y} - \frac{1}{(y^2 + a^2)^{1/2}} \right) \hat{i}$. If $y \gg a$

$$\vec{F} \approx \frac{1}{4\pi\epsilon_0} \frac{-2Qq}{ay} (1 - (1 - a^2/2y^2 + \dots)) \hat{i} = -\frac{1}{4\pi\epsilon_0} \frac{Qqa}{y^3} \hat{i}.$$

(b) If the point charge is now on the x -axis the two halves of the charge distribution provide different forces,

though still along the x -axis, as given in Problem 21.89: $\vec{F}_+ = q\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{Qq}{a} \left(\frac{1}{x-a} - \frac{1}{x} \right) \hat{i}$

and $\vec{F}_- = q\vec{E}_- = -\frac{1}{4\pi\epsilon_0} \frac{Qq}{a} \left(\frac{1}{x} - \frac{1}{x+a} \right) \hat{i}$. Therefore, $\vec{F} = \vec{F}_+ + \vec{F}_- = \frac{1}{4\pi\epsilon_0} \frac{Qq}{a} \left(\frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a} \right) \hat{i}$. For $x \gg a$,

$$\vec{F} \approx \frac{1}{4\pi\epsilon_0} \frac{Qq}{ax} \left(\left(1 + \frac{a}{x} + \frac{a^2}{x^2} + \dots \right) - 2 + \left(1 - \frac{a}{x} + \frac{a^2}{x^2} - \dots \right) \right) \hat{i} = \frac{1}{4\pi\epsilon_0} \frac{2Qqa}{x^3} \hat{i}.$$

EVALUATE: If the charge distributed along the x -axis were all positive or all negative, the force would be proportional to $1/y^2$ in part (a) and to $1/x^2$ in part (b), when y or x is very large.

21.96. IDENTIFY: Divide the semicircle into infinitesimal segments. Find the electric field $d\vec{E}$ due to each segment and integrate over the semicircle to find the total electric field.

SET UP: The electric fields along the x -direction from the left and right halves of the semicircle cancel. The remaining y -component points in the negative y -direction. The charge per unit length of the semicircle is

$$\lambda = \frac{Q}{\pi a} \text{ and } dE = \frac{k\lambda dl}{a^2} = \frac{k\lambda d\theta}{a}.$$

EXECUTE: $dE_y = dE \sin \theta = \frac{k\lambda \sin \theta d\theta}{a}$. Therefore, $E_y = \frac{2k\lambda}{a} \int_0^{\pi/2} \sin \theta d\theta = \frac{2k\lambda}{a} [-\cos \theta]_0^{\pi/2} = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}$, in the $-y$ -direction.

EVALUATE: For a full circle of charge the electric field at the center would be zero. For a quarter-circle of charge, in the first quadrant, the electric field at the center of curvature would have nonzero x and y components. The calculation for the semicircle is particularly simple, because all the charge is the same distance from point P .

21.97. IDENTIFY: Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x and y components of the total field.

SET UP: Consider the small segment shown in Figure 21.97a.

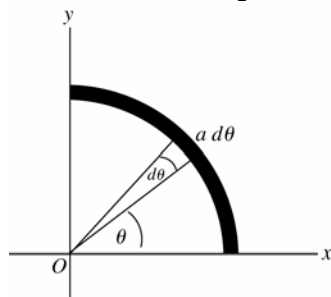


Figure 21.97a

EXECUTE: A small segment that subtends angle $d\theta$ has length $a d\theta$ and

$$\text{contains charge } dQ = \left(\frac{a d\theta}{\frac{1}{2}\pi a} \right) Q = \frac{2Q}{\pi} d\theta.$$

($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.97b, along with its components.

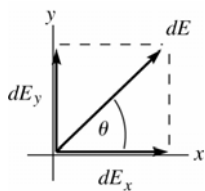


Figure 21.97b

$$dE = \frac{1}{4\pi\epsilon_0} \frac{|dQ|}{a^2}$$

$$dE = \frac{Q}{2\pi^2\epsilon_0 a^2} d\theta$$

$$dE_x = dE \cos \theta = \left(Q / 2\pi^2 \epsilon_0 a^2 \right) \cos \theta d\theta$$

$$E_x = \int dE_x = \frac{Q}{2\pi^2 \epsilon_0 a^2} \int_0^{\pi/2} \cos \theta d\theta = \frac{Q}{2\pi^2 \epsilon_0 a^2} \left(\sin \theta \Big|_0^{\pi/2} \right) = \frac{Q}{2\pi^2 \epsilon_0 a^2}$$

$$dE_y = dE \sin \theta = \left(Q / 2\pi^2 \epsilon_0 a^2 \right) \sin \theta d\theta$$

$$E_y = \int dE_y = \frac{Q}{2\pi^2 \epsilon_0 a^2} \int_0^{\pi/2} \sin \theta d\theta = \frac{Q}{2\pi^2 \epsilon_0 a^2} \left(-\cos \theta \Big|_0^{\pi/2} \right) = \frac{Q}{2\pi^2 \epsilon_0 a^2}$$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.98. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere, with x horizontal and y vertical.

SET UP: The free-body diagram for the sphere is given in Figure 21.98. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E = \frac{\sigma}{2\epsilon_0}$ (Eq. 21.12).

EXECUTE: $\sum F_y = 0$ gives $T \cos \alpha = mg$ and $T = \frac{mg}{\cos \alpha}$. $\sum F_x = 0$ gives $T \sin \alpha = \frac{q\sigma}{2\epsilon_0}$ and $T = \frac{q\sigma}{2\epsilon_0 \sin \alpha}$.

Combining these two equations we have $\frac{mg}{\cos \alpha} = \frac{q\sigma}{2\epsilon_0 \sin \alpha}$ and $\tan \alpha = \frac{q\sigma}{2\epsilon_0 mg}$. Therefore, $\alpha = \arctan \left(\frac{q\sigma}{2\epsilon_0 mg} \right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

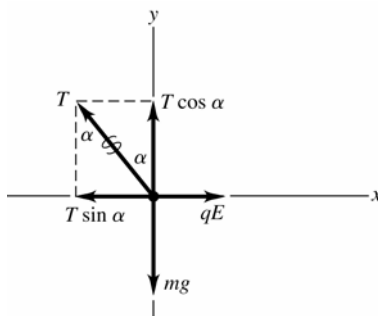


Figure 21.98

21.99. IDENTIFY: Each wire produces an electric field at P due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{\text{net}} = 2E_1 \cos 45^\circ = 2 \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}} \cos 45^\circ$. In

part (b), the electrical force on an electron at P is eE .

EXECUTE: (a) The net field is $E_{\text{net}} = 2 \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}} \cos 45^\circ$.

$$E_{\text{net}} = \frac{2(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.50 \times 10^{-6} \text{ C}) \cos 45^\circ}{(0.600 \text{ m})\sqrt{(0.600 \text{ m})^2 + (0.600 \text{ m})^2}} = 6.25 \times 10^4 \text{ N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right through the positive wire.

(b) $F = eE = (1.60 \times 10^{-19} \text{ C})(6.25 \times 10^4 \text{ N/C}) = 1.00 \times 10^{-14} \text{ N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

- 21.100. IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these,

$$E_{\text{net}} = \frac{\sigma_B}{2\epsilon_0} \pm \frac{\sigma_A}{2\epsilon_0} = \frac{\sigma_B \pm \sigma_A}{2\epsilon_0}, \text{ where we use the + or - sign depending on whether the fields are in the same or}$$

opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: (a) The two fields oppose and the field of B is stronger than that of A , so

$$E_{\text{net}} = \frac{\sigma_B}{2\epsilon_0} - \frac{\sigma_A}{2\epsilon_0} = \frac{\sigma_B - \sigma_A}{2\epsilon_0} = \frac{11.6 \mu\text{C/m}^2 - 9.50 \mu\text{C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 1.19 \times 10^5 \text{ N/C, to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}} = (11.6 \mu\text{C/m}^2 + 9.50 \mu\text{C/m}^2)/2\epsilon_0 = 1.19 \times 10^6 \text{ N/C, to the right}$$

(c) The fields add but now point to the left, so $E_{\text{net}} = 1.19 \times 10^6 \text{ N/C, to the left.}$

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

- 21.101. IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these,

$$E_{\text{net}} = \frac{\sigma_B}{2\epsilon_0} \pm \frac{\sigma_A}{2\epsilon_0} = \frac{\sigma_B \pm \sigma_A}{2\epsilon_0}, \text{ where we use the + or - sign depending on whether the fields are in the same or}$$

opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: (a) The fields add and point to the left, giving $E_{\text{net}} = 1.19 \times 10^6 \text{ N/C.}$

(b) The fields oppose and point to the left, so $E_{\text{net}} = 1.19 \times 10^5 \text{ N/C.}$

(c) The fields oppose but now point to the right, giving $E_{\text{net}} = 1.19 \times 10^5 \text{ N/C.}$

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

- 21.102. IDENTIFY:** The sheets produce an electric field in the region between them which is the vector sum of the fields from the two sheets.

SET UP: The force on the negative oil droplet must be upward to balance gravity. The net electric field between the sheets is $E = \sigma/\epsilon_0$, and the electrical force on the droplet must balance gravity, so $qE = mg$.

EXECUTE: (a) The electrical force on the drop must be upward, so the field should point downward since the drop is negative.

(b) The charge of the drop is $5e$, so $qE = mg$. $(5e)(\sigma/\epsilon_0) = mg$ and

$$\sigma = \frac{mg\epsilon_0}{5e} = \frac{(324 \times 10^{-9} \text{ kg})(9.80 \text{ m/s}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}{5(1.60 \times 10^{-19} \text{ C})} = 35.1 \text{ C/m}^2$$

EVALUATE: Balancing oil droplets between plates was the basis of the Milliken Oil-Drop Experiment which produced the first measurement of the mass of an electron.

- 21.103. IDENTIFY and SET UP:** Example 21.12 gives the electric field due to one infinite sheet. Add the two fields as vectors.

EXECUTE: The electric field due to the first sheet, which is in the xy -plane, is $\vec{E}_1 = (\sigma/2\epsilon_0)\hat{k}$ for $z > 0$ and

$\vec{E}_1 = -(\sigma/2\epsilon_0)\hat{k}$ for $z < 0$. We can write this as $\vec{E}_1 = (\sigma/2\epsilon_0)(z/|z|)\hat{k}$, since $z/|z| = +1$ for $z > 0$ and $z/|z| = -1$ for $z < 0$. Similarly, we can write the electric field due to the second sheet as $\vec{E}_2 = -(\sigma/2\epsilon_0)(x/|x|)\hat{i}$, since its

charge density is $-\sigma$. The net field is $\vec{E} = \vec{E}_1 + \vec{E}_2 = (\sigma/2\epsilon_0)(-(x/|x|)\hat{i} + (z/|z|)\hat{k})$.

EVALUATE: The electric field is independent of the y -component of the field point since displacement in the $\pm y$ -direction is parallel to both planes. The field depends on which side of each plane the field is located.

- 21.104. IDENTIFY:** Apply Eq.(21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2 .

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq.(21.11) is

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{(R/x)^2 + 1}} \right].$$

EXECUTE: (a) $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

$$(b) \vec{E}(x) = \frac{\sigma}{2\epsilon_0} \left(\left[1 - \frac{1}{\sqrt{(R_2/x)^2 + 1}} \right] - \left[1 - \frac{1}{\sqrt{(R_1/x)^2 + 1}} \right] \right) \frac{|x|}{x} \hat{i}. \quad \vec{E}(x) = \frac{-\sigma}{2\epsilon_0} \left(\frac{1}{\sqrt{(R_1/x)^2 + 1}} - \frac{1}{\sqrt{(R_2/x)^2 + 1}} \right) \frac{|x|}{x} \hat{i}.$$

The electric field is in the $+x$ direction at points above the disk and in the $-x$ direction at points below the disk, and the factor $\frac{|x|}{x} \hat{i}$ specifies these directions.

$$(c) \text{ Note that } \frac{1}{\sqrt{(R_1/x)^2 + 1}} = \frac{|x|}{R_1} (1 + (x/R_1)^2)^{-1/2} \approx \frac{|x|}{R_1}. \text{ This gives } \vec{E}(x) = \frac{\sigma}{2\epsilon_0} \left(\frac{x}{R_1} - \frac{x}{R_2} \right) \frac{|x|}{x} \hat{i} = \frac{\sigma}{2\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) x \hat{i}.$$

Sufficiently close means that $(x/R_1)^2 \ll 1$.

$$(d) F_x = qE_x = -\frac{q\sigma}{2\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) x. \text{ The force is in the form of Hooke's law: } F_x = -kx, \text{ with } k = \frac{q\sigma}{2\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{q\sigma}{2\epsilon_0 m} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.105. IDENTIFY: Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3 , and add these force vectors to get the net force.

SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.

EXECUTE: (a) The four possible force diagrams are sketched in Figure 21.105a.

Only the last picture can result in a net force in the $-x$ -direction.

(b) $q_1 = -2.00 \mu\text{C}$, $q_3 = +4.00 \mu\text{C}$, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.105b.

$$F_y = 0 = -\frac{1}{4\pi\epsilon_0} \frac{|q_1||q_3|}{(0.0400 \text{ m})^2} \sin\theta_1 + \frac{1}{4\pi\epsilon_0} \frac{|q_2||q_3|}{(0.0300 \text{ m})^2} \sin\theta_2. \text{ This gives}$$

$$q_2 = \frac{9}{16} |q_1| \frac{\sin\theta_1}{\sin\theta_2} = \frac{9}{16} |q_1| \frac{3/5}{4/5} = \frac{27}{64} |q_1| = 0.843 \mu\text{C}.$$

$$(d) F_x = F_{1x} + F_{2x} \text{ and } F_y = 0, \text{ so } F = |q_3| \frac{1}{4\pi\epsilon_0} \left(\frac{|q_1|}{(0.0400 \text{ m})^2} \frac{4}{5} + \frac{|q_2|}{(0.0300 \text{ m})^2} \frac{3}{5} \right) = 56.2 \text{ N}.$$

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2 .

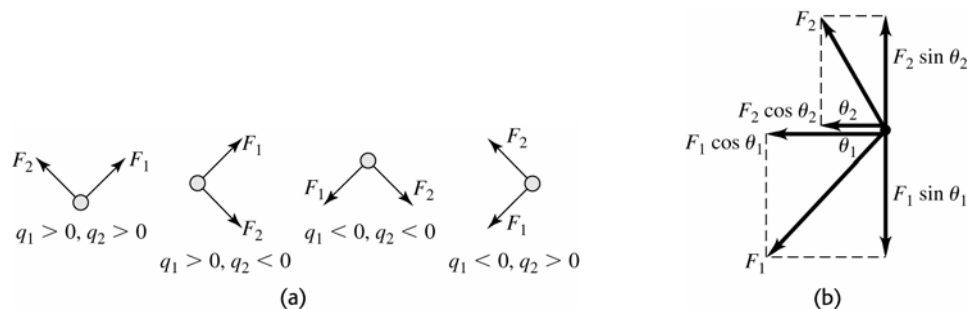


Figure 21.105

21.106. IDENTIFY: Calculate the electric field at P due to each charge and add these field vectors to get the net field.

SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.

EXECUTE: (a) The four possible diagrams are sketched in Figure 21.106a.

The first diagram is the only one in which the electric field must point in the negative y -direction.

(b) $q_1 = -3.00 \mu\text{C}$, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 24.106b. $\cos\theta_1 = \frac{5}{13}$, $\sin\theta_1 = \frac{12}{13}$, $\cos\theta_2 = \frac{12}{13}$ and $\sin\theta_2 = \frac{5}{13}$. $E_x = 0 = -\frac{k|q_1|}{(0.050\text{ m})^2} \frac{5}{13} + \frac{k|q_2|}{(0.120\text{ m})^2} \frac{12}{13}$. This gives $\frac{k|q_2|}{(0.120\text{ m})^2} = \frac{k|q_1|}{(0.050\text{ m})^2} \frac{5}{12}$. Solving for $|q_2|$ gives $|q_2| = 7.2\text{ }\mu\text{C}$, so $q_2 = -7.2\text{ }\mu\text{C}$. Then $E_y = -\frac{k|q_1|}{(0.050\text{ m})^2} \frac{12}{13} - \frac{kq_2}{(0.120\text{ m})^2} \frac{5}{13} = -1.17 \times 10^7\text{ N/C}$. $E = 1.17 \times 10^7\text{ N/C}$.

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E .

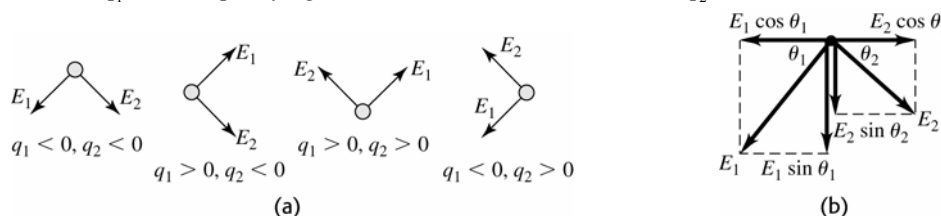


Figure 21.106

21.107. IDENTIFY: To find the electric field due to the second rod, divide that rod into infinitesimal segments of length dx , calculate the field dE due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F} = dq \vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.107. $dQ = (Q/L)dx'$.

EXECUTE: (a) $dE = \frac{k dQ}{(x + a/2 + L - x')^2} = \frac{kQ}{L} \frac{dx'}{(x + a/2 + L - x')^2}$.

$$E_x = \int_0^L dE_x = \frac{kQ}{L} \int_0^L \frac{dx'}{(x + a/2 + L - x')^2} = \frac{kQ}{L} \left[\frac{1}{x + a/2 + L - x'} \right]_0^L = \frac{kQ}{L} \left(\frac{1}{x + a/2} - \frac{1}{x + a/2 + L} \right).$$

$$E_x = \frac{2kQ}{L} \left(\frac{1}{2x + a} - \frac{1}{2L + 2x + a} \right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment dq of the first rod. This force is in the $+x$ -direction. $dF = dq E$.

$$F = \int E dq = \int_{a/2}^{L+a/2} \frac{EQ}{L} dx = \frac{2kQ^2}{L^2} \int_{a/2}^{L+a/2} \left(\frac{1}{2x + a} - \frac{1}{2L + 2x + a} \right) dx.$$

$$F = \frac{2kQ^2}{L^2} \frac{1}{2} \left([\ln(a + 2x)]_{a/2}^{L+a/2} - [\ln(2L + 2x + a)]_{a/2}^{L+a/2} \right) = \frac{kQ^2}{L^2} \ln \left(\left(\frac{a + 2L + a}{2a} \right) \left(\frac{2L + 2a}{4L + 2a} \right) \right).$$

$$F = \frac{kQ^2}{L^2} \ln \left(\frac{(a + L)^2}{a(a + 2L)} \right).$$

(c) For $a \gg L$, $F = \frac{kQ^2}{L^2} \ln \left(\frac{a^2(1 + L/a)^2}{a^2(1 + 2L/a)} \right) = \frac{kQ^2}{L^2} (2 \ln(1 + L/a) - \ln(1 + 2L/a))$.

For small z , $\ln(1 + z) \approx z - \frac{z^2}{2}$. Therefore, for $a \gg L$, $F \approx \frac{kQ^2}{L^2} \left(2 \left(\frac{L}{a} - \frac{L^2}{2a^2} + \dots \right) - \left(\frac{2L}{a} - \frac{2L^2}{a^2} + \dots \right) \right) \approx \frac{kQ^2}{a^2}$.

EVALUATE: The distance between adjacent ends of the rods is a . When $a \gg L$ the distance between the rods is much greater than their lengths and they interact as point charges.

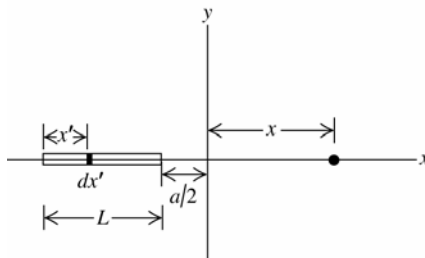


Figure 21.107

GAUSS'S LAW

- 22.1. (a) IDENTIFY and SET UP:** $\Phi_E = \int E \cos \phi dA$, where ϕ is the angle between the normal to the sheet $\hat{n}$ and the electric field $\vec{E}$.
EXECUTE: In this problem E and $\cos \phi$ are constant over the surface so
 $\Phi_E = E \cos \phi \int dA = E \cos \phi A = (14 \text{ N/C})(\cos 60^\circ)(0.250 \text{ m}^2) = 1.8 \text{ N} \cdot \text{m}^2/\text{C}$.
(b) EVALUATE: Φ_E is independent of the shape of the sheet as long as ϕ and E are constant at all points on the sheet.
(c) EXECUTE: (i) $\Phi_E = E \cos \phi A$. Φ_E is largest for $\phi = 0^\circ$, so $\cos \phi = 1$ and $\Phi_E = EA$.
(ii) Φ_E is smallest for $\phi = 90^\circ$, so $\cos \phi = 0$ and $\Phi_E = 0$.
EVALUATE: Φ_E is 0 when the surface is parallel to the field so no electric field lines pass through the surface.
- 22.2. IDENTIFY:** The field is uniform and the surface is flat, so use $\Phi_E = EA \cos \phi$.
SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{E}$, so $\phi = 70^\circ$.
EXECUTE: $\Phi_E = (75.0 \text{ N/C})(0.400 \text{ m})(0.600 \text{ m}) \cos 70^\circ = 6.16 \text{ N} \cdot \text{m}^2/\text{C}$
EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 18.0 \text{ N} \cdot \text{m}^2/\text{C}$. The flux in this problem is much less than this because only the component of $\vec{E}$ perpendicular to the surface contributes to the flux.
- 22.3. IDENTIFY:** The electric flux through an area is defined as the product of the component of the electric field perpendicular to the area times the area.
(a) SET UP: In this case, the electric field is perpendicular to the surface of the sphere, so $\Phi_E = EA = E(4\pi r^2)$.
EXECUTE: Substituting in the numbers gives

$$\Phi_E = (1.25 \times 10^6 \text{ N/C})4\pi(0.150 \text{ m})^2 = 3.53 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$$

(b) IDENTIFY: We use the electric field due to a point charge.
SET UP: $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
EXECUTE: Solving for q and substituting the numbers gives

$$q = 4\pi\epsilon_0 r^2 E = \frac{1}{9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} (0.150 \text{ m})^2 (1.25 \times 10^6 \text{ N/C}) = 3.13 \times 10^{-6} \text{ C}$$

EVALUATE: The flux would be the same no matter how large the circle, since the area is proportional to r^2 while the electric field is proportional to $1/r^2$.
- 22.4. IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface. Use Eq.(22.8) to calculate the total enclosed charge.
SET UP: $\vec{E} = (-5.00 \text{ N/C} \cdot \text{m})x\hat{i} + (3.00 \text{ N/C} \cdot \text{m})z\hat{k}$. The area of each face is L^2 , where $L = 0.300 \text{ m}$.
EXECUTE: $\hat{n}_{s_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E} \cdot \hat{n}_{s_1} A = 0$.
 $\hat{n}_{s_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E} \cdot \hat{n}_{s_2} A = (3.00 \text{ N/C} \cdot \text{m})(0.300 \text{ m})^2 z = (0.27 \text{ (N/C)} \cdot \text{m})z$.
 $\Phi_2 = (0.27 \text{ (N/C)} \cdot \text{m})(0.300 \text{ m}) = 0.081 \text{ (N/C)} \cdot \text{m}^2$.
 $\hat{n}_{s_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E} \cdot \hat{n}_{s_3} A = 0$.
 $\hat{n}_{s_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E} \cdot \hat{n}_{s_4} A = -(0.27 \text{ (N/C)} \cdot \text{m})z = 0$ (since $z = 0$).
 $\hat{n}_{s_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E} \cdot \hat{n}_{s_5} A = (-5.00 \text{ N/C} \cdot \text{m})(0.300 \text{ m})^2 x = -(0.45 \text{ (N/C)} \cdot \text{m})x$.
 $\Phi_5 = -(0.45 \text{ (N/C)} \cdot \text{m})(0.300 \text{ m}) = -(0.135 \text{ (N/C)} \cdot \text{m}^2)$.
 $\hat{n}_{s_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E} \cdot \hat{n}_{s_6} A = +(0.45 \text{ (N/C)} \cdot \text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_3 = (0.081 - 0.135)(\text{N/C}) \cdot \text{m}^2 = -0.054 \text{ N} \cdot \text{m}^2/\text{C}$. Therefore, $q = \epsilon_0 \Phi = -4.78 \times 10^{-13} \text{ C}$.

EVALUATE: Flux is positive when $\vec{E}$ is directed out of the volume and negative when it is directed into the volume.

- 22.5. IDENTIFY: The flux through the curved upper half of the hemisphere is the same as the flux through the flat circle defined by the bottom of the hemisphere because every electric field line that passes through the flat circle also must pass through the curved surface of the hemisphere.

SET UP: The electric field is perpendicular to the flat circle, so the flux is simply the product of E and the area of the flat circle of radius r .

EXECUTE: $\Phi_E = EA = E(\pi r^2) = \pi r^2 E$

EVALUATE: The flux would be the same if the hemisphere were replaced by any other surface bounded by the flat circle.

- 22.6. IDENTIFY: Use Eq.(22.3) to calculate the flux for each surface.

SET UP: $\Phi = \vec{E} \cdot \vec{A} = EA \cos \phi$ where $\vec{A} = A\hat{n}$.

EXECUTE: (a) $\hat{n}_{S_1} = -\hat{j}$ (left). $\Phi_{S_1} = -(4 \times 10^3 \text{ N/C})(0.10 \text{ m})^2 \cos(90^\circ - 36.9^\circ) = -24 \text{ N} \cdot \text{m}^2/\text{C}$.

$\hat{n}_{S_2} = +\hat{k}$ (top). $\Phi_{S_2} = -(4 \times 10^3 \text{ N/C})(0.10 \text{ m})^2 \cos 90^\circ = 0$.

$\hat{n}_{S_3} = +\hat{j}$ (right). $\Phi_{S_3} = +(4 \times 10^3 \text{ N/C})(0.10 \text{ m})^2 \cos(90^\circ - 36.9^\circ) = +24 \text{ N} \cdot \text{m}^2/\text{C}$.

$\hat{n}_{S_4} = -\hat{k}$ (bottom). $\Phi_{S_4} = (4 \times 10^3 \text{ N/C})(0.10 \text{ m})^2 \cos 90^\circ = 0$.

$\hat{n}_{S_5} = +\hat{i}$ (front). $\Phi_{S_5} = +(4 \times 10^3 \text{ N/C})(0.10 \text{ m})^2 \cos 36.9^\circ = 32 \text{ N} \cdot \text{m}^2/\text{C}$.

$\hat{n}_{S_6} = -\hat{i}$ (back). $\Phi_{S_6} = -(4 \times 10^3 \text{ N/C})(0.10 \text{ m})^2 \cos 36.9^\circ = -32 \text{ N} \cdot \text{m}^2/\text{C}$.

EVALUATE: (b) The total flux through the cube must be zero; any flux entering the cube must also leave it, since the field is uniform. Our calculation gives the result; the sum of the fluxes calculated in part (a) is zero.

- 22.7. (a) IDENTIFY: Use Eq.(22.5) to calculate the flux through the surface of the cylinder.

SET UP: The line of charge and the cylinder are sketched in Figure 22.7.

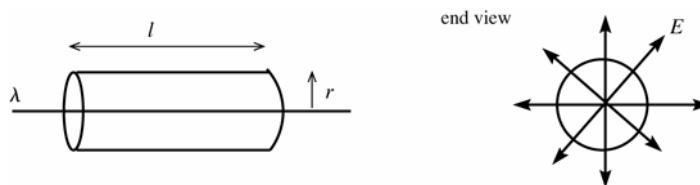


Figure 22.7

EXECUTE: The area of the curved part of the cylinder is $A = 2\pi r l$.

The electric field is parallel to the end caps of the cylinder, so $\vec{E} \cdot \vec{A} = 0$ for the ends and the flux through the cylinder end caps is zero.

The electric field is normal to the curved surface of the cylinder and has the same magnitude $E = \lambda / 2\pi\epsilon_0 r$ at all points on this surface. Thus $\phi = 0^\circ$ and

$$\Phi_E = EA \cos \phi = EA = (\lambda / 2\pi\epsilon_0 r)(2\pi r l) = \frac{\lambda l}{\epsilon_0} = \frac{(6.00 \times 10^{-6} \text{ C/m})(0.400 \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 2.71 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$$

(b) In the calculation in part (a) the radius r of the cylinder divided out, so the flux remains the same,

$$\Phi_E = 2.71 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}.$$

$$(c) \Phi_E = \frac{\lambda l}{\epsilon_0} = \frac{(6.00 \times 10^{-6} \text{ C/m})(0.800 \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 5.42 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \text{ (twice the flux calculated in parts (b) and (c)).}$$

EVALUATE: The flux depends on the number of field lines that pass through the surface of the cylinder.

- 22.8. IDENTIFY: Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: (a) $\Phi_{S_1} = q_1/\epsilon_0 = (4.00 \times 10^{-9} \text{ C})/\epsilon_0 = 452 \text{ N} \cdot \text{m}^2/\text{C}$.

(b) $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80 \times 10^{-9} \text{ C})/\epsilon_0 = -881 \text{ N} \cdot \text{m}^2/\text{C}$.

(c) $\Phi_{S_3} = (q_1 + q_2)/\epsilon_0 = ((4.00 - 7.80) \times 10^{-9} \text{ C})/\epsilon_0 = -429 \text{ N} \cdot \text{m}^2/\text{C}$.

(d) $\Phi_{S_4} = (q_1 + q_3)/\epsilon_0 = ((4.00 + 2.40) \times 10^{-9} \text{ C})/\epsilon_0 = 723 \text{ N} \cdot \text{m}^2/\text{C}$.

(e) $\Phi_{S_5} = (q_1 + q_2 + q_3)/\epsilon_0 = ((4.00 - 7.80 + 2.40) \times 10^{-9} \text{ C})/\epsilon_0 = -158 \text{ N} \cdot \text{m}^2/\text{C}$.

EVALUATE: (f) All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.9. IDENTIFY: Apply the results in Example 21.10 for the field of a spherical shell of charge.

SET UP: Example 22.10 shows that $E = 0$ inside a uniform spherical shell and that $E = k \frac{|q|}{r^2}$ outside the shell.

EXECUTE: (a) $E = 0$

(b) $r = 0.060 \text{ m}$ and $E = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{15.0 \times 10^{-6} \text{ C}}{(0.060 \text{ m})^2} = 3.75 \times 10^7 \text{ N/C}$

(c) $r = 0.110 \text{ m}$ and $E = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{15.0 \times 10^{-6} \text{ C}}{(0.110 \text{ m})^2} = 1.11 \times 10^7 \text{ N/C}$

EVALUATE: Outside the shell the electric field is the same as if all the charge were concentrated at the center of the shell. But inside the shell the field is not the same as for a point charge at the center of the shell, inside the shell the electric field is zero.

22.10. IDENTIFY: Apply Gauss's law to the spherical surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by the sphere.

EXECUTE: (a) No charge enclosed so $\Phi = 0$.

(b) $\Phi = \frac{q_2}{\epsilon_0} = \frac{-6.00 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = -678 \text{ N} \cdot \text{m}^2/\text{C}$.

(c) $\Phi = \frac{q_1 + q_2}{\epsilon_0} = \frac{(4.00 - 6.00) \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = -226 \text{ N} \cdot \text{m}^2/\text{C}$.

EVALUATE: Negative flux corresponds to flux directed into the enclosed volume. The net flux depends only on the net charge enclosed by the surface and is not affected by any charges outside the enclosed volume.

22.11. IDENTIFY: Apply Gauss's law.

SET UP: In each case consider a small Gaussian surface in the region of interest.

EXECUTE: (a) Since $\vec{E}$ is uniform, the flux through a closed surface must be zero. That is:

$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int \rho dV = 0 \Rightarrow \int \rho dV = 0$. But because we can choose any volume we want, ρ must be zero if the integral equals zero.

(b) If there is no charge in a region of space, that does NOT mean that the electric field is uniform. Consider a closed volume close to, but not including, a point charge. The field diverges there, but there is no charge in that region.

EVALUATE: The electric field within a region can depend on charges located outside the region. But the flux through a closed surface depends only on the net charge contained within that surface.

22.12. IDENTIFY: Apply Gauss's law.

SET UP: Use a small Gaussian surface located in the region of question.

EXECUTE: (a) If $\rho > 0$ and uniform, then q inside any closed surface is greater than zero. This implies $\Phi > 0$, so

$\oint \vec{E} \cdot d\vec{A} > 0$ and so the electric field cannot be uniform. That is, since an arbitrary surface of our choice encloses a non-zero amount of charge, E must depend on position.

(b) However, inside a small bubble of zero charge density within the material with density ρ , the field can be uniform. All that is important is that there be zero flux through the surface of the bubble (since it encloses no charge). (See Problem 22.61.)

EVALUATE: In a region of uniform field, the flux through any closed surface is zero.

22.13. (a) IDENTIFY and SET UP: It is rather difficult to calculate the flux directly from $\Phi = \oint \vec{E} \cdot d\vec{A}$ since the magnitude of $\vec{E}$ and its angle with $d\vec{A}$ varies over the surface of the cube. A much easier approach is to use Gauss's law to calculate the total flux through the cube. Let the cube be the Gaussian surface. The charge enclosed is the point charge.

EXECUTE: $\Phi_E = Q_{\text{encl}}/\epsilon_0 = \frac{9.60 \times 10^{-6} \text{ C}}{8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 1.084 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}$.

By symmetry the flux is the same through each of the six faces, so the flux through one face is

$\frac{1}{6}(1.084 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}) = 1.81 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$.

(b) **EVALUATE:** In part (a) the size of the cube did not enter into the calculations. The flux through one face depends only on the amount of charge at the center of the cube. So the answer to (a) would not change if the size of the cube were changed.

22.14. IDENTIFY: Apply the results of Examples 22.9 and 22.10.

SET UP: $E = k \frac{|q|}{r^2}$ outside the sphere. A proton has charge $+e$.

EXECUTE: (a) $E = k \frac{|q|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{92(1.60 \times 10^{-19} \text{ C})}{(7.4 \times 10^{-15} \text{ m})^2} = 2.4 \times 10^{21} \text{ N/C}$

(b) For $r = 1.0 \times 10^{-10} \text{ m}$, $E = (2.4 \times 10^{21} \text{ N/C}) \left(\frac{7.4 \times 10^{-15} \text{ m}}{1.0 \times 10^{-10} \text{ m}} \right)^2 = 1.3 \times 10^{13} \text{ N/C}$

(c) $E = 0$, inside a spherical shell.

EVALUATE: The electric field in an atom is very large.

22.15. IDENTIFY: The electric fields are produced by point charges.

SET UP: We use Coulomb's law, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$, to calculate the electric fields.

EXECUTE: (a) $E = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.00 \times 10^{-6} \text{ C}}{(1.00 \text{ m})^2} = 4.50 \times 10^4 \text{ N/C}$

(b) $E = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.00 \times 10^{-6} \text{ C}}{(7.00 \text{ m})^2} = 9.18 \times 10^2 \text{ N/C}$

(c) Every field line that enters the sphere on one side leaves it on the other side, so the net flux through the surface is zero.

EVALUATE: The flux would be zero no matter what shape the surface had, providing that no charge was inside the surface.

22.16. IDENTIFY: Apply the results of Example 22.5.

SET UP: At a point 0.100 m outside the surface, $r = 0.550 \text{ m}$.

EXECUTE: (a) $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{(2.50 \times 10^{-10} \text{ C})}{(0.550 \text{ m})^2} = 7.44 \text{ N/C}$.

(b) $E = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

EVALUATE: Outside the sphere its electric field is the same as would be produced by a point charge at its center, with the same charge.

22.17. IDENTIFY: The electric field required to produce a spark 6 in. long is 6 times as strong as the field needed to produce a spark 1 in. long.

SET UP: By Gauss's law, $q = \epsilon_0 EA$ and the electric field is the same as for a point-charge, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$.

EXECUTE: (a) The electric field for 6-inch sparks is $E = 6 \times 2.00 \times 10^4 \text{ N/C} = 1.20 \times 10^5 \text{ N/C}$

The charge to produce this field is

$q = \epsilon_0 EA = \epsilon_0 E(4\pi r^2) = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.20 \times 10^5 \text{ N/C})(4\pi)(0.15 \text{ m})^2 = 3.00 \times 10^{-7} \text{ C}$.

(b) Using Coulomb's law gives $E = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{3.00 \times 10^{-7} \text{ C}}{(0.150 \text{ m})^2} = 1.20 \times 10^5 \text{ N/C}$.

EVALUATE: It takes only about $0.3 \mu\text{C}$ to produce a field this strong.

22.18. IDENTIFY: According to Exercise 21.32, the Earth's electric field points towards its center. Since Mars's electric field is similar to that of Earth, we assume it points toward the center of Mars. Therefore the charge on Mars must be negative. We use Gauss's law to relate the electric flux to the charge causing it.

SET UP: Gauss's law is $\Phi_E = \frac{q}{\epsilon_0}$ and the electric flux is $\Phi_E = EA$.

EXECUTE: (a) Solving Gauss's law for q , putting in the numbers, and recalling that q is negative, gives

$q = -\epsilon_0 \Phi_E = -(3.63 \times 10^{16} \text{ N} \cdot \text{m}^2/\text{C})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = -3.21 \times 10^5 \text{ C}$.

(b) Use the definition of electric flux to find the electric field. The area to use is the surface area of Mars.

$E = \frac{\Phi_E}{A} = \frac{3.63 \times 10^{16} \text{ N} \cdot \text{m}^2/\text{C}}{4\pi(3.40 \times 10^6 \text{ m})^2} = 2.50 \times 10^2 \text{ N/C}$

(c) The surface charge density on Mars is therefore $\sigma = \frac{q}{A_{\text{Mars}}} = \frac{-3.21 \times 10^5 \text{ C}}{4\pi(3.40 \times 10^6 \text{ m})^2} = -2.21 \times 10^{-9} \text{ C/m}^2$

EVALUATE: Even though the charge on Mars is very large, it is spread over a large area, giving a small surface charge density.

- 22.19. IDENTIFY and SET UP:** Example 22.5 derived that the electric field just outside the surface of a spherical conductor that has net charge q is $E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$. Calculate q and from this the number of excess electrons.

EXECUTE: $q = \frac{R^2 E}{(1/4\pi\epsilon_0)} = \frac{(0.160 \text{ m})^2 (1150 \text{ N/C})}{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 3.275 \times 10^{-9} \text{ C}.$

Each electron has a charge of magnitude $e = 1.602 \times 10^{-19} \text{ C}$, so the number of excess electrons needed is

$$\frac{3.275 \times 10^{-9} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 2.04 \times 10^{10}.$$

EVALUATE: The result we obtained for q is a typical value for the charge of an object. Such net charges correspond to a large number of excess electrons since the charge of each electron is very small.

- 22.20. IDENTIFY:** Apply Gauss's law.

SET UP: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and is directed perpendicular to the surface.

EXECUTE: $\oint \vec{E} \cdot d\vec{A} = EA_{\text{cylinder}} = E(2\pi rL) = (840 \text{ N/C})(2\pi)(0.400 \text{ m})(0.0200 \text{ m}) = 42.2 \text{ N} \cdot \text{m}^2/\text{C}.$

The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E} \cdot d\vec{A} = 0$. From Gauss's law,

$$q = \epsilon_0 \Phi_E = (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(42.2 \text{ N} \cdot \text{m}^2/\text{C}) = 3.74 \times 10^{-10} \text{ C}.$$

EVALUATE: We could have applied the result in Example 22.6 and solved for λ . Then $q = \lambda L$.

- 22.21. IDENTIFY:** Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of charge and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.21.

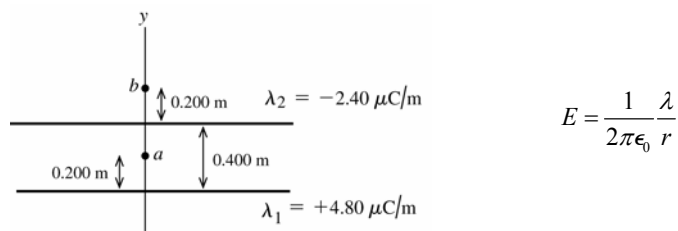


Figure 22.21

EXECUTE: (a) At point a , $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$ -direction (toward negative charge, away from positive charge).

$$E_1 = (1/2\pi\epsilon_0) \left[(4.80 \times 10^{-6} \text{ C/m}) / (0.200 \text{ m}) \right] = 4.314 \times 10^5 \text{ N/C}$$

$$E_2 = (1/2\pi\epsilon_0) \left[(2.40 \times 10^{-6} \text{ C/m}) / (0.200 \text{ m}) \right] = 2.157 \times 10^5 \text{ N/C}$$

$$E = E_1 + E_2 = 6.47 \times 10^5 \text{ N/C, in the } y\text{-direction.}$$

(b) At point b , $\vec{E}_1$ is in the $+y$ -direction and $\vec{E}_2$ is in the $-y$ -direction.

$$E_1 = (1/2\pi\epsilon_0) \left[(4.80 \times 10^{-6} \text{ C/m}) / (0.600 \text{ m}) \right] = 1.438 \times 10^5 \text{ N/C}$$

$$E_2 = (1/2\pi\epsilon_0) \left[(2.40 \times 10^{-6} \text{ C/m}) / (0.200 \text{ m}) \right] = 2.157 \times 10^5 \text{ N/C}$$

$$E = E_2 - E_1 = 7.2 \times 10^4 \text{ N/C, in the } -y\text{-direction.}$$

EVALUATION: At point a the two fields are in the same direction and the magnitudes add. At point b the two fields are in opposite directions and the magnitudes subtract.

- 22.22. IDENTIFY:** Apply the results of Examples 22.5, 22.6 and 22.7.

SET UP: Gauss's law can be used to show that the field outside a long conducting cylinder is the same as for a line of charge along the axis of the cylinder.

EXECUTE: (a) For points outside a uniform spherical charge distribution, all the charge can be considered to be concentrated at the center of the sphere. The field outside the sphere is thus inversely proportional to the square of the distance from the center. In this case,

$$E = (480 \text{ N/C}) \left(\frac{0.200 \text{ cm}}{0.600 \text{ cm}} \right)^2 = 53 \text{ N/C}$$

(b) For points outside a long cylindrically symmetrical charge distribution, the field is identical to that of a long line of charge: $E = \frac{\lambda}{2\pi\epsilon_0 r}$, that is, inversely proportional to the distance from the axis of the cylinder. In this case

$$E = (480 \text{ N/C}) \left(\frac{0.200 \text{ cm}}{0.600 \text{ cm}} \right) = 160 \text{ N/C}$$

(c) The field of an infinite sheet of charge is $E = \sigma/2\epsilon_0$; i.e., it is independent of the distance from the sheet. Thus in this case $E = 480 \text{ N/C}$.

EVALUATE: For each of these three distributions of charge the electric field has a different dependence on distance.
22.23. IDENTIFY: The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) **SET UP:** First find the initial positive charge on the outer surface of the conductor using $q_i = \sigma A$, where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally use the definition of surface charge density.

EXECUTE: The original positive charge on the outer surface is

$$q_i = \sigma A = \sigma(4\pi r^2) = (6.37 \times 10^{-6} \text{ C/m}^2)4\pi(0.250 \text{ m})^2 = 5.00 \times 10^{-6} \text{ C/m}^2$$

After the introduction of $-0.500 \mu\text{C}$ into the cavity, the outer charge is now

$$5.00 \mu\text{C} - 0.500 \mu\text{C} = 4.50 \mu\text{C}$$

The surface charge density is now $\sigma = \frac{q}{A} = \frac{q}{4\pi r^2} = \frac{4.50 \times 10^{-6} \text{ C}}{4\pi(0.250 \text{ m})^2} = 5.73 \times 10^{-6} \text{ C/m}^2$

(b) **SET UP:** Using Gauss's law, the electric field is $E = \frac{\Phi_E}{A} = \frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 4\pi r^2}$

EXECUTE: Substituting numbers gives

$$E = \frac{4.50 \times 10^{-6} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4\pi)(0.250 \text{ m})^2} = 6.47 \times 10^5 \text{ N/C}.$$

(c) **SET UP:** We use Gauss's law again to find the flux. $\Phi_E = \frac{q}{\epsilon_0}$.

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = -5.65 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C}^2.$$

EVALUATE: The excess charge on the conductor is still $+5.00 \mu\text{C}$, as it originally was. The introduction of the $-0.500 \mu\text{C}$ inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.24. IDENTIFY: We apply Gauss's law, taking the Gaussian surface beyond the cavity but inside the solid.

SET UP: Because of the symmetry of the charge, Gauss's law gives us $E = \frac{q_{\text{total}}}{\epsilon_0 A}$, where A is the surface area of a

sphere of radius $R = 9.50 \text{ cm}$ centered on the point-charge, and q_{total} is the total charge contained within that sphere. This charge is the sum of the $-2.00 \mu\text{C}$ point charge at the center of the cavity plus the charge within the solid between $r = 6.50 \text{ cm}$ and $R = 9.50 \text{ cm}$. The charge within the solid is $q_{\text{solid}} = \rho V = \rho([4/3]\pi R^3 - [4/3]\pi r^3) = ([4\pi/3]\rho)(R^3 - r^3)$

EXECUTE: First find the charge within the solid between $r = 6.50 \text{ cm}$ and $R = 9.50 \text{ cm}$:

$$q_{\text{solid}} = \frac{4\pi}{3}(7.35 \times 10^{-4} \text{ C/m}^3)[(0.0950 \text{ m})^3 - (0.0650 \text{ m})^3] = 1.794 \times 10^{-6} \text{ C},$$

Now find the total charge within the Gaussian surface:

$$q_{\text{total}} = q_{\text{solid}} + q_{\text{point}} = -2.00 \mu\text{C} + 1.794 \mu\text{C} = -0.2059 \mu\text{C}$$

Now find the magnitude of the electric field from Gauss's law:

$$E = \frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 4\pi r^2} = \frac{0.2059 \times 10^{-6} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4\pi)(0.0950 \text{ m})^2} = 2.05 \times 10^5 \text{ N/C}.$$

The fact that the charge is negative means that the electric field points radially inward.

EVALUATE: Because of the uniformity of the charge distribution, the charge beyond 9.50 cm does not contribute to the electric field.

- 22.25. IDENTIFY:** The magnitude of the electric field is constant at any given distance from the center because the charge density is uniform inside the sphere. We can use Gauss's law to relate the field to the charge causing it.

(a) SET UP: Gauss's law tells us that $EA = \frac{q}{\epsilon_0}$, and the charge density is given by $\rho = \frac{q}{V} = \frac{q}{(4/3)\pi R^3}$.

EXECUTE: Solving for q and substituting numbers gives

$$q = EA\epsilon_0 = E(4\pi r^2)\epsilon_0 = (1750 \text{ N/C})(4\pi)(0.500 \text{ m})^2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 4.866 \times 10^{-8} \text{ C}.$$

Using the formula for charge density we get $\rho = \frac{q}{V} = \frac{q}{(4/3)\pi R^3} = \frac{4.866 \times 10^{-8} \text{ C}}{(4/3)\pi(0.355 \text{ m})^3} = 2.60 \times 10^{-7} \text{ C/m}^3$.

(b) SET UP: Take a Gaussian surface of radius $r = 0.200 \text{ m}$, concentric with the insulating sphere. The charge enclosed within this surface is $q_{\text{encl}} = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$, and we can treat this charge as a point-charge, using

Coulomb's law $E = \frac{1}{4\pi\epsilon_0} \frac{q_{\text{encl}}}{r^2}$. The charge beyond $r = 0.200 \text{ m}$ makes no contribution to the electric field.

EXECUTE: First find the enclosed charge:

$$q_{\text{encl}} = \rho\left(\frac{4}{3}\pi r^3\right) = (2.60 \times 10^{-7} \text{ C/m}^3)\left[\frac{4}{3}\pi(0.200 \text{ m})^3\right] = 8.70 \times 10^{-9} \text{ C}$$

Now treat this charge as a point-charge and use Coulomb's law to find the field:

$$E = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{8.70 \times 10^{-9} \text{ C}}{(0.200 \text{ m})^2} = 1.96 \times 10^3 \text{ N/C}$$

EVALUATE: Outside this sphere, it behaves like a point-charge located at its center. Inside of it, at a distance r from the center, the field is due only to the charge between the center and r .

- 22.26. IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that lies wholly within the conducting material.

EXECUTE: **(a)** Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{\text{inner}} = +6.00 \text{ nC}$, since $E = 0$ inside a conductor and a Gaussian surface that lies wholly within the conductor must enclose zero net charge.

(b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00 \text{ nC}$ moved to the inner surface:

$$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00 \text{ nC} - 6.00 \text{ nC} = -1.00 \text{ nC}.$$

EVALUATE: The electric field outside the conductor is due to the charge on its surface.

- 22.27. IDENTIFY:** Apply Gauss's law to each surface.

SET UP: The field is zero within the plates. By symmetry the field is perpendicular to the plates outside the plates and can depend only on the distance from the plates. Flux into the enclosed volume is positive.

EXECUTE: S_2 and S_3 enclose no charge, so the flux is zero, and electric field outside the plates is zero. Between the plates, S_4 shows that $-EA = -q/\epsilon_0 = -\sigma A/\epsilon_0$ and $E = \sigma/\epsilon_0$.

EVALUATE: Our result for the field between the plates agrees with the result stated in Example 22.8.

- 22.28. IDENTIFY:** Close to a finite sheet the field is the same as for an infinite sheet. Very far from a finite sheet the field is that of a point charge.

SET UP: For an infinite sheet, $E = \frac{\sigma}{2\epsilon_0}$. For a point charge, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$.

EXECUTE: **(a)** At a distance of 0.1 mm from the center, the sheet appears "infinite," so

$$E \approx \frac{q}{2\epsilon_0 A} = \frac{7.50 \times 10^{-9} \text{ C}}{2\epsilon_0(0.800 \text{ m})^2} = 662 \text{ N/C}.$$

(b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{(7.50 \times 10^{-9} \text{ C})}{(100 \text{ m})^2} = 6.75 \times 10^{-3} \text{ N/C}.$$

(c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on either face as the insulator but the same electric field. Far away, they both look like points with the same charge.

EVALUATE: The sheet can be treated as infinite at points where the distance to the sheet is much less than the distance to the edge of the sheet. The sheet can be treated as a point charge at points for which the distance to the sheet is much greater than the dimensions of the sheet.

22.29. IDENTIFY: Apply Gauss's law to a Gaussian surface and calculate E .

(a) SET UP: Consider the charge on a length l of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) Apply Gauss's law to a Gaussian surface that is a cylinder of length l , radius r , and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.29.

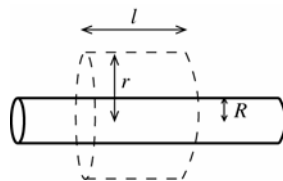


Figure 22.29

EXECUTE:

$$Q_{\text{encl}} = \sigma(2\pi Rl)$$

$$\Phi_E = 2\pi r l E$$

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } 2\pi r l E = \frac{\sigma(2\pi Rl)}{\epsilon_0}$$

$$E = \frac{\sigma R}{\epsilon_0 r}$$

(c) EVALUATE: Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda / 2\pi\epsilon_0 r$. $\sigma = \frac{\lambda}{2\pi R}$,

so $E = \frac{\sigma R}{\epsilon_0 r} = \frac{R}{\epsilon_0 r} \left(\frac{\lambda}{2\pi R} \right) = \frac{\lambda}{2\pi\epsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

22.30. IDENTIFY: The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma / 2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A : $E_A = \frac{|\sigma_2|}{2\epsilon_0} + \frac{|\sigma_3|}{2\epsilon_0} + \frac{|\sigma_4|}{2\epsilon_0} - \frac{|\sigma_1|}{2\epsilon_0} = \frac{1}{2\epsilon_0} (|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|)$.

$$E_A = \frac{1}{2\epsilon_0} (5 \mu\text{C}/\text{m}^2 + 2 \mu\text{C}/\text{m}^2 + 4 \mu\text{C}/\text{m}^2 - 6 \mu\text{C}/\text{m}^2) = 2.82 \times 10^5 \text{ N/C to the left.}$$

(b) $E_B = \frac{|\sigma_1|}{2\epsilon_0} + \frac{|\sigma_3|}{2\epsilon_0} + \frac{|\sigma_4|}{2\epsilon_0} - \frac{|\sigma_2|}{2\epsilon_0} = \frac{1}{2\epsilon_0} (|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|)$.

$$E_B = \frac{1}{2\epsilon_0} (6 \mu\text{C}/\text{m}^2 + 2 \mu\text{C}/\text{m}^2 + 4 \mu\text{C}/\text{m}^2 - 5 \mu\text{C}/\text{m}^2) = 3.95 \times 10^5 \text{ N/C to the left.}$$

(c) $E_C = \frac{|\sigma_4|}{2\epsilon_0} + \frac{|\sigma_1|}{2\epsilon_0} - \frac{|\sigma_2|}{2\epsilon_0} - \frac{|\sigma_3|}{2\epsilon_0} = \frac{1}{2\epsilon_0} (|\sigma_2| + |\sigma_3| - |\sigma_4| - |\sigma_1|)$.

$$E_C = \frac{1}{2\epsilon_0} (4 \mu\text{C}/\text{m}^2 + 6 \mu\text{C}/\text{m}^2 - 5 \mu\text{C}/\text{m}^2 - 2 \mu\text{C}/\text{m}^2) = 1.69 \times 10^5 \text{ N/C to the left}$$

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.31. IDENTIFY: Apply Gauss's law and conservation of charge.

SET UP: $E = 0$ in a conducting material.

EXECUTE: **(a)** Gauss's law says $+Q$ on inner surface, so $E = 0$ inside metal.

(b) The outside surface of the sphere is grounded, so no excess charge.

(c) Consider a Gaussian sphere with the $-Q$ charge at its center and radius less than the inner radius of the metal. This sphere encloses net charge $-Q$ so there is an electric field flux through it; there is electric field in the cavity.

(d) In an electrostatic situation $E = 0$ inside a conductor. A Gaussian sphere with the $-Q$ charge at its center and radius greater than the outer radius of the metal encloses zero net charge (the $-Q$ charge and the $+Q$ on the inner surface of the metal) so there is no flux through it and $E = 0$ outside the metal.

(e) No, $E = 0$ there. Yes, the charge has been shielded by the grounded conductor. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

EVALUATE: Field lines within the cavity terminate on the charges induced on the inner surface.

- 22.32. IDENTIFY and SET UP:** Eq.(22.3) to calculate the flux. Identify the direction of the normal unit vector $\hat{n}$ for each surface.

EXECUTE: (a) $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$; $A = L^2$

face S_1 : $\hat{n} = -\hat{j}$

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot (A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k}) \cdot (-A\hat{j}) = -CL^2.$$

face S_2 : $\hat{n} = +\hat{k}$

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot (A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k}) \cdot (A\hat{k}) = -DL^2.$$

face S_3 : $\hat{n} = +\hat{j}$

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot (A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k}) \cdot (A\hat{j}) = +CL^2.$$

face S_4 : $\hat{n} = -\hat{k}$

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot (A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k}) \cdot (-A\hat{k}) = +DL^2.$$

face S_5 : $\hat{n} = +\hat{i}$

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot (A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k}) \cdot (A\hat{i}) = -BL^2.$$

face S_6 : $\hat{n} = -\hat{i}$

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot (A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k}) \cdot (-A\hat{i}) = +BL^2.$$

(b) Add the flux through each of the six faces: $\Phi_E = -CL^2 - DL^2 + CL^2 + DL^2 - BL^2 + BL^2 = 0$

The total electric flux through all sides is zero.

EVALUATE: All electric field lines that enter one face of the cube leave through another face. No electric field lines terminate inside the cube and the net flux is zero.

- 22.33. IDENTIFY:** Use Eq.(22.3) to calculate the flux through each surface and use Gauss's law to relate the net flux to the enclosed charge.

SET UP: Flux into the enclosed volume is negative and flux out of the volume is positive.

EXECUTE: (a) $\Phi = EA = (125 \text{ N/C})(6.0 \text{ m}^2) = 750 \text{ N} \cdot \text{m}^2/\text{C}$.

(b) Since the field is parallel to the surface, $\Phi = 0$.

(c) Choose the Gaussian surface to equal the volume's surface. Then $750 \text{ N} \cdot \text{m}^2/\text{C} - EA = q/\epsilon_0$ and

$$E = \frac{1}{6.0 \text{ m}^2} (2.40 \times 10^{-8} \text{ C}/\epsilon_0 + 750 \text{ N} \cdot \text{m}^2/\text{C}) = 577 \text{ N/C}, \text{ in the positive } x\text{-direction. Since } q < 0 \text{ we must have some}$$

net flux flowing *in* so the flux is $-|EA|$ on second face.

EVALUATE: (d) $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E .

- 22.34. IDENTIFY:** Apply Gauss's law to a cube centered at the origin and with side length $2L$.

SET UP: The total surface area of a cube with side length $2L$ is $6(2L)^2 = 24L^2$.

EXECUTE: (a) The square is sketched in Figure 22.34.

(b) Imagine a charge q at the center of a cube of edge length $2L$. Then: $\Phi = q/\epsilon_0$. Here the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is, $\Phi = q/24\epsilon_0$.

EVALUATE: Calculating the flux directly from Eq.(22.5) would involve a complicated integral. Using Gauss's law and symmetry considerations is much simpler.

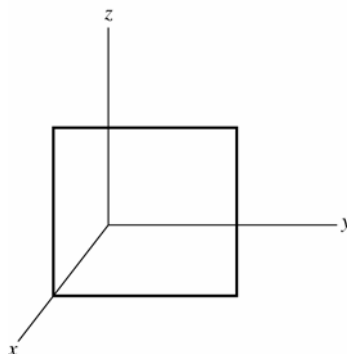


Figure 22.34

- 22.35. (a) **IDENTIFY:** Find the net flux through the parallelepiped surface and then use that in Gauss's law to find the net charge within. Flux out of the surface is positive and flux into the surface is negative.

SET UP: $\vec{E}_1$ gives flux out of the surface. See Figure 22.35a.

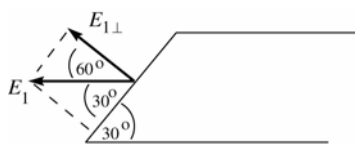


Figure 22.35a

EXECUTE: $\Phi_1 = +E_{1\perp}A$

$$A = (0.0600 \text{ m})(0.0500 \text{ m}) = 3.00 \times 10^{-3} \text{ m}^2$$

$$E_{1\perp} = E_1 \cos 60^\circ = (2.50 \times 10^4 \text{ N/C}) \cos 60^\circ$$

$$E_{1\perp} = 1.25 \times 10^4 \text{ N/C}$$

$$\Phi_{E_1} = +E_{1\perp}A = +(1.25 \times 10^4 \text{ N/C})(3.00 \times 10^{-3} \text{ m}^2) = 37.5 \text{ N} \cdot \text{m}^2/\text{C}$$

SET UP: $\vec{E}_2$ gives flux into the surface. See Figure 22.35b.

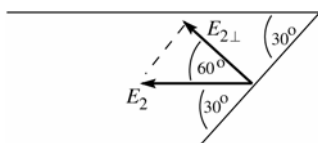


Figure 22.35b

EXECUTE: $\Phi_2 = -E_{2\perp}A$

$$A = (0.0600 \text{ m})(0.0500 \text{ m}) = 3.00 \times 10^{-3} \text{ m}^2$$

$$E_{2\perp} = E_2 \cos 60^\circ = (7.00 \times 10^4 \text{ N/C}) \cos 60^\circ$$

$$E_{2\perp} = 3.50 \times 10^4 \text{ N/C}$$

$$\Phi_{E_2} = -E_{2\perp}A = -(3.50 \times 10^4 \text{ N/C})(3.00 \times 10^{-3} \text{ m}^2) = -105.0 \text{ N} \cdot \text{m}^2/\text{C}$$

The net flux is $\Phi_E = \Phi_{E_1} + \Phi_{E_2} = +37.5 \text{ N} \cdot \text{m}^2/\text{C} - 105.0 \text{ N} \cdot \text{m}^2/\text{C} = -67.5 \text{ N} \cdot \text{m}^2/\text{C}$.

The net flux is negative (inward), so the net charge enclosed is negative.

Apply Gauss's law: $\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$

$$Q_{\text{encl}} = \Phi_E \epsilon_0 = (-67.5 \text{ N} \cdot \text{m}^2/\text{C})(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = -5.98 \times 10^{-10} \text{ C}.$$

(b) **EVALUATE:** If there were no charge within the parallelepiped the net flux would be zero. This is not the case, so there is charge inside. The electric field lines that pass out through the surface of the parallelepiped must terminate on charges, so there also must be charges outside the parallelepiped.

- 22.36. **IDENTIFY:** The α particle feels no force where the net electric field due to the two distributions of charge is zero.

SET UP: The fields can cancel only in the regions *A* and *B* shown in Figure 22.36, because only in these two regions are the two fields in opposite directions.

EXECUTE: $E_{\text{line}} = E_{\text{sheet}}$ gives $\frac{\lambda}{2\pi\epsilon_0 r} = \frac{\sigma}{2\epsilon_0}$ and $r = \lambda/\pi\sigma = \frac{50 \mu\text{C}/\text{m}}{\pi(100 \mu\text{C}/\text{m}^2)} = 0.16 \text{ m} = 16 \text{ cm}.$

The fields cancel 16 cm from the line in regions *A* and *B*.

EVALUATE: The result is independent of the distance between the line and the sheet. The electric field of an infinite sheet of charge is uniform, independent of the distance from the sheet.

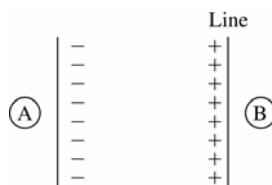


Figure 22.36

- 22.37. (a) **IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r , where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.37a.

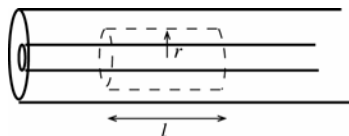


Figure 22.37a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(2\pi rl) = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}. \text{ The enclosed charge is positive so the direction of } \vec{E} \text{ is radially outward.}$$

(b) SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r , where $r > c$, as shown in Figure 22.37b.

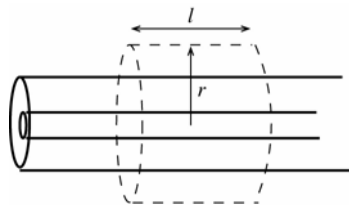


Figure 22.37b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(2\pi rl) = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}. \text{ The enclosed charge is positive so the direction of } \vec{E} \text{ is radially outward.}$$

(c) $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \text{ for } a < r < b; \quad E = 0 \text{ for } b < r < c;$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \text{ for } r > c. \text{ The graph of } E \text{ versus } r \text{ is sketched in Figure 22.37c.}$$

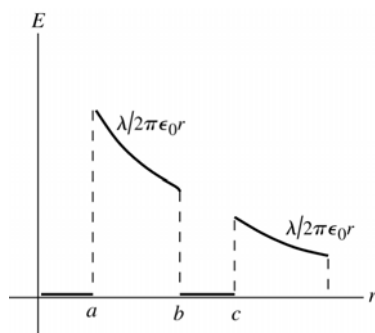


Figure 22.37c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and SET UP: inner surface: Apply Gauss's law to a Gaussian cylinder with radius r , where $b < r < c$. We know E on this surface; calculate Q_{encl} .

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

outer surface: The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per unit length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor. These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.38. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r , length l and that has the line of charge along its axis. The charge on a length l of the line of charge or of the tube is $q = \alpha l$.

EXECUTE: **(a)** (i) For $r < a$, Gauss's law gives $E(2\pi rl) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\alpha l}{\epsilon_0}$ and $E = \frac{\alpha}{2\pi\epsilon_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi rl) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{2\alpha l}{\epsilon_0}$ and $E = \frac{\alpha}{\pi\epsilon_0 r}$.

The graph of E versus r is sketched in Figure 22.38.

(b) (i) The Gaussian cylinder with radius r , for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.

EVALUATE: For $r > b$ the electric field is due to the charge on the outer surface of the tube.

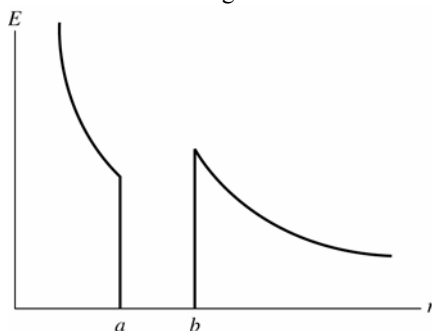


Figure 22.38

22.39. (a) IDENTIFY: Use Gauss's law to calculate $E(r)$.

(i) **SET UP:** $r < a$: Apply Gauss's law to a cylindrical Gaussian surface of length l and radius r , where $r < a$, as sketched in Figure 22.39a.

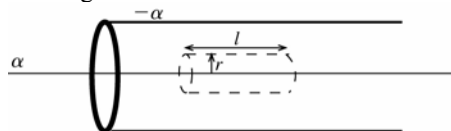


Figure 22.39a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \alpha l$ (the charge on the length l of the line of charge)

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(2\pi rl) = \frac{\alpha l}{\epsilon_0}$$

$$E = \frac{\alpha}{2\pi\epsilon_0 r}. \text{ The enclosed charge is positive so the direction of } \vec{E} \text{ is radially outward.}$$

(ii) $a < r < b$: Points in this region are within the conducting tube, so $E = 0$.

(iii) **SET UP:** $r > b$: Apply Gauss's law to a cylindrical Gaussian surface of length l and radius r , where $r > b$, as sketched in Figure 22.39b.

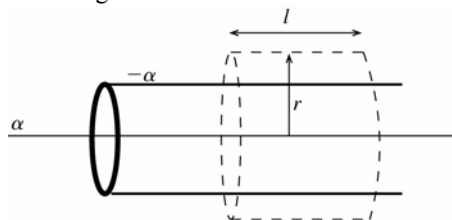


Figure 22.39b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \alpha l$ (the charge on length l of the line of charge) $-\alpha l$ (the charge on length l of the tube) Thus $Q_{\text{encl}} = 0$.

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = 0$ and $E = 0$. The graph of E versus r is sketched in Figure 22.39c.

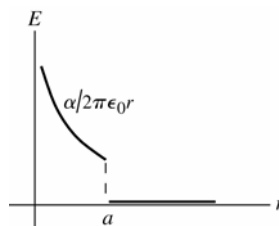


Figure 22.39c

(b) **IDENTIFY:** Apply Gauss's law to cylindrical surfaces that lie just outside the inner and outer surfaces of the tube. We know E so can calculate Q_{encl} .

(i) **SET UP:** inner surface

Apply Gauss's law to a cylindrical Gaussian surface of length l and radius r , where $a < r < b$.

EXECUTE: This surface lies within the conductor of the tube, where $E = 0$, so $\Phi_E = 0$. Then by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge αl on the line of charge so must enclose charge $-\alpha l$ on the inner surface of the tube. The charge per unit length on the inner surface of the tube is $-\alpha$.

(ii) outer surface

The net charge per unit length on the tube is $-\alpha$. We have shown in part (i) that this must all reside on the inner surface, so there is no net charge on the outer surface of the tube.

EVALUATE: For $r < a$ the electric field is due only to the line of charge. For $r > b$ the electric field of the tube is the same as for a line of charge along its axis. The fields of the line of charge and of the tube are equal in magnitude and opposite in direction and sum to zero. For $r < a$ the electric field lines originate on the line of charge and terminate on the surface charge on the inner surface of the tube. There is no electric field outside the tube and no surface charge on the outer surface of the tube.

22.40. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l , and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi r l$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: (a) For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi r l) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho\pi r^2 l}{\epsilon_0}$ and $E = \frac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi r l) = \frac{q}{\epsilon_0} = \frac{\rho\pi R^2 l}{\epsilon_0}$ and $E = \frac{\rho R^2}{2\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$, radially outward.

(c) At $r = R$, the electric field for BOTH regions is $E = \frac{\rho R}{2\epsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.40.

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

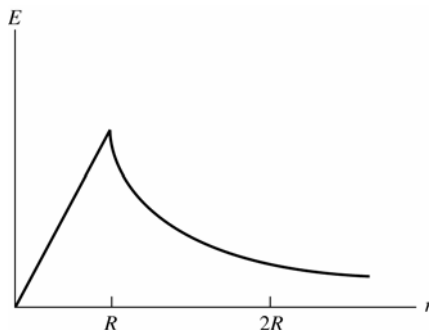


Figure 22.40

22.41. IDENTIFY: First make a free-body diagram of the sphere. The electric force acts to the left on it since the electric field due to the sheet is horizontal. Since it hangs at rest, the sphere is in equilibrium so the forces on it add to zero, by Newton's first law. Balance horizontal and vertical force components separately.

SET UP: Call T the tension in the thread and E the electric field. Balancing horizontal forces gives $T \sin \theta = qE$. Balancing vertical forces we get $T \cos \theta = mg$. Combining these equations gives $\tan \theta = qE/mg$, which means that $\theta = \arctan(qE/mg)$. The electric field for a sheet of charge is $E = \sigma/2\epsilon_0$.

EXECUTE: Substituting the numbers gives us $E = \frac{\sigma}{2\epsilon_0} = \frac{2.50 \times 10^{-7} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 1.41 \times 10^4 \text{ N/C}$. Then

$$\theta = \arctan \left[\frac{(5.00 \times 10^{-8} \text{ C})(1.41 \times 10^4 \text{ N/C})}{(2.00 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2)} \right] = 19.8^\circ$$

EVALUATE: Increasing the field, or decreasing the mass of the sphere, would cause the sphere to hang at a larger angle.

22.42. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: (a) For $r < a$, $E = 0$, since these points are within the conducting material.

For $a < r < b$, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$, since there is $+q$ inside a radius r .

For $b < r < c$, $E = 0$, since these points are within the conducting material

For $r > c$, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$, since again the total charge enclosed is $+q$.

(b) The graph of E versus r is sketched in Figure 22.42a.

(c) Since the Gaussian sphere of radius r , for $b < r < c$, must enclose zero net charge, the charge on inner shell surface is $-q$.

(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on outer shell surface is $+q$.

(e) The field lines are sketched in Figure 22.42b. Where the field is nonzero, it is radially outward.

EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.

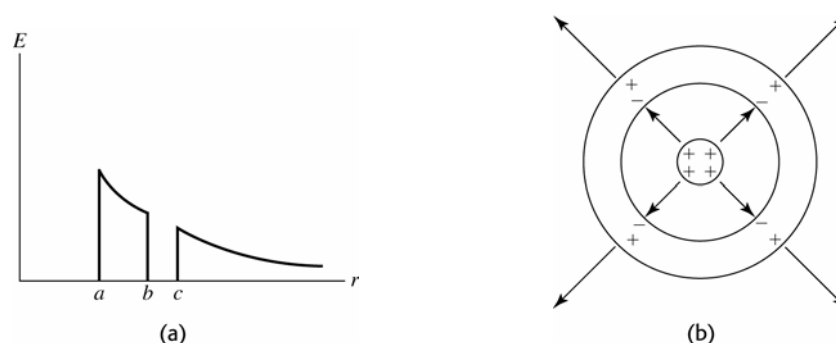


Figure 22.42

22.43. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charge distributions.

EXECUTE: (a) For $r < R$, $E = 0$, since these points are within the conducting material. For $R < r < 2R$,

$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$, since the charge enclosed is Q . For $r > 2R$, $E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$ since the charge enclosed is $2Q$.

(b) The graph of E versus r is sketched in Figure 22.43.

EVALUATE: For $r < 2R$ the electric field is unaffected by the presence of the charged shell.

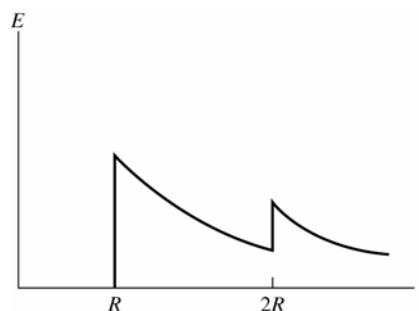


Figure 22.43

22.44. IDENTIFY: Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: (a) For $r < a$, $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$, radially outward, since the charge enclosed is Q , the charge of the point

charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

- (b) Since a Gaussian surface with radius r , for $a < r < b$, must enclose zero net charge, the total charge on the inner surface is $-Q$ and the surface charge density on inner surface is $\sigma = -\frac{Q}{4\pi a^2}$.
- (c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\frac{2Q}{4\pi b^2}$.
- (d) The field lines and the locations of the charges are sketched in Figure 22.44a.
- (e) The graph of E versus r is sketched in Figure 22.44b.

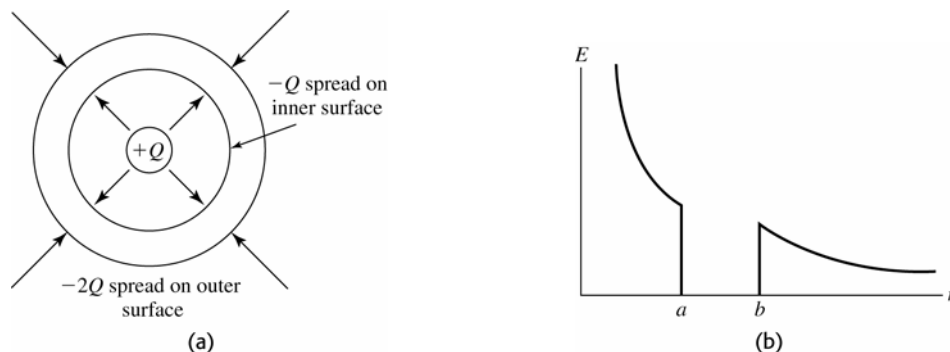


Figure 22.44

EVALUATE: For $r < a$ the electric field is due solely to the point charge Q . For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

- 22.45. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r . Calculate the electric field at the surface of the Gaussian sphere.

(a) **SET UP:** (i) $r < a$: The Gaussian surface is sketched in Figure 22.45a.



Figure 22.45a

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ says $E(4\pi r^2) = 0$ and $E = 0$.

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.45b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.

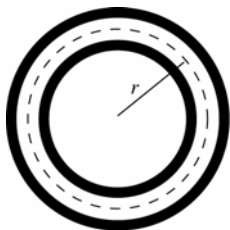


Figure 22.45b

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \frac{2q}{\epsilon_0}$ so $E = \frac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.45c.

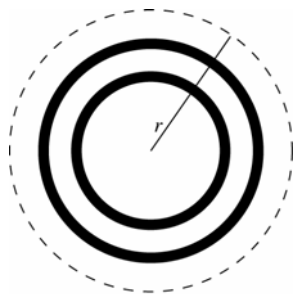


Figure 22.45c

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(4\pi r^2) = \frac{6q}{\epsilon_0}$$

$$E = \frac{6q}{4\pi\epsilon_0 r^2}. \text{ Since the enclosed charge is positive the electric field is radially outward.}$$

The graph of E versus r is sketched in Figure 22.45d.

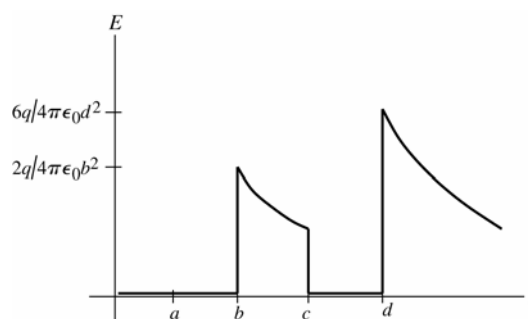


Figure 22.45d

(b) IDENTIFY and SET UP: Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.46. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charged shells.

EXECUTE: (a) (i) For $r < a$, $E = 0$, since the charge enclosed is zero. (ii) For $a < r < b$, $E = 0$, since the points are within the conducting material. (iii) For $b < r < c$, $E = \frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}$, outward, since charge enclosed is $+2q$.

(iv) For $c < r < d$, $E = 0$, since the points are within the conducting material. (v) For $r > d$, $E = 0$, since the net charge enclosed is zero. The graph of E versus r is sketched in Figure 22.46.

(b) (i) small shell inner surface: Since a Gaussian surface with radius r , for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius r , for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-2q$, the charge on this surface is zero.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

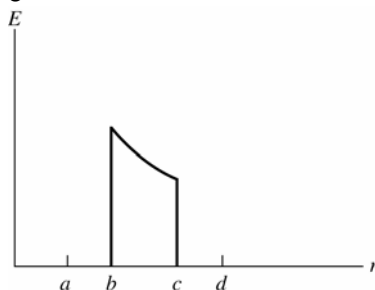


Figure 22.46

22.47. **IDENTIFY:** Apply Gauss's law

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charged shells.

EXECUTE: (a) (i) For $r < a$, $E = 0$, since charge enclosed is zero. (ii) $a < r < b$, $E = 0$, since the points are within the conducting material. (iii) For $b < r < c$, $E = \frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}$, outward, since charge enclosed is $+2q$.

(iv) For $c < r < d$, $E = 0$, since the points are within the conducting material. (v) For $r > d$, $E = \frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}$, inward, since charge enclosed is $-2q$. The graph of the radial component of the electric field versus r is sketched in Figure 22.47, where we use the convention that outward field is positive and inward field is negative.

(b) (i) small shell inner surface: Since a Gaussian surface with radius r , for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius r , for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-4q$, the charge on this surface is $-2q$.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

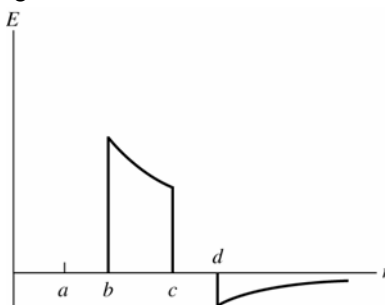


Figure 22.47

22.48. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the sphere and shell. The volume of the insulating shell is $V = \frac{4}{3}\pi([2R]^3 - R^3) = \frac{28\pi}{3}R^3$.

EXECUTE: (a) Zero net charge requires that $-Q = \frac{28\pi\rho R^3}{3}$, so $\rho = -\frac{3Q}{28\pi R^3}$.

(b) For $r < R$, $E = 0$ since this region is within the conducting sphere. For $r > 2R$, $E = 0$, since the net charge enclosed by the Gaussian surface with this radius is zero. For $R < r < 2R$, Gauss's law gives $E(4\pi r^2) = \frac{Q}{\epsilon_0} + \frac{4\pi\rho}{3\epsilon_0}(r^3 - R^3)$ and

$E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\rho}{3\epsilon_0 r^2}(r^3 - R^3)$. Substituting ρ from part (a) gives $E = \frac{2}{7\pi\epsilon_0} \frac{Q}{r^2} - \frac{Qr}{28\pi\epsilon_0 R^3}$. The net enclosed charge for each r in this range is positive and the electric field is outward.

(c) The graph is sketched in Figure 22.48. We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere. But we see a smooth transition from the uniform insulator to the surrounding space.

EVALUATE: The expression for E within the insulator gives $E = 0$ at $r = 2R$.

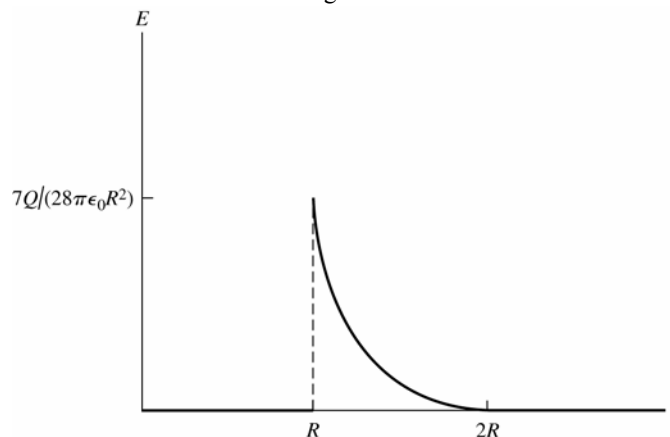


Figure 22.48

22.49. IDENTIFY: Use Gauss's law to find the electric field $\vec{E}$ produced by the shell for $r < R$ and $r > R$ and then use $\vec{F} = q\vec{E}$ to find the force the shell exerts on the point charge.

(a) **SET UP:** Apply Gauss's law to a spherical Gaussian surface that has radius $r > R$ and that is concentric with the shell, as sketched in Figure 22.49a.

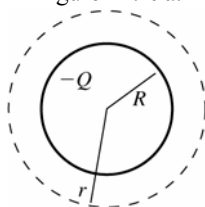


Figure 22.49a

EXECUTE: $\Phi_E = E(4\pi r^2)$

$Q_{\text{encl}} = -Q$

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(4\pi r^2) = \frac{-Q}{\epsilon_0}$$

The magnitude of the field is $E = \frac{Q}{4\pi\epsilon_0 r^2}$ and it is directed toward the center of the shell. Then $F = qE = \frac{qQ}{4\pi\epsilon_0 r^2}$,

directed toward the center of the shell. (Since q is positive, $\vec{E}$ and $\vec{F}$ are in the same direction.)

(b) **SET UP:** Apply Gauss's law to a spherical Gaussian surface that has radius $r < R$ and that is concentric with the shell, as sketched in Figure 22.49b.

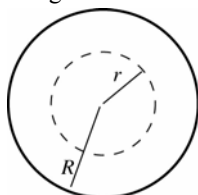


Figure 22.49b

EXECUTE: $\Phi_E = E(4\pi r^2)$

$Q_{\text{encl}} = 0$

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(4\pi r^2) = 0$$

Then $E = 0$ so $F = 0$.

EVALUATE: Outside the shell the electric field and the force it exerts is the same as for a point charge $-Q$ located at the center of the shell. Inside the shell $E = 0$ and there is no force.

22.50. IDENTIFY: The method of Example 22.9 shows that the electric field outside the sphere is the same as for a point charge of the same charge located at the center of the sphere.

SET UP: The charge of an electron has magnitude $e = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: (a) $E = k \frac{|q|}{r^2}$. For $r = R = 0.150$ m, $E = 1150$ N/C so $|q| = \frac{Er^2}{k} = \frac{(1150 \text{ N/C})(0.150 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 2.88 \times 10^{-9} \text{ C}$.

The number of excess electrons is $\frac{2.88 \times 10^{-9} \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 1.80 \times 10^{10}$ electrons.

(b) $r = R + 0.100 \text{ m} = 0.250 \text{ m}$. $E = k \frac{|q|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2.88 \times 10^{-9} \text{ C}}{(0.250 \text{ m})^2} = 414 \text{ N/C}$.

EVALUATE: The magnitude of the electric field decreases according to the square of the distance from the center of the sphere.

22.51. IDENTIFY: The net electric field is the vector sum of the fields due to the sheet of charge on each surface of the plate.

SET UP: The electric field due to the sheet of charge on each surface is $E = \sigma/2\epsilon_0$ and is directed away from the surface.

EXECUTE: (a) For the conductor the charge sheet on each surface produces fields of magnitude $\sigma/2\epsilon_0$ and in the same direction, so the total field is twice this, or σ/ϵ_0 .

(b) At points inside the plate the fields of the sheets of charge on each surface are equal in magnitude and opposite in direction, so their vector sum is zero. At points outside the plate, on either side, the fields of the two sheets of charge are in the same direction so their magnitudes add, giving $E = \sigma/\epsilon_0$.

EVALUATE: Gauss's law can also be used directly to determine the fields in these regions.

22.52. IDENTIFY: Example 22.9 gives the expression for the electric field both inside and outside a uniformly charged sphere. Use $\vec{F} = -e\vec{E}$ to calculate the force on the electron.

SET UP: The sphere has charge $Q = +e$.

EXECUTE: (a) Only at $r = 0$ is $E = 0$ for the uniformly charged sphere.

(b) At points inside the sphere, $E_r = \frac{er}{4\pi\epsilon_0 R^3}$. The field is radially outward. $F_r = -eE = -\frac{1}{4\pi\epsilon_0} \frac{e^2 r}{R^3}$. The minus sign

denotes that F_r is radially inward. For simple harmonic motion, $F_r = -kr = -m\omega^2 r$, where $\omega = \sqrt{k/m} = 2\pi f$.

$$F_r = -m\omega^2 r = -\frac{1}{4\pi\epsilon_0} \frac{e^2 r}{R^3} \text{ so } \omega = \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mR^3}} \text{ and } f = \frac{1}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mR^3}}.$$

(c) If $f = 4.57 \times 10^{14} \text{ Hz} = \frac{1}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mR^3}}$ then $R = \sqrt[3]{\frac{1}{4\pi\epsilon_0} \frac{(1.60 \times 10^{-19} \text{ C})^2}{4\pi^2 (9.11 \times 10^{-31} \text{ kg})(4.57 \times 10^{14} \text{ Hz})^2}} = 3.13 \times 10^{-10} \text{ m}$.

The atom radius in this model is the correct order of magnitude.

(d) If $r > R$, $E_r = \frac{e}{4\pi\epsilon_0 r^2}$ and $F_r = -\frac{e^2}{4\pi\epsilon_0 r^2}$. The electron would still oscillate because the force is directed toward

the equilibrium position at $r = 0$. But the motion would not be simple harmonic, since F_r is proportional to $1/r^2$ and simple harmonic motion requires that the restoring force be proportional to the displacement from equilibrium.

EVALUATE: As long as the initial displacement is less than R the frequency of the motion is independent of the initial displacement.

22.53. IDENTIFY: There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and Eq.(21.3) gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.53a.

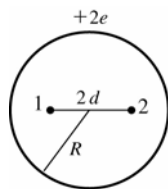


Figure 22.53a

If the electrons are in equilibrium the net force on each one is zero.

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \frac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$.

At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{cd} = eE = \frac{e(2e)d}{4\pi\epsilon_0 R^3}$ and is attractive.

The force diagram for electron 2 is given in Figure 22.53b.

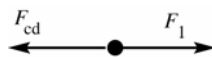


Figure 22.53b

Net force equals zero implies $F_1 = F_{cd}$ and $\frac{1}{4\pi\epsilon_0} \frac{e^2}{4d^2} = \frac{2e^2 d}{4\pi\epsilon_0 R^3}$

Thus $(1/4d^2) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

22.54. IDENTIFY: Use Gauss's law to find the electric field both inside and outside the slab.

SET UP: Use a Gaussian surface that has one face of area A in the yz plane at $x = 0$, and the other face at a general value x . The volume enclosed by such a Gaussian surface is Ax .

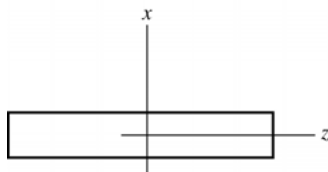
EXECUTE: (a) The electric field of the slab must be zero by symmetry. There is no preferred direction in the yz plane, so the electric field can only point in the x -direction. But at the origin, neither the positive nor negative x -directions should be singled out as special, and so the field must be zero.

(b) For $|x| \leq d$, Gauss's law gives $EA = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho A|x|}{\epsilon_0}$ and $E = \frac{\rho|x|}{\epsilon_0}$, with direction given by $\frac{x}{|x|}\hat{i}$ (away from the center of the slab). Note that this expression does give $E = 0$ at $x = 0$. Outside the slab, the enclosed charge does not depend on x and is equal to ρAd . For $|x| \geq d$, Gauss's law gives $EA = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho Ad}{\epsilon_0}$ and $E = \frac{\rho d}{\epsilon_0}$, again with

direction given by $\frac{x}{|x|}\hat{i}$.

EVALUATE: At the surfaces of the slab, $x = \pm d$. For these values of x the two expressions for E (for inside and outside the slab) give the same result. The charge per unit area σ of the slab is given by $\sigma A = \rho A(2d)$ and $\rho d = \sigma/2$. The result for E outside the slab can therefore be written as $E = \sigma/2\epsilon_0$ and is the same as for a thin sheet of charge.

22.55. (a) IDENTIFY and SET UP: Consider the direction of the field for x slightly greater than and slightly less than zero. The slab is sketched in Figure 22.55a.



$$\rho(x) = \rho_0 (x/d)^2$$

Figure 22.55a

EXECUTE: The charge distribution is symmetric about $x = 0$, so by symmetry $E(x) = E(-x)$. But for $x > 0$ the field is in the $+x$ direction and for $x < 0$ the field is in the $-x$ direction. At $x = 0$ the field can't be both in the $+x$ and $-x$ directions so must be zero. That is, $E_x(x) = -E_x(-x)$. At point $x = 0$ this gives $E_x(0) = -E_x(0)$ and this equation is satisfied only for $E_x(0) = 0$.

(b) IDENTIFY and SET UP: $|x| > d$ (outside the slab)

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| > d$ from $x = 0$, as shown in Figure 22.55b.

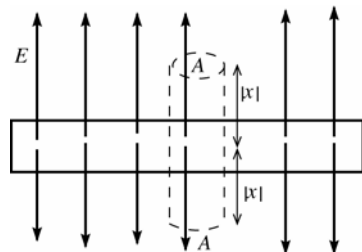


Figure 22.55b

EXECUTE: $\Phi_E = 2EA$

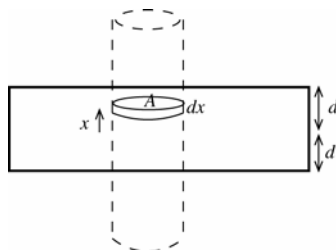


Figure 22.55c

To find Q_{encl} consider a thin disk at coordinate x and with thickness dx , as shown in Figure 22.55c. The charge within this disk is

$$dq = \rho dV = \rho A dx = (\rho_0 A / d^2) x^2 dx.$$

The total charge enclosed by the Gaussian cylinder is

$$Q_{\text{encl}} = 2 \int_0^d dq = (2\rho_0 A / d^2) \int_0^d x^2 dx = (2\rho_0 A / d^2) (d^3 / 3) = \frac{2}{3} \rho_0 A d.$$

Then $\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A d / 3\epsilon_0$.

$$E = \rho_0 d / 3\epsilon_0$$

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 d / 3\epsilon_0) (x / |x|) \hat{i}$.

IDENTIFY and SET UP: $|x| < d$ (inside the slab)

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| < d$ from $x = 0$, as shown in Figure 22.55d.

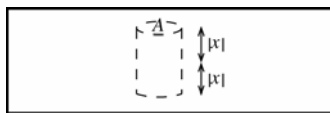


Figure 22.55d

EXECUTE: $\Phi_E = 2EA$

Q_{encl} is found as above, but now the integral on dx is only from 0 to x instead of 0 to d .

$$Q_{\text{encl}} = 2 \int_0^x dq = (2\rho_0 A / d^2) \int_0^x x^2 dx = (2\rho_0 A / d^2) (x^3 / 3).$$

Then $\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A x^3 / 3\epsilon_0 d^2$.

$$E = \rho_0 x^3 / 3\epsilon_0 d^2$$

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 x^3 / 3\epsilon_0 d^2) \hat{i}$.

EVALUATE: Note that $E = 0$ at $x = 0$ as stated in part (a). Note also that the expressions for $|x| > d$ and $|x| < d$ agree for $x = d$.

22.56. IDENTIFY: Apply $\vec{F} = q\vec{E}$ to relate the force on q to the electric field at the location of q .

SET UP: Flux is negative if the electric field is directed into the enclosed volume.

EXECUTE: (a) We could place two charges $+Q$ on either side of the charge $+q$, as shown in Figure 22.56.

(b) In order for the charge to be stable, the electric field in a neighborhood around it must always point back to the equilibrium position.

(c) If q is moved to infinity and we require there to be an electric field always pointing in to the region where q had been, we could draw a small Gaussian surface there. We would find that we need a negative flux into the surface. That is, there has to be a negative charge in that region. However, there is none, and so we cannot get such a stable equilibrium.

(d) For a negative charge to be in stable equilibrium, we need the electric field to always point away from the charge position. The argument in (c) carries through again, this time implying that a positive charge must be in the space where the negative charge was if stable equilibrium is to be attained.

EVALUATE: If q is displaced to the left or right in Figure 22.56, the net force is directed back toward the equilibrium position. But if q is displaced slightly in a direction perpendicular to the line connecting the two charges Q , then the net force on q is directed away from the equilibrium position and the equilibrium is not stable for such a displacement.

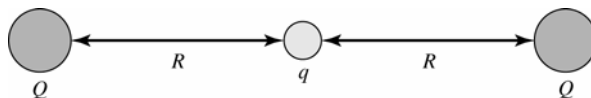


Figure 22.56

22.57. $\rho(r) = \rho_0(1 - r/R)$ for $r \leq R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r) = 0$ for $r \geq R$

(a) **IDENTIFY:** The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr . Find the charge dq in each shell and integrate to find the total charge.

SET UP: The thin shell is sketched in Figure 22.57a.

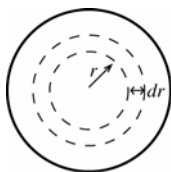


Figure 22.57a

EXECUTE: The volume of such

a shell is $dV = 4\pi r^2 dr$

The charge contained within the shell is

$$dq = \rho(r)dV = 4\pi r^2 \rho_0(1 - r/R)dr$$

The total charge Q in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q = \int dq = \int_0^R 4\pi r^2 \rho_0(1 - r/R)dr = 4\pi \rho_0 \int_0^R (r^2 - r^3/R)dr$$

$$Q = 4\pi \rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R} \right]_0^R = 4\pi \rho_0 \left(\frac{R^3}{3} - \frac{R^4}{4R} \right) = 4\pi \rho_0 (R^3/12) = 4\pi (3Q/\pi R^3) (R^3/12) = Q, \text{ as was to be shown.}$$

(b) **IDENTIFY:** Apply Gauss's law to a spherical surface of radius r , where $r > R$.

SET UP: The Gaussian surface is shown in Figure 22.57b.

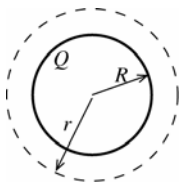


Figure 22.57b

EXECUTE: $\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2}; \text{ same as for point charge of charge } Q.$$

(c) **IDENTIFY:** Apply Gauss's law to a spherical surface of radius r , where $r < R$:

SET UP: The Gaussian surface is shown in Figure 22.57c.

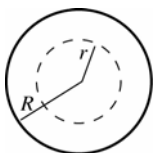


Figure 22.57c

EXECUTE: $\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$

$$\Phi_E = E(4\pi r^2)$$

To calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R . (We want the charge that is inside radius r .)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right) dr' = 4\pi \rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right) dr'$$

$$Q_{\text{encl}} = 4\pi \rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R} \right]_0^r = 4\pi \rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R} \right) = 4\pi \rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R} \right)$$

$$\rho_0 = \frac{3Q}{\pi R^3} \text{ so } Q_{\text{encl}} = 12Q \frac{r^3}{R^3} \left(\frac{1}{3} - \frac{r}{4R} \right) = Q \left(\frac{r^3}{R^3} \right) \left(4 - 3 \frac{r}{R} \right).$$

$$\text{Thus Gauss's law gives } E(4\pi r^2) = \frac{Q}{\epsilon_0} \left(\frac{r^3}{R^3} \right) \left(4 - 3 \frac{r}{R} \right)$$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3} \left(4 - \frac{3r}{R} \right), \quad r \leq R$$

(d) The graph of E versus r is sketched in Figure 22.57d.

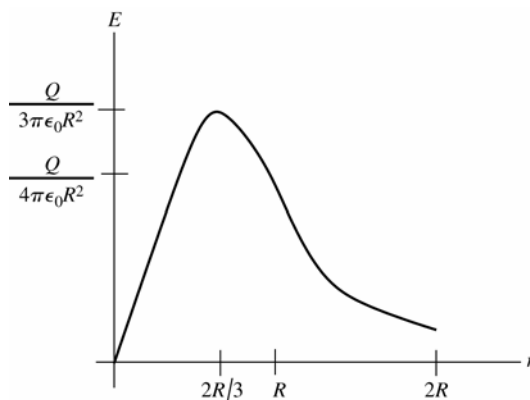


Figure 22.57d

(e) Where the electric field is a maximum, $\frac{dE}{dr} = 0$. Thus $\frac{d}{dr} \left(4r - \frac{3r^2}{R} \right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3$.

$$\text{At this value of } r, \quad E = \frac{Q}{4\pi\epsilon_0 R^3} \left(\frac{2R}{3} \right) \left(4 - \frac{3}{R} \frac{2R}{3} \right) = \frac{Q}{3\pi\epsilon_0 R^2}$$

EVALUATE: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R$. The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).

22.58. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the spherical distribution of charge. The volume of a thin spherical shell of radius r and thickness dr is $dV = 4\pi r^2 dr$.

$$\text{EXECUTE: (a) } Q = \int \rho(r) dV = 4\pi \int_0^\infty \rho(r) r^2 dr = 4\pi \rho_0 \int_0^R \left(1 - \frac{4r}{3R} \right) r^2 dr = 4\pi \rho_0 \left[\int_0^R r^2 dr - \frac{4}{3R} \int_0^R r^3 dr \right]$$

$$Q = 4\pi \rho_0 \left[\frac{R^3}{3} - \frac{4}{3R} \frac{R^4}{4} \right] = 0. \text{ The total charge is zero.}$$

$$\text{(b) For } r \geq R, \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} = 0, \text{ so } E = 0.$$

$$\text{(c) For } r \leq R, \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{4\pi}{\epsilon_0} \int_0^r \rho(r') r'^2 dr'. \quad E 4\pi r^2 = \frac{4\pi \rho_0}{\epsilon_0} \left[\int_0^r r'^2 dr' - \frac{4}{3R} \int_0^r r'^3 dr' \right] \text{ and}$$

$$E = \frac{\rho_0}{\epsilon_0} \frac{1}{r^2} \left[\frac{r^3}{3} - \frac{r^4}{3R} \right] = \frac{\rho_0}{3\epsilon_0} r \left[1 - \frac{r}{R} \right].$$

(d) The graph of E versus r is sketched in Figure 22.58.

(e) Where E is a maximum, $\frac{dE}{dr} = 0$. This gives $\frac{\rho_0}{3\epsilon_0} - \frac{2\rho_0 r_{\text{max}}}{3\epsilon_0 R} = 0$ and $r_{\text{max}} = \frac{R}{2}$. At this r , $E = \frac{\rho_0}{3\epsilon_0} \frac{R}{2} \left[1 - \frac{1}{2} \right] = \frac{\rho_0 R}{12\epsilon_0}$.

EVALUATE: The result in part (b) for $r \leq R$ gives $E = 0$ at $r = R$; the field is continuous at the surface of the charge distribution.

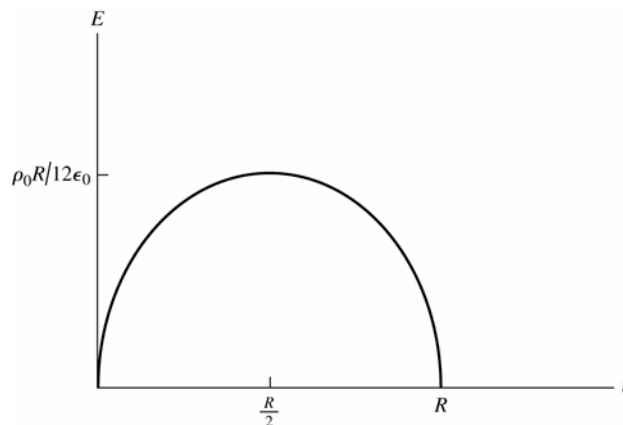


Figure 22.58

22.59. IDENTIFY: Follow the steps specified in the problem.

SET UP: In spherical polar coordinates $d\vec{A} = r^2 \sin \theta \, d\theta \, d\phi \, \hat{r}$. $\oint \sin \theta \, d\theta \, d\phi = 4\pi$.

EXECUTE: (a) $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -Gm \oint \frac{r^2 \sin \theta \, d\theta \, d\phi}{r^2} = -4\pi Gm$.

(b) For any closed surface, mass OUTSIDE the surface contributes zero to the flux passing through the surface. Thus the formula above holds for any situation where m is the mass enclosed by the Gaussian surface.

That is, $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -4\pi G M_{\text{encl}}$.

EVALUATE: The minus sign in the expression for the flux signifies that the flux is directed inward.

22.60. IDENTIFY: Apply $\oint \vec{g} \cdot d\vec{A} = -4\pi G M_{\text{encl}}$.

SET UP: Use a Gaussian surface that is a sphere of radius r , concentric with the mass distribution. Let Φ_g denote $\oint \vec{g} \cdot d\vec{A}$.

EXECUTE: (a) Use a Gaussian sphere with radius $r > R$, where R is the radius of the mass distribution. g is constant on this surface and the flux is inward. The enclosed mass is M . Therefore, $\Phi_g = -g 4\pi r^2 = -4\pi G M$ and $g = \frac{GM}{r^2}$, which is the same as for a point mass.

(b) For a Gaussian sphere of radius $r < R$, where R is the radius of the shell, $M_{\text{encl}} = 0$, so $g = 0$.

(c) Use a Gaussian sphere of radius $r < R$, where R is the radius of the planet. Then $M_{\text{encl}} = \rho \left(\frac{4}{3} \pi r^3 \right) = Mr^3 / R^3$.

This gives $\Phi_g = -g 4\pi r^2 = -4\pi G M_{\text{encl}} = -4\pi G \left(M \frac{r^3}{R^3} \right)$ and $g = \frac{GMr}{R^3}$, which is linear in r .

EVALUATE: The spherically symmetric distribution of mass results in an acceleration due to gravity $\vec{g}$ that is radial and that depends only on r , the distance from the center of the mass distribution.

22.61. (a) IDENTIFY: Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr' / 4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q / 4\pi R^3$ so this may be written as $E = \rho r' / 3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E} = \rho \vec{r}' / 3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.61.

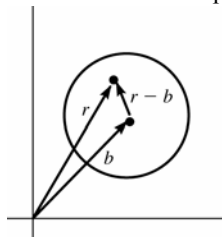


Figure 22.61

EXECUTE: Thus $\vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$

EVALUATE: When $b = 0$ this reduces to the result of Example 22.9. When $\vec{r} = \vec{b}$, this gives $E = 0$, which is correct since we know that $E = 0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r} = \vec{b}$.

SET UP: $\vec{E} = \vec{E}_{\text{uniform}} + \vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho - \rho = 0$.)

EXECUTE: $\vec{E}_{\text{uniform}} = \frac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$$\vec{E}_{\text{hole}} = \frac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}, \text{ at points inside the hole.}$$

$$\text{Then } \vec{E} = \frac{\rho\vec{r}}{3\epsilon_0} + \left(\frac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0} \right) = \frac{\rho\vec{b}}{3\epsilon_0}.$$

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.62. IDENTIFY: We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.62,

$$\vec{r}' = \vec{r} - \vec{b}. \text{ Problem 23.48 shows that at points within a long insulating cylinder, } \vec{E} = \frac{\rho\vec{r}}{2\epsilon_0}.$$

$$\text{EXECUTE: } \vec{E}_{\text{off-axis}} = \frac{\rho\vec{r}'}{2\epsilon_0} = \frac{\rho(\vec{r} - \vec{b})}{2\epsilon_0}. \quad \vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{off-axis}} = \frac{\rho\vec{r}}{2\epsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{2\epsilon_0} = \frac{\rho\vec{b}}{2\epsilon_0}.$$

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b = 0$ and $E_{\text{hole}} = 0$.

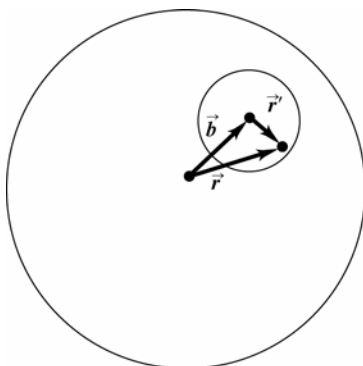


Figure 22.62

22.63. IDENTIFY: The electric field at each point is the vector sum of the fields of the two charge distributions.

SET UP: Inside a sphere of uniform positive charge, $E = \frac{\rho r}{3\epsilon_0}$.

$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$ so $E = \frac{Qr}{4\pi\epsilon_0 R^3}$, directed away from the center of the sphere. Outside a sphere of uniform

positive charge, $E = \frac{Q}{4\pi\epsilon_0 r^2}$, directed away from the center of the sphere.

EXECUTE: (a) $x = 0$. This point is inside sphere 1 and outside sphere 2. The fields are shown in Figure 22.63a.



Figure 22.63a

$$E_1 = \frac{Qr}{4\pi\epsilon_0 R^3} = 0, \text{ since } r = 0.$$

$E_2 = \frac{Q}{4\pi\epsilon_0 r^2}$ with $r = 2R$ so $E_2 = \frac{Q}{16\pi\epsilon_0 R^2}$, in the $-x$ -direction.

Thus $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{Q}{16\pi\epsilon_0 R^2} \hat{i}$.

(b) $x = R/2$. This point is inside sphere 1 and outside sphere 2. Each field is directed away from the center of the sphere that produces it. The fields are shown in Figure 22.63b.

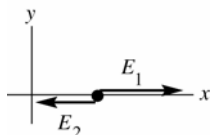


Figure 22.63b

$$E_1 = \frac{Qr}{4\pi\epsilon_0 R^3} \text{ with } r = R/2 \text{ so}$$

$$E_1 = \frac{Q}{8\pi\epsilon_0 R^2}$$

$E_2 = \frac{Q}{4\pi\epsilon_0 r^2}$ with $r = 3R/2$ so $E_2 = \frac{Q}{9\pi\epsilon_0 R^2}$

$E = E_1 - E_2 = \frac{Q}{72\pi\epsilon_0 R^2}$, in the $+x$ -direction and $\vec{E} = \frac{Q}{72\pi\epsilon_0 R^2} \hat{i}$

(c) $x = R$. This point is at the surface of each sphere. The fields have equal magnitudes and opposite directions, so $E = 0$.

(d) $x = 3R$. This point is outside both spheres. Each field is directed away from the center of the sphere that produces it. The fields are shown in Figure 22.63c.

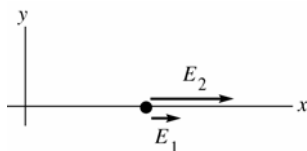


Figure 22.63c

$$E_1 = \frac{Q}{4\pi\epsilon_0 r^2} \text{ with } r = 3R \text{ so}$$

$$E_1 = \frac{Q}{36\pi\epsilon_0 R^2}$$

$E_2 = \frac{Q}{4\pi\epsilon_0 r^2}$ with $r = R$ so $E_2 = \frac{Q}{4\pi\epsilon_0 R^2}$

$E = E_1 + E_2 = \frac{5Q}{18\pi\epsilon_0 R^2}$, in the $+x$ -direction and $\vec{E} = \frac{5Q}{18\pi\epsilon_0 R^2} \hat{i}$

EVALUATE: The field of each sphere is radially outward from the center of the sphere. We must use the correct expression for $E(r)$ for each sphere, depending on whether the field point is inside or outside that sphere.

22.64. IDENTIFY: The net electric field at any point is the vector sum of the fields at each sphere.

SET UP: Example 22.9 gives the electric field inside and outside a uniformly charged sphere. For the positively charged sphere the field is radially outward and for the negatively charged sphere the electric field is radially inward.

EXECUTE: (a) At this point the field of the left-hand sphere is zero and the field of the right-hand sphere is toward the center of that sphere, in the $+x$ -direction. This point is outside the right-hand sphere, a distance $r = 2R$ from its center. $\vec{E} = +\frac{1}{4\pi\epsilon_0} \frac{Q}{4R^2} \hat{i}$.

(b) This point is inside the left-hand sphere, at $r = R/2$, and is outside the right-hand sphere, at $r = 3R/2$. Both fields are in the $+x$ -direction.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left[\frac{Q(R/2)}{R^3} + \frac{Q}{(3R/2)^2} \right] \hat{i} = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{2R^2} + \frac{4Q}{9R^2} \right] \hat{i} = \frac{1}{4\pi\epsilon_0} \frac{17Q}{18R^2} \hat{i}.$$

(c) This point is outside both spheres, at a distance $r = R$ from their centers. Both fields are in the $+x$ -direction.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{R^2} + \frac{Q}{R^2} \right] \hat{i} = \frac{Q}{2\pi\epsilon_0 R^2} \hat{i}.$$

(d) This point is outside both spheres, a distance $r = 3R$ from the center of the left-hand sphere and a distance $r = R$ from the center of the right-hand sphere. The field of the left-hand sphere is in the $+x$ -direction and the field of the right-hand sphere is in the $-x$ -direction.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{(3R)^2} - \frac{Q}{R^2} \right] \hat{i} = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{9R^2} - \frac{Q}{R^2} \right] \hat{i} = \frac{-1}{4\pi\epsilon_0} \frac{8Q}{9R^2} \hat{i}.$$

EVALUATE: At all points on the x -axis the net field is parallel to the x -axis.

22.65. IDENTIFY: Let $-dQ$ be the electron charge contained within a spherical shell of radius r' and thickness dr' . Integrate r' from 0 to r to find the electron charge within a sphere of radius r . Apply Gauss's law to a sphere of radius r to find the electric field $E(r)$.

SET UP: The volume of the spherical shell is $dV = 4\pi r'^2 dr'$.

EXECUTE: (a) $Q(r) = Q - \int \rho dV = Q - \frac{Q4\pi}{\pi a_0^3} \int_0^r e^{-2r'/a_0} r'^2 dr' = Q - \frac{4Q}{a_0^3} \int_0^r r'^2 e^{-2r'/a_0} dr'$.

$$Q(r) = Q - \frac{4Qe^{-\alpha r}}{a_0^3 \alpha^3} (2e^{\alpha r} - \alpha^2 r^2 - 2\alpha r - 2) = Qe^{-2r/a_0} [2(r/a_0)^2 + 2(r/a_0) + 1].$$

Note if $r \rightarrow \infty$, $Q(r) \rightarrow 0$; the total net charge of the atom is zero.

(b) The electric field is radially outward. Gauss's law gives $E(4\pi r^2) = \frac{Q(r)}{\epsilon_0}$ and

$$E = \frac{kQe^{-2r/a_0}}{r^2} (2(r/a_0)^2 + 2(r/a_0) + 1).$$

(c) The graph of E versus r is sketched in Figure 22.65. What is plotted is the scaled E , equal to $E/E_{\text{pt charge}}$, versus scaled r , equal to r/a_0 . $E_{\text{pt charge}} = \frac{kQ}{r^2}$ is the field of a point charge.

EVALUATE: As $r \rightarrow 0$, the field approaches that of the positive point charge (the proton). For increasing r the electric field falls to zero more rapidly than the $1/r^2$ dependence for a point charge.

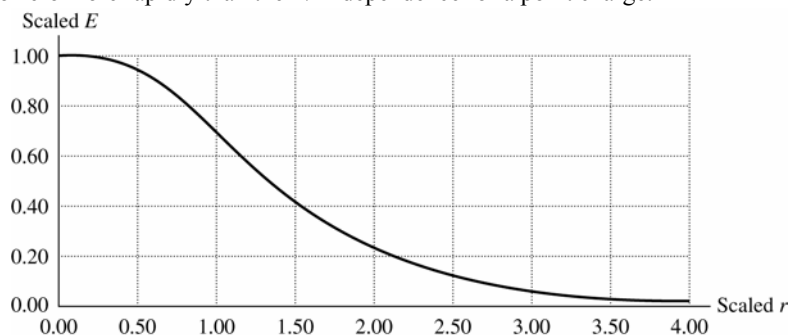


Figure 22.65

22.66. IDENTIFY: The charge in a spherical shell of radius r and thickness dr is $dQ = \rho(r)4\pi r^2 dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r . Let Q_i be the charge in the region $r \leq R/2$ and let Q_o be the charge in the region where $R/2 \leq r \leq R$.

EXECUTE: (a) The total charge is $Q = Q_i + Q_0$, where $Q_i = \alpha \frac{4\pi(R/2)^3}{3} = \frac{\alpha\pi R^3}{6}$ and

$$Q_0 = 4\pi(2\alpha) \int_{R/2}^R (r^2 - r^3/R) dr = 8\alpha\pi \left(\frac{(R^3 - R^3/8)}{3} - \frac{(R^4 - R^4/16)}{4R} \right) = \frac{11\alpha\pi R^3}{24}. \text{ Therefore, } Q = \frac{15\alpha\pi R^3}{24} \text{ and } \alpha = \frac{8Q}{5\pi R^3}.$$

(b) For $r \leq R/2$, Gauss's law gives $E4\pi r^2 = \frac{\alpha 4\pi r^3}{3\epsilon_0}$ and $E = \frac{\alpha r}{3\epsilon_0} = \frac{8Qr}{15\pi\epsilon_0 R^3}$. For $R/2 \leq r \leq R$,

$$E4\pi r^2 = \frac{Q_i}{\epsilon_0} + \frac{1}{\epsilon_0} \left(8\alpha\pi \left(\frac{(r^3 - R^3/8)}{3} - \frac{(r^4 - R^4/16)}{4R} \right) \right) \text{ and}$$

$$E = \frac{\alpha\pi R^3}{24\epsilon_0(4\pi r^2)} (64(r/R)^3 - 48(r/R)^4 - 1) = \frac{kQ}{15r^2} (64(r/R)^3 - 48(r/R)^4 - 1).$$

$$\text{For } r \geq R, E(4\pi r^2) = \frac{Q}{\epsilon_0} \text{ and } E = \frac{Q}{4\pi\epsilon_0 r^2}.$$

$$(c) \frac{Q_i}{Q} = \frac{(4Q/15)}{Q} = \frac{4}{15} = 0.267.$$

(d) For $r \leq R/2$, $F_r = -eE = -\frac{8eQ}{15\pi\epsilon_0 R^3} r$, so the restoring force depends upon displacement to the first power, and we have simple harmonic motion.

$$(e) \text{ Comparing to } F = -kr, k = \frac{8eQ}{15\pi\epsilon_0 R^3}. \text{ Then } \omega = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{8eQ}{15\pi\epsilon_0 R^3 m_e}} \text{ and } T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{15\pi\epsilon_0 R^3 m_e}{8eQ}}.$$

EVALUATE: (f) If the amplitude of oscillation is greater than $R/2$, the force is no longer linear in r , and is thus no longer simple harmonic.

22.67. IDENTIFY: The charge in a spherical shell of radius r and thickness dr is $dQ = \rho(r)4\pi r^2 dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r . Let Q_i be the charge in the region $r \leq R/2$ and let Q_0 be the charge in the region where $R/2 \leq r \leq R$.

EXECUTE: (a) The total charge is $Q = Q_i + Q_0$, where $Q_i = 4\pi \int_0^{R/2} \frac{3ar^3}{2R} dr = \frac{6\pi a}{R} \frac{1}{4} \frac{R^4}{16} = \frac{3}{32}\pi a R^3$ and

$$Q_0 = 4\pi a \int_{R/2}^R (1 - (r/R)^2) r^2 dr = 4\pi a R^3 \left(\frac{7}{24} - \frac{31}{160} \right) = \frac{47}{120}\pi a R^3. \text{ Therefore, } Q = \left(\frac{3}{32} + \frac{47}{120} \right) \pi a R^3 = \frac{233}{480}\pi a R^3 \text{ and}$$

$$\alpha = \frac{480Q}{233\pi R^3}.$$

(b) For $r \leq R/2$, Gauss's law gives $E4\pi r^2 = \frac{4\pi}{\epsilon_0} \int_0^r \frac{3ar'^3}{2R} dr' = \frac{3\pi ar^4}{2\epsilon_0 R}$ and $E = \frac{6ar^2}{16\epsilon_0 R} = \frac{180Qr^2}{233\pi\epsilon_0 R^4}$. For $R/2 \leq r \leq R$,

$$E4\pi r^2 = \frac{Q_i}{\epsilon_0} + \frac{4\pi a}{\epsilon_0} \int_{R/2}^r (1 - (r'/R)^2) r'^2 dr' = \frac{Q_i}{\epsilon_0} + \frac{4\pi a}{\epsilon_0} \left(\frac{r^3}{3} - \frac{R^3}{24} - \frac{r^5}{5R^2} + \frac{R^3}{160} \right).$$

$$E4\pi r^2 = \frac{3}{128} \frac{4\pi a R^3}{\epsilon_0} + \frac{4\pi a R^3}{\epsilon_0} \left(\frac{1}{3} \left(\frac{r}{R} \right)^3 - \frac{1}{5} \left(\frac{r}{R} \right)^5 - \frac{17}{480} \right) \text{ and } E = \frac{480Q}{233\pi\epsilon_0 r^2} \left(\frac{1}{3} \left(\frac{r}{R} \right)^3 - \frac{1}{5} \left(\frac{r}{R} \right)^5 - \frac{23}{1920} \right). \text{ For } r \geq R,$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}, \text{ since all the charge is enclosed.}$$

(c) The fraction of Q between $R/2 \leq r \leq R$ is $\frac{Q_0}{Q} = \frac{47}{120} \frac{480}{233} = 0.807$.

(d) $E = \frac{180}{233} \frac{Q}{4\pi\epsilon_0 R^4}$ using either of the electric field expressions above, evaluated at $r = R/2$.

EVALUATE: (e) The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be simple harmonic motion.

ELECTRIC POTENTIAL

23.1. IDENTIFY: Apply Eq.(23.2) to calculate the work. The electric potential energy of a pair of point charges is given by Eq.(23.9).

SET UP: Let the initial position of q_2 be point a and the final position be point b , as shown in Figure 23.1.

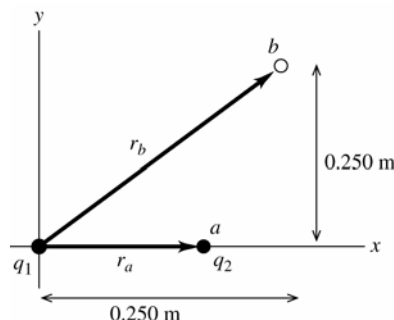


Figure 23.1

$$r_a = 0.150 \text{ m}$$

$$r_b = \sqrt{(0.250 \text{ m})^2 + (0.250 \text{ m})^2}$$

$$r_b = 0.3536 \text{ m}$$

EXECUTE: $W_{a \rightarrow b} = U_a - U_b$

$$U_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_a} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(+2.40 \times 10^{-6} \text{ C})(-4.30 \times 10^{-6} \text{ C})}{0.150 \text{ m}}$$

$$U_a = -0.6184 \text{ J}$$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_b} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(+2.40 \times 10^{-6} \text{ C})(-4.30 \times 10^{-6} \text{ C})}{0.3536 \text{ m}}$$

$$U_b = -0.2623 \text{ J}$$

$$W_{a \rightarrow b} = U_a - U_b = -0.6184 \text{ J} - (-0.2623 \text{ J}) = -0.356 \text{ J}$$

EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger r .

23.2. IDENTIFY: Apply $W_{a \rightarrow b} = U_a - U_b$.

SET UP: $U_a = +5.4 \times 10^{-8} \text{ J}$. Solve for U_b .

EXECUTE: $W_{a \rightarrow b} = -1.9 \times 10^{-8} \text{ J} = U_a - U_b$. $U_b = U_a - W_{a \rightarrow b} = 1.9 \times 10^{-8} \text{ J} - (-5.4 \times 10^{-8} \text{ J}) = 7.3 \times 10^{-8} \text{ J}$.

EVALUATE: When the electric force does negative work the electrical potential energy increases.

23.3. IDENTIFY: The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is e and the charges are equidistant from each other, so the total

$$\text{potential energy is } U = \frac{1}{4\pi\epsilon_0} \left(\frac{e^2}{r} + \frac{e^2}{r} + \frac{e^2}{r} \right) = \frac{3e^2}{4\pi\epsilon_0 r}.$$

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60 \times 10^{-19} \text{ C})^2 (9.00 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)}{2.00 \times 10^{-15} \text{ m}} = 3.46 \times 10^{-13} \text{ J} = 2.16 \text{ MeV}$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.4. IDENTIFY: The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity, $U = (1/4\pi\epsilon_0)(qq_0/r)$, we have

$$W = \Delta U = U_2 - U_1 = \frac{1}{4\pi\epsilon_0} \left(\frac{e^2}{r_2} - \frac{e^2}{r_1} \right).$$

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (1.60 \times 10^{-19} \text{ C})^2 \left(\frac{1}{3.00 \times 10^{-15} \text{ m}} - \frac{1}{2.00 \times 10^{-15} \text{ m}} \right) = 7.68 \times 10^{-14} \text{ J}$$

(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\frac{1}{2}mv^2\right) = mv^2$.

EXECUTE: Solving for v gives $v = \sqrt{\frac{\Delta U}{m}} = \sqrt{\frac{7.68 \times 10^{-14} \text{ J}}{1.67 \times 10^{-27} \text{ kg}}} = 6.78 \times 10^6 \text{ m/s}$

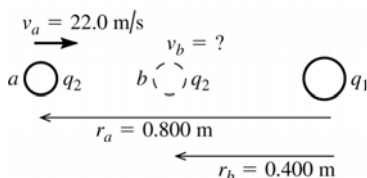
EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.5. (a) IDENTIFY: Use conservation of energy:

$$K_a + U_a + W_{\text{other}} = K_b + U_b$$

U for the pair of point charges is given by Eq.(23.9).

SET UP:



Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1 , as shown in Figure 23.5a.

Figure 23.5a

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(1.50 \times 10^{-3} \text{ kg})(22.0 \text{ m/s})^2 = 0.3630 \text{ J}$$

$$U_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_a} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{0.800 \text{ m}} = +0.2454 \text{ J}$$

$$K_b = \frac{1}{2}mv_b^2$$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_b} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{0.400 \text{ m}} = +0.4907 \text{ J}$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$

$$\frac{1}{2}mv_b^2 = +0.3630 \text{ J} + (0.2454 \text{ J} - 0.4907 \text{ J}) = 0.1177 \text{ J}$$

$$v_b = \sqrt{\frac{2(0.1177 \text{ J})}{1.50 \times 10^{-3} \text{ kg}}} = 12.5 \text{ m/s}$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c : $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.

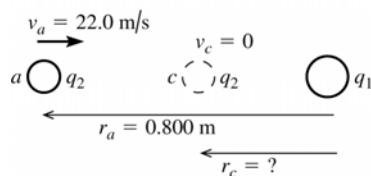


Figure 23.5b

EXECUTE: $K_a = +0.3630 \text{ J}$ (from part (a))

$U_a = +0.2454 \text{ J}$ (from part (a))

$K_c = 0$ (at distance of closest approach the speed is zero)

$$U_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_c}$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_c} = +0.3630 \text{ J} + 0.2454 \text{ J} = 0.6084 \text{ J}$

$$r_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{0.6084 \text{ J}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{+0.6084 \text{ J}} = 0.323 \text{ m}.$$

EVALUATE: $U \rightarrow \infty$ as $r \rightarrow 0$ so q_2 will stop no matter what its initial speed is.

23.6. IDENTIFY: Apply $U = k \frac{q_1 q_2}{r}$ and solve for r .

SET UP: $q_1 = -7.2 \times 10^{-6} \text{ C}$, $q_2 = +2.3 \times 10^{-6} \text{ C}$

EXECUTE: $r = \frac{k q_1 q_2}{U} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-7.20 \times 10^{-6} \text{ C})(+2.30 \times 10^{-6} \text{ C})}{-0.400 \text{ J}} = 0.372 \text{ m}$

EVALUATE: The potential energy U is a scalar and can take positive and negative values.

23.7. (a) IDENTIFY and SET UP: U is given by Eq.(23.9).

EXECUTE: $U = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r}$

$$U = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(+4.60 \times 10^{-6} \text{ C})(+1.20 \times 10^{-6} \text{ C})}{0.250 \text{ m}} = +0.198 \text{ J}$$

EVALUATE: The two charges are both of the same sign so their electric potential energy is positive.

(b) IDENTIFY: Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$

SET UP: Let point a be where q is released and point b be at its final position, as shown in Figure 23.7.

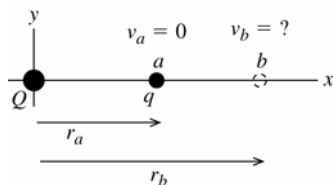


Figure 23.7

EXECUTE: $K_a = 0$ (released from rest)

$U_a = +0.198 \text{ J}$ (from part (a))

$$K_b = \frac{1}{2} m v_b^2$$

Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r}$.

(i) $r_b = 0.500 \text{ m}$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(+4.60 \times 10^{-6} \text{ C})(+1.20 \times 10^{-6} \text{ C})}{0.500 \text{ m}} = +0.0992 \text{ J}$$

Then $K_a + U_a + W_{\text{other}} = K_b + U_b$ gives $K_b = U_a - U_b$ and $\frac{1}{2} m v_b^2 = U_a - U_b$ and

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.0992 \text{ J})}{2.80 \times 10^{-4} \text{ kg}}} = 26.6 \text{ m/s}.$$

(ii) $r_b = 5.00 \text{ m}$ r_b is now ten times larger than in (i) so U_b is ten times smaller: $U_b = +0.0992 \text{ J}/10 = +0.00992 \text{ J}$.

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.00992 \text{ J})}{2.80 \times 10^{-4} \text{ kg}}} = 36.7 \text{ m/s}.$$

(iii) $r_b = 50.0$ m r_b is now ten times larger than in (ii) so U_b is ten times smaller:

$$U_b = +0.00992 \text{ J}/10 = +0.000992 \text{ J}.$$

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.000992 \text{ J})}{2.80 \times 10^{-4} \text{ kg}}} = 37.5 \text{ m/s}.$$

EVALUATE: The force between the two charges is repulsive and provides an acceleration to q . This causes the speed of q to increase as it moves away from Q .

23.8. IDENTIFY: Call the three charges 1, 2 and 3. $U = U_{12} + U_{13} + U_{23}$

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.500 m.

EXECUTE: $U = \frac{3kq^2}{0.500 \text{ m}} = \frac{3k(1.2 \times 10^{-6} \text{ C})^2}{0.500 \text{ m}} = 0.078 \text{ J}.$

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.9. IDENTIFY: $U = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$

SET UP: In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate x , so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200 - x$.

EXECUTE: (a) $U = k \left(\frac{(4.00 \text{ nC})(-3.00 \text{ nC})}{(0.200 \text{ m})} + \frac{(4.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})} + \frac{(-3.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})} \right) = 3.60 \times 10^{-7} \text{ J}$

(b) If $U = 0$, then $0 = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{x} + \frac{q_2 q_3}{r_{12} - x} \right)$. Solving for x we find:

$$0 = -60 + \frac{8}{x} - \frac{6}{0.2 - x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074 \text{ m}, 0.360 \text{ m}. \text{ Therefore, } x = 0.074 \text{ m} \text{ since it is the only value between the two charges.}$$

EVALUATE: U_{13} is positive and both U_{23} and U_{12} are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$. For

$x = 0.074$ m, $U_{13} = +9.7 \times 10^{-7} \text{ J}$, $U_{23} = -4.3 \times 10^{-7} \text{ J}$ and $U_{12} = -5.4 \times 10^{-7} \text{ J}$. It is true that $U = 0$ at this x .

23.10. IDENTIFY: The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a \rightarrow b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center} \rightarrow \text{side}} = U_{\text{center}} - U_{\text{side}}$.

There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00$ nm from the two nearest electrons and a distance $r_3 = \sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\epsilon_0)(qq_0/r)$, the total work is

$$W_{\text{center} \rightarrow \text{side}} = U_{\text{center}} - U_{\text{side}} = 4 \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_e}{r_1} - \left(2 \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_e}{r_2} + 2 \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_e}{r_3} \right)$$

Substituting $q_e = e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center} \rightarrow \text{side}} = -4e^2 \frac{1}{4\pi\epsilon_0} \left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3} \right) \right]$$

EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4(1.60 \times 10^{-19} \text{ C})^2 \left[\frac{2}{\sqrt{50} \text{ m}} - \left(\frac{1}{5.00 \text{ nm}} + \frac{1}{\sqrt{125} \text{ nm}} \right) \right] = 6.08 \times 10^{-21} \text{ J} ???$$

EVALUATE: Since the work is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side.

- 23.11. IDENTIFY:** Apply Eq.(23.2). The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from Eq.(23.9).

SET UP: Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.

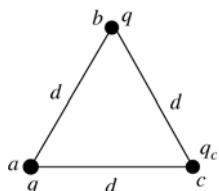


Figure 23.11

Let q_c be the third, unknown charge.

EXECUTE: $W = -\Delta U = -(U_2 - U_1)$

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\epsilon_0 d} (q^2 + 2qq_c)$$

Want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$

$$0 = \frac{1}{4\pi\epsilon_0 d} (q^2 + 2qq_c)$$

$$q^2 + 2qq_c = 0 \text{ and } q_c = -q/2.$$

EVALUATE: The potential energy for the two charges q is positive and for each q with q_c it is negative. There are two of the q, q_c terms so must have $q_c < q$.

- 23.12. IDENTIFY:** Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b . The

maximum force is at the distance of closest approach, $F = k \frac{|q_1 q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass 1.67×10^{-27} kg and charge $q = +e = +1.60 \times 10^{-19}$ C.

EXECUTE: $K_a = U_b$. $2(\frac{1}{2}mv_a^2) = k \frac{q_1 q_2}{r_b}$. $mv_a^2 = k \frac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.67 \times 10^{-27} \text{ kg})(1.00 \times 10^6 \text{ m/s})^2} = 1.38 \times 10^{-13} \text{ m}.$$

$$F = k \frac{e^2}{r_b^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(1.38 \times 10^{-13} \text{ C})^2} = 0.012 \text{ N}.$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

- 23.13. IDENTIFY:** $\vec{E}$ points from high potential to low potential. $\frac{W_{a \rightarrow b}}{q_0} = V_a - V_b$.

SET UP: The force on a positive test charge is in the direction of $\vec{E}$.

EXECUTE: V decreases in the eastward direction. A is east of B , so $V_B > V_A$. C is east of A , so $V_C < V_A$. The force on a positive test charge is east, so no work is done on it by the electric force when it moves due south (the force and displacement are perpendicular), and $V_D = V_A$.

EVALUATE: The electric potential is constant in a direction perpendicular to the electric field.

- 23.14. IDENTIFY:** $\frac{W_{a \rightarrow b}}{q_0} = V_a - V_b$. For a point charge, $V = \frac{kq}{r}$.

SET UP: Each vacant corner is the same distance, 0.200 m, from each point charge.

EXECUTE: Taking the origin at the center of the square, the symmetry means that the potential is the same at the two corners not occupied by the $+5.00 \mu\text{C}$ charges. This means that no net work is done in moving from one corner to the other.

EVALUATE: If the charge q_0 moves along a diagonal of the square, the electrical force does positive work for part of the path and negative work for another part of the path, but the net work done is zero.

23.15. IDENTIFY and SET UP: Apply conservation of energy to points A and B .

EXECUTE: $K_A + U_A = K_B + U_B$

$U = qV$, so $K_A + qV_A = K_B + qV_B$

$K_B = K_A + q(V_A - V_B) = 0.00250 \text{ J} + (-5.00 \times 10^{-6} \text{ C})(200 \text{ V} - 800 \text{ V}) = 0.00550 \text{ J}$

$v_B = \sqrt{2K_B/m} = 7.42 \text{ m/s}$

EVALUATE: It is faster at B ; a negative charge gains speed when it moves to higher potential.

23.16. IDENTIFY: The work-energy theorem says $W_{a \rightarrow b} = K_b - K_a$. $\frac{W_{a \rightarrow b}}{q} = V_a - V_b$.

SET UP: Point a is the starting and point b is the ending point. Since the field is uniform,

$W_{a \rightarrow b} = Fs \cos \phi = E|q|s \cos \phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0^\circ$ and the work $W_{a \rightarrow b}$ is positive.

EXECUTE: (a) $W_{a \rightarrow b} = K_b - K_a = 1.50 \times 10^{-6} \text{ J} - 0 = 1.50 \times 10^{-6} \text{ J}$

(b) $V_a - V_b = \frac{W_{a \rightarrow b}}{q} = \frac{1.50 \times 10^{-6} \text{ J}}{4.20 \times 10^{-9} \text{ C}} = 357 \text{ V}$. Point a is at higher potential than point b .

(c) $E|q|s = W_{a \rightarrow b}$, so $E = \frac{W_{a \rightarrow b}}{|q|s} = \frac{V_a - V_b}{s} = \frac{357 \text{ V}}{6.00 \times 10^{-2} \text{ m}} = 5.95 \times 10^3 \text{ V/m}$.

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.17. IDENTIFY: Apply the equation that precedes Eq.(23.17): $W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l}$.

SET UP: Use coordinates where $+y$ is upward and $+x$ is to the right. Then $\vec{E} = E\hat{j}$ with $E = 4.00 \times 10^4 \text{ N/C}$.

(a) The path is sketched in Figure 23.17a.

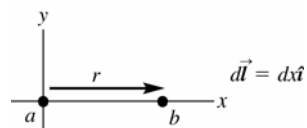


Figure 23.17a

EXECUTE: $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i}) = 0$ so $W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = 0$.

EVALUATE: The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) **SET UP:** The path is sketched in Figure 23.17b.

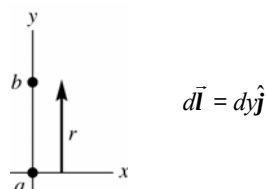


Figure 23.17b

EXECUTE: $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dy\hat{j}) = E dy$

$$W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q'E \int_a^b dy = q'E(y_b - y_a)$$

$y_b - y_a = +0.670 \text{ m}$, positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a \rightarrow b} = q'E(y_b - y_a) = (+28.0 \times 10^{-9} \text{ C})(4.00 \times 10^4 \text{ N/C})(+0.670 \text{ m}) = +7.50 \times 10^{-4} \text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) **SET UP:** The path is sketched in Figure 23.17c.

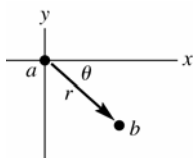


Figure 23.17c

$$y_a = 0$$

$$y_b = -r \sin \theta = -(2.60 \text{ m}) \sin 45^\circ = -1.838 \text{ m}$$

The vertical component of the 2.60 m displacement is 1.838 m downward.

EXECUTE: $d\vec{l} = dx\hat{i} + dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = E dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q'E \int_a^b dy = q'E(y_b - y_a)$$

$$W_{a \rightarrow b} = q'E(y_b - y_a) = (+28.0 \times 10^{-9} \text{ C})(4.00 \times 10^4 \text{ N/C})(-1.838 \text{ m}) = -2.06 \times 10^{-3} \text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.18. IDENTIFY: Apply $K_a + U_a = K_b + U_b$.

SET UP: Let $q_1 = +3.00 \text{ nC}$ and $q_2 = +2.00 \text{ nC}$. At point a , $r_{1a} = r_{2a} = 0.250 \text{ m}$. At point b , $r_{1b} = 0.100 \text{ m}$ and $r_{2b} = 0.400 \text{ m}$. The electron has $q = -e$ and $m_e = 9.11 \times 10^{-31} \text{ kg}$. $K_a = 0$ since the electron is released from rest.

$$\text{EXECUTE: } -\frac{keq_1}{r_{1a}} - \frac{keq_2}{r_{2a}} = -\frac{keq_1}{r_{1b}} - \frac{keq_2}{r_{2b}} + \frac{1}{2}m_e v_b^2.$$

$$E_a = K_a + U_a = k(-1.60 \times 10^{-19} \text{ C}) \left(\frac{(3.00 \times 10^{-9} \text{ C})}{0.250 \text{ m}} + \frac{(2.00 \times 10^{-9} \text{ C})}{0.250 \text{ m}} \right) = -2.88 \times 10^{-17} \text{ J}.$$

$$E_b = K_b + U_b = k(-1.60 \times 10^{-19} \text{ C}) \left(\frac{(3.00 \times 10^{-9} \text{ C})}{0.100 \text{ m}} + \frac{(2.00 \times 10^{-9} \text{ C})}{0.400 \text{ m}} \right) + \frac{1}{2}m_e v_b^2 = -5.04 \times 10^{-17} \text{ J} + \frac{1}{2}m_e v_b^2$$

$$\text{Setting } E_a = E_b \text{ gives } v_b = \sqrt{\frac{2}{9.11 \times 10^{-31} \text{ kg}} (5.04 \times 10^{-17} \text{ J} - 2.88 \times 10^{-17} \text{ J})} = 6.89 \times 10^6 \text{ m/s}.$$

EVALUATE: $V_a = V_{1a} + V_{2a} = 180 \text{ V}$. $V_b = V_{1b} + V_{2b} = 315 \text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

23.19. IDENTIFY and SET UP: For a point charge $V = \frac{kq}{r}$. Solve for r .

$$\text{EXECUTE: (a) } r = \frac{kq}{V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.50 \times 10^{-11} \text{ C})}{90.0 \text{ V}} = 2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm}$$

$$\text{(b) } Vr = kq = \text{constant so } V_1 r_1 = V_2 r_2. \quad r_2 = r_1 \left(\frac{V_1}{V_2} \right) = (2.50 \text{ mm}) \left(\frac{90.0 \text{ V}}{30.0 \text{ V}} \right) = 7.50 \text{ mm}.$$

EVALUATE: The potential of a positive charge is positive and decreases as the distance from the point charge increases.

23.20. IDENTIFY: The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

SET UP: The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

EXECUTE: (a) (i) Since both charges have the same sign, there are no points for which the potential is zero.

(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but V is never zero.

(b) (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.

(ii) Between the two charges, the fields point in the same direction, so E cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence E is not zero anywhere.

EVALUATE: It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

23.21. **IDENTIFY:** $V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$

SET UP: The locations of the charges and points A and B are sketched in Figure 23.21.

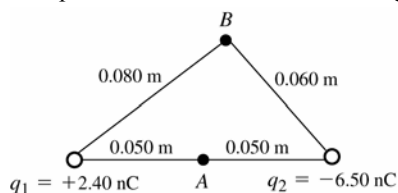


Figure 23.21

EXECUTE: (a) $V_A = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{A1}} + \frac{q_2}{r_{A2}} \right)$

$$V_A = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{+2.40 \times 10^{-9} \text{ C}}{0.050 \text{ m}} + \frac{-6.50 \times 10^{-9} \text{ C}}{0.050 \text{ m}} \right) = -737 \text{ V}$$

(b) $V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{B1}} + \frac{q_2}{r_{B2}} \right)$

$$V_B = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{+2.40 \times 10^{-9} \text{ C}}{0.080 \text{ m}} + \frac{-6.50 \times 10^{-9} \text{ C}}{0.060 \text{ m}} \right) = -704 \text{ V}$$

(c) **IDENTIFY and SET UP:** Use Eq.(23.13) and the results of parts (a) and (b) to calculate W .

EXECUTE: $W_{B \rightarrow A} = q'(V_B - V_A) = (2.50 \times 10^{-9} \text{ C})(-704 \text{ V} - (-737 \text{ V})) = +8.2 \times 10^{-8} \text{ J}$

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point B) to lower potential (point A).

23.22. **IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: (a) The positions of the two charges are shown in Figure 23.22a. $r = \sqrt{a^2 + x^2}$.

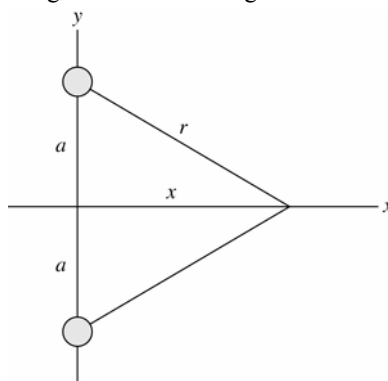


Figure 23.22a

EXECUTE: (b) $V_0 = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{a}$

(c) $V(x) = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{r} = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{a^2 + x^2}}$

(d) The graph of V versus x is sketched in Figure 23.22b.

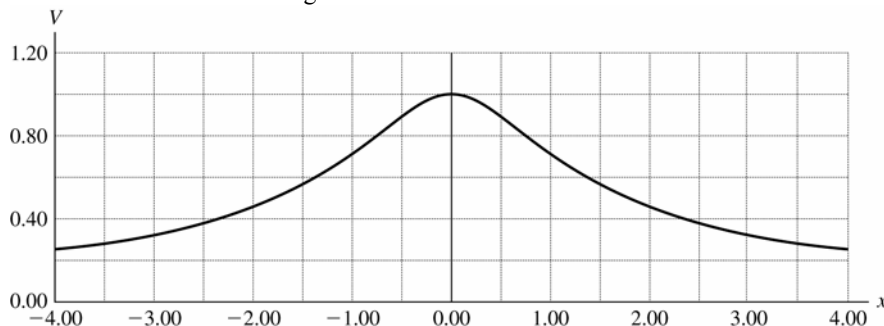


Figure 23.22b

EVALUATE: (e) When $x \gg a$, $V = \frac{1}{4\pi\epsilon_0} \frac{2q}{x}$, just like a point charge of charge $+2q$. At distances from the charges much greater than their separation, the two charges act like a single point charge.

23.23. IDENTIFY: For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: (a) The positions of the two charges are shown in Figure 23.23.

EXECUTE: (b) $V = \frac{kq}{r} + \frac{k(-q)}{r} = 0$.

(c) The potential along the x -axis is always zero, so a graph would be flat.

(d) If the two charges are interchanged, then the results of (b) and (c) still hold. The potential is zero.

EVALUATE: The potential is zero at any point on the x -axis because any point on the x -axis is equidistant from the two charges.

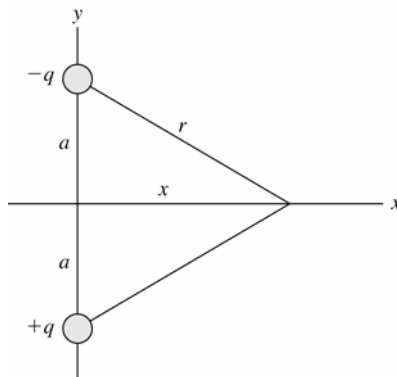


Figure 23.23

23.24. IDENTIFY: For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: Consider the distances from the point on the y -axis to each charge for the three regions $-a \leq y \leq a$ (between the two charges), $y > a$ (above both charges) and $y < -a$ (below both charges).

EXECUTE: (a) $|y| < a: V = \frac{kq}{(a+y)} - \frac{kq}{(a-y)} = \frac{2kqy}{y^2 - a^2}$. $y > a: V = \frac{kq}{(a+y)} - \frac{kq}{y-a} = \frac{-2kqa}{y^2 - a^2}$.

$y < -a: V = \frac{-kq}{(a+y)} - \frac{kq}{(-y+a)} = \frac{2kqa}{y^2 - a^2}$.

A general expression valid for any y is $V = k \left(\frac{-q}{|y-a|} + \frac{q}{|y+a|} \right)$.

(b) The graph of V versus y is sketched in Figure 23.24.

(c) $y \gg a: V = \frac{-2kqa}{y^2 - a^2} \approx \frac{-2kqa}{y^2}$.

(d) If the charges are interchanged, then the potential is of the opposite sign.

EVALUATE: $V = 0$ at $y = 0$. $V \rightarrow +\infty$ as the positive charge is approached and $V \rightarrow -\infty$ as the negative charge is approached.

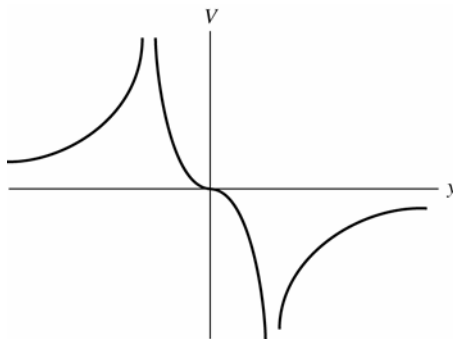


Figure 23.24

- 23.25. IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: (a) The positions of the two charges are shown in Figure 23.25a.

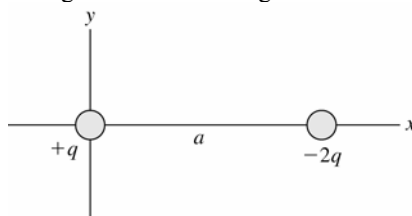


Figure 23.25a

(b) $x > a: V = \frac{kq}{x} - \frac{2kq}{x-a} = \frac{-kq(x+a)}{x(x-a)}$. $0 < x < a: V = \frac{kq}{x} - \frac{2kq}{a-x} = \frac{kq(3x-a)}{x(x-a)}$.

$x < 0: V = \frac{-kq}{x} + \frac{2kq}{x-a} = \frac{kq(x+a)}{x(x-a)}$. A general expression valid for any x is $V = k \left(\frac{q}{|x|} - \frac{2q}{|x-a|} \right)$.

(c) The potential is zero at $x = -a$ and $a/3$.

(d) The graph of V versus x is sketched in Figure 23.25b.

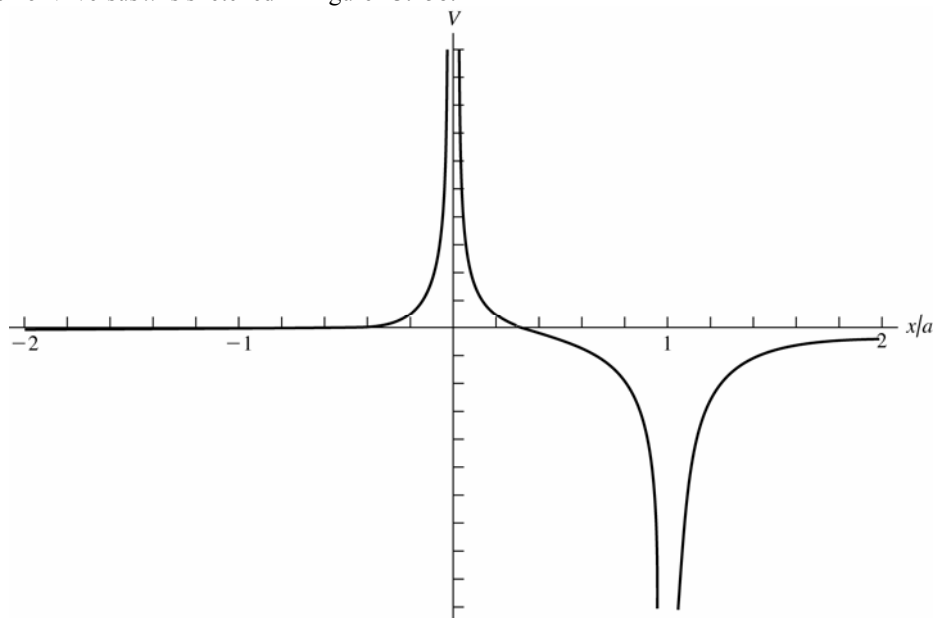


Figure 23.25b

EVALUATE: (e) For $x \gg a: V \approx \frac{-kqx}{x^2} = \frac{-kq}{x}$, which is the same as the potential of a point charge $-q$. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

- 23.26. IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: The distance of a point with coordinate y from the positive charge is $|y|$ and the distance from the negative charge is $r = \sqrt{a^2 + y^2}$.

EXECUTE: (a) $V = \frac{kq}{|y|} - \frac{2kq}{r} = kq \left(\frac{1}{|y|} - \frac{2}{\sqrt{a^2 + y^2}} \right)$.

(b) $V = 0$, when $y^2 = \frac{a^2 + y^2}{4} \Rightarrow 3y^2 = a^2 \Rightarrow y = \pm \frac{a}{\sqrt{3}}$.

(c) The graph of V versus y is sketched in Figure 23.26. $V \rightarrow \infty$ as the positive charge at the origin is approached.

EVALUATE: (d) $y \gg a: V \approx kq \left(\frac{1}{y} - \frac{2}{y} \right) = -\frac{kq}{y}$, which is the potential of a point charge $-q$. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

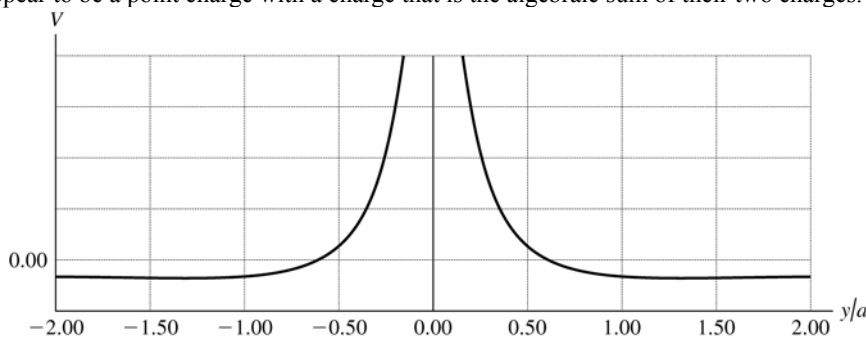


Figure 23.26

- 23.27. IDENTIFY:** $K_a + qV_a = K_b + qV_b$.

SET UP: Let point a be at the cathode and let point b be at the anode. $K_a = 0$. $V_b - V_a = 295$ V. An electron has $q = -e$ and $m = 9.11 \times 10^{-31}$ kg.

EXECUTE: $K_b = q(V_a - V_b) = -(1.60 \times 10^{-19} \text{ C})(-295 \text{ V}) = 4.72 \times 10^{-17} \text{ J}$. $K_b = \frac{1}{2}mv_b^2$, so

$$v_b = \sqrt{\frac{2(4.72 \times 10^{-17} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 1.01 \times 10^7 \text{ m/s}.$$

EVALUATE: The negatively charged electron gains kinetic energy when it moves to higher potential.

- 23.28. IDENTIFY:** For a point charge, $E = \frac{k|q|}{r^2}$ and $V = \frac{kq}{r}$.

SET UP: The electric field is directed toward a negative charge and away from a positive charge.

EXECUTE: (a) $V > 0$ so $q > 0$. $\frac{V}{E} = \frac{kq/r}{k|q|/r^2} = \left(\frac{kq}{r} \right) \left(\frac{r^2}{kq} \right) = r$. $r = \frac{4.98 \text{ V}}{12.0 \text{ V/m}} = 0.415 \text{ m}$.

(b) $q = \frac{rV}{k} = \frac{(0.415 \text{ m})(4.98 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 2.30 \times 10^{-10} \text{ C}$

(c) $q > 0$, so the electric field is directed away from the charge.

EVALUATE: The ratio of V to E due to a point charge increases as the distance r from the charge increases, because E falls off as $1/r^2$ and V falls off as $1/r$.

- 23.29. (a) IDENTIFY and SET UP:** The direction of $\vec{E}$ is always from high potential to low potential so point b is at higher potential.

(b) Apply Eq.(23.17) to relate $V_b - V_a$ to E .

EXECUTE: $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E dx = E(x_b - x_a)$.

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240 \text{ V}}{0.90 \text{ m} - 0.60 \text{ m}} = 800 \text{ V/m}$$

(c) $W_{b \rightarrow a} = q(V_b - V_a) = (-0.200 \times 10^{-6} \text{ C})(+240 \text{ V}) = -4.80 \times 10^{-5} \text{ J}.$

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point b) to low potential (point a).

- 23.30. IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges. For a point charge, $E = \frac{k|q|}{r^2}$. The net electric field is the vector sum of the electric fields of the two charges.

SET UP: $\vec{E}$ produced by a point charge is directed away from the point charge if it is positive and toward the charge if it is negative.

EXECUTE: (a) $V = V_Q + V_{2Q} > 0$, so V is zero nowhere except for infinitely far from the charges. The fields can cancel only between the charges, because only there are the fields of the two charges in opposite directions. Consider a point a distance x from Q and $d - x$ from $2Q$, as shown in Figure 23.30a. $E_Q = E_{2Q} \rightarrow \frac{kQ}{x^2} = \frac{k(2Q)}{(d - x)^2} \rightarrow (d - x)^2 = 2x^2$.

$x = \frac{d}{1 + \sqrt{2}}$. The other root, $x = \frac{d}{1 - \sqrt{2}}$, does not lie between the charges.

(b) V can be zero in 2 places, A and B , as shown in Figure 23.30b. Point A is a distance x from $-Q$ and $d - x$ from $2Q$. B is a distance y from $-Q$ and $d + y$ from $2Q$. At A : $\frac{k(-Q)}{x} + \frac{k(2Q)}{d - x} = 0 \rightarrow x = d/3$.

At B : $\frac{k(-Q)}{y} + \frac{k(2Q)}{d + y} = 0 \rightarrow y = d$.

The two electric fields are in opposite directions to the left of $-Q$ or to the right of $2Q$ in Figure 23.30c. But for the magnitudes to be equal, the point must be closer to the charge with smaller magnitude of charge. This can be the case only in the region to the left of $-Q$. $E_Q = E_{2Q}$ gives $\frac{kQ}{x^2} = \frac{k(2Q)}{(d + x)^2}$ and $x = \frac{d}{\sqrt{2} - 1}$.

EVALUATE: (d) E and V are not zero at the same places. $\vec{E}$ is a vector and V is a scalar. E is proportional to $1/r^2$ and V is proportional to $1/r$. $\vec{E}$ is related to the force on a test charge and ΔV is related to the work done on a test charge when it moves from one point to another.



Figure 23.30

- 23.31. IDENTIFY and SET UP:** Apply conservation of energy, Eq.(23.3). Use Eq.(23.12) to express U in terms of V .

(a) **EXECUTE:** $K_1 + qV_1 = K_2 + qV_2$

$q(V_1 - V_2) = K_2 - K_1$; $q = -1.602 \times 10^{-19} \text{ C}$

$K_1 = \frac{1}{2} m_e v_1^2 = 4.099 \times 10^{-18} \text{ J}$; $K_2 = \frac{1}{2} m_e v_2^2 = 2.915 \times 10^{-17} \text{ J}$

$V_1 - V_2 = \frac{K_2 - K_1}{q} = -156 \text{ V}$

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) **EXECUTE:** Now $K_1 = 2.915 \times 10^{-17} \text{ J}$, $K_2 = 0$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = +182 \text{ V}$$

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

- 23.32. IDENTIFY and SET UP:** Expressions for the electric potential inside and outside a solid conducting sphere are derived in Example 23.8.

EXECUTE: (a) This is outside the sphere, so $V = \frac{kq}{r} = \frac{k(3.50 \times 10^{-9} \text{ C})}{0.480 \text{ m}} = 65.6 \text{ V}.$

(b) This is at the surface of the sphere, so $V = \frac{k(3.50 \times 10^{-9} \text{ C})}{0.240 \text{ m}} = 131 \text{ V}.$

(c) This is inside the sphere. The potential has the same value as at the surface, 131 V.

EVALUATE: All points of a conductor are at the same potential.

23.33. (a) IDENTIFY and SET UP: The electric field on the ring's axis is calculated in Example 21.10. The force on the electron exerted by this field is given by Eq.(21.3).

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring. From

Example 23.11, $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + R^2}}$, where R is the radius of the ring.

EXECUTE: $x_a = 30.0$ cm, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$

Thus $\frac{1}{2}mv^2 = U_a - U_b$

And $U = qV = -eV$ so $v = \sqrt{\frac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x_a^2 + R^2}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{24.0 \times 10^{-9} \text{ C}}{\sqrt{(0.300 \text{ m})^2 + (0.150 \text{ m})^2}}$$

$$V_a = 643 \text{ V}$$

$$V_b = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x_b^2 + R^2}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{24.0 \times 10^{-9} \text{ C}}{0.150 \text{ m}} = 1438 \text{ V}$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602 \times 10^{-19} \text{ C})(1438 \text{ V} - 643 \text{ V})}{9.109 \times 10^{-31} \text{ kg}}} = 1.67 \times 10^7 \text{ m/s}$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.34. IDENTIFY: Example 23.10 shows that for a line of charge, $V_a - V_b = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

SET UP: Let point a be 18.0 cm from the line and let point b be at the distance of closest approach, where $K_b = 0$.

EXECUTE: (a) $K_a = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(1.50 \times 10^3 \text{ m/s})^2 = 1.88 \times 10^{-21} \text{ J}$.

(b) $K_a + qV_a = K_b + qV_b$. $V_a - V_b = \frac{K_b - K_a}{q} = \frac{-1.88 \times 10^{-21} \text{ J}}{1.60 \times 10^{-19} \text{ C}} = -0.01175 \text{ V}$. $\ln(r_b/r_a) = \left(\frac{2\pi\epsilon_0}{\lambda}\right)(-0.01175 \text{ V})$.

$$r_b = r_a \exp\left(\frac{2\pi\epsilon_0(-0.01175 \text{ V})}{\lambda}\right) = (0.180 \text{ m}) \exp\left(-\frac{2\pi\epsilon_0(0.01175 \text{ V})}{5.00 \times 10^{-12} \text{ C/m}}\right) = 0.158 \text{ m}.$$

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

23.35. IDENTIFY: The voltmeter measures the potential difference between the two points. We must relate this quantity to the linear charge density on the wire.

SET UP: For a very long (infinite) wire, the potential difference between two points is $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$.

EXECUTE: (a) Solving for λ gives

$$\lambda = \frac{(\Delta V)2\pi\epsilon_0}{\ln(r_b/r_a)} = \frac{575 \text{ V}}{(18 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \ln\left(\frac{3.50 \text{ cm}}{2.50 \text{ cm}}\right)} = 9.49 \times 10^{-8} \text{ C/m}$$

(b) The meter will read less than 575 V because the electric field is weaker over this 1.00-cm distance than it was over the 1.00-cm distance in part (a).

(c) The potential difference is zero because both probes are at the same distance from the wire, and hence at the same potential.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV , even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each r is the distance from the *center* of the cylinder, not from the surface.

- 23.36. IDENTIFY:** The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.

SET UP: Using $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$ and solving for r_b , we have $r_b = r_a e^{2\pi\epsilon_0 \Delta V / \lambda}$.

EXECUTE: The exponent is $\frac{\left(\frac{1}{2 \times 9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}\right)(175 \text{ V})}{15.0 \times 10^{-9} \text{ C/m}} = 0.648$, which gives
 $r_b = (2.50 \text{ cm}) e^{0.648} = 4.78 \text{ cm}$.

The distance above the *surface* is $4.78 \text{ cm} - 2.50 \text{ cm} = 2.28 \text{ cm}$.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV , even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each r is the distance from the *center* of the cylinder, not from the surface.

- 23.37. IDENTIFY:** For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$.

EXECUTE: (a) Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a) = (8.50 \times 10^{-6} \text{ C/m}) \left(2 \times 9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \ln\left(\frac{10.0 \text{ cm}}{6.00 \text{ cm}}\right)$$

$$\Delta V = 7.82 \times 10^4 \text{ V} = 78,200 \text{ V} = 78.2 \text{ kV}$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

- 23.38. IDENTIFY:** The work required is equal to the change in the electrical potential energy of the charge-ring system. We need only look at the beginning and ending points, since the potential difference is independent of path for a conservative field.

SET UP: (a) $W = \Delta U = q\Delta V = q(V_{\text{center}} - V_{\infty}) = q\left(\frac{1}{4\pi\epsilon_0} \frac{Q}{a} - 0\right)$

EXECUTE: Substituting numbers gives

$$\Delta U = (3.00 \times 10^{-6} \text{ C})(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C})/(0.0400 \text{ m}) = 3.38 \text{ J}$$

(b) We can take any path since the potential is independent of path.

(c) **SET UP:** The net force is away from the ring, so the ball will accelerate away. Energy conservation gives

$$U_0 = K_{\text{max}} = \frac{1}{2}mv^2.$$

EXECUTE: Solving for v gives

$$v = \sqrt{\frac{2U_0}{m}} = \sqrt{\frac{2(3.38 \text{ J})}{0.00150 \text{ kg}}} = 67.1 \text{ m/s}$$

EVALUATE: Direct calculation of the work from the electric field would be extremely difficult, and we would need to know the path followed by the charge. But, since the electric field is conservative, we can bypass all this calculation just by looking at the end points (infinity and the center of the ring) using the potential.

- 23.39. IDENTIFY:** The electric field is zero everywhere except between the plates, and in this region it is uniform and points from the positive to the negative plate (to the left in Figure 23.32).

SET UP: Since the field is uniform between the plates, the potential increases linearly as we go from left to right starting at b .

EXECUTE: Since the potential is taken to be zero at the left surface of the negative plate (a in Figure 23.32), it is zero everywhere to the left of b . It increases linearly from b to c , and remains constant (since $E = 0$) past c . The graph is sketched in Figure 23.39.

EVALUATE: When the electric field is zero, the potential remains constant but not necessarily zero (as to the right of c). When the electric field is constant, the potential is linear.

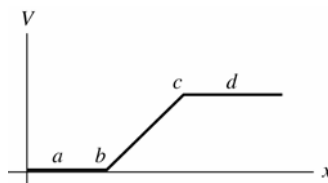


Figure 23.39

- 23.40. IDENTIFY and SET UP:** For oppositely charged parallel plates, $E = \sigma / \epsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

EXECUTE: (a) $E = \frac{\sigma}{\epsilon_0} = \frac{47.0 \times 10^{-9} \text{ C/m}^2}{\epsilon_0} = 5310 \text{ N/C}$.

(b) $V = Ed = (5310 \text{ N/C})(0.0220 \text{ m}) = 117 \text{ V}$.

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

EVALUATE: The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles the work, which is force times distance, doubles and the potential difference doubles.

- 23.41. IDENTIFY and SET UP:** Use the result of Example 23.9 to relate the electric field between the plates to the potential difference between them and their separation. The force this field exerts on the particle is given by Eq.(21.3). Use the equation that precedes Eq.(23.17) to calculate the work.

EXECUTE: (a) From Example 23.9, $E = \frac{V_{ab}}{d} = \frac{360 \text{ V}}{0.0450 \text{ m}} = 8000 \text{ V/m}$

(b) $F = |q|E = (2.40 \times 10^{-9} \text{ C})(8000 \text{ V/m}) = +1.92 \times 10^{-5} \text{ N}$

(c) The electric field between the plates is shown in Figure 23.41.

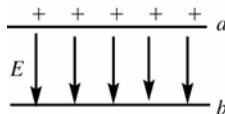


Figure 23.41

The plate with positive charge (plate a) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from $+$ charge toward $-$ charge), so $\vec{E}$ points from a to b . Hence the force that $\vec{E}$ exerts on the positive charge is from a to b , so it does positive work.

$W = \int_a^b \vec{F} \cdot d\vec{l} = Fd$, where d is the separation between the plates.

$W = Fd = (1.92 \times 10^{-5} \text{ N})(0.0450 \text{ m}) = +8.64 \times 10^{-7} \text{ J}$

(d) $V_a - V_b = +360 \text{ V}$ (plate a is at higher potential)

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40 \times 10^{-9} \text{ C})(-360 \text{ V}) = -8.64 \times 10^{-7} \text{ J}$.

EVALUATE: We see that $W_{a \rightarrow b} = -(U_b - U_a) = U_a - U_b$.

- 23.42. IDENTIFY:** The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the field is that of a point-charge,

$E = Q / (4\pi\epsilon_0 R^2)$.

EXECUTE: (a) Solving for Q and substituting the numbers gives

$Q = 4\pi\epsilon_0 R^2 E = (0.125 \text{ m})(1500 \text{ V}) / (9.00 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) = 2.08 \times 10^{-8} \text{ C} = 20.8 \text{ nC}$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 1.50 kV.

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

- 23.43. IDENTIFY and SET UP:** Consider the electric field outside and inside the shell and use that to deduce the potential.

EXECUTE: (a) The electric field outside the shell is the same as for a point charge at the center of the shell, so the potential outside the shell is the same as for a point charge:

$$V = \frac{q}{4\pi\epsilon_0 r} \text{ for } r > R.$$

The electric field is zero inside the shell, so no work is done on a test charge as it moves inside the shell and all

points inside the shell are at the same potential as the surface of the shell: $V = \frac{q}{4\pi\epsilon_0 R}$ for $r \leq R$.

(b) $V = \frac{kq}{R}$ so $q = \frac{RV}{k} = \frac{(0.15 \text{ m})(-1200 \text{ V})}{k} = -20 \text{ nC}$

(c) **EVALUATE:** No, the amount of charge on the sphere is very small. Since $U = qV$ the total amount of electric energy stored on the balloon is only $(20 \text{ nC})(1200 \text{ V}) = 2.4 \times 10^{-5} \text{ J}$.

- 23.44. IDENTIFY:** Example 23.8 shows that the potential of a solid conducting sphere is the same at every point inside the sphere and is equal to its value $V = q/2\pi\epsilon_0 R$ at the surface. Use the given value of E to find q .

SET UP: For negative charge the electric field is directed toward the charge.

For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \frac{q}{4\pi\epsilon_0 r^2}$ and $q = 4\pi\epsilon_0 E r^2 = \frac{(3800 \text{ N/C})(0.200 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.69 \times 10^{-8} \text{ C}$.

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-1.69 \times 10^{-8} \text{ C})}{0.200 \text{ m}} = -760 \text{ V} \text{ at the surface of the sphere. Since the charge all resides}$$

on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be -760 V .

EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

- 23.45. IDENTIFY:** Example 23.9 shows that $V(y) = Ey$, where y is the distance from the negatively charged plate, whose potential is zero. The electric field between the plates is uniform and perpendicular to the plates.

SET UP: V increases toward the positively charged plate. $\vec{E}$ is directed from the positively charged plate toward the negatively charged plate.

EXECUTE: (a) $E = \frac{V}{d} = \frac{480 \text{ V}}{0.0170 \text{ m}} = 2.82 \times 10^4 \text{ V/m}$ and $y = \frac{V}{E}$. $V = 0$ at $y = 0$, $V = 120 \text{ V}$ at $y = 0.43 \text{ cm}$,

$V = 240 \text{ V}$ at $y = 0.85 \text{ cm}$, $V = 360 \text{ V}$ at $y = 1.28 \text{ cm}$ and $V = 480 \text{ V}$ at $y = 1.70 \text{ cm}$. The equipotential surfaces are sketched in Figure 23.45. The surfaces are planes parallel to the plates.

(b) The electric field lines are also shown in Figure 23.45. The field lines are perpendicular to the plates and the equipotential lines are parallel to the plates, so the electric field lines and the equipotential lines are mutually perpendicular.

EVALUATE: Only differences in potential have physical significance. Letting $V = 0$ at the negative plate is a choice we are free to make.

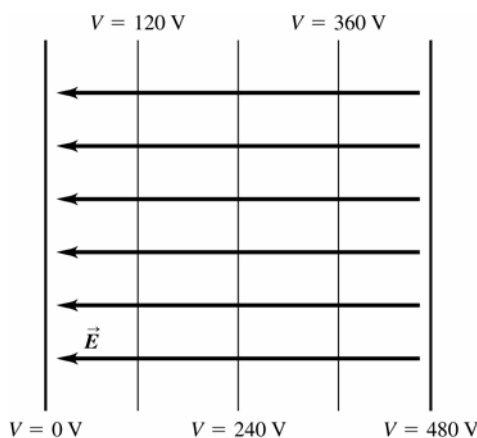


Figure 23.45

- 23.46. IDENTIFY:** By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.

(a) SET UP: No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.

EXECUTE: Since we must do work on the positive charge, it gains potential energy, so the potential increases.

(b) SET UP: Since the electric field is uniform and is equal to $\sigma/2\epsilon_0$, we have $\Delta V = Ed = \frac{\sigma}{2\epsilon_0} d$.

EXECUTE: Solving for d gives

$$d = \frac{2\epsilon_0 \Delta V}{\sigma} = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.00 \text{ V})}{6.00 \times 10^{-9} \text{ C/m}^2} = 0.00295 \text{ m} = 2.95 \text{ mm}$$

EVALUATE: Since the spacing of the equipotential surfaces ($d = 2.95 \text{ mm}$) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

23.47 IDENTIFY and SET UP: Use Eq.(23.19) to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$

(a) $E_x = -\frac{\partial V}{\partial x} = -Ay + 2Bx$

$E_y = -\frac{\partial V}{\partial y} = -Ax - C$

$E_z = -\frac{\partial V}{\partial z} = 0$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

And E_x is also equal zero for this x , any value of z , and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: V doesn't depend on z so $E_z = 0$ everywhere.

23.48. IDENTIFY: Apply Eq.(21.19).

SET UP: Eq.(21.7) says $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \hat{r}$ is the electric field due to a point charge q .

EXECUTE: **(a)** $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x} \left(\frac{kQ}{\sqrt{x^2 + y^2 + z^2}} \right) = \frac{kQx}{(x^2 + y^2 + z^2)^{3/2}} = \frac{kQx}{r^3}$.

Similarly, $E_y = \frac{kQy}{r^3}$ and $E_z = \frac{kQz}{r^3}$.

(b) From part (a), $E = \frac{kQ}{r^2} \left(\frac{x\hat{i}}{r} + \frac{y\hat{j}}{r} + \frac{z\hat{k}}{r} \right) = \frac{kQ}{r^2} \hat{r}$, which agrees with Equation (21.7).

EVALUATE: V is a scalar. $\vec{E}$ is a vector and has components.

23.49. IDENTIFY and SET UP: For a solid metal sphere or for a spherical shell, $V = \frac{kq}{r}$ outside the sphere and $V = \frac{kq}{R}$ at all points inside the sphere, where R is the radius of the sphere. When the electric field is radial, $E = -\frac{\partial V}{\partial r}$.

EXECUTE: **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \frac{kq}{r_a} - \frac{kq}{r_b} = kq \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$.

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell. $V = \frac{kq}{r} - \frac{kq}{r_b} = kq \left(\frac{1}{r} - \frac{1}{r_b} \right)$.

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_a} - \frac{q}{r_b} \right)$ and $V_b = 0$, so $V_{ab} = \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$.

(c) Between the spheres $r_a < r < r_b$ and $V = kq \left(\frac{1}{r} - \frac{1}{r_b} \right)$. $E = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial r} \left(\frac{1}{r} - \frac{1}{r_b} \right) = +\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \left(\frac{1}{r_a} - \frac{1}{r_b} \right) \frac{1}{r^2} V_{ab}$.

(d) From Equation (23.23): $E = 0$, since V is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is

$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} - \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = \frac{1}{4\pi\epsilon_0} \frac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount

$V = -\frac{1}{4\pi\epsilon_0} \frac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change.

However, now that the potential does vary outside the spheres, there is an electric field there:

$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r} \left(\frac{kq}{r} + \frac{-kQ}{r} \right) = \frac{kq}{r^2} \left(1 - \frac{Q}{q} \right) = \frac{k}{r^2} (q - Q)$.

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.

23.50. IDENTIFY: Exercise 23.49 shows that $V = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$ for $r_a < r < r_b$ and

$$V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right).$$

SET UP: $E = \frac{kq}{r^2}$, radially outward, for $r_a \leq r \leq r_b$

EXECUTE: (a) $V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500 \text{ V}$ gives $q = \frac{500 \text{ V}}{k\left(\frac{1}{0.012 \text{ m}} - \frac{1}{0.096 \text{ m}}\right)} = 7.62 \times 10^{-10} \text{ C}$.

(b) $V_b = 0$ so $V_a = 500 \text{ V}$. The inner metal sphere is an equipotential with $V = 500 \text{ V}$. $\frac{1}{r} = \frac{1}{r_a} + \frac{V}{kq}$. $V = 400 \text{ V}$ at

$r = 1.45 \text{ cm}$, $V = 300 \text{ V}$ at $r = 1.85 \text{ cm}$, $V = 200 \text{ V}$ at $r = 2.53 \text{ cm}$, $V = 100 \text{ V}$ at $r = 4.00 \text{ cm}$, $V = 0$ at

$r = 9.60 \text{ cm}$. The equipotential surfaces are sketched in Figure 23.50.

EVALUATE: (c) The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller r , where the electric field is largest.

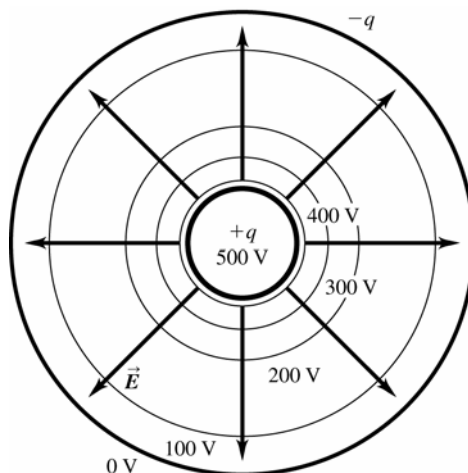


Figure 23.50

23.51. IDENTIFY: Outside the cylinder it is equivalent to a line of charge at its center.

SET UP: The difference in potential between the surface of the cylinder (a distance R from the central axis) and a general point a distance r from the central axis is given by $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r/R)$.

EXECUTE: (a) The potential difference depends only on r , and not direction. Therefore all points at the same value of r will be at the same potential. Thus the equipotential surfaces are cylinders coaxial with the given cylinder.

(b) Solving $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r/R)$ for r , gives $r = R e^{2\pi\epsilon_0 \Delta V / \lambda}$.

For 10 V, the exponent is $(10 \text{ V}) / [(2 \times 9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.50 \times 10^{-9} \text{ C/m})] = 0.370$, which gives $r = (2.00 \text{ cm}) e^{0.370} = 2.90 \text{ cm}$. Likewise, the other radii are 4.20 cm (for 20 V) and 6.08 cm (for 30 V).

(c) $\Delta r_1 = 2.90 \text{ cm} - 2.00 \text{ cm} = 0.90 \text{ cm}$; $\Delta r_2 = 4.20 \text{ cm} - 2.90 \text{ cm} = 1.30 \text{ cm}$; $\Delta r_3 = 6.08 \text{ cm} - 4.20 \text{ cm} = 1.88 \text{ cm}$

EVALUATE: As we can see, Δr increases, so the surfaces get farther apart. This is very different from a sheet of charge, where the surfaces are equally spaced planes.

23.52. IDENTIFY: The electric field is the negative gradient of the potential.

SET UP: $E_x = -\frac{\partial V}{\partial x}$, so E_x is the negative slope of the graph of V as a function of x .

EXECUTE: The graph is sketched in Figure 23.52. Up to a , V is constant, so $E_x = 0$. From a to b , V is linear with a positive slope, so E_x is a negative constant. Past b , the V - x graph has a decreasing positive slope which approaches zero, so E_x is negative and approaches zero.

EVALUATE: Notice that an increasing potential does not necessarily mean that the electric field is increasing.

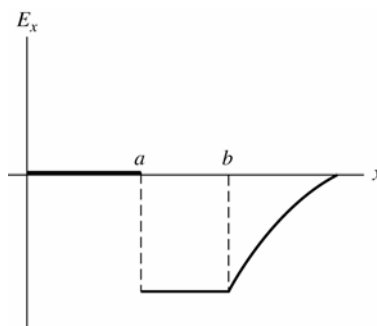


Figure 23.52

23.53. (a) IDENTIFY: Apply the work-energy theorem, Eq.(6.6).

SET UP: Points a and b are shown in Figure 23.53a.

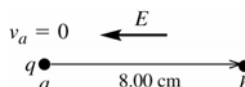


Figure 23.53a

EXECUTE: $W_{\text{tot}} = \Delta K = K_b - K_a = K_b = 4.35 \times 10^{-5} \text{ J}$

The electric force F_E and the additional force F both do work, so that $W_{\text{tot}} = W_{F_E} + W_F$.

$$W_{F_E} = W_{\text{tot}} - W_F = 4.35 \times 10^{-5} \text{ J} - 6.50 \times 10^{-5} \text{ J} = -2.15 \times 10^{-5} \text{ J}$$

EVALUATE: The forces on the charged particle are shown in Figure 23.53b.



Figure 23.53b

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force F is in the direction of the displacement, so it does positive work.

(b) IDENTIFY and SET UP: For the work done by the electric force, $W_{a \rightarrow b} = q(V_a - V_b)$

$$\text{EXECUTE: } V_a - V_b = \frac{W_{a \rightarrow b}}{q} = \frac{-2.15 \times 10^{-5} \text{ J}}{7.60 \times 10^{-9} \text{ C}} = -2.83 \times 10^3 \text{ V.}$$

EVALUATE: The starting point (point a) is at $2.83 \times 10^3 \text{ V}$ lower potential than the ending point (point b). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

(c) IDENTIFY: Calculate E from $V_a - V_b$ and the separation d between the two points.

SET UP: Since the electric field is uniform and directed opposite to the displacement $W_{a \rightarrow b} = -F_E d = -qEd$, where $d = 8.00 \text{ cm}$ is the displacement of the particle.

$$\text{EXECUTE: } E = -\frac{W_{a \rightarrow b}}{qd} = -\frac{V_a - V_b}{d} = \frac{-2.83 \times 10^3 \text{ V}}{0.0800 \text{ m}} = 3.54 \times 10^4 \text{ V/m.}$$

EVALUATE: In part (a), W_{tot} is the total work done by both forces. In parts (b) and (c) $W_{a \rightarrow b}$ is the work done just by the electric force.

23.54. IDENTIFY: The electric force between the electron and proton is attractive and has magnitude $F = \frac{ke^2}{r^2}$. For

circular motion the acceleration is $a_{\text{rad}} = v^2/r$. $U = -k \frac{e^2}{r}$.

SET UP: $e = 1.60 \times 10^{-19} \text{ C}$. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

$$\text{EXECUTE: (a) } \frac{mv^2}{r} = \frac{ke^2}{r^2} \text{ and } v = \sqrt{\frac{ke^2}{mr}}.$$

$$\text{(b) } K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{ke^2}{r} = -\frac{1}{2}U$$

$$(c) E = K + U = \frac{1}{2}U = -\frac{1}{2}\frac{ke^2}{r} = -\frac{1}{2}\frac{k(1.60 \times 10^{-19} \text{ C})^2}{5.29 \times 10^{-11} \text{ m}} = -2.17 \times 10^{-18} \text{ J} = -13.6 \text{ eV}.$$

EVALUATE: The total energy is negative, so the electron is bound to the proton. Work must be done on the electron to take it far from the proton.

23.55. IDENTIFY and SET UP: Calculate the components of $\vec{E}$ from Eq.(23.19). Eq.(21.3) gives $\vec{F}$ from $\vec{E}$.

EXECUTE: (a) $V = Cx^{4/3}$

$$C = V/x^{4/3} = 240 \text{ V}/(13.0 \times 10^{-3} \text{ m})^{4/3} = 7.85 \times 10^4 \text{ V/m}^{4/3}$$

$$(b) E_x = -\frac{\partial V}{\partial x} = -\frac{4}{3}Cx^{1/3} = -(1.05 \times 10^5 \text{ V/m}^{4/3})x^{1/3}$$

The minus sign means that E_x is in the $-x$ -direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

$$(c) \vec{F} = q\vec{E} \text{ so } F_x = -eE_x = \frac{4}{3}eCx^{1/3}$$

Halfway between the electrodes means $x = 6.50 \times 10^{-3} \text{ m}$.

$$F_x = \frac{4}{3}(1.602 \times 10^{-19} \text{ C})(7.85 \times 10^4 \text{ V/m}^{4/3})(6.50 \times 10^{-3} \text{ m})^{1/3} = 3.13 \times 10^{-15} \text{ N}$$

F_x is positive, so the force is directed toward the positive anode.

EVALUATE: V depends only on x , so $E_y = E_z = 0$. $\vec{E}$ is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

23.56. IDENTIFY: At each point (a and b), the potential is the sum of the potentials due to *both* spheres. The voltmeter reads the difference between these two potentials. The spheres behave like a point-charges since the meter is connected to the surface of each one.

SET UP: (a) Call a the point on the surface of one sphere and b the point on the surface of the other sphere, call r the radius of each sphere, and call d the center-to-center distance between the spheres. The potential difference V_{ab} between points a and b is then

$$V_b - V_a = V_{ab} = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{r} + \frac{q}{d-r} - \left(\frac{q}{r} + \frac{-q}{d-r} \right) \right] = \frac{2q}{4\pi\epsilon_0} \left(\frac{1}{d-r} - \frac{1}{r} \right)$$

EXECUTE: Substituting the numbers gives

$$V_b - V_a = 2(175 \mu\text{C}) \left(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left(\frac{1}{0.750 \text{ m}} - \frac{1}{0.250 \text{ m}} \right) = -8.40 \times 10^6 \text{ V}$$

The meter reads 8.40 MV.

(b) Since $V_b - V_a$ is negative, $V_a > V_b$, so point a is at the higher potential.

EVALUATE: An easy way to see that the potential at a is higher than the potential at b is that it would take positive work to move a positive test charge from b to a since this charge would be attracted by the negative sphere and repelled by the positive sphere.

23.57. IDENTIFY: $U = \frac{kq_1q_2}{r}$

SET UP: Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of q and $-q$ separated by d , 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of q and $-q$ separated by $\sqrt{3}d$.

$$\text{EXECUTE: (a) } U = kq^2 \left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d} \right) = -\frac{12kq^2}{d} \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) = -1.46q^2/\pi\epsilon_0 d$$

EVALUATE: (b) The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

23.58. IDENTIFY: For two small spheres, $U = \frac{kq_1q_2}{r}$. For part (b) apply conservation of energy.

SET UP: Let $q_1 = 2.00 \mu\text{C}$ and $q_2 = -3.50 \mu\text{C}$. Let $r_a = 0.250 \text{ m}$ and $r_b \rightarrow \infty$.

$$\text{EXECUTE: (a) } U = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.00 \times 10^{-6} \text{ C})(-3.50 \times 10^{-6} \text{ C})}{0.250 \text{ m}} = -0.252 \text{ J}$$

(b) $K_b = 0$. $U_b = 0$. $U_a = -0.252 \text{ J}$. $K_a + U_a = K_b + U_b$ gives $K_a = 0.252 \text{ J}$. $K_a = \frac{1}{2}mv_a^2$, so

$$v_a = \sqrt{\frac{2K_a}{m}} = \sqrt{\frac{2(0.252 \text{ J})}{1.50 \times 10^{-3} \text{ kg}}} = 18.3 \text{ m/s}$$

EVALUATE: As the sphere moves away, the attractive electrical force exerted by the other sphere does negative work and removes all the kinetic energy it initially had. Note that it doesn't matter which sphere is held fixed and which is shot away; the answer to part (b) is unaffected.

23.59. (a) IDENTIFY: Use Eq.(23.10) for the electron and each proton.

SET UP: The positions of the particles are shown in Figure 23.59a.

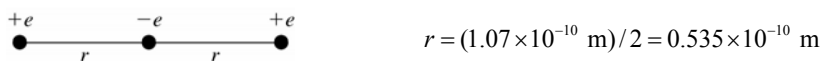


Figure 23.59a

EXECUTE: The potential energy of interaction of the electron with each proton is

$$U = \frac{1}{4\pi\epsilon_0} \frac{(-e^2)}{r}, \text{ so the total potential energy is}$$

$$U = -\frac{2e^2}{4\pi\epsilon_0 r} = -\frac{2(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{0.535 \times 10^{-10} \text{ m}} = -8.60 \times 10^{-18} \text{ J}$$

$$U = -8.60 \times 10^{-18} \text{ J} (1 \text{ eV} / 1.602 \times 10^{-19} \text{ J}) = -53.7 \text{ eV}$$

EVALUATE: The electron and proton have charges of opposite signs, so the potential energy of the system is negative.

(b) IDENTIFY and SET UP: The positions of the protons and points a and b are shown in Figure 23.59b.

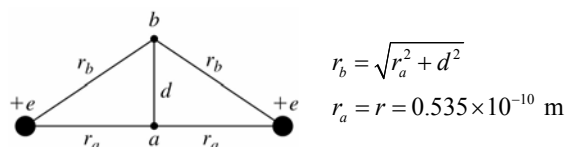


Figure 23.59b

Apply $K_a + U_a + W_{\text{other}} = K_b + U_b$ with point a midway between the protons and point b where the electron instantaneously has $v = 0$ (at its maximum displacement d from point a).

EXECUTE: Only the Coulomb force does work, so $W_{\text{other}} = 0$.

$$U_a = -8.60 \times 10^{-18} \text{ J (from part (a))}$$

$$K_a = \frac{1}{2}mv^2 = \frac{1}{2}(9.109 \times 10^{-31} \text{ kg})(1.50 \times 10^6 \text{ m/s})^2 = 1.025 \times 10^{-18} \text{ J}$$

$$K_b = 0$$

$$U_b = -2ke^2/r_b$$

$$\text{Then } U_b = K_a + U_a - K_b = 1.025 \times 10^{-18} \text{ J} - 8.60 \times 10^{-18} \text{ J} = -7.575 \times 10^{-18} \text{ J}.$$

$$r_b = -\frac{2ke^2}{U_b} = -\frac{2(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{-7.575 \times 10^{-18} \text{ J}} = 6.075 \times 10^{-11} \text{ m}$$

$$\text{Then } d = \sqrt{r_b^2 - r_a^2} = \sqrt{(6.075 \times 10^{-11} \text{ m})^2 - (5.35 \times 10^{-11} \text{ m})^2} = 2.88 \times 10^{-11} \text{ m}.$$

EVALUATE: The force on the electron pulls it back toward the midpoint. The transverse distance the electron moves is about 0.27 times the separation of the protons.

23.60. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.56.

EXECUTE: $T \cos \theta = mg$ and $T \sin \theta = F_e$ gives $F_e = mg \tan \theta = (1.50 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan(30^\circ) = 0.0085 \text{ N}$.

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085 \text{ N})(0.0500 \text{ m})}{8.90 \times 10^{-6} \text{ C}} = 47.8 \text{ V}.$$

EVALUATE: $E = V/d = 956 \text{ V/m}$. $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46 \times 10^{-9} \text{ C/m}^2$.

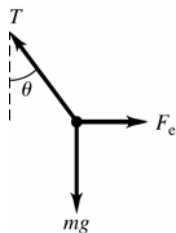


Figure 23.60

- 23.61. (a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.
SET UP: From Example 23.10, for a conducting cylinder with charge per unit length λ the potential outside the cylinder is given by $V = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$ where r is the distance from the cylinder axis and r_0 is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for V applies to both. This problem says to take $r_0 = b$.

EXECUTE: For the hollow tube of radius b and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi\epsilon_0)\ln(b/r)$; inside $V = 0$ since $V = 0$ at $r = b$.

For the metal cylinder of radius a and charge per unit length λ :

outside $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$, inside $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$, the value at $r = a$.

(i) $r < a$; inside both $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$

(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$

(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0$.

(b) For $r = a$, $V_a = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

For $r = b$, $V_b = 0$.

Thus $V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

(c) IDENTIFY and SET UP: Use Eq.(23.23) to calculate E .

EXECUTE: $E = -\frac{\partial V}{\partial r} = -\frac{\lambda}{2\pi\epsilon_0} \frac{\partial}{\partial r} \ln\left(\frac{b}{r}\right) = -\frac{\lambda}{2\pi\epsilon_0} \left(\frac{r}{b}\right) \left(-\frac{b}{r^2}\right) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$.

(d) The electric field between the cylinders is due only to the inner cylinder, so V_{ab} is not changed,

$V_{ab} = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

EVALUATE: The electric field is not uniform between the cylinders, so $V_{ab} \neq E(b-a)$.

- 23.62. IDENTIFY:** The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$.

SET UP: $a = 145 \times 10^{-6} \text{ m}$, $b = 0.0180 \text{ m}$.

EXECUTE: $E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$ and $V_{ab} = E \ln(b/a)r = (2.00 \times 10^4 \text{ N/C})(\ln(0.018 \text{ m}/145 \times 10^{-6} \text{ m}))(0.012 \text{ m}) = 1157 \text{ V}$.

EVALUATE: The electric field at any r is directly proportional to the potential difference between the wire and the cylinder.

- 23.63. IDENTIFY and SET UP:** Use Eq.(21.3) to calculate $\vec{F}$ and then $\vec{F} = m\vec{a}$ gives $\vec{a}$.

EXECUTE: (a) $\vec{F}_E = q\vec{E}$. Since $q = -e$ is negative $\vec{F}_E$ and $\vec{E}$ are in opposite directions; $\vec{E}$ is upward so $\vec{F}_E$ is downward. The magnitude of F_E is

$$F_E = |q|E = eE = (1.602 \times 10^{-19} \text{ C})(1.10 \times 10^3 \text{ N/C}) = 1.76 \times 10^{-16} \text{ N}.$$

(b) Calculate the acceleration of the electron produced by the electric force:

$$a = \frac{F}{m} = \frac{1.76 \times 10^{-16} \text{ N}}{9.109 \times 10^{-31} \text{ kg}} = 1.93 \times 10^{14} \text{ m/s}^2$$

EVALUATE: This is much larger than $g = 9.80 \text{ m/s}^2$, so the gravity force on the electron can be neglected. $\vec{F}_E$ is downward, so $\vec{a}$ is downward.

(c) IDENTIFY and SET UP: The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

EXECUTE: x-component

$$v_{0x} = 6.50 \times 10^6 \text{ m/s}; \quad a_x = 0; \quad x - x_0 = 0.060 \text{ m}; \quad t = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ and the } a_x \text{ term is zero, so}$$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.060 \text{ m}}{6.50 \times 10^6 \text{ m/s}} = 9.231 \times 10^{-9} \text{ s}$$

y-component

$$v_{0y} = 0; \quad a_y = 1.93 \times 10^{14} \text{ m/s}^2; \quad t = 9.231 \times 10^{-9} \text{ s}; \quad y - y_0 = ?$$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$y - y_0 = \frac{1}{2}(1.93 \times 10^{14} \text{ m/s}^2)(9.231 \times 10^{-9} \text{ s})^2 = 0.00822 \text{ m} = 0.822 \text{ cm}$$

(d) The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

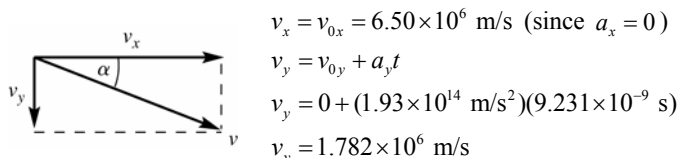


Figure 23.63

$$\tan \alpha = \frac{v_y}{v_x} = \frac{1.782 \times 10^6 \text{ m/s}}{6.50 \times 10^6 \text{ m/s}} = 0.2742 \text{ so } \alpha = 15.3^\circ.$$

EVALUATE: The greater the electric field or the smaller the initial speed the greater the downward deflection.

(e) **IDENTIFY and SET UP:** Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

EXECUTE: x-component

$$v_{0x} = 6.50 \times 10^6 \text{ m/s}; \quad a_x = 0; \quad x - x_0 = 0.120 \text{ m}; \quad t = ?$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ and the } a_x \text{ term is zero, so}$$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.120 \text{ m}}{6.50 \times 10^6 \text{ m/s}} = 1.846 \times 10^{-8} \text{ s}$$

y-component

$$v_{0y} = 1.782 \times 10^6 \text{ m/s (from part (b)); } a_y = 0; \quad t = 1.846 \times 10^{-8} \text{ s}; \quad y - y_0 = ?$$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782 \times 10^6 \text{ m/s})(1.846 \times 10^{-8} \text{ s}) = 0.0329 \text{ m} = 3.29 \text{ cm}$$

EVALUATE: The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is 0.822 cm + 3.29 cm = 4.11 cm.

The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

23.64. IDENTIFY: The charge on the plates and the electric field between them depend on the potential difference across the plates. Since we do not know the numerical potential, we shall call this potential V and find the answers in terms of V .

(a) **SET UP:** For two parallel plates, the potential difference between them is $V = Ed = \frac{\sigma}{\epsilon_0} d = \frac{Qd}{\epsilon_0 A}$.

EXECUTE: Solving for Q gives

$$Q = \epsilon_0 AV / d = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.030 \text{ m})^2 V / (0.0050 \text{ m})$$

$$Q = 1.59V \times 10^{-12} \text{ C} = 1.59V \text{ pC, when } V \text{ is in volts.}$$

(b) $E = V/d = V/(0.0050 \text{ m}) = 200V \text{ V/m, with } V \text{ in volts.}$

(c) **SET UP:** Energy conservation gives $\frac{1}{2}mv^2 = eV$.

EXECUTE: Solving for v gives

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})V}{9.11 \times 10^{-31} \text{ kg}}} = 5.93 \times 10^5 V^{1/2} \text{ m/s, with } V \text{ in volts}$$

EVALUATE: Typical voltages in student laboratory work run up to around 25 V, so the charge on the plates is typically about around 40 pC, the electric field is about 5000 V/m, and the electron speed would be about 3 million m/s.

- 23.65. (a) IDENTIFY and SET UP:** Problem 23.61 derived that $E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$, where a is the radius of the inner cylinder

(wire) and b is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab} .

Use this expression to calculate E at the specified r .

EXECUTE: Midway between the wire and the cylinder wall is at a radius of

$$r = (a + b)/2 = (90.0 \times 10^{-6} \text{ m} + 0.140 \text{ m})/2 = 0.07004 \text{ m}.$$

$$E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r} = \frac{50.0 \times 10^3 \text{ V}}{\ln(0.140 \text{ m}/90.0 \times 10^{-6} \text{ m})(0.07004 \text{ m})} = 9.71 \times 10^4 \text{ V/m}$$

(b) IDENTIFY and SET UP: The electric force is given by Eq.(21.3). Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0 \times 10^{-9} \text{ kg})(9.80 \text{ m/s}^2)}{9.71 \times 10^4 \text{ V/m}} = 3.03 \times 10^{-11} \text{ C}$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

- 23.66. (a) IDENTIFY:** Calculate the potential due to each thin ring and integrate over the disk to find the potential. V is a scalar so no components are involved.

SET UP: Consider a thin ring of radius y and width dy . The ring has area $2\pi y dy$ so the charge on the ring is $dq = \sigma(2\pi y dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance x from the ring is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2 + y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0} \frac{y dy}{\sqrt{x^2 + y^2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{y dy}{\sqrt{x^2 + y^2}} = \frac{\sigma}{2\epsilon_0} \left[\sqrt{x^2 + y^2} \right]_0^R = \frac{\sigma}{2\epsilon_0} (\sqrt{x^2 + R^2} - x)$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2 + R^2} = x(1 + R^2/x^2)^{1/2} \approx x(1 + R^2/2x^2) \text{ so } \sqrt{x^2 + R^2} - x \approx R^2/2x$$

$$\text{Then } V \approx \frac{\sigma}{2\epsilon_0} \frac{R^2}{2x} = \frac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \frac{Q}{4\pi\epsilon_0 x}, \text{ as expected.}$$

(b) IDENTIFY and SET UP: Use Eq.(23.19) to calculate E_x .

$$\text{EXECUTE: } E_x = -\frac{\partial V}{\partial x} = -\frac{\sigma}{2\epsilon_0} \left(\frac{x}{\sqrt{x^2 + R^2}} - 1 \right) = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right).$$

EVALUATE: Our result agrees with Eq.(21.11) in Example 21.12.

- 23.67. (a) IDENTIFY:** Use $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$.

SET UP: From Problem 22.48, $E(r) = \frac{\lambda r}{2\pi\epsilon_0 R^2}$ for $r \leq R$ (inside the cylindrical charge distribution) and

$$E(r) = \frac{\lambda R}{2\pi\epsilon_0 r} \text{ for } r \geq R. \text{ Let } V = 0 \text{ at } r = R \text{ (at the surface of the cylinder).}$$

EXECUTE: $r > R$

Take point a to be at R and point b to be at r , where $r > R$. Let $d\vec{l} = d\vec{r}$. $\vec{E}$ and $d\vec{r}$ are both radially outward, so

$\vec{E} \cdot d\vec{r} = E dr$. Thus $V_R - V_r = \int_R^r E dr$. Then $V_R = 0$ gives $V_r = -\int_R^r E dr$. In this interval ($r > R$), $E(r) = \lambda/2\pi\epsilon_0 r$, so

$$V_r = -\int_R^r \frac{\lambda}{2\pi\epsilon_0 r} dr = -\frac{\lambda}{2\pi\epsilon_0} \int_R^r \frac{dr}{r} = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{R}\right).$$

EVALUATE: This expression gives $V_r = 0$ when $r = R$ and the potential decreases (becomes a negative number of larger magnitude) with increasing distance from the cylinder.

EXECUTE: $r < R$

Take point a at r , where $r < R$, and point b at R . $\vec{E} \cdot d\vec{r} = E dr$ as before. Thus $V_r - V_R = \int_r^R E dr$. Then $V_R = 0$ gives $V_r = \int_r^R E dr$. In this interval ($r < R$), $E(r) = \lambda r / 2\pi\epsilon_0 R^2$, so

$$V_r = \int_r^R \frac{\lambda}{2\pi\epsilon_0 R^2} dr = \frac{\lambda}{2\pi\epsilon_0 R^2} \int_r^R r dr = \frac{\lambda}{2\pi\epsilon_0 R^2} \left(\frac{R^2}{2} - \frac{r^2}{2} \right).$$

$$V_r = \frac{\lambda}{4\pi\epsilon_0} \left(1 - \left(\frac{r}{R} \right)^2 \right).$$

EVALUATE: This expression also gives $V_r = 0$ when $r = R$. The potential is $\lambda / 4\pi\epsilon_0$ at $r = 0$ and decreases with increasing r .

(b) EXECUTE: Graphs of V and E as functions of r are sketched in Figure 23.67.

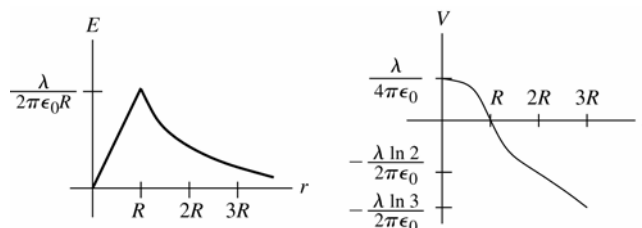


Figure 23.67

EVALUATE: E at any r is the negative of the slope of $V(r)$ at that r (Eq. 23.23).

23.68. IDENTIFY: The alpha particles start out with kinetic energy and wind up with electrical potential energy at closest approach to the nucleus.

SET UP: (a) The energy of the system is conserved, with $U = (1/4\pi\epsilon_0)(qq_0/r)$ being the electric potential energy. With the charge of the alpha particle being $2e$ and that of the gold nucleus being Ze , we have

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{R}$$

EXECUTE: Solving for v and using $Z = 79$ for gold gives

$$v = \sqrt{\left(\frac{1}{4\pi\epsilon_0} \right) \frac{4Ze^2}{mR}} = \sqrt{\frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4)(79)(1.60 \times 10^{-19} \text{ C})^2}{(6.7 \times 10^{-27} \text{ kg})(5.6 \times 10^{-15} \text{ m})}} = 4.4 \times 10^7 \text{ m/s}$$

We have neglected any relativistic effects.

(b) Outside the atom, it is neutral. Inside the atom, we can model the 79 electrons as a uniform spherical shell, which produces no electric field inside of itself, so the only electric field is that of the nucleus.

EVALUATE: Neglecting relativistic effects was not such a good idea since the speed in part (a) is over 10% the speed of light. Modeling 79 electrons as a uniform spherical shell is reasonable, but we would not want to do this with small atoms.

23.69. IDENTIFY: $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$.

SET UP: From Example 21.10, we have: $E_x = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}$. $\vec{E} \cdot d\vec{l} = E_x dx$. Let $a = \infty$ so $V_a = 0$.

EXECUTE: $V = -\frac{Q}{4\pi\epsilon_0} \int_{\infty}^x \frac{x'}{(x'^2 + a^2)^{3/2}} dx' = \frac{Q}{4\pi\epsilon_0} u^{-1/2} \bigg|_{u=\infty}^{u=x^2+a^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$.

EVALUATE: Our result agrees with Eq. (23.16) in Example 23.11.

23.70. IDENTIFY: Divide the rod into infinitesimal segments with charge dq . The potential dV due to the segment is

$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$. Integrate over the rod to find the total potential.

SET UP: $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a d\theta$.

EXECUTE: $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{a} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi a} \frac{dl}{a} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi a} \frac{d\theta}{\pi a}$. $V = \frac{1}{4\pi\epsilon_0} \int_0^\pi \frac{Q}{\pi a} \frac{d\theta}{\pi a} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a}$.

EVALUATE: All the charge of the ring is the same distance a from the center of curvature.

- 23.71. IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell of radius r and thickness dr is $dq = \rho 4\pi r^2 dr$, and $\rho = Q/(4/3 \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3 \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq , where V is the potential due to q , which is $q/4\pi\epsilon_0 r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int Vdq = \int \frac{q}{4\pi\epsilon_0 r} dq = \int_0^R \frac{\rho \frac{4}{3} \pi r^3}{4\pi\epsilon_0 r} (\rho 4\pi r^2 dr) = \frac{3}{5} \left(\frac{1}{4\pi\epsilon_0} \frac{Q^2}{R} \right)$$

where we have substituted $\rho = Q/(4/3 \pi R^3)$ and simplified the result.

EVALUATE: For a point-charge, $R \rightarrow 0$ so $U \rightarrow \infty$, which means that a point-charge should have infinite self-energy. This suggests that either point-charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

- 23.72. IDENTIFY:** $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$. The electric field is radially outward, so $\vec{E} \cdot d\vec{l} = E dr$.

SET UP: Let $a = \infty$, so $V_a = 0$.

EXECUTE: From Example 22.9, we have the following. For $r > R$: $E = \frac{kQ}{r^2}$ and $V = -kQ \int_{\infty}^r \frac{dr'}{r'^2} = \frac{kQ}{r}$.

For $r < R$: $E = \frac{kQr}{R^3}$ and $V = -\int_{\infty}^R \vec{E} \cdot d\vec{r}' - \int_R^r \vec{E} \cdot d\vec{r}' = \frac{kQ}{R} - \frac{kQ}{R^3} \int_R^r r' dr' = \frac{kQ}{R} - \frac{kQ}{R^3} \frac{1}{2} r'^2 \Big|_R^r = \frac{kQ}{R} + \frac{kQ}{2R} - \frac{kQr^2}{2R^3} = \frac{kQ}{2R} \left[3 - \frac{r^2}{R^2} \right]$.

(b) The graphs of V and E versus r are sketched in Figure 23.72.

EVALUATE: For $r < R$ the potential depends on the electric field in the region r to ∞ .

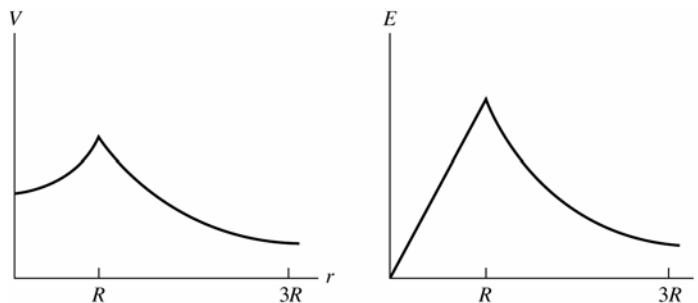


Figure 23.72

- 23.73. IDENTIFY:** Problem 23.70 shows that $V_r = \frac{Q}{8\pi\epsilon_0 R} (3 - r^2/R^2)$ for $r \leq R$ and $V_r = \frac{Q}{4\pi\epsilon_0 r}$ for $r \geq R$.

SET UP: $V_0 = \frac{3Q}{8\pi\epsilon_0 R}$, $V_R = \frac{Q}{4\pi\epsilon_0 R}$

EXECUTE: **(a)** $V_0 - V_R = \frac{Q}{8\pi\epsilon_0 R}$

(b) If $Q > 0$, V is higher at the center. If $Q < 0$, V is higher at the surface.

EVALUATE: For $Q > 0$ the electric field is radially outward, $\vec{E}$ is directed toward lower potential, so V is higher at the center. If $Q < 0$, the electric field is directed radially inward and V is higher at the surface.

- 23.74. IDENTIFY:** For $r < c$, $E = 0$ and the potential is constant. For $r > c$, E is the same as for a point charge and $V = \frac{kq}{r}$.

SET UP: $V_\infty = 0$

EXECUTE: **(a)** Points a , b , and c are all at the same potential, so $V_a - V_b = V_b - V_c = V_a - V_c = 0$.

$$V_c - V_\infty = \frac{kq}{R} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(150 \times 10^{-6} \text{ C})}{0.60 \text{ m}} = 2.25 \times 10^6 \text{ V}$$

(b) They are all at the same potential.

(c) Only $V_c - V_\infty$ would change; it would be $-2.25 \times 10^6 \text{ V}$.

EVALUATE: The voltmeter reads the potential difference between the two points to which it is connected.

23.75. IDENTIFY and SET UP: Apply $F_r = -dU/dr$ and Newton's third law.

EXECUTE: (a) The electrical potential energy for a spherical shell with uniform surface charge density and a point charge q outside the shell is the same as if the shell is replaced by a point charge at its center. Since $F_r = -dU/dr$, this means the force the shell exerts on the point charge is the same as if the shell were replaced by a point charge at its center. But by Newton's 3rd law, the force q exerts on the shell is the same as if the shell were a point charge. But q can be replaced by a spherical shell with uniform surface charge and the force is the same, so the force between the shells is the same as if they were both replaced by point charges at their centers. And since the force is the same as for point charges, the electrical potential energy for the pair of spheres is the same as for a pair of point charges.

(b) The potential for solid insulating spheres with uniform charge density is the same outside of the sphere as for a spherical shell, so the same result holds.

(c) The result doesn't hold for conducting spheres or shells because when two charged conductors are brought close together, the forces between them causes the charges to redistribute and the charges are no longer distributed uniformly over the surfaces.

EVALUATE: For the insulating shells or spheres, $F = k \frac{|q_1 q_2|}{r^2}$ and $U = \frac{kq_1 q_2}{r}$, where q_1 and q_2 are the charges of the objects and r is the distance between their centers.

23.76. IDENTIFY: Apply Newton's second law to calculate the acceleration. Apply conservation of energy and conservation of momentum to the motions of the spheres.

SET UP: Problem 23.75 shows that $F = k \frac{|q_1 q_2|}{r^2}$ and $U = \frac{kq_1 q_2}{r}$, where q_1 and q_2 are the charges of the objects and r is the distance between their centers.

EXECUTE: Maximum speed occurs when the spheres are very far apart. Energy conservation gives

$$\frac{kq_1 q_2}{r} = \frac{1}{2} m_{50} v_{50}^2 + \frac{1}{2} m_{150} v_{150}^2. \text{ Momentum conservation gives } m_{50} v_{50} = m_{150} v_{150} \text{ and } v_{50} = 3v_{150}. r = 0.50 \text{ m. Solve for } v_{50}$$

and v_{150} : $v_{50} = 12.7 \text{ m/s}$, $v_{150} = 4.24 \text{ m/s}$. Maximum acceleration occurs just after spheres are released. $\Sigma F = ma$

$$\text{gives } \frac{kq_1 q_2}{r^2} = m_{150} a_{150}. \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10^{-5} \text{ C})(3 \times 10^{-5} \text{ C})}{(0.50 \text{ m})^2} = (0.15 \text{ kg}) a_{150}. a_{150} = 72.0 \text{ m/s}^2 \text{ and}$$

$$a_{50} = 3a_{150} = 216 \text{ m/s}^2.$$

EVALUATE: The more massive sphere has a smaller acceleration and a smaller final speed.

23.77. IDENTIFY: Use Eq.(23.17) to calculate V_{ab} .

SET UP: From Problem 22.43, for $R \leq r \leq 2R$ (between the sphere and the shell) $E = Q/4\pi\epsilon_0 r^2$. Take a at R and b at $2R$.

$$\begin{aligned} \text{EXECUTE: } V_{ab} &= V_a - V_b = \int_R^{2R} E dr = \frac{Q}{4\pi\epsilon_0} \int_R^{2R} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_R^{2R} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{2R} \right) \\ V_{ab} &= \frac{Q}{8\pi\epsilon_0 R} \end{aligned}$$

EVALUATE: The electric field is radially outward and points in the direction of decreasing potential, so the sphere is at higher potential than the shell.

23.78. IDENTIFY: $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$

SET UP: $\vec{E}$ is radially outward, so $\vec{E} \cdot d\vec{l} = E dr$. Problem 22.42 shows that $E(r) = 0$ for $r \leq a$, $E(r) = kq/r^2$ for $a < r < b$, $E(r) = 0$ for $b < r < c$ and $E(r) = kq/r^2$ for $r > c$.

$$\text{EXECUTE: (a) At } r = c: V_c = -\int_{\infty}^c \frac{kq}{r^2} dr = \frac{kq}{c}.$$

$$\text{(b) At } r = b: V_b = -\int_{\infty}^b \vec{E} \cdot d\vec{r} = -\int_c^b \vec{E} \cdot d\vec{r} = \frac{kq}{c} - 0 = \frac{kq}{c}.$$

$$\text{(c) At } r = a: V_a = -\int_{\infty}^a \vec{E} \cdot d\vec{r} = -\int_c^a \vec{E} \cdot d\vec{r} - \int_b^a \vec{E} \cdot d\vec{r} = \frac{kq}{c} - kq \int_b^a \frac{dr}{r^2} = kq \left[\frac{1}{c} - \frac{1}{b} + \frac{1}{a} \right]$$

$$\text{(d) At } r = 0: V_0 = kq \left[\frac{1}{c} - \frac{1}{b} + \frac{1}{a} \right] \text{ since it is inside a metal sphere, and thus at the same potential as its surface.}$$

$$\text{EVALUATE: The potential difference between the two conductors is } V_a - V_b = kq \left[\frac{1}{a} - \frac{1}{b} \right].$$

23.79. IDENTIFY: Slice the rod into thin slices and use Eq.(23.14) to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point P are shown in Figure 23.79a.

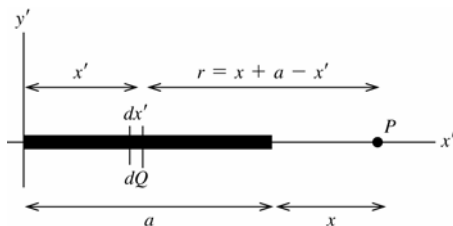


Figure 23.79a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance x given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx' . Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point P . The potential at P due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0} \left(\frac{dQ}{r} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left(\frac{dx'}{x + a - x'} \right).$$

Compute the total V at P due to the entire rod by integrating dV over the length of the rod ($x' = 0$ to $x' = a$):

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a} \int_0^a \frac{dx'}{(x + a - x')} = \frac{Q}{4\pi\epsilon_0 a} [-\ln(x + a - x')]_0^a = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \rightarrow \infty$, $V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x}{x}\right) = 0$.

(b) SET UP: An infinitesimal slice of the rod and its distance from point R are shown in Figure 23.79b.

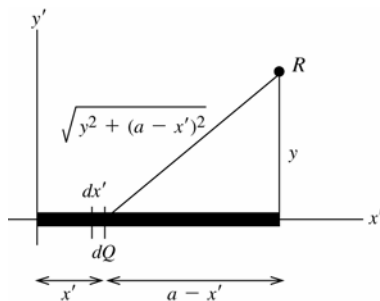


Figure 23.79b

$dQ = (Q/a)dx'$ as in part (a)

Each slice dQ is a distance $r = \sqrt{y^2 + (a - x')^2}$ from point R .

EXECUTE: The potential dV at R due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0} \left(\frac{dQ}{r} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a} \int_0^a \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

In the integral make the change of variable $u = a - x'$; $du = -dx'$

$$V = -\frac{Q}{4\pi\epsilon_0 a} \int_a^0 \frac{du}{\sqrt{y^2 + u^2}} = -\frac{Q}{4\pi\epsilon_0 a} \left[\ln(u + \sqrt{y^2 + u^2}) \right]_a^0$$

$$V = -\frac{Q}{4\pi\epsilon_0 a} [\ln y - \ln(a + \sqrt{y^2 + a^2})] = \frac{Q}{4\pi\epsilon_0 a} \left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right) \right].$$

(The expression for the integral was found in appendix B.)

EVALUATE: As $y \rightarrow \infty$, $V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{y}{y}\right) = 0$.

(c) SET UP: $part (a): V = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x+a}{x}\right) = \frac{Q}{4\pi\epsilon_0 a} \ln\left(1 + \frac{a}{x}\right).$

From Appendix B, $\ln(1+u) = u - u^2/2 \dots$, so $\ln(1+a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \left(\frac{a}{x}\right) = \frac{Q}{4\pi\epsilon_0 x}$. For large x , V becomes the potential of a point charge.

$part (b): V = \frac{Q}{4\pi\epsilon_0 a} \left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right) \right] = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{a}{y} + \sqrt{1 + \frac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1 + a^2/y^2} = (1 + a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \dots$

Thus $a/y + \sqrt{1 + a^2/y^2} \rightarrow 1 + a/y + a^2/2y^2 + \dots \rightarrow 1 + a/y$. And then using $\ln(1+u) \approx u$ gives

$$V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \ln(1 + a/y) \rightarrow \frac{Q}{4\pi\epsilon_0 a} \left(\frac{a}{y}\right) = \frac{Q}{4\pi\epsilon_0 y}.$$

EVALUATE: For large y , V becomes the potential of a point charge.

23.80. IDENTIFY: The potential at the surface of a uniformly charged sphere is $V = \frac{kQ}{R}$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume is conserved.

EXECUTE: (a) $V = \frac{kQ}{R} = \frac{k(-1.20 \times 10^{-12} \text{ C})}{6.50 \times 10^{-4} \text{ m}} = -16.6 \text{ V}.$

(b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2} R = 8.19 \times 10^{-4} \text{ m}$ and the new charge is $Q_{\text{new}} = 2Q = -2.40 \times 10^{-12} \text{ C}$. The new potential is $V_{\text{new}} = \frac{kQ_{\text{new}}}{R_{\text{new}}} = \frac{k(-2.40 \times 10^{-12} \text{ C})}{8.19 \times 10^{-4} \text{ m}} = -26.4 \text{ V}.$

EVALUATE: The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\frac{2}{2^{1/3}} = 2^{2/3}$.

23.81. (a) IDENTIFY and SET UP: The potential at the surface of a charged conducting sphere is given by Example 23.8:

$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}.$ For spheres A and B this gives

$$V_A = \frac{Q_A}{4\pi\epsilon_0 R_A} \text{ and } V_B = \frac{Q_B}{4\pi\epsilon_0 R_B}.$$

EXECUTE: $V_A = V_B$ gives $Q_A/4\pi\epsilon_0 R_A = Q_B/4\pi\epsilon_0 R_B$ and $Q_B/Q_A = R_B/R_A$. And then $R_A = 3R_B$ implies $Q_B/Q_A = 1/3$.

(b) IDENTIFY and SET UP: The electric field at the surface of a charged conducting sphere is given in Example 22.5:

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{R^2}.$$

EXECUTE: For spheres A and B this gives

$$E_A = \frac{|Q_A|}{4\pi\epsilon_0 R_A^2} \text{ and } E_B = \frac{|Q_B|}{4\pi\epsilon_0 R_B^2}.$$

$$\frac{E_B}{E_A} = \left(\frac{|Q_B|}{4\pi\epsilon_0 R_B^2} \right) \left(\frac{4\pi\epsilon_0 R_A^2}{|Q_A|} \right) = |Q_B/Q_A| (R_A/R_B)^2 = (1/3)(3)^2 = 3.$$

EVALUATE: The sphere with the larger radius needs more net charge to produce the same potential. We can write $E = V/R$ for a sphere, so with equal potentials the sphere with the smaller R has the larger V .

23.82. IDENTIFY: Apply conservation of energy, $K_a + U_a = K_b + U_b$.

SET UP: Assume the particles initially are far apart, so $U_a = 0$. The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

EXECUTE: Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$$11.0 \text{ MeV} = \frac{k(2e)(82e)}{r} \quad \text{and} \quad r = \frac{k(164)(1.60 \times 10^{-19} \text{ C})^2}{(11.0 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 2.15 \times 10^{-14} \text{ m}.$$

EVALUATE: The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

23.83. IDENTIFY and SET UP: The potential at the surface is given by Example 23.8 and the electric field at the surface is given by Example 22.5. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

EXECUTE: (a) $E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_1^2}$, $V_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_1} = R_1 E_1$

(b) Two conditions must be met:

1) Let q_1 and q_2 be the final potentials of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation)

2) Let V_1 and V_2 be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$

$$q_1 R_2 = q_2 R_1 = (Q_1 - q_1) R_1$$

This gives $q_1 = (R_1/[R_1 + R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1 + R_2]) = Q_1(R_2/[R_1 + R_2])$

(c) $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1} = \frac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$ and $V_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{R_2} = \frac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$, which equals V_1 as it should.

(d) $E_1 = \frac{V_1}{R_1} = \frac{Q_1}{4\pi\epsilon_0 R_1(R_1 + R_2)}$, $E_2 = \frac{V_2}{R_2} = \frac{Q_1}{4\pi\epsilon_0 R_2(R_1 + R_2)}$.

EVALUATE: Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

23.84. IDENTIFY: Apply $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$

SET UP: From Problem 22.57, for $r \geq R$, $E = \frac{kQ}{r^2}$. For $r \leq R$, $E = \frac{kQ}{r^2} \left[4 \frac{r^3}{R^3} - 3 \frac{r^4}{R^4} \right]$.

EXECUTE: (a) $r \geq R$: $E = \frac{kQ}{r^2} \Rightarrow V = -\int_{\infty}^r \frac{kQ}{r'^2} dr' = \frac{kQ}{r}$, which is the potential of a point charge.

(b) $r \leq R$: $E = \frac{kQ}{r^2} \left[4 \frac{r^3}{R^3} - 3 \frac{r^4}{R^4} \right]$ and $V = -\int_{\infty}^R E dr' - \int_R^r E dr' = \frac{kQ}{R} \left[1 - 2 \frac{r^2}{R^2} + 2 \frac{R^2}{R^2} + \frac{r^3}{R^3} - \frac{R^3}{R^3} \right] = \frac{kQ}{R} \left[\frac{r^3}{R^3} - 2 \frac{r^2}{R^2} + 2 \right]$.

EVALUATE: At $r = R$, $V = \frac{kQ}{R}$. At $r = 0$, $V = \frac{2kQ}{R}$. The electric field is radially outward and V increases as r decreases.

23.85. IDENTIFY: Apply conservation of energy: $E_i = E_f$.

SET UP: In the collision the initial kinetic energy of the two particles is converted into potential energy at the distance of closest approach.

EXECUTE: (a) The two protons must approach to a distance of $2r_p$, where r_p is the radius of a proton.

$E_i = E_f$ gives $2 \left[\frac{1}{2} m_p v^2 \right] = \frac{ke^2}{2r_p}$ and $v = \sqrt{\frac{k(1.60 \times 10^{-19} \text{ C})^2}{2(1.2 \times 10^{-15} \text{ m})(1.67 \times 10^{-27} \text{ kg})}} = 7.58 \times 10^6 \text{ m/s}$.

(b) For a helium-helium collision, the charges and masses change from (a) and

$v = \sqrt{\frac{k(2(1.60 \times 10^{-19} \text{ C}))^2}{3(3.5 \times 10^{-15} \text{ m})(2.99)(1.67 \times 10^{-27} \text{ kg})}} = 7.26 \times 10^6 \text{ m/s}$.

(c) $K = \frac{3kT}{2} = \frac{mv^2}{2}$. $T_p = \frac{m_p v^2}{3k} = \frac{(1.67 \times 10^{-27} \text{ kg})(7.58 \times 10^6 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 2.3 \times 10^9 \text{ K}$.

$$T_{\text{He}} = \frac{m_{\text{He}} v^2}{3k} = \frac{(2.99)(1.67 \times 10^{-27} \text{ kg})(7.26 \times 10^6 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 6.4 \times 10^9 \text{ K}.$$

(d) These calculations were based on the particles' average speed. The distribution of speeds ensures that there are always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

EVALUATE: The kinetic energies required for fusion correspond to very high temperatures.

23.86. IDENTIFY and SET UP: Apply Eq.(23.20). $\frac{W_{a \rightarrow b}}{q_0} = V_a - V_b$ and $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$.

EXECUTE: (a) $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -2Ax\hat{i} + 6Ay\hat{j} - 2Az\hat{k}$

(b) A charge is moved in along the z -axis. The work done is given by $W = q \int_{z_0}^0 \vec{E} \cdot \hat{k} dz = q \int_{z_0}^0 (-2Az) dz = +(Aq)z_0^2$.

Therefore, $A = \frac{W_{a \rightarrow b}}{qz_0^2} = \frac{6.00 \times 10^{-5} \text{ J}}{(1.5 \times 10^{-6} \text{ C})(0.250 \text{ m})^2} = 640 \text{ V/m}^2$.

(c) $\vec{E}(0,0,0.250) = -2(640 \text{ V/m}^2)(0.250 \text{ m})\hat{k} = -(320 \text{ V/m})\hat{k}$.

(d) In every plane parallel to the xz -plane, y is constant, so $V(x,y,z) = Ax^2 + Az^2 - C$, where $C = 3Ay^2$.

$x^2 + z^2 = \frac{V+C}{A} = R^2$, which is the equation for a circle since R is constant as long as we have constant potential on those planes.

(e) $V = 1280 \text{ V}$ and $y = 2.00 \text{ m}$, so $x^2 + z^2 = \frac{1280 \text{ V} + 3(640 \text{ V/m}^2)(2.00 \text{ m})^2}{640 \text{ V/m}^2} = 14.0 \text{ m}^2$ and the radius of the circle is 3.74 m .

EVALUATE: In any plane parallel to the xz -plane, $\vec{E}$ projected onto the plane is radial and hence perpendicular to the equipotential circles.

23.87. IDENTIFY: Apply conservation of energy to the motion of the daughter nuclei.

SET UP: Problem 23.73 shows that the electrical potential energy of the two nuclei is the same as if all their charge was concentrated at their centers.

EXECUTE: (a) The two daughter nuclei have half the volume of the original uranium nucleus, so their radii are smaller by a factor of the cube root of 2: $r = \frac{7.4 \times 10^{-15} \text{ m}}{\sqrt[3]{2}} = 5.9 \times 10^{-15} \text{ m}$.

(b) $U = \frac{k(46e)^2}{2r} = \frac{k(46)^2(1.60 \times 10^{-19} \text{ C})^2}{1.18 \times 10^{-14} \text{ m}} = 4.14 \times 10^{-11} \text{ J}$. $U = 2K$, where K is the final kinetic energy of each nucleus. $K = U/2 = (4.14 \times 10^{-11} \text{ J})/2 = 2.07 \times 10^{-11} \text{ J}$.

(c) If we have 10.0 kg of uranium, then the number of nuclei is $n = \frac{10.0 \text{ kg}}{(236 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 2.55 \times 10^{25}$ nuclei.

And each releases energy U , so $E = nU = (2.55 \times 10^{25})(4.14 \times 10^{-11} \text{ J}) = 1.06 \times 10^{15} \text{ J} = 253 \text{ kilotons of TNT}$.

(d) We could call an atomic bomb an "electric" bomb since the electric potential energy provides the kinetic energy of the particles.

EVALUATE: This simple model considers only the electrical force between the daughter nuclei and neglects the nuclear force.

23.88. IDENTIFY and SET UP: In part (a) apply $E = -\frac{\partial V}{\partial r}$. In part (b) apply Gauss's law.

EXECUTE: (a) For $r \leq a$, $E = -\frac{\partial V}{\partial r} = -\frac{\rho_0 a^2}{18\epsilon_0} \left[-6\frac{r}{a^2} + 6\frac{r^2}{a^3} \right] = \frac{\rho_0 a}{3\epsilon_0} \left[\frac{r}{a} - \frac{r^2}{a^2} \right]$. For $r \geq a$, $E = -\frac{\partial V}{\partial r} = 0$. $\vec{E}$ has only a radial component because V depends only on r .

(b) For $r \leq a$, Gauss's law gives $E_r 4\pi r^2 = \frac{Q_r}{\epsilon_0} = \frac{\rho_0 a}{3\epsilon_0} \left[\frac{r}{a} - \frac{r^2}{a^2} \right] 4\pi r^2$ and

$E_{r+dr} 4\pi(r^2 + 2rdr) = \frac{Q_{r+dr}}{\epsilon_0} = \frac{\rho_0 a}{3\epsilon_0} \left[\frac{r+dr}{a} - \frac{(r^2 + 2rdr)}{a^2} \right] 4\pi(r^2 + 2rdr)$. Therefore,

$\frac{Q_{r+dr} - Q_r}{\epsilon_0} = \frac{\rho(r) 4\pi r^2 dr}{\epsilon_0} \approx \frac{\rho_0 a 4\pi r^2 dr}{3\epsilon_0} \left[-\frac{2r}{a^2} + \frac{2}{a} - \frac{2r}{a^2} + \frac{1}{a} \right]$ and $\rho(r) = \frac{\rho_0}{3} \left[3 - \frac{4r}{a} \right] = \rho_0 \left[1 - \frac{4r}{3a} \right]$.

(c) For $r \geq a$, $\rho(r) = 0$, so the total charge enclosed will be given by

$$Q = 4\pi \int_0^a \rho(r) r^2 dr = 4\pi \rho_0 \int_0^a \left[r^2 - \frac{4r^3}{3a} \right] dr = 4\pi \rho_0 \left[\frac{1}{3} r^3 - \frac{r^4}{4a} \right]_0^a = 0.$$

EVALUATE: Apply Gauss's law to a sphere of radius $r > R$. The result of part (c) says that $Q_{\text{encl}} = 0$, so $E = 0$.

This agrees with the result we calculated in part (a).

23.89. IDENTIFY: Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

EXECUTE: $mv_1 b = mv_2 r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2} m v_2^2 + \frac{k q_1 q_2}{r_2}$. $E_1 = 11 \text{ MeV} = 1.76 \times 10^{-12} \text{ J}$. r_2 is the distance of

closest approach. Substituting in for $v_2 = v_1 \left(\frac{b}{r_2} \right)$ we find $E_1 = E_1 \frac{b^2}{r_2^2} + \frac{k q_1 q_2}{r_2}$.

$(E_1) r_2^2 - (k q_1 q_2) r_2 - E_1 b^2 = 0$. For $b = 10^{-12} \text{ m}$, $r_2 = 1.01 \times 10^{-12} \text{ m}$. For $b = 10^{-13} \text{ m}$, $r_2 = 1.11 \times 10^{-13} \text{ m}$. And for $b = 10^{-14} \text{ m}$, $r_2 = 2.54 \times 10^{-14} \text{ m}$.

EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases.

Problem 23.82 shows that the distance of closest approach is $2.15 \times 10^{-14} \text{ m}$ when $b = 0$.

23.90. IDENTIFY: Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\frac{\partial V}{\partial x}$.

SET UP: Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate z along the x -axis, relative to the center of the tube.

EXECUTE: (a) For an infinitesimal slice of the finite cylinder, we have the potential

$$dV = \frac{k dQ}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L} \frac{dz}{\sqrt{(x-z)^2 + R^2}}. \text{ Integrating gives}$$

$$V = \frac{kQ}{L} \int_{-L/2}^{L/2} \frac{dz}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L} \int_{-L/2-x}^{L/2-x} \frac{du}{\sqrt{u^2 + R^2}} \text{ where } u = x - z. \text{ Therefore,}$$

$$V = \frac{kQ}{L} \ln \left[\frac{\sqrt{(L/2-x)^2 + R^2} + (L/2-x)}{\sqrt{(L/2+x)^2 + R^2} - L/2-x} \right] \text{ on the cylinder axis.}$$

$$(b) \text{ For } L \ll R, V \approx \frac{kQ}{L} \ln \left[\frac{\sqrt{(L/2-x)^2 + R^2} + L/2-x}{\sqrt{(L/2+x)^2 + R^2} - L/2-x} \right] \approx \frac{kQ}{L} \ln \left[\frac{\sqrt{x^2 - xL + R^2} + L/2-x}{\sqrt{x^2 + xL + R^2} - L/2-x} \right].$$

$$V \approx \frac{kQ}{L} \ln \left[\frac{\sqrt{1 - xL/(R^2 + x^2)} + (L/2-x)/\sqrt{R^2 + x^2}}{\sqrt{1 + xL/(R^2 + x^2)} + (-L/2-x)/\sqrt{R^2 + x^2}} \right] = \frac{kQ}{L} \ln \left[\frac{1 - xL/2(R^2 + x^2) + (L/2-x)/\sqrt{R^2 + x^2}}{1 + xL/2(R^2 + x^2) + (-L/2-x)/\sqrt{R^2 + x^2}} \right].$$

$$V \approx \frac{kQ}{L} \ln \left[\frac{1 + L/2\sqrt{R^2 + x^2}}{1 - L/2\sqrt{R^2 + x^2}} \right] = \frac{kQ}{L} \left(\ln \left[1 + \frac{L}{2\sqrt{R^2 + x^2}} \right] - \ln \left[1 - \frac{L}{2\sqrt{R^2 + x^2}} \right] \right).$$

$$V \approx \frac{kQ}{L} \frac{2L}{2\sqrt{x^2 + R^2}} = \frac{kQ}{\sqrt{x^2 + R^2}}, \text{ which is the same as for a ring.}$$

$$(c) E_x = -\frac{\partial V}{\partial x} = \frac{2kQ \left(\sqrt{(L-2x)^2 + 4R^2} - \sqrt{(L+2x)^2 + 4R^2} \right)}{\sqrt{(L-2x)^2 + 4R^2} \sqrt{(L+2x)^2 + 4R^2}}$$

EVALUATE: For $L \ll R$ the expression for E_x reduces to that for a ring of charge, as given in Example 23.14.

23.91. IDENTIFY: When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

SET UP: The volume of the drop is related to its radius r by $V = \frac{4}{3} \pi r^3$.

EXECUTE: (a) $F_g = mg = \frac{4\pi r^3}{3} \rho g$. $F_e = |q|E = |q|V_{AB}/d$. $F_e = F_g$ gives $|q| = \frac{4\pi \rho r^3 g d}{3 V_{AB}}$.

(b) $\frac{4\pi r^3}{3} \rho g = 6\pi \eta r v_t$ gives $r = \sqrt{\frac{9\eta v_t}{2\rho g}}$. Using this result to replace r in the expression in part (a) gives

$$|q| = \frac{4\pi}{3} \frac{\rho g d}{V_{AB}} \left[\sqrt{\frac{9\eta v_t}{2\rho g}} \right]^3 = 18\pi \frac{d}{V_{AB}} \sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

(c) $|q| = 18\pi \frac{10^{-3} \text{ m}}{9.16 \text{ V}} \sqrt{\frac{(1.81 \times 10^{-5} \text{ N} \cdot \text{s}/\text{m}^2)^3 (1.00 \times 10^{-3} \text{ m}/39.3 \text{ s})^3}{2(824 \text{ kg}/\text{m}^3)(9.80 \text{ m}/\text{s}^2)}} = 4.80 \times 10^{-19} \text{ C} = 3e$. The drop has acquired three excess electrons.

$$r = \sqrt{\frac{9(1.81 \times 10^{-5} \text{ N} \cdot \text{s}/\text{m}^2)(1.00 \times 10^{-3} \text{ m}/39.3 \text{ s})}{2(824 \text{ kg}/\text{m}^3)(9.80 \text{ m}/\text{s}^2)}} = 5.07 \times 10^{-7} \text{ m} = 0.507 \text{ } \mu\text{m}.$$

EVALUATE: The weight of the drop is $\left(\frac{4\pi r^3}{3}\right) \rho g = 4.4 \times 10^{-15} \text{ N}$. The density of air at room temperature is $1.2 \text{ kg}/\text{m}^3$, so the buoyancy force is $\rho_{\text{air}} V g = 6.4 \times 10^{-18} \text{ N}$ and can be neglected.

23.92. IDENTIFY: $v_{\text{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

SET UP: $E = K_1 + K_2 + U$, where $U = \frac{kq_1 q_2}{r}$.

EXECUTE: (a) $v_{\text{cm}} = \frac{(6 \times 10^{-5} \text{ kg})(400 \text{ m/s}) + (3 \times 10^{-5} \text{ kg})(1300 \text{ m/s})}{6.0 \times 10^{-5} \text{ kg} + 3.0 \times 10^{-5} \text{ kg}} = 700 \text{ m/s}$

(b) $E_{\text{rel}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{kq_1 q_2}{r} - \frac{1}{2} (m_1 + m_2) v_{\text{cm}}^2$. After expanding the center of mass velocity and collecting like

terms $E_{\text{rel}} = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} [v_1^2 + v_2^2 - 2v_1 v_2] + \frac{kq_1 q_2}{r} = \frac{1}{2} \mu (v_1 - v_2)^2 + \frac{kq_1 q_2}{r}$.

(c) $E_{\text{rel}} = \frac{1}{2} (2.0 \times 10^{-5} \text{ kg})(900 \text{ m/s})^2 + \frac{k(2.0 \times 10^{-6} \text{ C})(-5.0 \times 10^{-6} \text{ C})}{0.0090 \text{ m}} = -1.9 \text{ J}$

(d) Since the energy is less than zero, the system is “bound.”

(e) The maximum separation is when the velocity is zero: $-1.9 \text{ J} = \frac{kq_1 q_2}{r}$ gives

$$r = \frac{k(2.0 \times 10^{-6} \text{ C})(-5.0 \times 10^{-6} \text{ C})}{-1.9 \text{ J}} = 0.047 \text{ m}.$$

(f) Now using $v_1 = 400 \text{ m/s}$ and $v_2 = 1800 \text{ m/s}$, we find $E_{\text{rel}} = +9.6 \text{ J}$. The particles do escape, and the final relative velocity is $|v_1 - v_2| = \sqrt{\frac{2E_{\text{rel}}}{\mu}} = \sqrt{\frac{2(9.6 \text{ J})}{2.0 \times 10^{-5} \text{ kg}}} = 980 \text{ m/s}$.

EVALUATE: For an isolated system the velocity of the center of mass is constant and the system must retain the kinetic energy associated with the motion of the center of mass.

CAPACITANCE AND DIELECTRICS

24.1. IDENTIFY: $C = \frac{Q}{V_{ab}}$

SET UP: $1 \mu\text{F} = 10^{-6} \text{ F}$

EXECUTE: $Q = CV_{ab} = (7.28 \times 10^{-6} \text{ F})(25.0 \text{ V}) = 1.82 \times 10^{-4} \text{ C} = 182 \mu\text{C}$

EVALUATE: One plate has charge $+Q$ and the other has charge $-Q$.

24.2. IDENTIFY and SET UP: $C = \frac{\epsilon_0 A}{d}$, $C = \frac{Q}{V}$ and $V = Ed$.

(a) $C = \epsilon_0 \frac{A}{d} = \epsilon_0 \frac{0.00122 \text{ m}^2}{0.00328 \text{ m}} = 3.29 \text{ pF}$

(b) $V = \frac{Q}{C} = \frac{4.35 \times 10^{-8} \text{ C}}{3.29 \times 10^{-12} \text{ F}} = 13.2 \text{ kV}$

(c) $E = \frac{V}{d} = \frac{13.2 \times 10^3 \text{ V}}{0.00328 \text{ m}} = 4.02 \times 10^6 \text{ V/m}$

EVALUATE: The electric field is uniform between the plates, at points that aren't close to the edges.

24.3. IDENTIFY and SET UP: It is a parallel-plate air capacitor, so we can apply the equations of Sections 24.1.

EXECUTE: (a) $C = \frac{Q}{V_{ab}}$ so $V_{ab} = \frac{Q}{C} = \frac{0.148 \times 10^{-6} \text{ C}}{245 \times 10^{-12} \text{ F}} = 604 \text{ V}$

(b) $C = \frac{\epsilon_0 A}{d}$ so $A = \frac{Cd}{\epsilon_0} = \frac{(245 \times 10^{-12} \text{ F})(0.328 \times 10^{-3} \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 9.08 \times 10^{-3} \text{ m}^2 = 90.8 \text{ cm}^2$

(c) $V_{ab} = Ed$ so $E = \frac{V_{ab}}{d} = \frac{604 \text{ V}}{0.328 \times 10^{-3} \text{ m}} = 1.84 \times 10^6 \text{ V/m}$

(d) $E = \frac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84 \times 10^6 \text{ V/m})(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 1.63 \times 10^{-5} \text{ C/m}^2$

EVALUATE: We could also calculate σ directly as Q/A . $\sigma = \frac{Q}{A} = \frac{0.148 \times 10^{-6} \text{ C}}{9.08 \times 10^{-3} \text{ m}^2} = 1.63 \times 10^{-5} \text{ C/m}^2$, which checks.

24.4. IDENTIFY: $C = \epsilon_0 \frac{A}{d}$ when there is air between the plates.

SET UP: $A = (3.0 \times 10^{-2} \text{ m})^2$ is the area of each plate.

EXECUTE: $C = \frac{(8.854 \times 10^{-12} \text{ F/m})(3.0 \times 10^{-2} \text{ m})^2}{5.0 \times 10^{-3} \text{ m}} = 1.59 \times 10^{-12} \text{ F} = 1.59 \text{ pF}$

EVALUATE: C increases when A increases and C increases when d decreases.

24.5. IDENTIFY: $C = \frac{Q}{V_{ab}}$. $C = \frac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0 \text{ V}$.

EXECUTE: (a) $Q = CV_{ab} = (10.0 \times 10^{-6} \text{ F})(12.0 \text{ V}) = 1.20 \times 10^{-4} \text{ C} = 120 \mu\text{C}$

(b) When d is doubled C is halved, so Q is halved. $Q = 60 \mu\text{C}$.

(c) If r is doubled, A increases by a factor of 4. C increases by a factor of 4 and Q increases by a factor of 4. $Q = 480 \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ . To produce the same σ , more charge is required when the area increases.

24.6. IDENTIFY: $C = \frac{Q}{V_{ab}}$. $C = \frac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab} = 12.0$ V.

EXECUTE: (a) 12.0 V

(b) (i) When d is doubled, C is halved. $V_{ab} = \frac{Q}{C}$ and Q is constant, so V doubles. $V = 24.0$ V.

(ii) When r is doubled, A increases by a factor of 4. V decreases by a factor of 4 and $V = 3.0$ V.

EVALUATE: The electric field between the plates is $E = Q/\epsilon_0 A$. $V_{ab} = Ed$. When d is doubled E is unchanged and V doubles. When A is increased by a factor of 4, E decreases by a factor of 4 so V decreases by a factor of 4.

24.7. IDENTIFY: $C = \frac{\epsilon_0 A}{d}$. Solve for d .

SET UP: Estimate $r = 1.0$ cm. $A = \pi r^2$.

EXECUTE: $C = \frac{\epsilon_0 A}{d}$ so $d = \frac{\epsilon_0 \pi r^2}{C} = \frac{\epsilon_0 \pi (0.010 \text{ m})^2}{1.00 \times 10^{-12} \text{ F}} = 2.8 \text{ mm}$.

EVALUATE: The separation between the pennies is nearly a factor of 10 smaller than the diameter of a penny, so it is a reasonable approximation to treat them as infinite sheets.

24.8. INCREASE: $C = \frac{Q}{V_{ab}}$. $V_{ab} = Ed$. $C = \frac{\epsilon_0 A}{d}$.

SET UP: We want $E = 1.00 \times 10^4$ N/C when $V = 100$ V.

EXECUTE: (a) $d = \frac{V_{ab}}{E} = \frac{1.00 \times 10^2 \text{ V}}{1.00 \times 10^4 \text{ N/C}} = 1.00 \times 10^{-2} \text{ m} = 1.00 \text{ cm}$.

$A = \frac{Cd}{\epsilon_0} = \frac{(5.00 \times 10^{-12} \text{ F})(1.00 \times 10^{-2} \text{ m})}{8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 5.65 \times 10^{-3} \text{ m}^2$. $A = \pi r^2$ so $r = \sqrt{\frac{A}{\pi}} = 4.24 \times 10^{-2} \text{ m} = 4.24 \text{ cm}$.

(b) $Q = CV_{ab} = (5.00 \times 10^{-12} \text{ F})(1.00 \times 10^2 \text{ V}) = 5.00 \times 10^{-10} \text{ C} = 500 \text{ pC}$

EVALUATE: $C = \frac{\epsilon_0 A}{d}$. We could have a larger d , along with a larger A , and still achieve the required C without exceeding the maximum allowed E .

24.9. IDENTIFY: Apply the results of Example 24.4. $C = Q/V$.

SET UP: $r_a = 0.50$ mm, $r_b = 5.00$ mm

EXECUTE: (a) $C = \frac{L2\pi\epsilon_0}{\ln(r_b/r_a)} = \frac{(0.180 \text{ m})2\pi\epsilon_0}{\ln(5.00/0.50)} = 4.35 \times 10^{-12} \text{ F}$.

(b) $V = Q/C = (10.0 \times 10^{-12} \text{ C})/(4.35 \times 10^{-12} \text{ F}) = 2.30 \text{ V}$

EVALUATE: $\frac{C}{L} = 24.2 \text{ pF}$. This value is similar to those in Example 24.4. The capacitance is determined entirely by the dimensions of the cylinders.

24.10. IDENTIFY: Capacitance depends on the geometry of the object.

(a) **SET UP:** The capacitance of a cylindrical capacitor is $C = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$. Solving for r_b gives $r_b = r_a e^{2\pi\epsilon_0 L/C}$.

EXECUTE: Substituting in the numbers for the exponent gives

$$\frac{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.120 \text{ m})}{3.67 \times 10^{-11} \text{ F}} = 0.182$$

Now use this value to calculate r_b : $r_b = r_a e^{0.182} = (0.250 \text{ cm})e^{0.182} = 0.300 \text{ cm}$

(b) **SET UP:** For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the numbers gives $\lambda = Q/L = CV/L$.

EXECUTE: Numerically we get

$$\lambda = \frac{CV}{L} = \frac{(3.67 \times 10^{-11} \text{ F})(125 \text{ V})}{0.120 \text{ m}} = 3.82 \times 10^{-8} \text{ C/m} = 38.2 \text{ nC/m}$$

EVALUATE: The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just 0.50 mm. These cylinders would have to be carefully constructed.

24.11. IDENTIFY and SET UP: Use the expression for C/L derived in Example 24.4. Then use Eq.(24.1) to calculate Q .

EXECUTE: (a) From Example 24.4, $\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$

$$\frac{C}{L} = \frac{2\pi(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}{\ln(3.5 \text{ mm}/1.5 \text{ mm})} = 6.57 \times 10^{-11} \text{ F/m} = 66 \text{ pF/m}$$

$$(b) C = (6.57 \times 10^{-11} \text{ F/m})(2.8 \text{ m}) = 1.84 \times 10^{-10} \text{ F}.$$

$$Q = CV = (1.84 \times 10^{-10} \text{ F})(350 \times 10^{-3} \text{ V}) = 6.4 \times 10^{-11} \text{ C} = 64 \text{ pC}$$

The conductor at higher potential has the positive charge, so there is +64 pC on the inner conductor and -64 pC on the outer conductor.

EVALUATE: C depends only on the dimensions of the capacitor. Q and V are proportional.

24.12. IDENTIFY: Apply the results of Example 24.3. $C = Q/V$.

SET UP: $r_a = 15.0 \text{ cm}$. Solve for r_b .

EXECUTE: (a) For two concentric spherical shells, the capacitance is $C = \frac{1}{k} \left(\frac{r_a r_b}{r_b - r_a} \right)$. $kCr_b - kCr_a = r_a r_b$ and

$$r_b = \frac{kCr_a}{kC - r_a} = \frac{k(116 \times 10^{-12} \text{ F})(0.150 \text{ m})}{k(116 \times 10^{-12} \text{ F}) - 0.150 \text{ m}} = 0.175 \text{ m}.$$

$$(b) V = 220 \text{ V} \text{ and } Q = CV = (116 \times 10^{-12} \text{ F})(220 \text{ V}) = 2.55 \times 10^{-8} \text{ C}.$$

EVALUATE: A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33 \text{ m}^2$ and $d = r_b - r_a = 2.5 \times 10^{-2} \text{ m}$ has

$$C = \frac{\epsilon_0 A}{d} = 117 \text{ pF}, \text{ in excellent agreement with the value of } C \text{ for the spherical capacitor.}$$

24.13. IDENTIFY: We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

(a) **SET UP:** By the definition of capacitance, $C = Q/V$.

$$\text{EXECUTE: } C = \frac{Q}{V} = \frac{3.30 \times 10^{-9} \text{ C}}{2.20 \times 10^2 \text{ V}} = 1.50 \times 10^{-11} \text{ F} = 15.0 \text{ pF}$$

(b) **SET UP:** The capacitance of a spherical capacitor is $C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$.

EXECUTE: Solve for r_a and evaluate using $C = 15.0 \text{ pF}$ and $r_b = 4.00 \text{ cm}$, giving $r_a = 3.09 \text{ cm}$.

(c) **SET UP:** We can treat the inner sphere as a point-charge located at its center and use Coulomb's law,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}.$$

$$\text{EXECUTE: } E = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.30 \times 10^{-9} \text{ C})}{(0.0309 \text{ m})^2} = 3.12 \times 10^4 \text{ N/C}$$

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.14. IDENTIFY: The capacitors between b and c are in parallel. This combination is in series with the 15 pF capacitor.

SET UP: Let $C_1 = 15 \text{ pF}$, $C_2 = 9.0 \text{ pF}$ and $C_3 = 11 \text{ pF}$.

EXECUTE: (a) For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \dots$ so $C_{23} = C_2 + C_3 = 20 \text{ pF}$

(b) $C_1 = 15 \text{ pF}$ is in series with $C_{23} = 20 \text{ pF}$. For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ so $\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_{23}}$ and

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{(15 \text{ pF})(20 \text{ pF})}{15 \text{ pF} + 20 \text{ pF}} = 8.6 \text{ pF}.$$

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.15. IDENTIFY: Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

SET UP: Do parts (a) and (b) together. The capacitor network is drawn in Figure 24.15a.

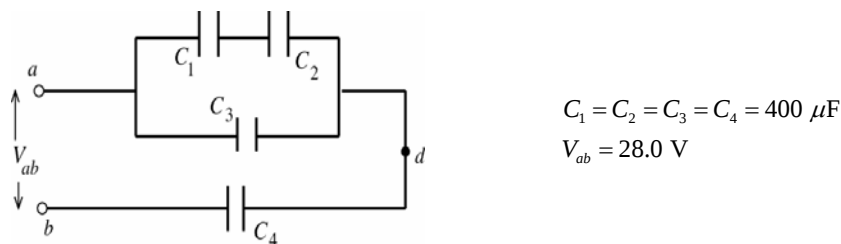


Figure 24.15a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.15b).

$$\text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} = \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} \quad \frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Figure 24.15b

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00 \times 10^{-6} \text{ F})(4.00 \times 10^{-6} \text{ F})}{4.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}} = 2.00 \times 10^{-6} \text{ F}$$

C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.15c).

$$\begin{aligned} \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} &= \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} \\ C_{123} &= C_{12} + C_3 \\ C_{123} &= 2.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F} \\ C_{123} &= 6.00 \times 10^{-6} \text{ F} \end{aligned}$$

Figure 24.15c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.15d).

$$\begin{aligned} \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} &= \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} \\ \frac{1}{C_{1234}} &= \frac{1}{C_{123}} + \frac{1}{C_4} \end{aligned}$$

Figure 24.15d

$$C_{1234} = \frac{C_{123} C_4}{C_{123} + C_4} = \frac{(6.00 \times 10^{-6} \text{ F})(4.00 \times 10^{-6} \text{ F})}{6.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}} = 2.40 \times 10^{-6} \text{ F}$$

The circuit is equivalent to the circuit shown in Figure 24.15e.

$$\begin{aligned} \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} &= \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} \\ V_{1234} &= V = 28.0 \text{ V} \\ Q_{1234} &= C_{1234} V = (2.40 \times 10^{-6} \text{ F})(28.0 \text{ V}) = 67.2 \mu\text{C} \end{aligned}$$

Figure 24.15e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.15f).

$$\begin{aligned} \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} &= \text{---} \parallel \text{---} \parallel \text{---} \parallel \text{---} \\ Q_{123} &= Q_4 - Q_{1234} = 67.2 \mu\text{C} \\ &(\text{charge same for capacitors in series}) \end{aligned}$$

Figure 24.15f

$$\text{Then } V_{123} = \frac{Q_{123}}{C_{123}} = \frac{67.2 \mu\text{C}}{6.00 \mu\text{F}} = 11.2 \text{ V}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{67.2 \mu\text{C}}{4.00 \mu\text{F}} = 16.8 \text{ V}$$

Note that $V_4 + V_{123} = 16.8 \text{ V} + 11.2 \text{ V} = 28.0 \text{ V}$, as it should.

Next consider the circuit as written in Figure 24.15g.

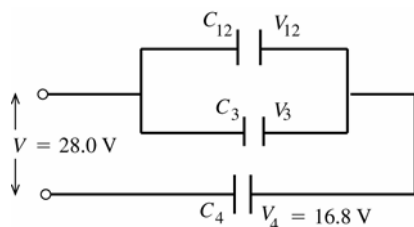


Figure 24.15g

$$\begin{aligned} V_3 &= V_{12} = 28.0 \text{ V} - V_4 \\ V_3 &= 11.2 \text{ V} \\ Q_3 &= C_3 V_3 = (4.00 \mu\text{F})(11.2 \text{ V}) \\ Q_3 &= 44.8 \mu\text{C} \\ Q_{12} &= C_{12} V_{12} = (2.00 \mu\text{F})(11.2 \text{ V}) \\ Q_{12} &= 22.4 \mu\text{C} \end{aligned}$$

Finally, consider the original circuit, as shown in Figure 24.15h.

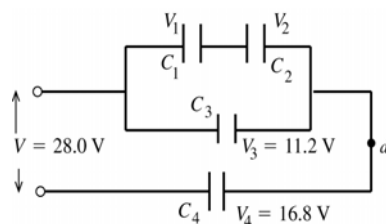


Figure 24.15h

$$\begin{aligned} Q_1 &= Q_2 = Q_{12} = 22.4 \mu\text{C} \\ &\text{(charge same for capacitors in series)} \\ V_1 &= \frac{Q_1}{C_1} = \frac{22.4 \mu\text{C}}{4.00 \mu\text{F}} = 5.6 \text{ V} \\ V_2 &= \frac{Q_2}{C_2} = \frac{22.4 \mu\text{C}}{4.00 \mu\text{F}} = 5.6 \text{ V} \end{aligned}$$

Note that $V_1 + V_2 = 11.2 \text{ V}$, which equals V_3 as it should.

Summary: $Q_1 = 22.4 \mu\text{C}$, $V_1 = 5.6 \text{ V}$

$Q_2 = 22.4 \mu\text{C}$, $V_2 = 5.6 \text{ V}$

$Q_3 = 44.8 \mu\text{C}$, $V_3 = 11.2 \text{ V}$

$Q_4 = 67.2 \mu\text{C}$, $V_4 = 16.8 \text{ V}$

(c) $V_{ad} = V_3 = 11.2 \text{ V}$

EVALUATE: $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2$, $Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

- 24.16. IDENTIFY:** The two capacitors are in series. The equivalent capacitance is given by $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$.

SET UP: For capacitors in series the charges are the same and the potentials add to give the potential across the network.

EXECUTE: (a) $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{(3.0 \times 10^{-6} \text{ F})} + \frac{1}{(5.0 \times 10^{-6} \text{ F})} = 5.33 \times 10^5 \text{ F}^{-1}$. $C_{\text{eq}} = 1.88 \times 10^{-6} \text{ F}$. Then

$Q = VC_{\text{eq}} = (52.0 \text{ V})(1.88 \times 10^{-6} \text{ F}) = 9.75 \times 10^{-5} \text{ C}$. Each capacitor has charge $9.75 \times 10^{-5} \text{ C}$.

(b) $V_1 = Q/C_1 = 9.75 \times 10^{-5} \text{ C} / 3.0 \times 10^{-6} \text{ F} = 32.5 \text{ V}$.

$V_2 = Q/C_2 = 9.75 \times 10^{-5} \text{ C} / 5.0 \times 10^{-6} \text{ F} = 19.5 \text{ V}$.

EVALUATE: $V_1 + V_2 = 52.0 \text{ V}$, which is equal to the applied potential V_{ab} . The capacitor with the smaller C has the larger V .

- 24.17. IDENTIFY:** The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage V_{ab} .

SET UP: Do parts (a) and (b) together. The network is sketched in Figure 24.17.

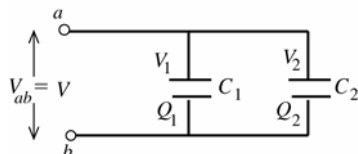


Figure 24.17

EXECUTE: $V_1 = V_2 = V$

$V_1 = 52.0 \text{ V}$

$V_2 = 52.0 \text{ V}$

$C = Q/V$ so $Q = CV$

$Q_1 = C_1 V_1 = (3.00 \mu\text{F})(52.0 \text{ V}) = 156 \mu\text{C}$. $Q_2 = C_2 V_2 = (5.00 \mu\text{F})(52.0 \text{ V}) = 260 \mu\text{C}$.

EVALUATE: To produce the same potential difference, the capacitor with the larger C has the larger Q .

24.18. IDENTIFY: For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2 .

EXECUTE: (a) C_1 and C_2 are in parallel and so have the same potential across them:

$$V_1 = V_2 = \frac{Q_2}{C_2} = \frac{40.0 \times 10^{-6} \text{ C}}{3.00 \times 10^{-6} \text{ F}} = 13.33 \text{ V}. \text{ Therefore, } Q_1 = V_1 C_1 = (13.33 \text{ V})(3.00 \times 10^{-6} \text{ F}) = 80.0 \times 10^{-6} \text{ C}. \text{ Since } C_3 \text{ is}$$

in series with the parallel combination of C_1 and C_2 , its charge must be equal to their combined charge:

$$C_3 = 40.0 \times 10^{-6} \text{ C} + 80.0 \times 10^{-6} \text{ C} = 120.0 \times 10^{-6} \text{ C}.$$

(b) The total capacitance is found from $\frac{1}{C_{\text{tot}}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{9.00 \times 10^{-6} \text{ F}} + \frac{1}{5.00 \times 10^{-6} \text{ F}}$ and $C_{\text{tot}} = 3.21 \mu\text{F}$.

$$V_{ab} = \frac{Q_{\text{tot}}}{C_{\text{tot}}} = \frac{120.0 \times 10^{-6} \text{ C}}{3.21 \times 10^{-6} \text{ F}} = 37.4 \text{ V}.$$

EVALUATE: $V_3 = \frac{Q_3}{C_3} = \frac{120.0 \times 10^{-6} \text{ C}}{5.00 \times 10^{-6} \text{ F}} = 24.0 \text{ V}$. $V_{ab} = V_1 + V_3$.

24.19. IDENTIFY and SET UP: Use the rules for V for capacitors in series and parallel: for capacitors in parallel the voltages are the same and for capacitors in series the voltages add.

EXECUTE: $V_1 = Q_1 / C_1 = (150 \mu\text{C}) / (3.00 \mu\text{F}) = 50 \text{ V}$

C_1 and C_2 are in parallel, so $V_2 = 50 \text{ V}$

$$V_3 = 120 \text{ V} - V_1 = 70 \text{ V}$$

EVALUATE: Now that we know the voltages, we could also calculate Q for the other two capacitors.

24.20. IDENTIFY and SET UP: $C = \frac{\epsilon_0 A}{d}$. For two capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$.

EXECUTE: $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left(\frac{d_1}{\epsilon_0 A} + \frac{d_2}{\epsilon_0 A} \right)^{-1} = \frac{\epsilon_0 A}{d_1 + d_2}$. This shows that the combined capacitance for two

capacitors in series is the same as that for a capacitor of area A and separation $(d_1 + d_2)$.

EVALUATE: C_{eq} is smaller than either C_1 or C_2 .

24.21. IDENTIFY and SET UP: $C = \frac{\epsilon_0 A}{d}$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: $C_{\text{eq}} = C_1 + C_2 = \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d} = \frac{\epsilon_0 (A_1 + A_2)}{d}$. So the combined capacitance for two capacitors in parallel is that of a single capacitor of their combined area $(A_1 + A_2)$ and common plate separation d .

EVALUATE: C_{eq} is larger than either C_1 or C_2 .

24.22. IDENTIFY: Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ For capacitors

in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \dots$ $C = \frac{Q}{V}$.

EXECUTE: (a) The equivalent capacitance of the $5.0 \mu\text{F}$ and $8.0 \mu\text{F}$ capacitors in parallel is $13.0 \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.22. The equivalent capacitance of these three capacitors in series is $3.47 \mu\text{F}$.

(b) $Q_{\text{tot}} = C_{\text{tot}} V = (3.47 \mu\text{F})(50.0 \text{ V}) = 174 \mu\text{C}$

(c) Q_{tot} is the same as Q for each of the capacitors in the series combination shown in Figure 24.22, so Q for each of the capacitors is $174 \mu\text{C}$.

EVALUATE: The voltages across each capacitor in Figure 24.22 are $V_{10} = \frac{Q_{\text{tot}}}{C_{10}} = 17.4 \text{ V}$, $V_{13} = \frac{Q_{\text{tot}}}{C_{13}} = 13.4 \text{ V}$ and

$V_9 = \frac{Q_{\text{tot}}}{C_9} = 19.3 \text{ V}$. $V_{10} + V_{13} + V_9 = 17.4 \text{ V} + 13.4 \text{ V} + 19.3 \text{ V} = 50.1 \text{ V}$. The sum of the voltages equals the applied

voltage, apart from a small difference due to rounding.



Figure 24.22

- 24.23. IDENTIFY:** Refer to Figure 24.10b in the textbook. For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \dots$. For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$.

SET UP: The $11\ \mu\text{F}$, $4\ \mu\text{F}$ and replacement capacitor are in parallel and this combination is in series with the $9.0\ \mu\text{F}$ capacitor.

EXECUTE: $\frac{1}{C_{\text{eq}}} = \frac{1}{8.0\ \mu\text{F}} = \left(\frac{1}{(11 + 4.0 + x)\ \mu\text{F}} + \frac{1}{9.0\ \mu\text{F}} \right)$. $(15 + x)\ \mu\text{F} = 72\ \mu\text{F}$ and $x = 57\ \mu\text{F}$.

EVALUATE: Increasing the capacitance of the one capacitor by a large amount makes a small increase in the equivalent capacitance of the network.

- 24.24. IDENTIFY:** Apply $C = Q/V$. $C = \frac{\epsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

SET UP: $U = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$.

EXECUTE: (a) $V = Q/C = (2.55\ \mu\text{C})/(920 \times 10^{-12}\ \text{F}) = 2770\ \text{V}$

(b) $C = \frac{\epsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to $5540\ \text{V}$.

(c) $U = \frac{Q^2}{2C} = \frac{(2.55 \times 10^{-6}\ \text{C})^2}{2(920 \times 10^{-12}\ \text{F})} = 3.53 \times 10^{-3}\ \text{J}$. When if the separation is doubled while Q stays the same, the capacitance halves, and the energy stored doubles. So the amount of work done to move the plates equals the difference in energy stored in the capacitor, which is $3.53 \times 10^{-3}\ \text{J}$.

EVALUATE: The oppositely charged plates attract each other and positive work must be done by an external force to pull them farther apart.

- 24.25. IDENTIFY and SET UP:** The energy density is given by Eq.(24.11): $u = \frac{1}{2} \epsilon_0 E^2$. Use $V = Ed$ to solve for E .

EXECUTE: Calculate E : $E = \frac{V}{d} = \frac{400\ \text{V}}{5.00 \times 10^{-3}\ \text{m}} = 8.00 \times 10^4\ \text{V/m}$.

Then $u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} (8.854 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2) (8.00 \times 10^4\ \text{V/m})^2 = 0.0283\ \text{J/m}^3$

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

- 24.26. IDENTIFY:** $C = \frac{Q}{V_{ab}}$. $C = \frac{\epsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2} QV$.

SET UP: $d = 1.50 \times 10^{-3}\ \text{m}$. $1\ \mu\text{C} = 10^{-6}\ \text{C}$

EXECUTE: (a) $C = \frac{0.0180 \times 10^{-6}\ \text{C}}{200\ \text{V}} = 9.00 \times 10^{-11}\ \text{F} = 90.0\ \text{pF}$

(b) $C = \frac{\epsilon_0 A}{d}$ so $A = \frac{Cd}{\epsilon_0} = \frac{(9.00 \times 10^{-11}\ \text{F})(1.50 \times 10^{-3}\ \text{m})}{8.854 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = 0.0152\ \text{m}^2$.

(c) $V = Ed = (3.0 \times 10^6\ \text{V/m})(1.50 \times 10^{-3}\ \text{m}) = 4.5 \times 10^3\ \text{V}$

(d) Energy $= \frac{1}{2} QV = \frac{1}{2} (0.0180 \times 10^{-6}\ \text{C})(200\ \text{V}) = 1.80 \times 10^{-6}\ \text{J} = 1.80\ \mu\text{J}$

EVALUATE: We could also calculate the stored energy as $\frac{Q^2}{2C} = \frac{(0.0180 \times 10^{-6}\ \text{C})^2}{2(9.00 \times 10^{-11}\ \text{F})} = 1.80\ \mu\text{J}$.

- 24.27. IDENTIFY:** The energy stored in a charged capacitor is $\frac{1}{2} CV^2$.

SET UP: $1\ \mu\text{F} = 10^{-6}\ \text{F}$

EXECUTE: $\frac{1}{2} CV^2 = \frac{1}{2} (450 \times 10^{-6}\ \text{F})(295\ \text{V})^2 = 19.6\ \text{J}$

EVALUATE: Thermal energy is generated in the wire at the rate $I^2 R$, where I is the current in the wire. When the capacitor discharges there is a flow of charge that corresponds to current in the wire.

- 24.28. IDENTIFY:** After the two capacitors are connected they must have equal potential difference, and their combined charge must add up to the original charge.

SET UP: $C = Q/V$. The stored energy is $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$

EXECUTE: (a) $Q = CV_0$.

(b) $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ and also $Q_1 + Q_2 = Q = CV_0$. $C_1 = C$ and $C_2 = \frac{C}{2}$ so $\frac{Q_1}{C} = \frac{Q_2}{(C/2)}$ and $Q_2 = \frac{Q_1}{2}$. $Q = \frac{3}{2}Q_1$.

$Q_1 = \frac{2}{3}Q$ and $V = \frac{Q_1}{C} = \frac{\frac{2}{3}Q}{C} = \frac{2}{3}V_0$.

(c) $U = \frac{1}{2} \left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} \right) = \frac{1}{2} \left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{2(\frac{1}{3}Q)^2}{C} \right] = \frac{1}{3} \frac{Q^2}{C} = \frac{1}{3}CV_0^2$

(d) The original U was $U = \frac{1}{2}CV_0^2$, so $\Delta U = -\frac{1}{6}CV_0^2$.

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

EVALUATE: The original charge of the charged capacitor must distribute between the two capacitors to make the potential the same across each capacitor. The voltage V for each after they are connected is less than the original voltage V_0 of the charged capacitor.

- 24.29. IDENTIFY and SET UP:** Combine Eqs. (24.9) and (24.2) to write the stored energy in terms of the separation between the plates.

EXECUTE: (a) $U = \frac{Q^2}{2C}$; $C = \frac{\epsilon_0 A}{x}$ so $U = \frac{xQ^2}{2\epsilon_0 A}$

(b) $x \rightarrow x + dx$ gives $U = \frac{(x + dx)Q^2}{2\epsilon_0 A}$

$dU = \frac{(x + dx)Q^2}{2\epsilon_0 A} - \frac{xQ^2}{2\epsilon_0 A} = \left(\frac{Q^2}{2\epsilon_0 A} \right) dx$

(c) $dW = F dx = dU$, so $F = \frac{Q^2}{2\epsilon_0 A}$

(d) **EVALUATE:** The capacitor plates and the field between the plates are shown in Figure 24.29a.

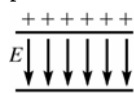


Figure 24.29a

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$F = \frac{1}{2}QE, \text{ not } QE$$

The reason for the difference is that E is the field due to both plates. If we consider the positive plate only and calculate its electric field using Gauss's law (Figure 24.29b):

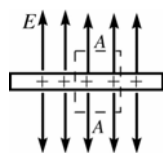


Figure 24.29b

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

The force this field exerts on the other plate, that has charge $-Q$, is $F = \frac{Q^2}{2\epsilon_0 A}$.

- 24.30. IDENTIFY:** $C = \frac{\epsilon_0 A}{d}$. The stored energy can be expressed either as $\frac{Q^2}{2C}$ or as $\frac{CV^2}{2}$, whichever is more convenient for the calculation.

SET UP: Since d is halved, C doubles.

EXECUTE: (a) If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

(b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when V is held constant throughout.

EVALUATE: When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

24.31. IDENTIFY and SET UP: $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

EXECUTE: (a) $Q = CV = (5.0 \mu\text{F})(1.5 \text{ V}) = 7.5 \mu\text{C}$. $U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0 \mu\text{F})(1.5 \text{ V})^2 = 5.62 \mu\text{J}$

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$. $Q = \sqrt{2CU} = \sqrt{2(5.0 \times 10^{-6} \text{ F})(1.0 \text{ J})} = 3.2 \times 10^{-3} \text{ C}$.

$V = \frac{Q}{C} = \frac{3.2 \times 10^{-3} \text{ C}}{5.0 \times 10^{-6} \text{ F}} = 640 \text{ V}$.

EVALUATE: The stored energy is proportional to Q^2 and to V^2 .

24.32. IDENTIFY: The two capacitors are in series. $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$. $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: (a) $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$ so $C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(150 \text{ nF})(120 \text{ nF})}{150 \text{ nF} + 120 \text{ nF}} = 66.7 \text{ nF}$.

$Q = CV = (66.7 \text{ nF})(36 \text{ V}) = 2.4 \times 10^{-6} \text{ C} = 2.4 \mu\text{C}$

(b) $Q = 2.4 \mu\text{C}$ for each capacitor.

(c) $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(66.7 \times 10^{-9} \text{ F})(36 \text{ V})^2 = 43.2 \mu\text{J}$

(d) We know C and Q for each capacitor so rewrite U in terms of these quantities. $U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$

150 nF: $U = \frac{(2.4 \times 10^{-6} \text{ C})^2}{2(150 \times 10^{-9} \text{ F})} = 19.2 \mu\text{J}$; 120 nF: $U = \frac{(2.4 \times 10^{-6} \text{ C})^2}{2(120 \times 10^{-9} \text{ F})} = 24.0 \mu\text{J}$

Note that $19.2 \mu\text{J} + 24.0 \mu\text{J} = 43.2 \mu\text{J}$, the total stored energy calculated in part (c).

(e) 150 nF: $V = \frac{Q}{C} = \frac{2.4 \times 10^{-6} \text{ C}}{150 \times 10^{-9} \text{ F}} = 16 \text{ V}$; 120 nF: $V = \frac{Q}{C} = \frac{2.4 \times 10^{-6} \text{ C}}{120 \times 10^{-9} \text{ F}} = 20 \text{ V}$

Note that these two voltages sum to 36 V, the voltage applied across the network.

EVALUATE: Since Q is the same the capacitor with smaller C stores more energy ($U = Q^2/2C$) and has a larger voltage ($V = Q/C$).

24.33. IDENTIFY: The two capacitors are in parallel. $C_{\text{eq}} = C_1 + C_2$. $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in parallel, the voltages are the same and the charges add.

EXECUTE: (a) $C_{\text{eq}} = C_1 + C_2 = 35 \text{ nF} + 75 \text{ nF} = 110 \text{ nF}$. $Q_{\text{tot}} = C_{\text{eq}}V = (110 \times 10^{-9} \text{ F})(220 \text{ V}) = 24.2 \mu\text{C}$

(b) $V = 220 \text{ V}$ for each capacitor.

35 nF: $Q_{35} = C_{35}V = (35 \times 10^{-9} \text{ F})(220 \text{ V}) = 7.7 \mu\text{C}$; 75 nF: $Q_{75} = C_{75}V = (75 \times 10^{-9} \text{ F})(220 \text{ V}) = 16.5 \mu\text{C}$. Note that $Q_{35} + Q_{75} = Q_{\text{tot}}$.

(c) $U_{\text{tot}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(110 \times 10^{-9} \text{ F})(220 \text{ V})^2 = 2.66 \text{ mJ}$

(d) 35 nF: $U_{35} = \frac{1}{2}C_{35}V^2 = \frac{1}{2}(35 \times 10^{-9} \text{ F})(220 \text{ V})^2 = 0.85 \text{ mJ}$;

75 nF: $U_{75} = \frac{1}{2}C_{75}V^2 = \frac{1}{2}(75 \times 10^{-9} \text{ F})(220 \text{ V})^2 = 1.81 \text{ mJ}$. Since V is the same the capacitor with larger C stores more energy.

(e) 220 V for each capacitor.

EVALUATE: The capacitor with the larger C has the larger Q .

24.34. IDENTIFY: Capacitance depends on the geometry of the object.

(a) **SET UP:** The potential difference between the core and tube is $V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$. Solving for the linear charge

density gives $\lambda = \frac{2\pi\epsilon_0 V}{\ln(r_b/r_a)} = \frac{4\pi\epsilon_0 V}{2 \ln(r_b/r_a)}$.

EXECUTE: Using the given values gives $\lambda = \frac{6.00 \text{ V}}{2(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \ln\left(\frac{2.00}{1.20}\right)} = 6.53 \times 10^{-10} \text{ C/m}$

(b) SET UP: $Q = \lambda L$

EXECUTE: $Q = (6.53 \times 10^{-10} \text{ C/m})(0.350 \text{ m}) = 2.29 \times 10^{-10} \text{ C}$

(c) SET UP: The definition of capacitance is $C = Q/V$.

EXECUTE: $C = \frac{2.29 \times 10^{-10} \text{ C}}{6.00 \text{ V}} = 3.81 \times 10^{-11} \text{ F}$

(d) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: $U = \frac{1}{2}(3.81 \times 10^{-11} \text{ F})(6.00 \text{ V})^2 = 6.85 \times 10^{-10} \text{ J}$

EVALUATE: The stored energy could be converted to heat or other forms of energy.

24.35. IDENTIFY: $U = \frac{1}{2}QV$. Solve for Q . $C = Q/V$.

SET UP: Example 24.4 shows that for a cylindrical capacitor, $\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$.

EXECUTE: **(a)** $U = \frac{1}{2}QV$ gives $Q = \frac{2U}{V} = \frac{2(3.20 \times 10^{-9} \text{ J})}{4.00 \text{ V}} = 1.60 \times 10^{-9} \text{ C}$.

(b) $\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)} \cdot \frac{r_b}{r_a} = \exp(2\pi\epsilon_0 L/C) = \exp(2\pi\epsilon_0 LV/Q) = \exp(2\pi\epsilon_0 (15.0 \text{ m})(4.00 \text{ V})/(1.60 \times 10^{-9} \text{ C})) = 8.05$.

The radius of the outer conductor is 8.05 times the radius of the inner conductor.

EVALUATE: When the ratio r_b/r_a increases, C/L decreases and less charge is stored for a given potential difference.

24.36. IDENTIFY: Apply Eq.(24.11).

SET UP: Example 24.3 shows that $E = \frac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that $\frac{Q}{4\pi\epsilon_0} = \left(\frac{r_a r_b}{r_b - r_a}\right)V_{ab}$.

EXECUTE: $E = \left(\frac{r_a r_b}{r_b - r_a}\right) \frac{V_{ab}}{r^2} = \left(\frac{[0.125 \text{ m}][0.148 \text{ m}]}{0.148 \text{ m} - 0.125 \text{ m}}\right) \frac{120 \text{ V}}{r^2} = \frac{96.5 \text{ V} \cdot \text{m}}{r^2}$

(a) For $r = 0.126 \text{ m}$, $E = 6.08 \times 10^3 \text{ V/m}$. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64 \times 10^{-4} \text{ J/m}^3$.

(b) For $r = 0.147 \text{ m}$, $E = 4.47 \times 10^3 \text{ V/m}$. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85 \times 10^{-5} \text{ J/m}^3$.

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. E decreases as r increases, so u decreases also.

24.37. IDENTIFY: Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express U , Q , and E in terms of the capacitor voltage.

SET UP: Let the applied voltage be V . Let each capacitor have capacitance C . $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V .

EXECUTE: **(a) series**

Voltage across each capacitor is $V/2$. The total energy stored is $U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$

parallel

Voltage across each capacitor is V . The total energy stored is $U_p = 2\left(\frac{1}{2}CV^2\right) = CV^2$

$U_p = 4U_s$

(b) $Q = CV$ for a single capacitor with voltage V . $Q_s = 2(C[V/2]) = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$

(c) $E = V/d$ for a capacitor with voltage V . $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.38. IDENTIFY: $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: **(a)** $V = Ed = (3.00 \times 10^4 \text{ V/m})(1.50 \times 10^{-3} \text{ m}) = 45.0 \text{ V}$.

$Q = C_0 V = (5.00 \times 10^{-12} \text{ F})(45.0 \text{ V}) = 2.25 \times 10^{-10} \text{ C}$.

(b) With the dielectric, $C = KC_0 = (2.70)(5.00 \text{ pF}) = 13.5 \text{ pF}$. V is still 45.0 V , so

$Q = CV = (13.5 \times 10^{-12} \text{ F})(45.0 \text{ V}) = 6.08 \times 10^{-10} \text{ C}$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. Q increases by a factor of K .

- 24.39. IDENTIFY and SET UP:** Q is constant so we can apply Eq.(24.14). The charge density on each surface of the dielectric is given by Eq.(24.16).

EXECUTE: $E = \frac{E_0}{K}$ so $K = \frac{E_0}{E} = \frac{3.20 \times 10^5 \text{ V/m}}{2.50 \times 10^5 \text{ V/m}} = 1.28$

(a) $\sigma_i = \sigma(1 - 1/K)$

$$\sigma = \epsilon_0 E_0 = (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.20 \times 10^5 \text{ N/C}) = 2.833 \times 10^{-6} \text{ C/m}^2$$

$$\sigma_i = (2.833 \times 10^{-6} \text{ C/m}^2)(1 - 1/1.28) = 6.20 \times 10^{-7} \text{ C/m}^2$$

(b) As calculated above, $K = 1.28$.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

- 24.40. IDENTIFY:** Capacitance depends on geometry, and the introduction of a dielectric increases the capacitance.

SET UP: For a parallel-plate capacitor, $C = K\epsilon_0 A/d$.

EXECUTE: **(a)** Solving for d gives

$$d = \frac{K\epsilon_0 A}{C} = \frac{(3.0)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.22 \text{ m})(0.28 \text{ m})}{1.0 \times 10^{-9} \text{ F}} = 1.64 \times 10^{-3} \text{ m} = 1.64 \text{ mm}.$$

Dividing this result by the thickness of a sheet of paper gives $\frac{1.64 \text{ mm}}{0.20 \text{ mm/sheet}} \approx 8$ sheets.

(b) Solving for the area of the plates gives $A = \frac{Cd}{K\epsilon_0} = \frac{(1.0 \times 10^{-9} \text{ F})(0.012 \text{ m})}{(3.0)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 0.45 \text{ m}^2$.

(c) Teflon has a smaller dielectric constant (2.1) than the posterboard, so she will need more area to achieve the same capacitance.

EVALUATE: The use of dielectric makes it possible to construct reasonable-sized capacitors since the dielectric increases the capacitance by a factor of K .

- 24.41. IDENTIFY and SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C = K\epsilon_0 A/d$.

Minimum A means smallest possible d . d is limited by the requirement that E be less than $1.60 \times 10^7 \text{ V/m}$ when V is as large as 5500 V.

EXECUTE: $V = Ed$ so $d = \frac{V}{E} = \frac{5500 \text{ V}}{1.60 \times 10^7 \text{ V/m}} = 3.44 \times 10^{-4} \text{ m}$

Then $A = \frac{Cd}{K\epsilon_0} = \frac{(1.25 \times 10^{-9} \text{ F})(3.44 \times 10^{-4} \text{ m})}{(3.60)(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 0.0135 \text{ m}^2$.

EVALUATE: The relation $V = Ed$ applies with or without a dielectric present. A would have to be larger if there were no dielectric.

- 24.42. IDENTIFY and SET UP:** Adapt the derivation of Eq.(24.1) to the situation where a dielectric is present.

EXECUTE: Placing a dielectric between the plates just results in the replacement of ϵ for ϵ_0 in the derivation of Equation (24.20). One can follow exactly the procedure as shown for Equation (24.11).

EVALUATE: The presence of the dielectric increases the energy density for a given electric field.

- 24.43. IDENTIFY:** The permittivity ϵ of a material is related to its dielectric constant by $\epsilon = K\epsilon_0$. The maximum voltage is

related to the maximum possible electric field before dielectric breakdown by $V_{\text{max}} = E_{\text{max}} d$. $E = \frac{E_0}{K} = \frac{\sigma}{K\epsilon_0}$, where

σ is the surface charge density on each plate. The induced surface charge density on the surface of the dielectric is given by $\sigma_i = \sigma(1 - 1/K)$.

SET UP: From Table 24.2, for polystyrene $K = 2.6$ and the dielectric strength (maximum allowed electric field) is $2 \times 10^7 \text{ V/m}$.

EXECUTE: **(a)** $\epsilon = K\epsilon_0 = (2.6)\epsilon_0 = 2.3 \times 10^{-11} \text{ C}^2/\text{N} \cdot \text{m}^2$

(b) $V_{\text{max}} = E_{\text{max}} d = (2.0 \times 10^7 \text{ V/m})(2.0 \times 10^{-3} \text{ m}) = 4.0 \times 10^4 \text{ V}$

(c) $E = \frac{\sigma}{K\epsilon_0}$ and $\sigma = \epsilon E = (2.3 \times 10^{-11} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.0 \times 10^7 \text{ V/m}) = 0.46 \times 10^{-3} \text{ C/m}^2$.

$$\sigma_i = \sigma \left(1 - \frac{1}{K}\right) = (0.46 \times 10^{-3} \text{ C/m}^2)(1 - 1/2.6) = 2.8 \times 10^{-4} \text{ C/m}^2.$$

EVALUATE: The net surface charge density is $\sigma_{\text{net}} = \sigma - \sigma_i = 1.8 \times 10^{-4} \text{ C/m}^2$ and the electric field between the plates is $E = \sigma_{\text{net}} / \epsilon_0$.

24.44. IDENTIFY: $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: (a) $\Delta Q = Q - Q_0 = (K - 1)Q_0 = (K - 1)C_0V_0 = (2.1)(2.5 \times 10^{-7} \text{ F})(12 \text{ V}) = 6.3 \times 10^{-6} \text{ C}$.

(b) $\sigma_i = \sigma(1 - 1/K)$ so $Q_i = Q(1 - 1/K) = (9.3 \times 10^{-6} \text{ C})(1 - 1/3.1) = 6.3 \times 10^{-6} \text{ C}$.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.45. (a) IDENTIFY and SET UP: Since the capacitor remains connected to the power supply the potential difference doesn't change when the dielectric is inserted. Use Eq.(24.9) to calculate V and combine it with Eq.(24.12) to obtain a relation between the stored energies and the dielectric constant and use this to calculate K .

EXECUTE: Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\frac{2U_0}{C_0}} = \sqrt{\frac{2(1.85 \times 10^{-5} \text{ J})}{360 \times 10^{-9} \text{ F}}} = 10.1 \text{ V}$

(b) $K = C/C_0$

$U_0 = \frac{1}{2}C_0V^2$, $U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$

$K = \frac{U}{U_0} = \frac{1.85 \times 10^{-5} \text{ J} + 2.32 \times 10^{-5} \text{ J}}{1.85 \times 10^{-5} \text{ J}} = 2.25$

EVALUATE: K increases the capacitance and then from $U = \frac{1}{2}CV^2$, with V constant an increase in C gives an increase in U .

24.46. IDENTIFY: $C = KC_0$. $C = Q/V$. $V = Ed$.

SET UP: Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

EXECUTE: (a) The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (the battery is still connected), $\frac{C_{\text{after}}}{C_{\text{before}}} = \frac{Q_{\text{after}}}{Q_{\text{before}}} = \frac{45.0 \text{ pC}}{25.0 \text{ pC}} = K = 1.80$.

(b) The area of the plates is $\pi r^2 = \pi(0.0300 \text{ m})^2 = 2.827 \times 10^{-3} \text{ m}^2$ and the separation between them is thus

$d = \frac{\epsilon_0 A}{C} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)}{12.5 \times 10^{-12} \text{ F}} = 2.00 \times 10^{-3} \text{ m}$. Before the dielectric is inserted, $C = \frac{\epsilon_0 A}{d} = \frac{Q}{V}$

and $V = \frac{Qd}{\epsilon_0 A} = \frac{(25.0 \times 10^{-12} \text{ C})(2.00 \times 10^{-3} \text{ m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)} = 2.00 \text{ V}$. The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

(c) Before the dielectric is inserted, $E = \frac{Q}{\epsilon_0 A} = \frac{25.0 \times 10^{-12} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)} = 1000 \text{ N/C}$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

EVALUATE: $E = \frac{V}{d} = \frac{2.00 \text{ V}}{2.00 \times 10^{-3} \text{ m}} = 1000 \text{ N/C}$, whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need d to calculate E because V is the potential difference between points separated by distance d .

24.47. IDENTIFY: $C = KC_0$. $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5 \mu\text{F}$ is the value of the capacitance without the dielectric present.

EXECUTE: (a) With the dielectric, $C = (3.75)(12.5 \mu\text{F}) = 46.9 \mu\text{F}$.

before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5 \times 10^{-6} \text{ F})(24.0 \text{ V})^2 = 3.60 \text{ mJ}$

after: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9 \times 10^{-6} \text{ F})(24.0 \text{ V})^2 = 13.5 \text{ mJ}$

(b) $\Delta U = 13.5 \text{ mJ} - 3.6 \text{ mJ} = 9.9 \text{ mJ}$. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

24.48. IDENTIFY: Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of K and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

(a) **SET UP:** The capacitance of a parallel-plate capacitor is $C = K\epsilon_0 A/d$ and the charge on its plates is $Q = CV$.

EXECUTE: First find the capacitance:

$$C = \frac{K\epsilon_0 A}{d} = \frac{(2.1)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0225 \text{ m}^2)}{1.00 \times 10^{-3} \text{ m}} = 4.18 \times 10^{-10} \text{ F}.$$

Now find the charge on the plates: $Q = CV = (4.18 \times 10^{-10} \text{ F})(12.0 \text{ V}) = 5.02 \times 10^{-9} \text{ C}.$

(b) SET UP: Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\epsilon_0.$

EXECUTE: Solving for E gives

$$E = \frac{Q_{\text{free}}}{KA\epsilon_0} = \frac{5.02 \times 10^{-9} \text{ C}}{(2.1)(0.0225 \text{ m}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 1.20 \times 10^4 \text{ N/C}$$

(c) SET UP: Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \epsilon_0 A/d$ and Gauss's law now gives $EA = Q/\epsilon_0.$

EXECUTE: First find the capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0225 \text{ m}^2)}{1.00 \times 10^{-3} \text{ m}} = 1.99 \times 10^{-10} \text{ F}.$$

The potential difference is $V = \frac{Q}{C} = \frac{5.02 \times 10^{-9} \text{ C}}{1.99 \times 10^{-10} \text{ F}} = 25.2 \text{ V}.$ From Gauss's law, the electric field is

$$E = \frac{Q}{\epsilon_0 A} = \frac{5.02 \times 10^{-9} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0225 \text{ m}^2)} = 2.52 \times 10^4 \text{ N/C}.$$

EVALUATE: The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

24.49. IDENTIFY: Apply Eq.(24.23) to calculate E . $V = Ed$ and $C = Q/V$ apply whether there is a dielectric between the plates or not.

(a) SET UP: Apply Eq.(24.23) to the dashed surface in Figure 24.49:

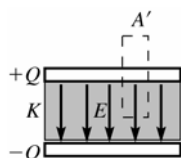


Figure 24.49

$$\text{EXECUTE: } \oint K\vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$$

$$\oint K\vec{E} \cdot d\vec{A} = KEA'$$

since $E = 0$ outside the plates

$$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'$$

$$\text{Thus } KEA' = \frac{(Q/A)A'}{\epsilon_0} \text{ and } E = \frac{Q}{\epsilon_0 AK}$$

$$\text{(b) } V = Ed = \frac{Qd}{\epsilon_0 AK}$$

$$\text{(c) } C = \frac{Q}{V} = \frac{Q}{(Qd/\epsilon_0 AK)} = K \frac{\epsilon_0 A}{d} = KC_0.$$

EVALUATE: Our result shows that $K = C/C_0$, which is Eq.(24.12).

24.50. IDENTIFY: $C = \frac{\epsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2} CV^2$.

SET UP: With the battery disconnected, Q is constant. When the separation d is doubled, C is halved.

$$\text{EXECUTE: (a) } C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 (0.16 \text{ m})^2}{4.7 \times 10^{-3} \text{ m}} = 4.8 \times 10^{-11} \text{ F}$$

$$\text{(b) } Q = CV = (4.8 \times 10^{-11} \text{ F})(12 \text{ V}) = 0.58 \times 10^{-9} \text{ C}$$

$$\text{(c) } E = V/d = (12 \text{ V})/(4.7 \times 10^{-3} \text{ m}) = 2550 \text{ V/m}$$

$$\text{(d) } U = \frac{1}{2} CV^2 = \frac{1}{2} (4.8 \times 10^{-11} \text{ F})(12 \text{ V})^2 = 3.46 \times 10^{-9} \text{ J}$$

(e) If the battery is disconnected, so the charge remains constant, and the plates are pulled further apart to 0.0094 m, then the calculations above can be carried out just as before, and we find: (a) $C = 2.41 \times 10^{-11} \text{ F}$ (b) $Q = 0.58 \times 10^{-9} \text{ C}$

$$\text{(c) } E = 2550 \text{ V/m} \quad \text{(d) } U = \frac{Q^2}{2C} = \frac{(0.58 \times 10^{-9} \text{ C})^2}{2(2.41 \times 10^{-11} \text{ F})} = 6.91 \times 10^{-9} \text{ J}$$

EVALUATE: Q is unchanged. $E = \frac{Q}{\epsilon_0 A}$ so E is unchanged. U doubles because C is halved. The additional stored energy comes from the work done by the force that pulled the plates apart.

- 24.51. IDENTIFY and SET UP:** If the capacitor remains connected to the battery, the battery keeps the potential difference between the plates constant by changing the charge on the plates.

EXECUTE: (a) $C = \frac{\epsilon_0 A}{d}$

$$C = \frac{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.16 \text{ m})^2}{9.4 \times 10^{-3} \text{ m}} = 2.4 \times 10^{-11} \text{ F} = 24 \text{ pF}$$

(b) Remains connected to the battery says that V stays 12 V. $Q = CV = (2.4 \times 10^{-11} \text{ F})(12 \text{ V}) = 2.9 \times 10^{-10} \text{ C}$

(c) $E = \frac{V}{d} = \frac{12 \text{ V}}{9.4 \times 10^{-3} \text{ m}} = 1.3 \times 10^3 \text{ V/m}$

(d) $U = \frac{1}{2}QV = \frac{1}{2}(2.9 \times 10^{-10} \text{ C})(12.0 \text{ V}) = 1.7 \times 10^{-9} \text{ J}$

EVALUATE: Increasing the separation decreases C . With V constant, this means that Q decreases and U decreases. Q decreases and $E = Q/\epsilon_0 A$ so E decreases. We come to the same conclusion from $E = V/d$.

- 24.52. IDENTIFY:** $C = KC_0 = K\epsilon_0 \frac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0 \text{ cm}^2 = 1.0 \times 10^{-4} \text{ m}^2$.

EXECUTE: (a) $C = (10) \frac{(8.85 \times 10^{-12} \text{ F/m})(1.0 \times 10^{-4} \text{ m}^2)}{7.5 \times 10^{-9} \text{ m}} = 1.18 \text{ } \mu\text{F}$ per cm^2 .

(b) $E = \frac{V}{K} = \frac{85 \text{ mV}}{7.5 \times 10^{-9} \text{ m}} = 1.13 \times 10^7 \text{ V/m}$.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

- 24.53. IDENTIFY:** $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

SET UP: $E = 0.95U$

EXECUTE: (a) The power output is 600 W, and 95% of the original energy is converted, so

$$E = Pt = (2.70 \times 10^5 \text{ W})(1.48 \times 10^{-3} \text{ s}) = 400 \text{ J}. E_0 = \frac{400 \text{ J}}{0.95} = 421 \text{ J}.$$

(b) $U = \frac{1}{2}CV^2$ so $C = \frac{2U}{V^2} = \frac{2(421 \text{ J})}{(125 \text{ V})^2} = 0.054 \text{ F}$.

EVALUATE: For a given V , the stored energy increases linearly with C .

- 24.54. IDENTIFY:** $C = \frac{\epsilon_0 A}{d}$

SET UP: $A = 4.2 \times 10^{-5} \text{ m}^2$. The original separation between the plates is $d = 0.700 \times 10^{-3} \text{ m}$. d' is the separation between the plates at the new value of C .

EXECUTE: $C_0 = \frac{A\epsilon_0}{d} = \frac{(4.20 \times 10^{-5} \text{ m}^2)\epsilon_0}{7.00 \times 10^{-4} \text{ m}} = 5.31 \times 10^{-13} \text{ F}$. The new value of C is $C = C_0 + 0.25 \text{ pF} = 7.81 \times 10^{-13} \text{ F}$.

But $C = \frac{A\epsilon_0}{d'}$, so $d' = \frac{A\epsilon_0}{C} = \frac{(4.20 \times 10^{-5} \text{ m}^2)\epsilon_0}{7.81 \times 10^{-13} \text{ F}} = 4.76 \times 10^{-4} \text{ m}$. Therefore the key must be depressed by a distance of $7.00 \times 10^{-4} \text{ m} - 4.76 \times 10^{-4} \text{ m} = 0.224 \text{ mm}$.

EVALUATE: When the key is depressed, d decreases and C increases.

- 24.55. IDENTIFY:** Example 24.4 shows that $C = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

SET UP: $\ln(1+x) \approx x$ when x is small. The area of each conductor is approximately $A = 2\pi r_a L$.

EXECUTE: (a) $d \ll r_a$: $C = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)} = \frac{2\pi\epsilon_0 L}{\ln((d+r_a)/r_a)} = \frac{2\pi\epsilon_0 L}{\ln(1+d/r_a)} \approx \frac{2\pi r_a L \epsilon_0}{d} = \frac{\epsilon_0 A}{d}$

EVALUATE: (b) At the scale of part (a) the cylinders appear to be flat, and so the capacitance should appear like that of flat plates.

- 24.56. IDENTIFY:** Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and

$$Q_2 \text{ on each. After they are reconnected to each other the total charge is } Q = Q_2 - Q_1. U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{\text{eq}} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1 V_1 = (9.0 \mu\text{F})(28 \text{ V}) = 2.52 \times 10^{-4} \text{ C}$ and $Q_2 = C_2 V_2 = (4.0 \mu\text{F})(28 \text{ V}) = 1.12 \times 10^{-4} \text{ C}$.

$C_{\text{eq}} = C_1 + C_2 = 13.0 \mu\text{F}$. The original energy stored is $U = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2} (13.0 \times 10^{-6} \text{ F})(28 \text{ V})^2 = 5.10 \times 10^{-3} \text{ J}$.

Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.4 \times 10^{-4} \text{ C}$ and the equivalent capacitance

is still the same, $C_{\text{eq}} = 13.0 \mu\text{F}$. The new energy stored is $U = \frac{Q^2}{2C_{\text{eq}}} = \frac{(1.4 \times 10^{-4} \text{ C})^2}{2(13.0 \times 10^{-6} \text{ F})} = 7.54 \times 10^{-4} \text{ J}$. The change in

stored energy is $\Delta U = 7.45 \times 10^{-4} \text{ J} - 5.10 \times 10^{-3} \text{ J} = -4.35 \times 10^{-3} \text{ J}$.

EVALUATE: When they are reconnected, charge flows and thermal energy is generated and energy is radiated as electromagnetic waves.

- 24.57. IDENTIFY:** Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \frac{1}{2} CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$. For capacitors

in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \dots$. $C = \frac{Q}{V}$. $U = \frac{1}{2} CV^2$.

EXECUTE: (a) Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.57a-c.

$$U_{\text{tot}} = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2} (2.19 \times 10^{-6} \text{ F})(12.0 \text{ V})^2 = 1.58 \times 10^{-4} \text{ J} = 158 \mu\text{J}$$

(b) From Figure 24.57c, $Q_{\text{tot}} = C_{\text{eq}} V = (2.19 \times 10^{-6} \text{ F})(12.0 \text{ V}) = 2.63 \times 10^{-5} \text{ C}$. From Figure 24.57b, $Q_{\text{tot}} = 2.63 \times 10^{-5} \text{ C}$.

$$V_{4.8} = \frac{Q_{4.8}}{C_{4.8}} = \frac{2.63 \times 10^{-5} \text{ C}}{4.80 \times 10^{-6} \text{ F}} = 5.48 \text{ V}. U_{4.8} = \frac{1}{2} CV^2 = \frac{1}{2} (4.80 \times 10^{-6} \text{ F})(5.48 \text{ V})^2 = 7.21 \times 10^{-5} \text{ J} = 72.1 \mu\text{J}$$

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U = \frac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.

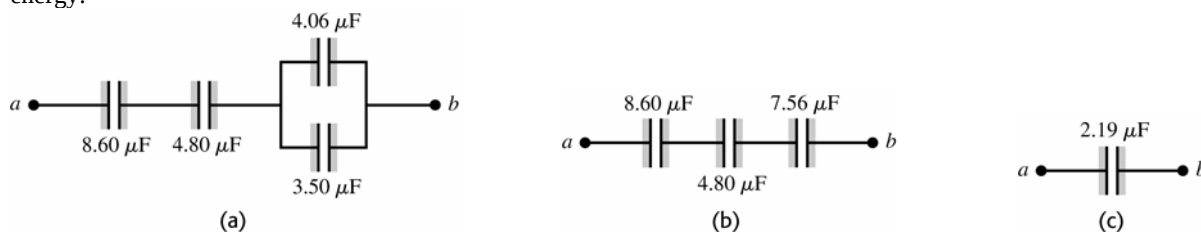


Figure 24.57

- 24.58. IDENTIFY:** Apply the rules for combining capacitors in series and parallel. For capacitors in series the voltages add and in parallel the voltages are the same.

SET UP: When a capacitor is a moderately good conductor it can be replaced by a wire and the potential across it is zero.

EXECUTE: (a) A network that has the desired properties is sketched in Figure 24.58a. $C_{\text{eq}} = \frac{C}{2} + \frac{C}{2} = C$. The total capacitance is the same as each individual capacitor, and the voltage is split over each so that $V = 480 \text{ V}$.

(b) If one capacitor is a moderately good conductor, then it can be treated as a “short” and thus removed from the circuit, and one capacitor will have greater than 600 V across it.

EVALUATE: An alternative solution is two in parallel in series with two in parallel, as sketched in Figure 24.58b.

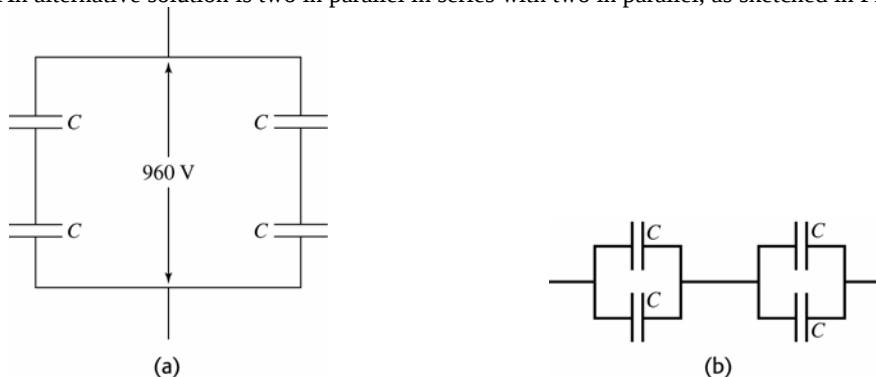
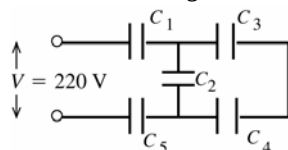


Figure 24.58

24.59. (a) IDENTIFY: Replace series and parallel combinations of capacitors by their equivalents.

SET UP: The network is sketched in Figure 24.59a.



$$C_1 = C_5 = 8.4 \mu\text{F}$$

$$C_2 = C_3 = C_4 = 4.2 \mu\text{F}$$

Figure 24.59a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.59b):

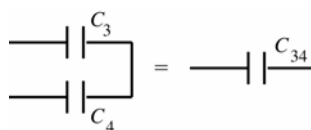


Figure 24.59b

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$

$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}$$

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2 \mu\text{F})(4.2 \mu\text{F})}{4.2 \mu\text{F} + 4.2 \mu\text{F}} = 2.1 \mu\text{F}$$

C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.59c):

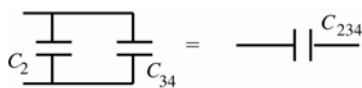


Figure 24.59c

$$C_{234} = C_2 + C_{34}$$

$$C_{234} = 4.2 \mu\text{F} + 2.1 \mu\text{F}$$

$$C_{234} = 6.3 \mu\text{F}$$

C_1 , C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.59d):

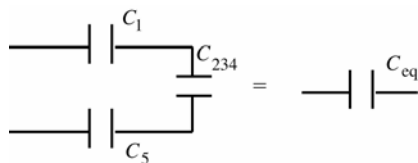


Figure 24.59d

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$

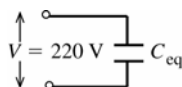
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4 \mu\text{F}} + \frac{1}{6.3 \mu\text{F}}$$

$$C_{\text{eq}} = 2.5 \mu\text{F}$$

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.

(b) IDENTIFY and SET UP: In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

EXECUTE: The equivalent circuit is drawn in Figure 24.59e.



$$Q_{\text{eq}} = C_{\text{eq}} V$$

$$Q_{\text{eq}} = (2.5 \mu\text{F})(220 \text{ V}) = 550 \mu\text{C}$$

Figure 24.59e

$$Q_1 = Q_5 = Q_{234} = 550 \mu\text{C} \text{ (capacitors in series have same charge)}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{550 \mu\text{C}}{8.4 \mu\text{F}} = 65 \text{ V}$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550 \mu\text{C}}{8.4 \mu\text{F}} = 65 \text{ V}$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550 \mu\text{C}}{6.3 \mu\text{F}} = 87 \text{ V}$$

Now draw the network as in Figure 24.59f.

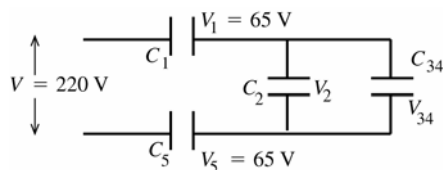


Figure 24.59f

$$V_2 = V_{34} = V_{234} = 87 \text{ V}$$

capacitors in parallel have the same potential

$$Q_2 = C_2 V_2 = (4.2 \mu\text{F})(87 \text{ V}) = 370 \mu\text{C}$$

$$Q_{34} = C_{34} V_{34} = (2.1 \mu\text{F})(87 \text{ V}) = 180 \mu\text{C}$$

Finally, consider the original circuit (Figure 24.59g).

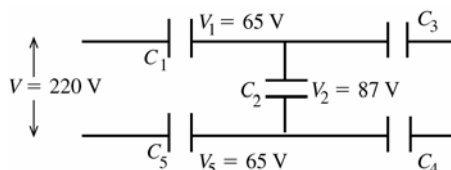


Figure 24.59g

$$Q_3 = Q_4 = Q_{34} = 180 \mu\text{C}$$

capacitors in series have the same charge

$$V_3 = \frac{Q_3}{C_3} = \frac{180 \mu\text{C}}{4.2 \mu\text{F}} = 43 \text{ V}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180 \mu\text{C}}{4.2 \mu\text{F}} = 43 \text{ V}$$

Summary: $Q_1 = 550 \mu\text{C}$, $V_1 = 65 \text{ V}$

$$Q_2 = 370 \mu\text{C}, V_2 = 87 \text{ V}$$

$$Q_3 = 180 \mu\text{C}, V_3 = 43 \text{ V}$$

$$Q_4 = 180 \mu\text{C}, V_4 = 43 \text{ V}$$

$$Q_5 = 550 \mu\text{C}, V_5 = 65 \text{ V}$$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220 \text{ V}$ (apart from some small rounding error)

$$Q_1 = Q_2 + Q_3 \text{ and } Q_5 = Q_2 + Q_4$$

24.60. IDENTIFY: Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open each pair of $3.00 \mu\text{F}$ and $6.00 \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed each pair of $3.00 \mu\text{F}$ and $6.00 \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: (a) With the switch open $C_{\text{eq}} = \left(\left(\frac{1}{3 \mu\text{F}} + \frac{1}{6 \mu\text{F}} \right)^{-1} + \left(\frac{1}{3 \mu\text{F}} + \frac{1}{6 \mu\text{F}} \right)^{-1} \right) = 4.00 \mu\text{F}$.

$Q_{\text{total}} = C_{\text{eq}} V = (4.00 \mu\text{F})(210 \text{ V}) = 8.40 \times 10^{-4} \text{ C}$. By symmetry, each capacitor carries $4.20 \times 10^{-4} \text{ C}$. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140 \text{ V}$ and $V_{ac} = Q/C_6 = 70 \text{ V}$.

$$V_{cd} = V_{ad} - V_{ac} = 70 \text{ V}.$$

(b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is

$$C_{\text{eq}} = \left(\frac{1}{(3.00 + 6.00) \mu\text{F}} + \frac{1}{(3.00 + 6.00) \mu\text{F}} \right)^{-1} = 4.5 \mu\text{F}. \quad Q_{\text{total}} = C_{\text{eq}} V = (4.50 \mu\text{F})(210 \text{ V}) = 9.5 \times 10^{-4} \text{ C}, \text{ and each}$$

capacitor has the same potential difference of 105 V (again, by symmetry).

(c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315 \mu\text{C}$, so $315 \mu\text{C}$ of charge flowed through the switch.

EVALUATE: When the switch is closed the charge must redistribute to make points c and d be at the same potential.

- 24.61. (a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.61a.

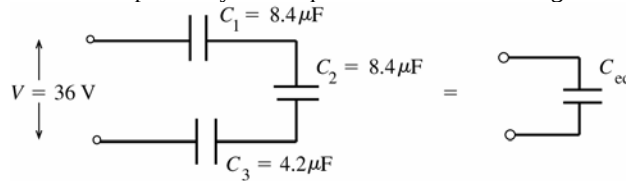


Figure 24.61a

EXECUTE: $C_3 = C_1/2$ so $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{4}{8.4 \mu\text{F}}$ and $C_{eq} = 8.4 \mu\text{F}/4 = 2.1 \mu\text{F}$

$$Q = C_{eq}V = (2.1 \mu\text{F})(36 \text{ V}) = 76 \mu\text{C}$$

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76 \mu\text{C}$

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and SET UP: Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36 \text{ V}$.

$$\text{EXECUTE: } U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$$

$$\text{But } Q_1 = Q_2 = Q_3 = Q \text{ so } U = \frac{1}{2}Q(V_1 + V_2 + V_3)$$

$$\text{But also } V_1 + V_2 + V_3 = V = 36 \text{ V, so } U = \frac{1}{2}QV = \frac{1}{2}(76 \mu\text{C})(36 \text{ V}) = 1.4 \times 10^{-3} \text{ J.}$$

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.61b.

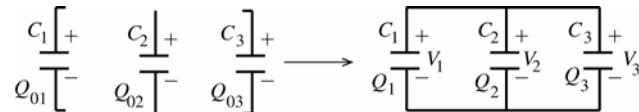


Figure 24.61b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76 \mu\text{C}) = 228 \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228 \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4 \mu\text{F}/4.2 \mu\text{F}) = 2Q_3$

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228 \mu\text{C}$.

$$5Q_3 = 228 \mu\text{C} \text{ and } Q_3 = 45.6 \mu\text{C}, Q_1 = Q_2 = 91.2 \mu\text{C}$$

$$\text{Then } V_1 = \frac{Q_1}{C_1} = \frac{91.2 \mu\text{C}}{8.4 \mu\text{F}} = 11 \text{ V}, V_2 = \frac{Q_2}{C_2} = \frac{91.2 \mu\text{C}}{8.4 \mu\text{F}} = 11 \text{ V}, \text{ and } V_3 = \frac{Q_3}{C_3} = \frac{45.6 \mu\text{C}}{4.2 \mu\text{F}} = 11 \text{ V.}$$

The voltage across each capacitor in the parallel combination is 11 V.

$$\text{(d) } U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3.$$

$$\text{But } V_1 = V_2 = V_3 \text{ so } U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11 \text{ V})(228 \mu\text{C}) = 1.3 \times 10^{-3} \text{ J.}$$

EVALUATE: This is less than the original energy of $1.4 \times 10^{-3} \text{ J}$. The stored energy has decreased, as in Example 24.7.

- 24.62. IDENTIFY:** $C = \frac{\epsilon_0 A}{d}$. $C = \frac{Q}{V}$. $V = Ed$. $U = \frac{1}{2}QV$.

SET UP: $d = 3.0 \times 10^3 \text{ m}$. $A = \pi r^2$, with $r = 1.0 \times 10^3 \text{ m}$.

$$\text{EXECUTE: (a) } C = \frac{\epsilon_0 A}{d} = \frac{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)\pi(1.0 \times 10^3 \text{ m})^2}{3.0 \times 10^3 \text{ m}} = 9.3 \times 10^{-9} \text{ F.}$$

$$\text{(b) } V = \frac{Q}{C} = \frac{20 \text{ C}}{9.3 \times 10^{-9} \text{ F}} = 2.2 \times 10^9 \text{ V}$$

$$\text{(c) } E = \frac{V}{d} = \frac{2.2 \times 10^9 \text{ V}}{3.0 \times 10^3 \text{ m}} = 7.3 \times 10^5 \text{ V/m}$$

$$\text{(d) } U = \frac{1}{2}QV = \frac{1}{2}(20 \text{ C})(2.2 \times 10^9 \text{ V}) = 2.2 \times 10^{10} \text{ J}$$

EVALUATE: Thunderclouds involve very large potential differences and large amounts of stored energy.

24.63. IDENTIFY: Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

(a) SET UP: The network is sketched in Figure 24.63a.

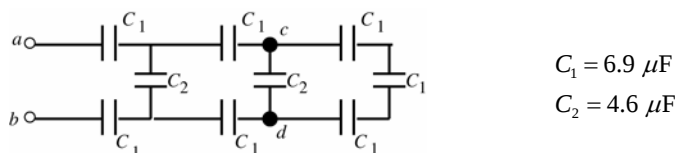


Figure 24.63a

EXECUTE: Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.63b.

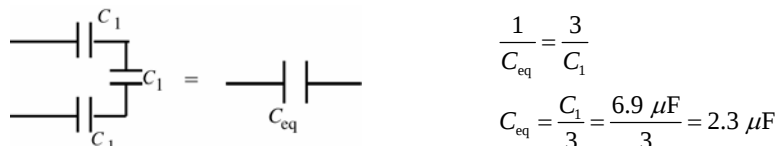


Figure 24.63b

Next make the replacement shown in Figure 24.63c.

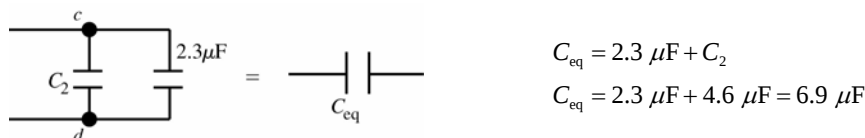


Figure 24.63c

Make the replacement shown in Figure 24.63d.

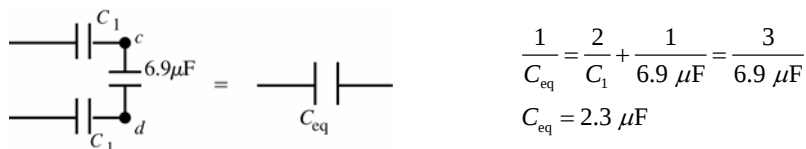


Figure 24.63d

Make the replacement shown in Figure 24.63e.



Figure 24.63e

Make the replacement shown in Figure 24.63f.

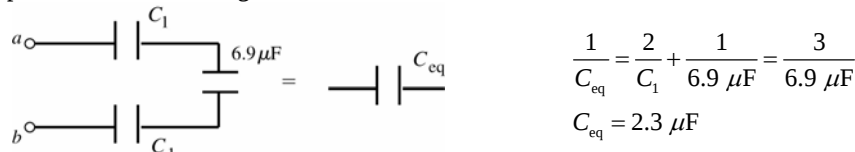


Figure 24.63f

(b) Consider the network as drawn in Figure 24.63g.

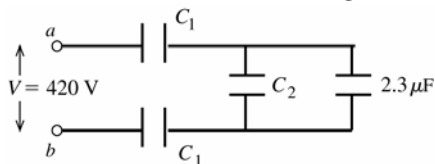


Figure 24.63g

From part (a) $2.3 \mu\text{F}$ is the equivalent capacitance of the rest of the network.

The equivalent network is shown in Figure 24.63h.

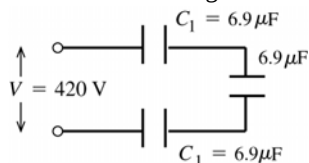


Figure 24.63h

The capacitors are in series, so all three capacitors have the same Q .

But here all three have the same C , so by $V = Q/C$ all three must have the same V . The three voltages must add to 420 V, so each capacitor has $V = 140$ V. The $6.9 \mu\text{F}$ to the right is the equivalent of C_2 and the $2.3 \mu\text{F}$ capacitor in parallel, so $V_2 = 140$ V. (Capacitors in parallel have the same potential difference.) Hence

$$Q_1 = C_1 V_1 = (6.9 \mu\text{F})(140 \text{ V}) = 9.7 \times 10^{-4} \text{ C} \text{ and } Q_2 = C_2 V_2 = (4.6 \mu\text{F})(140 \text{ V}) = 6.4 \times 10^{-4} \text{ C}.$$

(c) From the potentials deduced in part (b) we have the situation shown in Figure 24.63i.

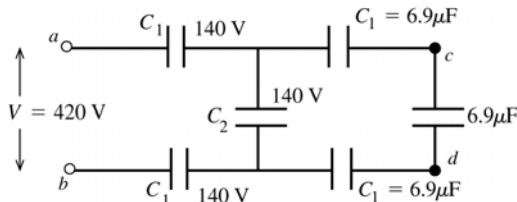


Figure 24.63i

From part (a) $6.9 \mu\text{F}$ is the equivalent capacitance of the rest of the network.

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by $V = Q/C$ they each have the same potential difference. Their potentials must sum to 140 V, so the potential across each is 47 V and $V_{cd} = 47$ V.

EVALUATE: In each capacitor network the rules for combining V for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140 \text{ V}) - 2(47 \text{ V}) = V_{cd}$.

- 24.64. IDENTIFY:** Find the total charge on the capacitor network when it is connected to the battery. This is the amount of charge that flows through the signal device when the switch is closed.

SET UP: For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + C_3 + \dots$

EXECUTE: $C_{\text{equiv}} = C_1 + C_2 + C_3 = 60.0 \mu\text{F}$. $Q = CV = (60.0 \mu\text{F})(120 \text{ V}) = 7200 \mu\text{C}$.

EVALUATE: More charge is stored by the three capacitors in parallel than would be stored in each capacitor used alone.

- 24.65. (a) IDENTIFY and SET UP:** Q is constant. $C = KC_0$; use Eq.(24.1) to relate the dielectric constant K to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$$V_0/V = K, \text{ so } K = (45.0 \text{ V})/(11.5 \text{ V}) = 3.91$$

EVALUATE: Our analysis agrees with Eq.(24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area $2A/3$ and air between the plates and one with area $A/3$ and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.65.

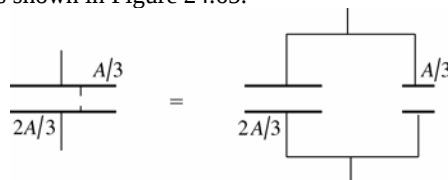


Figure 24.65

EXECUTE: Let $C_0 = \epsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3$, $C_2 = 2C_0/3$;

$$C_{\text{eq}} = C_1 + C_2 = (C_0/3)(K + 2)$$

$$V = \frac{Q}{C_{\text{eq}}} = \frac{Q}{C_0} \left(\frac{3}{K + 2} \right) = V_0 \left(\frac{3}{K + 2} \right) = (45.0 \text{ V}) \left(\frac{3}{5.91} \right) = 22.8 \text{ V}$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.66. IDENTIFY: This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d - a)$.

SET UP: For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$.

EXECUTE: (a) $C = \left(\frac{1}{C_1} + \frac{1}{C_1} \right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2} \frac{\epsilon_0 A}{(d - a)/2} = \frac{\epsilon_0 A}{d - a}$

(b) $C = \frac{\epsilon_0 A}{d - a} = \frac{\epsilon_0 A}{d} \frac{d}{d - a} = C_0 \frac{d}{d - a}$

(c) As $a \rightarrow 0$, $C \rightarrow C_0$. The metal slab has no effect if it is very thin. And as $a \rightarrow d$, $C \rightarrow \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large C .

24.67. (a) IDENTIFY: The conductor can be at some potential V , where $V = 0$ far from the conductor. This potential depends on the charge Q on the conductor so we can define $C = Q/V$ where C will not depend on V or Q .

(b) **SET UP:** Use the expression for the potential at the surface of the sphere in the analysis in part (a).

EXECUTE: For any point on a solid conducting sphere $V = Q/4\pi\epsilon_0 R$ if $V = 0$ at $r \rightarrow \infty$.

$$C = \frac{Q}{V} = Q \left(\frac{4\pi\epsilon_0 R}{Q} \right) = 4\pi\epsilon_0 R$$

(c) $C = 4\pi\epsilon_0 R = 4\pi(8.854 \times 10^{-12} \text{ F/m})(6.38 \times 10^6 \text{ m}) = 7.10 \times 10^{-4} \text{ F} = 710 \mu\text{F}$.

EVALUATE: The capacitance of the earth is about seven times larger than the largest capacitances in this range. The capacitance of the earth is quite small, in view of its large size.

24.68. IDENTIFY: The electric field energy density is $\frac{1}{2}\epsilon_0 E^2$. For a capacitor $U = \frac{Q^2}{2C}$.

SET UP: For a solid conducting sphere of radius R , $E = 0$ for $r < R$ and $E = \frac{Q}{4\pi\epsilon_0 r^2}$ for $r > R$.

EXECUTE: (a) $r < R$: $u = \frac{1}{2}\epsilon_0 E^2 = 0$.

(b) $r > R$: $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left(\frac{Q}{4\pi\epsilon_0 r^2} \right)^2 = \frac{Q^2}{32\pi^2\epsilon_0 r^4}$.

(c) $U = \int u dV = 4\pi \int_R^\infty r^2 u dr = \frac{Q^2}{8\pi\epsilon_0} \int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\epsilon_0 R}$.

(d) This energy is equal to $\frac{1}{2} \frac{Q^2}{4\pi\epsilon_0 R}$ which is just the energy required to assemble all the charge into a spherical

distribution. (Note that being aware of double counting gives the factor of 1/2 in front of the familiar potential energy formula for a charge Q a distance R from another charge Q .)

EVALUATE: (e) From Equation (24.9), $U = \frac{Q^2}{2C}$. $U = \frac{Q^2}{8\pi\epsilon_0 R}$ from part (c), $C = 4\pi\epsilon_0 R$, as in Problem (24.67).

24.69. IDENTIFY: We model the earth as a spherical capacitor.

SET UP: The capacitance of the earth is $C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$ and, the charge on it is $Q = CV$, and its stored energy is

$$U = \frac{1}{2} CV^2.$$

EXECUTE: (a) $C = \frac{1}{9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \frac{(6.38 \times 10^6 \text{ m})(6.45 \times 10^6 \text{ m})}{6.45 \times 10^6 \text{ m} - 6.38 \times 10^6 \text{ m}} = 6.5 \times 10^{-2} \text{ F}$

(b) $Q = CV = (6.54 \times 10^{-2} \text{ F})(350,000 \text{ V}) = 2.3 \times 10^4 \text{ C}$

(c) $U = \frac{1}{2} CV^2 = \frac{1}{2} (6.54 \times 10^{-2} \text{ F})(350,000 \text{ V})^2 = 4.0 \times 10^9 \text{ J}$

EVALUATE: While the capacitance of the earth is larger than ordinary laboratory capacitors, capacitors much larger than this, such as 1 F, are readily available.

24.70. IDENTIFY: The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \frac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \frac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\frac{U}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: (a) $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \frac{\lambda^2}{8\pi^2\epsilon_0 r^2}$

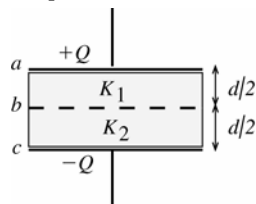
(b) $U = \int u dV = 2\pi L \int u r dr = \frac{L\lambda^2}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{dr}{r}$ and $\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0} \ln(r_b/r_a)$.

(c) Using Equation (24.9), $U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\epsilon_0 L} \ln(r_b/r_a) = \frac{\lambda^2 L}{4\pi\epsilon_0} \ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \frac{Q^2}{2C}$ to calculate U/L and would obtain the same result as in Example 24.4.

24.71. IDENTIFY: $C = Q/V$, so we need to calculate the effect of the dielectrics on the potential difference between the plates.

SET UP: Let the potential of the positive plate be V_a , the potential of the negative plate be V_c , and the potential midway between the plates where the dielectrics meet be V_b , as shown in Figure 24.71.



$$C = \frac{Q}{V_a - V_c} = \frac{Q}{V_{ac}}$$

$$V_{ac} = V_{ab} + V_{bc}.$$

Figure 24.71

EXECUTE: The electric field in the absence of any dielectric is $E_0 = \frac{Q}{\epsilon_0 A}$. In the first dielectric the electric field is

reduced to $E_1 = \frac{E_0}{K_1} = \frac{Q}{K_1\epsilon_0 A}$ and $V_{ab} = E_1 \left(\frac{d}{2}\right) = \frac{Qd}{K_1 2\epsilon_0 A}$. In the second dielectric the electric field is reduced to

$E_2 = \frac{E_0}{K_2} = \frac{Q}{K_2\epsilon_0 A}$ and $V_{bc} = E_2 \left(\frac{d}{2}\right) = \frac{Qd}{K_2 2\epsilon_0 A}$. Thus $V_{ac} = V_{ab} + V_{bc} = \frac{Qd}{K_1 2\epsilon_0 A} + \frac{Qd}{K_2 2\epsilon_0 A} = \frac{Qd}{2\epsilon_0 A} \left(\frac{1}{K_1} + \frac{1}{K_2}\right)$.

$V_{ac} = \frac{Qd}{2\epsilon_0 A} \left(\frac{K_1 + K_2}{K_1 K_2}\right)$. This gives $C = \frac{Q}{V_{ac}} = Q \left(\frac{2\epsilon_0 A}{Qd}\right) \left(\frac{K_1 K_2}{K_1 + K_2}\right) = \frac{2\epsilon_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2}\right)$.

EVALUATE: An equivalent way to calculate C is to consider the capacitor to be two in series, one with dielectric constant K_1 and the other with dielectric constant K_2 and both with plate separation $d/2$. (Can imagine inserting a thin conducting plate between the dielectric slabs.)

$$C_1 = K_1 \frac{\epsilon_0 A}{d/2} = 2K_1 \frac{\epsilon_0 A}{d}$$

$$C_2 = K_2 \frac{\epsilon_0 A}{d/2} = 2K_2 \frac{\epsilon_0 A}{d}$$

Since they are in series the total capacitance C is given by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ so $C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2\epsilon_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2}\right)$

24.72. IDENTIFY: This situation is analogous to having two capacitors in parallel, each with an area $A/2$.

SET UP: For capacitors in parallel, $C_{eq} = C_1 + C_2$. For a parallel-plate capacitor with plates of area $A/2$, $C = \frac{\epsilon_0 (A/2)}{d}$.

EXECUTE: $C_{eq} = C_1 + C_2 = \frac{\epsilon_0 A/2}{d} + \frac{\epsilon_0 A/2}{d} = \frac{\epsilon_0 A}{2d} (K_1 + K_2)$

EVALUATE: If $K_1 = K_2 = K$, $C_{eq} = K \frac{\epsilon_0 A}{d}$, which is Eq.(24.19).

24.73. IDENTIFY and SET UP: Show the transformation from one circuit to the other:

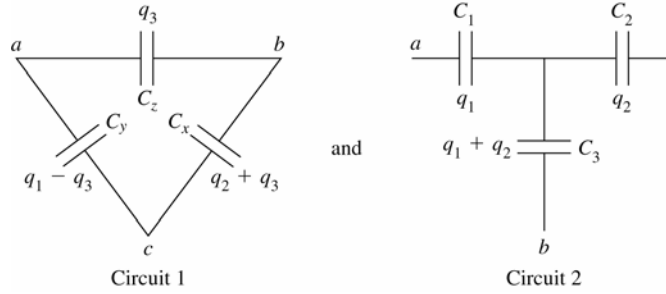


Figure 24.73a

EXECUTE: (a) Consider the two networks shown in Figure 24.73a. From Circuit 1: $V_{ac} = \frac{q_1 - q_3}{C_y}$ and $V_{bc} = \frac{q_2 + q_3}{C_x}$.

$$q_3 \text{ is derived from } V_{ab}: V_{ab} = \frac{q_3}{C_z} = \frac{q_1 - q_3}{C_y} = \frac{q_2 - q_3}{C_x}. \text{ This gives } q_3 = \frac{C_x C_y C_z}{C_x + C_y + C_z} \left(\frac{q_1}{C_y} - \frac{q_2}{C_x} \right) \equiv K \left(\frac{q_1}{C_y} - \frac{q_2}{C_x} \right).$$

From Circuit 2: $V_{ac} = \frac{q_1}{C_1} + \frac{q_1 + q_2}{C_3} = q_1 \left(\frac{1}{C_1} + \frac{1}{C_3} \right) + q_2 \frac{1}{C_3}$ and $V_{bc} = \frac{q_2}{C_2} + \frac{q_1 + q_2}{C_3} = q_1 \frac{1}{C_3} + q_2 \left(\frac{1}{C_2} + \frac{1}{C_3} \right)$. Setting the coefficients of the charges equal to each other in matching potential equations from the two circuits results in three independent equations relating the two sets of capacitances. The set of equations are $\frac{1}{C_1} = \frac{1}{C_y} \left(1 - \frac{1}{KC_y} - \frac{1}{KC_x} \right)$,

$$\frac{1}{C_2} = \frac{1}{C_x} \left(1 - \frac{1}{KC_y} - \frac{1}{KC_x} \right) \text{ and } \frac{1}{C_3} = \frac{1}{KC_y C_x}. \text{ From these, subbing in the expression for } K, \text{ we get}$$

$$C_1 = (C_x C_y + C_y C_z + C_z C_x) / C_x, \quad C_2 = (C_x C_y + C_y C_z + C_z C_x) / C_y \text{ and } C_3 = (C_x C_y + C_y C_z + C_z C_x) / C_z.$$

(b) Using the transformation of part (a) we have the equivalent networks shown in Figure 24.73b:

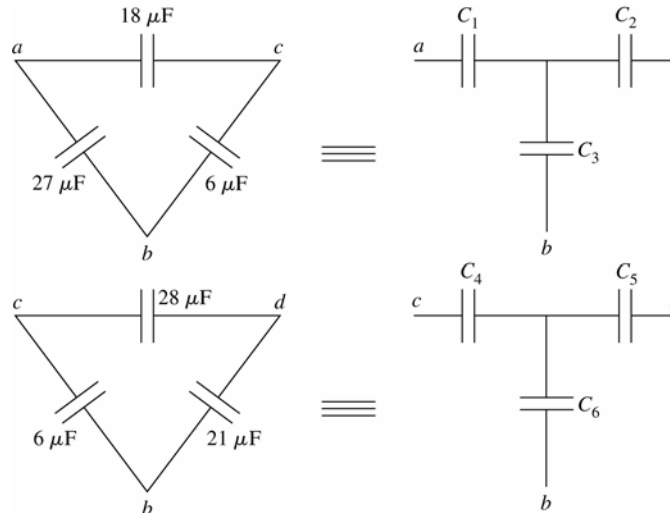


Figure 24.73b

$C_1 = 126 \mu\text{F}$, $C_2 = 28 \mu\text{F}$, $C_3 = 42 \mu\text{F}$, $C_4 = 42 \mu\text{F}$, $C_5 = 147 \mu\text{F}$ and $C_6 = 32 \mu\text{F}$. The total equivalent capacitance

$$\text{is } C_{\text{eq}} = \left(\frac{1}{72 \mu\text{F}} + \frac{1}{126 \mu\text{F}} + \frac{1}{34.8 \mu\text{F}} + \frac{1}{147 \mu\text{F}} + \frac{1}{72 \mu\text{F}} \right)^{-1} = 14.0 \mu\text{F}, \text{ where the } 34.8 \mu\text{F} \text{ comes from}$$

$$34.8 \mu\text{F} = \left(\left(\frac{1}{42 \mu\text{F}} + \frac{1}{32 \mu\text{F}} \right)^{-1} + \left(\frac{1}{28 \mu\text{F}} + \frac{1}{42 \mu\text{F}} \right)^{-1} \right).$$

(c) The circuit diagram can be redrawn as shown in Figure 25.73c. The overall charge is given by $Q = C_{\text{eq}}V = (14.0 \mu\text{F})(36 \text{ V}) = 5.04 \times 10^{-4} \text{ C}$. And this is also the charge on the $72 \mu\text{F}$ capacitors, so

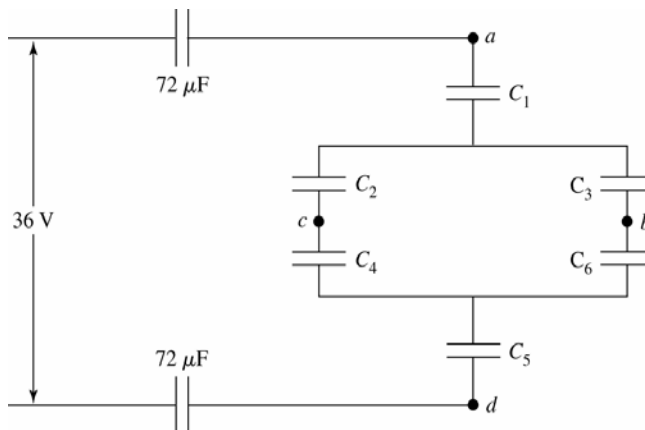
$$V_{72} = \frac{5.04 \times 10^{-4} \text{ C}}{72 \times 10^{-6} \text{ F}} = 7.0 \text{ V}.$$


Figure 24.73c

Next we will find the voltage over the numbered capacitors, and their associated voltages. Then those voltages will be changed back into voltage of the original capacitors, and then their charges. $Q_{C_1} = Q_{C_5} = Q_{72} = 5.04 \times 10^{-4} \text{ C}$.

$$V_{C_5} = \frac{5.04 \times 10^{-4} \text{ C}}{147 \times 10^{-6} \text{ F}} = 3.43 \text{ V} \text{ and } V_{C_1} = \frac{5.04 \times 10^{-4} \text{ C}}{126 \times 10^{-6} \text{ F}} = 4.00 \text{ V}.$$
 Therefore,

$$V_{C_2C_4} = V_{C_3C_6} = (36.0 - 7.00 - 7.00 - 4.00 - 3.43) \text{ V} = 14.6 \text{ V}.$$
 But $C_{\text{eq}}(C_2C_4) = \left(\frac{1}{C_2} + \frac{1}{C_4} \right)^{-1} = 16.8 \mu\text{F}$ and

$$C_{\text{eq}}(C_3C_6) = \left(\frac{1}{C_3} + \frac{1}{C_6} \right)^{-1} = 18.2 \mu\text{F},$$
 so $Q_{C_2} = Q_{C_4} = V_{C_2C_4} C_{\text{eq}}(C_2C_4) = 2.45 \times 10^{-4} \text{ C}$ and

$$Q_{C_3} = Q_{C_6} = V_{C_3C_6} C_{\text{eq}}(C_3C_6) = 2.64 \times 10^{-4} \text{ C}.$$
 Then $V_{C_2} = \frac{Q_{C_2}}{C_2} = 8.8 \text{ V}$, $V_{C_3} = \frac{Q_{C_3}}{C_3} = 6.3 \text{ V}$, $V_{C_4} = \frac{Q_{C_4}}{C_4} = 5.8 \text{ V}$ and

$$V_{C_6} = \frac{Q_{C_6}}{C_6} = 8.3 \text{ V}.$$
 $V_{ac} = V_{C_1} + V_{C_2} = V_{18} = 13 \text{ V}$ and $Q_{18} = C_{18}V_{18} = 2.3 \times 10^{-4} \text{ C}$. $V_{ab} = V_{C_1} + V_{C_3} = V_{27} = 10 \text{ V}$ and

$$Q_{27} = C_{27}V_{27} = 2.8 \times 10^{-4} \text{ C}.$$
 $V_{cd} = V_{C_4} + V_{C_5} = V_{28} = 9 \text{ V}$ and $Q_{28} = C_{28}V_{28} = 2.6 \times 10^{-4} \text{ C}$. $V_{bd} = V_{C_5} + V_{C_6} = V_{21} = 12 \text{ V}$ and $Q_{21} = C_{21}V_{21} = 2.5 \times 10^{-4} \text{ C}$. $V_{bc} = V_{C_3} - V_{C_2} = V_6 = 2.5 \text{ V}$ and $Q_6 = C_6V_6 = 1.5 \times 10^{-5} \text{ C}$.

EVALUATE: Note that $2V_{72} + V_{18} + V_{28} = 2(7.0 \text{ V}) + 13 \text{ V} + 9 \text{ V} = 36 \text{ V}$, as it should.

24.74. IDENTIFY: The force on one plate is due to the electric field of the other plate. The electrostatic force must be balanced by the forces from the springs.

SET UP: The electric field due to one plate is $E = \frac{\sigma}{2\epsilon_0}$. The force exerted by a spring compressed a distance $z_0 - z$ from equilibrium is $k(z_0 - z)$.

EXECUTE: (a) The force between the two parallel plates is $F = qE = \frac{q\sigma}{2\epsilon_0} = \frac{q^2}{2\epsilon_0 A} = \frac{(CV)^2}{2\epsilon_0 A} = \frac{\epsilon_0^2 A^2}{z^2} \frac{V^2}{2\epsilon_0 A} = \frac{\epsilon_0 AV^2}{2z^2}$.

(b) When $V = 0$, the separation is just z_0 . When $V \neq 0$, the total force from the four springs must equal the electrostatic force calculated in part (a). $F_{4\text{springs}} = 4k(z_0 - z) = \frac{\epsilon_0 AV^2}{2z^2}$ and $2z^3 - 2z^3 z_0 + \frac{\epsilon_0 AV^2}{4k} = 0$.

(c) For $A = 0.300 \text{ m}^2$, $z_0 = 1.2 \times 10^{-3} \text{ m}$, $k = 25 \text{ N/m}$ and $V = 120 \text{ V}$, so $2z^3 - (2.4 \times 10^{-3} \text{ m})z^2 + 3.82 \times 10^{-10} \text{ m}^3 = 0$. The physical solutions to this equation are $z = 0.537 \text{ mm}$ and 1.014 mm .

EVALUATE: (d) Stable equilibrium occurs if a slight displacement from equilibrium yields a force back toward the equilibrium point. If one evaluates the forces at small displacements from the equilibrium positions above, the 1.014 mm separation is seen to be stable, but not the 0.537 mm separation.

24.75. IDENTIFY: The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C = \frac{K\epsilon_0 A}{d}$ for a parallel-plate capacitor with plate area A .

EXECUTE: (a) $C = \frac{\epsilon_0}{D}((L-x)L + xKL) = \frac{\epsilon_0 L}{D}(L + (K-1)x)$

(b) $dU = \frac{1}{2}(dC)V^2$, where $C = C_0 + \frac{\epsilon_0 L}{D}(-dx + dxK)$, with $C_0 = \frac{\epsilon_0 L}{D}(L + (K-1)x)$. This gives

$$dU = \frac{1}{2}\left(\frac{\epsilon_0 L dx}{D}(K-1)\right)V^2 = \frac{(K-1)\epsilon_0 V^2 L}{2D}dx.$$

(c) If the charge is kept constant on the plates, then $Q = \frac{\epsilon_0 LV}{D}(L + (K-1)x)$ and $U = \frac{1}{2}CV^2 = \frac{1}{2}C_0V^2\left(\frac{C}{C_0}\right)$.

$$U \approx \frac{C_0 V^2}{2}\left(1 - \frac{\epsilon_0 L}{DC_0}(K-1)dx\right) \text{ and } \Delta U = U - U_0 = -\frac{(K-1)\epsilon_0 V^2 L}{2D}dx.$$

(d) Since $dU = -Fdx = -\frac{(K-1)\epsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx , meaning that the slab feels a force pushing it out.

EVALUATE: (e) When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU = -Fdx$ gives $F = \frac{(K-1)\epsilon_0 V^2 L}{2D}$.

24.76. IDENTIFY: $C = Q/V$. Apply Gauss's law and the relation between potential difference and electric field.

SET UP: Each conductor is an equipotential surface. $V_a - V_b = \int_{r_a}^{r_b} \vec{E}_U \cdot d\vec{r} = \int_{r_a}^{r_b} \vec{E}_L \cdot d\vec{r}$, so $E_U = E_L$, where these are the fields between the upper and lower hemispheres. The electric field is the same in the air space as in the dielectric.

EXECUTE: (a) For a normal spherical capacitor with air between the plates, $C_0 = 4\pi\epsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right)$. The capacitor in

this problem is equivalent to two parallel capacitors, C_L and C_U , each with half the plate area of the normal

capacitor. $C_L = \frac{KC_0}{2} = 2\pi K\epsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right)$ and $C_U = \frac{C_0}{2} = 2\pi\epsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right)$. $C = C_U + C_L = 2\pi\epsilon_0(1+K)\left(\frac{r_a r_b}{r_b - r_a}\right)$.

(b) Using a hemispherical Gaussian surface for each respective half, $E_L \frac{4\pi r^2}{2} = \frac{Q_L}{K\epsilon_0}$, so $E_L = \frac{Q_L}{2\pi K\epsilon_0 r^2}$, and

$E_U \frac{4\pi r^2}{2} = \frac{Q_U}{\epsilon_0}$, so $E_U = \frac{Q_U}{2\pi\epsilon_0 r^2}$. But $Q_L = VC_L$ and $Q_U = VC_U$. Also, $Q_L + Q_U = Q$. Therefore, $Q_L = \frac{VC_0 K}{2} = KQ_U$

and $Q_U = \frac{Q}{1+K}$, $Q_L = \frac{KQ}{1+K}$. This gives $E_L = \frac{KQ}{1+K} \frac{1}{2\pi K\epsilon_0 r^2} = \frac{2}{1+K} \frac{Q}{4\pi\epsilon_0 r^2}$ and

$E_U = \frac{Q}{1+K} \frac{1}{2\pi\epsilon_0 r^2} = \frac{2}{1+K} \frac{Q}{4\pi\epsilon_0 r^2}$. We do find that $E_U = E_L$.

(c) The free charge density on upper and lower hemispheres are: $(\sigma_{f,r_a})_U = \frac{Q_U}{2\pi r_a^2} = \frac{Q}{2\pi r_a^2(1+K)}$ and

$(\sigma_{f,r_b})_U = \frac{Q_U}{2\pi r_b^2} = \frac{Q}{2\pi r_b^2(1+K)}$; $(\sigma_{f,r_a})_L = \frac{Q_L}{2\pi r_a^2} = \frac{KQ}{2\pi r_a^2(1+K)}$ and $(\sigma_{f,r_b})_L = \frac{Q_L}{2\pi r_b^2} = \frac{KQ}{2\pi r_b^2(1+K)}$.

(d) $\sigma_{i,r_a} = \sigma_{f,r_a}(1-1/K) = \left(\frac{K-1}{K}\right)\frac{Q}{2\pi r_a^2}\left(\frac{K}{K+1}\right) = \left(\frac{K-1}{K+1}\right)\frac{Q}{2\pi r_a^2}$

$\sigma_{i,r_b} = \sigma_{f,r_b}(1-1/K) = \left(\frac{K-1}{K}\right)\frac{Q}{2\pi r_b^2}\left(\frac{K}{K+1}\right) = \left(\frac{K-1}{K+1}\right)\frac{Q}{2\pi r_b^2}$

(e) There is zero bound charge on the flat surface of the dielectric-air interface, or else that would imply a circumferential electric field, or that the electric field changed as we went around the sphere.

EVALUATE: The charge is not equally distributed over the surface of each conductor. There must be more charge on the lower half, by a factor of K , because the polarization of the dielectric means more free charge is needed on the lower half to produce the same electric field.

- 24.77. IDENTIFY:** The object is equivalent to two identical capacitors in parallel, where each has the same area A , plate separation d and dielectric with dielectric constant K .

SET UP: For each capacitor in the parallel combination, $C = \frac{\epsilon_0 A}{d}$.

EXECUTE: (a) The charge distribution on the plates is shown in Figure 24.77.

(b) $C = 2 \left(\frac{\epsilon_0 A}{d} \right) = \frac{2(4.2)\epsilon_0(0.120 \text{ m})^2}{4.5 \times 10^{-4} \text{ m}} = 2.38 \times 10^{-9} \text{ F}.$

EVALUATE: If two of the plates are separated by both sheets of paper to form a capacitor, $C = \frac{\epsilon_0 A}{2d} = \frac{2.38 \times 10^{-9} \text{ F}}{4}$, smaller by a factor of 4 compared to the capacitor in the problem.

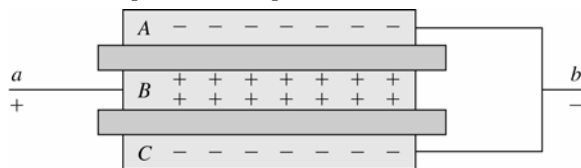


Figure 24.77

- 24.78. IDENTIFY:** As in Problem 24.72, the system is equivalent to two capacitors in parallel. One of the capacitors has plate separation d , plate area $w(L-h)$ and air between the plates. The other has the same plate separation d , plate area wh and dielectric constant K .

SET UP: Define K_{eff} by $C_{\text{eq}} = \frac{K_{\text{eff}} \epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: (a) The capacitors are in parallel, so $C = \frac{\epsilon_0 w(L-h)}{d} + \frac{K \epsilon_0 wh}{d} = \frac{\epsilon_0 wL}{d} \left(1 + \frac{Kh}{L} - \frac{h}{L} \right)$. This gives

$$K_{\text{eff}} = \left(1 + \frac{Kh}{L} - \frac{h}{L} \right).$$

(b) For gasoline, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}} \left(h = \frac{L}{4} \right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}} \left(h = \frac{L}{2} \right) = 1.48$;

$\frac{3}{4}$ full: $K_{\text{eff}} \left(h = \frac{3L}{4} \right) = 1.71$.

(c) For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}} \left(h = \frac{L}{4} \right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}} \left(h = \frac{L}{2} \right) = 17$; $\frac{3}{4}$ full: $K_{\text{eff}} \left(h = \frac{3L}{4} \right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

EVALUATE: When $h = 0$, $K_{\text{eff}} = 1$. When $h = L$, $K_{\text{eff}} = K$.

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

25.1. **IDENTIFY:** $I = Q/t$.

SET UP: $1.0 \text{ h} = 3600 \text{ s}$

EXECUTE: $Q = It = (3.6 \text{ A})(3600 \text{ s}) = 3.89 \times 10^4 \text{ C}$.

EVALUATE: Compared to typical charges of objects in electrostatics, this is a huge amount of charge.

25.2. **IDENTIFY:** $I = Q/t$. Use $I = n|q|v_d A$ to calculate the drift velocity v_d .

SET UP: $n = 5.8 \times 10^{28} \text{ m}^{-3}$. $|q| = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: (a) $I = \frac{Q}{t} = \frac{420 \text{ C}}{80(60 \text{ s})} = 8.75 \times 10^{-2} \text{ A}$.

(b) $I = n|q|v_d A$. This gives $v_d = \frac{I}{nqA} = \frac{8.75 \times 10^{-2} \text{ A}}{(5.8 \times 10^{28})(1.60 \times 10^{-19} \text{ C})(\pi(1.3 \times 10^{-3} \text{ m})^2)} = 1.78 \times 10^{-6} \text{ m/s}$.

EVALUATE: v_d is smaller than in Example 25.1, because I is smaller in this problem.

25.3. **IDENTIFY:** $I = Q/t$. $J = I/A$. $J = n|q|v_d$

SET UP: $A = (\pi/4)D^2$, with $D = 2.05 \times 10^{-3} \text{ m}$. The charge of an electron has magnitude $+e = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: (a) $Q = It = (5.00 \text{ A})(1.00 \text{ s}) = 5.00 \text{ C}$. The number of electrons is $\frac{Q}{e} = 3.12 \times 10^{19}$.

(b) $J = \frac{I}{(\pi/4)D^2} = \frac{5.00 \text{ A}}{(\pi/4)(2.05 \times 10^{-3} \text{ m})^2} = 1.51 \times 10^6 \text{ A/m}^2$.

(c) $v_d = \frac{J}{n|q|} = \frac{1.51 \times 10^6 \text{ A/m}^2}{(8.5 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})} = 1.11 \times 10^{-4} \text{ m/s} = 0.111 \text{ mm/s}$.

EVALUATE: (a) If I is the same, $J = I/A$ would decrease and v_d would decrease. The number of electrons passing through the light bulb in 1.00 s would not change.

25.4. (a) **IDENTIFY:** By definition, $J = I/A$ and radius is one-half the diameter.

SET UP: Solve for the current: $I = JA = J\pi(D/2)^2$

EXECUTE: $I = (1.50 \times 10^6 \text{ A/m}^2)(\pi)[(0.00102 \text{ m})/2]^2 = 1.23 \text{ A}$

EVALUATE: This is a realistic current.

(b) **IDENTIFY:** The current density is $J = nqv_d$

SET UP: Solve for the drift velocity: $v_d = J/nq$

EXECUTE: Since most laboratory wire is copper, we use the value of n for copper, giving

$v_d = (1.50 \times 10^6 \text{ A/m}^2)/[(8.5 \times 10^{28} \text{ el/m}^3)(1.60 \times 10^{-19} \text{ C})] = 1.1 \times 10^{-4} \text{ m/s} = 0.11 \text{ mm/s}$

EVALUATE: This is a typical drift velocity for ordinary currents and wires.

25.5. **IDENTIFY and SET UP:** Use Eq. (25.3) to calculate the drift speed and then use that to find the time to travel the length of the wire.

EXECUTE: (a) Calculate the drift speed v_d :

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4.85 \text{ A}}{\pi(1.025 \times 10^{-3} \text{ m})^2} = 1.469 \times 10^6 \text{ A/m}^2$$

$$v_d = \frac{J}{n|q|} = \frac{1.469 \times 10^6 \text{ A/m}^2}{(8.5 \times 10^{28} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})} = 1.079 \times 10^{-4} \text{ m/s}$$

$$t = \frac{L}{v_d} = \frac{0.710 \text{ m}}{1.079 \times 10^{-4} \text{ m/s}} = 6.58 \times 10^3 \text{ s} = 110 \text{ min.}$$

$$(b) v_d = \frac{I}{\pi r^2 n |q|}$$

$$t = \frac{L}{v_d} = \frac{\pi r^2 n |q| L}{I}$$

t is proportional to r^2 and hence to d^2 where $d = 2r$ is the wire diameter.

$$t = (6.58 \times 10^3 \text{ s}) \left(\frac{4.12 \text{ mm}}{2.05 \text{ mm}} \right)^2 = 2.66 \times 10^4 \text{ s} = 440 \text{ min.}$$

(c) EVALUATE: The drift speed is proportional to the current density and therefore it is inversely proportional to the square of the diameter of the wire. Increasing the diameter by some factor decreases the drift speed by the square of that factor.

- 25.6. IDENTIFY:** The number of moles of copper atoms is the mass of 1.00 m^3 divided by the atomic mass of copper. There are $N_A = 6.023 \times 10^{23}$ atoms per mole.

SET UP: The atomic mass of copper is 63.55 g/mole , and its density is 8.96 g/cm^3 . Example 25.1 says there are 8.5×10^{28} free electrons per m^3 .

EXECUTE: The number of copper atoms in 1.00 m^3 is

$$\frac{(8.96 \text{ g/cm}^3)(1.00 \times 10^6 \text{ cm}^3/\text{m}^3)(6.023 \times 10^{23} \text{ atoms/mole})}{63.55 \text{ g/mole}} = 8.49 \times 10^{28} \text{ atoms/m}^3.$$

EVALUATE: Since there are the same number of free electrons/ m^3 as there are atoms of copper/ m^3 , the number of free electrons per copper atom is one.

- 25.7. IDENTIFY and SET UP:** Apply Eq. (25.1) to find the charge dQ in time dt . Integrate to find the total charge in the whole time interval.

EXECUTE: (a) $dQ = I dt$

$$Q = \int_0^{8.0 \text{ s}} (55 \text{ A} - (0.65 \text{ A/s}^2)t^2) dt = \left[(55 \text{ A})t - (0.217 \text{ A/s}^2)t^3 \right]_0^{8.0 \text{ s}}$$

$$Q = (55 \text{ A})(8.0 \text{ s}) - (0.217 \text{ A/s}^2)(8.0 \text{ s})^3 = 330 \text{ C}$$

$$(b) I = \frac{Q}{t} = \frac{330 \text{ C}}{8.0 \text{ s}} = 41 \text{ A}$$

EVALUATE: The current decreases from 55 A to 13.4 A during the interval. The decrease is not linear and the average current is not equal to $(55 \text{ A} + 13.4 \text{ A}) / 2$.

- 25.8. IDENTIFY:** $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$

EXECUTE: (a) $Q_{\text{total}} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92 \times 10^{16} + 2.68 \times 10^{16})(1.60 \times 10^{-19} \text{ C}) = 0.0106 \text{ C}$. Then

$$I = \frac{Q_{\text{total}}}{t} = \frac{0.0106 \text{ C}}{1.00 \text{ s}} = 0.0106 \text{ A} = 10.6 \text{ mA}.$$

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

- 25.9. IDENTIFY:** The number of moles of silver atoms is the mass of 1.00 m^3 divided by the atomic mass of silver. There are $N_A = 6.023 \times 10^{23}$ atoms per mole.

SET UP: For silver, density $= 10.5 \times 10^3 \text{ kg/m}^3$ and the atomic mass is $M = 107.868 \times 10^{-3} \text{ kg/mol}$.

EXECUTE: Consider 1 m^3 of silver. $m = (\text{density})V = 10.5 \times 10^3 \text{ kg}$. $n = m/M = 9.734 \times 10^4 \text{ mol}$ and the number of atoms is $N = nN_A = 5.86 \times 10^{28}$ atoms. If there is one free electron per atom, there are 5.86×10^{28} free electrons/ m^3 . This agrees with the value given in Exercise 25.2.

EVALUATE: Our result verifies that for silver there is approximately one free electron per atom. Exercise 25.6 showed that for copper there is also one free electron per atom.

- 25.10. (a) IDENTIFY:** Start with the definition of resistivity and solve for E .

SET UP: $E = \rho J = \rho I / \pi r^2$

EXECUTE: $E = (1.72 \times 10^{-8} \Omega \cdot \text{m})(2.75 \text{ A}) / [\pi(0.001025 \text{ m})^2] = 1.43 \times 10^{-2} \text{ V/m}$

EVALUATE: The field is quite weak, since the potential would drop only a volt in 70 m of wire.

(b) IDENTIFY: Take the ratio of the field in silver to the field in copper.

SET UP: Take the ratio and solve for the field in silver: $E_s = E_c(\rho_s/\rho_c)$

EXECUTE: $E_s = (0.0143 \text{ V/m})[(1.47)/(1.72)] = 1.22 \times 10^{-2} \text{ V/m}$

EVALUATE: Since silver is a better conductor than copper, the field in silver is smaller than the field in copper.

- 25.11. IDENTIFY:** First use Ohm's law to find the resistance at 20.0°C ; then calculate the resistivity from the resistance. Finally use the dependence of resistance on temperature to calculate the temperature coefficient of resistance.

SET UP: Ohm's law is $R = V/I$, $R = \rho L/A$, $R = R_0[1 + \alpha(T - T_0)]$, and the radius is one-half the diameter.

EXECUTE: **(a)** At 20.0°C , $R = V/I = (15.0 \text{ V})/(18.5 \text{ A}) = 0.811 \Omega$. Using $R = \rho L/A$ and solving for ρ gives $\rho = RA/L = R\pi(D/2)^2/L = (0.811 \Omega)\pi[(0.00500 \text{ m})/2]^2/(1.50 \text{ m}) = 1.06 \times 10^{-6} \Omega \cdot \text{m}$.

(b) At 92.0°C , $R = V/I = (15.0 \text{ V})/(17.2 \text{ A}) = 0.872 \Omega$. Using $R = R_0[1 + \alpha(T - T_0)]$ with T_0 taken as 20.0°C , we have $0.872 \Omega = (0.811 \Omega)[1 + \alpha(92.0^\circ\text{C} - 20.0^\circ\text{C})]$. This gives $\alpha = 0.00105 \text{ (}^\circ\text{C)}^{-1}$

EVALUATE: The results are typical of ordinary metals.

- 25.12. IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_d A$.

SET UP: For copper, $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$. $n = 8.5 \times 10^{28} / \text{m}^3$.

EXECUTE: **(a)** $J = \frac{I}{A} = \frac{3.6 \text{ A}}{(2.3 \times 10^{-3} \text{ m})^2} = 6.81 \times 10^5 \text{ A/m}^2$.

(b) $E = \rho J = (1.72 \times 10^{-8} \Omega \cdot \text{m})(6.81 \times 10^5 \text{ A/m}^2) = 0.012 \text{ V/m}$.

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_d} = \frac{ln|q|A}{I} = \frac{(4.0 \text{ m})(8.5 \times 10^{28} / \text{m}^3)(1.6 \times 10^{-19} \text{ C})(2.3 \times 10^{-3} \text{ m})^2}{3.6 \text{ A}} = 8.0 \times 10^4 \text{ s}.$$

$$t = 1333 \text{ min} \approx 22 \text{ hrs!}$$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

- 25.13. IDENTIFY:** $E = \rho J$, where $J = I/A$.

SET UP: For tungsten $\rho = 5.25 \times 10^{-8} \Omega \cdot \text{m}$ and for aluminum $\rho = 2.75 \times 10^{-8} \Omega \cdot \text{m}$.

EXECUTE: **(a)** tungsten: $E = \rho J = \frac{\rho I}{A} = \frac{(5.25 \times 10^{-8} \Omega \cdot \text{m})(0.820 \text{ A})}{(\pi/4)(3.26 \times 10^{-3} \text{ m})^2} = 5.16 \times 10^{-3} \text{ V/m}$.

(b) aluminum: $E = \rho J = \frac{\rho I}{A} = \frac{(2.75 \times 10^{-8} \Omega \cdot \text{m})(0.820 \text{ A})}{(\pi/4)(3.26 \times 10^{-3} \text{ m})^2} = 2.70 \times 10^{-3} \text{ V/m}$.

EVALUATE: A larger electric field is required for tungsten, because it has a larger resistivity.

- 25.14. IDENTIFY:** The resistivity of the wire should identify what the material is.

SET UP: $R = \rho L/A$ and the radius of the wire is half its diameter.

EXECUTE: Solve for ρ and substitute the numerical values.

$$\rho = AR/L = \pi(D/2)^2 R/L = \frac{\pi([0.00205 \text{ m})/2]^2 (0.0290 \Omega)}{6.50 \text{ m}} = 1.47 \times 10^{-8} \Omega \cdot \text{m}$$

EVALUATE: This result is the same as the resistivity of silver, which implies that the material is silver.

- 25.15. (a) IDENTIFY:** Start with the definition of resistivity and use its dependence on temperature to find the electric field.

SET UP: $E = \rho J = \rho_{20}[1 + \alpha(T - T_0)] \frac{I}{\pi r^2}$

EXECUTE: $E = (5.25 \times 10^{-8} \Omega \cdot \text{m})[1 + (0.0045/^\circ\text{C})(120^\circ\text{C} - 20^\circ\text{C})](12.5 \text{ A})/[\pi(0.000500 \text{ m})^2] = 1.21 \text{ V/m}$.

(Note that the resistivity at 120°C turns out to be $7.61 \times 10^{-8} \Omega \cdot \text{m}$.)

EVALUATE: This result is fairly large because tungsten has a larger resistivity than copper.

(b) IDENTIFY: Relate resistance and resistivity.

SET UP: $R = \rho L/A = \rho L/\pi r^2$

EXECUTE: $R = (7.61 \times 10^{-8} \Omega \cdot \text{m})(0.150 \text{ m})/[\pi(0.000500 \text{ m})^2] = 0.0145 \Omega$

EVALUATE: Most metals have very low resistance.

(c) IDENTIFY: The potential difference is proportional to the length of wire.

SET UP: $V = EL$

EXECUTE: $V = (1.21 \text{ V/m})(0.150 \text{ m}) = 0.182 \text{ V}$

EVALUATE: We could also calculate $V = IR = (12.5 \text{ A})(0.0145 \Omega) = 0.181 \text{ V}$, in agreement with part (c).

- 25.16. IDENTIFY:** Apply $R = \frac{\rho L}{A}$ and solve for L .

SET UP: $A = \pi D^2 / 4$, where $D = 0.462$ mm.

EXECUTE: $L = \frac{RA}{\rho} = \frac{(1.00 \Omega)(\pi/4)(0.462 \times 10^{-3} \text{ m})^2}{1.72 \times 10^{-8} \Omega \cdot \text{m}} = 9.75 \text{ m}.$

EVALUATE: The resistance is proportional to the length of the wire.

- 25.17. IDENTIFY:** $R = \frac{\rho L}{A}.$

SET UP: For copper, $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$. $A = \pi r^2$.

EXECUTE: $R = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(24.0 \text{ m})}{\pi(1.025 \times 10^{-3} \text{ m})^2} = 0.125 \Omega$

EVALUATE: The resistance is proportional to the length of the piece of wire.

- 25.18. IDENTIFY:** $R = \frac{\rho L}{A} = \frac{\rho L}{\pi d^2 / 4}.$

SET UP: For aluminum, $\rho_{\text{al}} = 2.63 \times 10^{-8} \Omega \cdot \text{m}$. For copper, $\rho_{\text{c}} = 1.72 \times 10^{-8} \Omega \cdot \text{m}$.

EXECUTE: $\frac{\rho}{d^2} = \frac{R\pi}{4L} = \text{constant}$, so $\frac{\rho_{\text{al}}}{d_{\text{al}}^2} = \frac{\rho_{\text{c}}}{d_{\text{c}}^2}$. $d_{\text{c}} = d_{\text{al}} \sqrt{\frac{\rho_{\text{c}}}{\rho_{\text{al}}}} = (3.26 \text{ mm}) \sqrt{\frac{1.72 \times 10^{-8} \Omega \cdot \text{m}}{2.63 \times 10^{-8} \Omega \cdot \text{m}}} = 2.64 \text{ mm}.$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

- 25.19. IDENTIFY and SET UP:** Use Eq. (25.10) to calculate A . Find the volume of the wire and use the density to calculate the mass.

EXECUTE: Find the volume of one of the wires:

$R = \frac{\rho L}{A}$ so $A = \frac{\rho L}{R}$ and

$\text{volume} = AL = \frac{\rho L^2}{R} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(3.50 \text{ m})^2}{0.125 \Omega} = 1.686 \times 10^{-6} \text{ m}^3$

$m = (\text{density})V = (8.9 \times 10^3 \text{ kg/m}^3)(1.686 \times 10^{-6} \text{ m}^3) = 15 \text{ g}$

EVALUATE: The mass we calculated is reasonable for a wire.

- 25.20. IDENTIFY:** $R = \frac{\rho L}{A}.$

SET UP: The length of the wire in the spring is the circumference πd of each coil times the number of coils.

EXECUTE: $L = (75)\pi d = (75)\pi(3.50 \times 10^{-2} \text{ m}) = 8.25 \text{ m}.$

$A = \pi r^2 = \pi d^2 / 4 = \pi(3.25 \times 10^{-3} \text{ m})^2 / 4 = 8.30 \times 10^{-6} \text{ m}^2.$

$\rho = \frac{RA}{L} = \frac{(1.74 \Omega)(8.30 \times 10^{-6} \text{ m}^2)}{8.25 \text{ m}} = 1.75 \times 10^{-6} \Omega \cdot \text{m}.$

EVALUATE: The value of ρ we calculated is about a factor of 100 times larger than ρ for copper. The metal of the spring is not a very good conductor.

- 25.21. IDENTIFY:** $R = \frac{\rho L}{A}.$

SET UP: $L = 1.80 \text{ m}$, the length of one side of the cube. $A = L^2$.

EXECUTE: $R = \frac{\rho L}{A} = \frac{\rho L}{L^2} = \frac{\rho}{L} = \frac{2.75 \times 10^{-8} \Omega \cdot \text{m}}{1.80 \text{ m}} = 1.53 \times 10^{-8} \Omega$

EVALUATE: The resistance is very small because A is very much larger than the typical value for a wire.

- 25.22. IDENTIFY:** Apply $R_T = R_0(1 + \alpha(T - T_0))$.

SET UP: Since $V = IR$ and V is the same, $\frac{R_T}{R_{20}} = \frac{I_{20}}{I_T}$. For tungsten, $\alpha = 4.5 \times 10^{-3} (\text{C}^\circ)^{-1}$.

EXECUTE: The ratio of the current at 20°C to that at the higher temperature is $(0.860\text{ A})/(0.220\text{ A}) = 3.909$.

$$\frac{R_T}{R_{20}} = 1 + \alpha(T - T_0) = 3.909, \text{ where } T_0 = 20^{\circ}\text{C}.$$

$$T = T_0 + \frac{R_T/R_{20} - 1}{\alpha} = 20^{\circ}\text{C} + \frac{3.909 - 1}{4.5 \times 10^{-3} (\text{C}^{\circ})^{-1}} = 666^{\circ}\text{C}.$$

EVALUATE: As the temperature increases, the resistance increases and for constant applied voltage the current decreases. The resistance increases by nearly a factor of four.

25.23. IDENTIFY: Relate resistance to resistivity.

SET UP: $R = \rho L/A$

EXECUTE: (a) $R = \rho L/A = (0.60\ \Omega \cdot \text{m})(0.25\text{ m})/(0.12\text{ m})^2 = 10.4\ \Omega$

(b) $R = \rho L/A = (0.60\ \Omega \cdot \text{m})(0.12\text{ m})/(0.12\text{ m})(0.25\text{ m}) = 2.4\ \Omega$

EVALUATE: The resistance is greater for the faces that are farther apart.

25.24. IDENTIFY: Apply $R = \frac{\rho L}{A}$ and $V = IR$.

SET UP: $A = \pi r^2$

EXECUTE: $\rho = \frac{RA}{L} = \frac{VA}{IL} = \frac{(4.50\text{ V})\pi(6.54 \times 10^{-4}\text{ m})^2}{(17.6\text{ A})(2.50\text{ m})} = 1.37 \times 10^{-7}\ \Omega \cdot \text{m}.$

EVALUATE: Our result for ρ shows that the wire is made of a metal with resistivity greater than that of good metallic conductors such as copper and aluminum.

25.25. IDENTIFY and SET UP: Eq. (25.5) relates the electric field that is given to the current density. $V = EL$ gives the potential difference across a length L of wire and Eq. (25.11) allows us to calculate R .

EXECUTE: (a) Eq. (25.5): $\rho = E/J$ so $J = E/\rho$

From Table 25.1 the resistivity for gold is $2.44 \times 10^{-8}\ \Omega \cdot \text{m}$.

$$J = \frac{E}{\rho} = \frac{0.49\text{ V/m}}{2.44 \times 10^{-8}\ \Omega \cdot \text{m}} = 2.008 \times 10^7\text{ A/m}^2$$

$$I = JA = J\pi r^2 = (2.008 \times 10^7\text{ A/m}^2)\pi(0.41 \times 10^{-3}\text{ m})^2 = 11\text{ A}$$

(b) $V = EL = (0.49\text{ V/m})(6.4\text{ m}) = 3.1\text{ V}$

(c) We can use Ohm's law (Eq. (25.11)): $V = IR$.

$$R = \frac{V}{I} = \frac{3.1\text{ V}}{11\text{ A}} = 0.28\ \Omega$$

EVALUATE: We can also calculate R from the resistivity and the dimensions of the wire (Eq. 25.10):

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{(2.44 \times 10^{-8}\ \Omega \cdot \text{m})(6.4\text{ m})}{\pi(0.42 \times 10^{-3}\text{ m})^2} = 0.28\ \Omega, \text{ which checks.}$$

25.26. IDENTIFY and SET UP: Use $V = EL$ to calculate E and then $\rho = E/J$ to calculate ρ .

EXECUTE: (a) $E = \frac{V}{L} = \frac{0.938\text{ V}}{0.750\text{ m}} = 1.25\text{ V/m}$

(b) $E = \rho J$ so $\rho = \frac{E}{J} = \frac{1.25\text{ V/m}}{4.40 \times 10^7\text{ A/m}^2} = 2.84 \times 10^{-8}\ \Omega \cdot \text{m}$

EVALUATE: This value of ρ is similar to that for the good metallic conductors in Table 25.1.

25.27. IDENTIFY: Apply $R = R_0[1 + \alpha(T - T_0)]$ to calculate the resistance at the second temperature.

(a) **SET UP:** $\alpha = 0.0004\ (\text{C}^{\circ})^{-1}$ (Table 25.1). Let T_0 be 0.0°C and T be 11.5°C .

EXECUTE: $R_0 = \frac{R}{1 + \alpha(T - T_0)} = \frac{100.0\ \Omega}{1 + (0.0004\ (\text{C}^{\circ})^{-1})(11.5\text{ C}^{\circ})} = 99.54\ \Omega$

(b) **SET UP:** $\alpha = -0.0005\ (\text{C}^{\circ})^{-1}$ (Table 25.2). Let T_0 be 0.0°C and $T = 25.8^{\circ}\text{C}$.

EXECUTE: $R = R_0[1 + \alpha(T - T_0)] = 0.0160\ \Omega[1 + (-0.0005\ (\text{C}^{\circ})^{-1})(25.8\text{ C}^{\circ})] = 0.0158\ \Omega$

EVALUATE: Nichrome, like most metallic conductors, has a positive α and its resistance increases with temperature. For carbon, α is negative and its resistance decreases as T increases.

25.28. IDENTIFY: $R_T = R_0[1 + \alpha(T - T_0)]$

SET UP: $R_0 = 217.3 \, \Omega$. $R_T = 215.8 \, \Omega$. For carbon, $\alpha = -0.00050 \, (\text{C}^\circ)^{-1}$.

EXECUTE: $T - T_0 = \frac{(R_T/R_0) - 1}{\alpha} = \frac{(215.8 \, \Omega/217.3 \, \Omega) - 1}{-0.00050 \, (\text{C}^\circ)^{-1}} = 13.8 \, \text{C}^\circ$. $T = 13.8 \, \text{C}^\circ + 4.0^\circ\text{C} = 17.8^\circ\text{C}$.

EVALUATE: For carbon, α is negative so R decreases as T increases.

25.29. IDENTIFY and SET UP: Apply $R = \frac{\rho L}{A}$ to determine the effect of increasing A and L .

EXECUTE: (a) If 120 strands of wire are placed side by side, we are effectively increasing the area of the current carrier by 120. So the resistance is smaller by that factor: $R = (5.60 \times 10^{-6} \, \Omega)/120 = 4.67 \times 10^{-8} \, \Omega$.

(b) If 120 strands of wire are placed end to end, we are effectively increasing the length of the wire by 120, and so $R = (5.60 \times 10^{-6} \, \Omega)120 = 6.72 \times 10^{-4} \, \Omega$.

EVALUATE: Placing the strands side by side decreases the resistance and placing them end to end increases the resistance.

25.30. IDENTIFY: When the ohmmeter is connected between the opposite faces, the current flows along its length, but when the meter is connected between the inner and outer surfaces, the current flows radially outward.

(a) **SET UP:** For a hollow cylinder, $R = \rho L/A$, where $A = \pi(b^2 - a^2)$.

EXECUTE: $R = \rho L/A = \frac{\rho L}{\pi(b^2 - a^2)} = \frac{(2.75 \times 10^{-8} \, \Omega \cdot \text{m})(2.50 \, \text{m})}{\pi[(0.0460 \, \text{m})^2 - (0.0320 \, \text{m})^2]} = 2.00 \times 10^{-5} \, \Omega$

(b) **SET UP:** For radial current flow from $r = a$ to $r = b$, $R = (\rho/2\pi L) \ln(b/a)$ (Example 25.4)

EXECUTE: $R = \frac{\rho}{2\pi L} \ln(b/a) = \frac{2.75 \times 10^{-8} \, \Omega \cdot \text{m}}{2\pi(2.50 \, \text{m})} \ln\left(\frac{4.60 \, \text{cm}}{3.20 \, \text{cm}}\right) = 6.35 \times 10^{-10} \, \Omega$

EVALUATE: The resistance is much smaller for the radial flow because the current flows through a much smaller distance and the area through which it flows is much larger.

25.31. IDENTIFY: Use $R = \frac{\rho L}{A}$ to calculate R and then apply $V = IR$. $P = VI$ and energy $= Pt$

SET UP: For copper, $\rho = 1.72 \times 10^{-8} \, \Omega \cdot \text{m}$. $A = \pi r^2$, where $r = 0.050 \, \text{m}$.

EXECUTE: (a) $R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8} \, \Omega \cdot \text{m})(100 \times 10^3 \, \text{m})}{\pi(0.050 \, \text{m})^2} = 0.219 \, \Omega$. $V = IR = (125 \, \text{A})(0.219 \, \Omega) = 27.4 \, \text{V}$.

(b) $P = VI = (27.4 \, \text{V})(125 \, \text{A}) = 3422 \, \text{W} = 3422 \, \text{J/s}$ and energy $= Pt = (3422 \, \text{J/s})(3600 \, \text{s}) = 1.23 \times 10^7 \, \text{J}$.

EVALUATE: The rate of electrical energy loss in the cable is large, over 3 kW.

25.32. IDENTIFY: When current passes through a battery in the direction from the $-$ terminal toward the $+$ terminal, the terminal voltage V_{ab} of the battery is $V_{ab} = \mathcal{E} - Ir$. Also, $V_{ab} = IR$, the potential across the circuit resistor.

SET UP: $\mathcal{E} = 24.0 \, \text{V}$. $I = 4.00 \, \text{A}$.

EXECUTE: (a) $V_{ab} = \mathcal{E} - Ir$ gives $r = \frac{\mathcal{E} - V_{ab}}{I} = \frac{24.0 \, \text{V} - 21.2 \, \text{V}}{4.00 \, \text{A}} = 0.700 \, \Omega$.

(b) $V_{ab} - IR = 0$ so $R = \frac{V_{ab}}{I} = \frac{21.2 \, \text{V}}{4.00 \, \text{A}} = 5.30 \, \Omega$.

EVALUATE: The voltage drop across the internal resistance of the battery causes the terminal voltage of the battery to be less than its emf. The total resistance in the circuit is $R + r = 6.00 \, \Omega$. $I = \frac{24.0 \, \text{V}}{6.00 \, \Omega} = 4.00 \, \text{A}$, which agrees with the value specified in the problem.

25.33. IDENTIFY: $V = \mathcal{E} - Ir$.

SET UP: The graph gives $V = 9.0 \, \text{V}$ when $I = 0$ and $I = 2.0 \, \text{A}$ when $V = 0$.

EXECUTE: (a) $\mathcal{E}$ is equal to the terminal voltage when the current is zero. From the graph, this is $9.0 \, \text{V}$.

(b) When the terminal voltage is zero, the potential drop across the internal resistance is just equal in magnitude to the internal emf, so $rI = \mathcal{E}$, which gives $r = \mathcal{E}/I = (9.0 \, \text{V})/(2.0 \, \text{A}) = 4.5 \, \Omega$.

EVALUATE: The terminal voltage decreases as the current through the battery increases.

25.34. (a) IDENTIFY: The idealized ammeter has no resistance so there is no potential drop across it. Therefore it acts like a short circuit across the terminals of the battery and removes the $4.00\text{-}\Omega$ resistor from the circuit. Thus the only resistance in the circuit is the $2.00\text{-}\Omega$ internal resistance of the battery.

SET UP: Use Ohm's law: $I = \mathcal{E}/r$.

EXECUTE: $I = (10.0 \, \text{V})/(2.00 \, \Omega) = 5.00 \, \text{A}$.

(b) The zero-resistance ammeter is in parallel with the $4.00\text{-}\Omega$ resistor, so all the current goes through the ammeter. If no current goes through the $4.00\text{-}\Omega$ resistor, the potential drop across it must be zero.

(c) The terminal voltage is zero since there is no potential drop across the ammeter.

EVALUATE: An ammeter should *never* be connected this way because it would seriously alter the circuit!

- 25.35. **IDENTIFY:** The terminal voltage of the battery is $V_{ab} = \mathcal{E} - Ir$. The voltmeter reads the potential difference between its terminals.

SET UP: An ideal voltmeter has infinite resistance.

EXECUTE: (a) Since an ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\text{-}\Omega$ resistor.

(b) $V_{ab} = \mathcal{E} = 5.0\text{ V}$; since there is no current there is no voltage lost over the internal resistance.

(c) The voltmeter reading is therefore 5.0 V since with no current flowing there is no voltage drop across either resistor.

EVALUATE: This not the proper way to connect a voltmeter. If we wish to measure the terminal voltage of the battery in a circuit that does not include the voltmeter, then connect the voltmeter across the terminals of the battery.

- 25.36. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by IR when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to $+$ terminal.

SET UP: The current is counterclockwise, because the 16 V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V} - 8.0\text{ V} - I(1.6\text{ }\Omega + 5.0\text{ }\Omega + 1.4\text{ }\Omega + 9.0\text{ }\Omega) = 0$

$$I = \frac{16.0\text{ V} - 8.0\text{ V}}{1.6\text{ }\Omega + 5.0\text{ }\Omega + 1.4\text{ }\Omega + 9.0\text{ }\Omega} = 0.47\text{ A}$$

(b) $V_b + 16.0\text{ V} - I(1.6\text{ }\Omega) = V_a$, so $V_a - V_b = V_{ab} = 16.0\text{ V} - (1.6\text{ }\Omega)(0.47\text{ A}) = 15.2\text{ V}$.

(c) $V_c + 8.0\text{ V} + I(1.4\text{ }\Omega + 5.0\text{ }\Omega) = V_a$ so $V_{ac} = (5.0\text{ }\Omega)(0.47\text{ A}) + (1.4\text{ }\Omega)(0.47\text{ A}) + 8.0\text{ V} = 11.0\text{ V}$.

(d) The graph is sketched in Figure 25.36.

EVALUATE: $V_{cb} = (0.47\text{ A})(9.0\text{ }\Omega) = 4.2\text{ V}$. The potential at point b is 15.2 V below the potential at point a and the potential at point c is 11.0 V below the potential at point a , so the potential of point c is $15.2\text{ V} - 11.0\text{ V} = 4.2\text{ V}$ above the potential of point b .

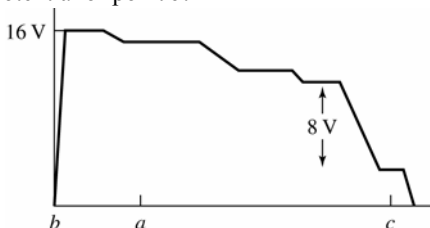


Figure 25.36

- 25.37. **IDENTIFY:** The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

SET UP: open circuit $I = 0$. The circuit is sketched in Figure 25.37a.

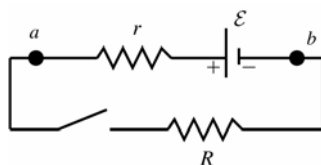


Figure 25.37a

EXECUTE: $V_{ab} = \mathcal{E} = 3.08\text{ V}$

SET UP: switch closed The circuit is sketched in Figure 35.37b.

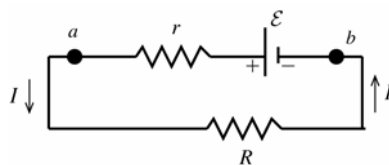


Figure 25.37b

EXECUTE:

$$V_{ab} = \mathcal{E} - Ir = 2.97\text{ V}$$

$$r = \frac{\mathcal{E} - 2.97\text{ V}}{I}$$

$$r = \frac{3.08\text{ V} - 2.97\text{ V}}{1.65\text{ A}} = 0.067\text{ }\Omega$$

And $V_{ab} = IR$ so $R = \frac{V_{ab}}{I} = \frac{2.97\text{ V}}{1.65\text{ A}} = 1.80\text{ }\Omega$.

EVALUATE: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage V is less than its emf.

25.38. IDENTIFY: The sum of the potential changes around the loop is zero.

SET UP: The voltmeter reads the IR voltage across the $9.0\ \Omega$ resistor. The current in the circuit is counterclockwise because the 16 V battery determines the direction of the current flow.

EXECUTE: (a) $V_{bc} = 1.9\text{ V}$ gives $I = V_{bc} / R_{bc} = 1.9\text{ V} / 9.0\ \Omega = 0.21\text{ A}$.

(b) $16.0\text{ V} - 8.0\text{ V} = (1.6\ \Omega + 9.0\ \Omega + 1.4\ \Omega + R)(0.21\text{ A})$ and $R = \frac{5.48\text{ V}}{0.21\text{ A}} = 26.1\ \Omega$.

(c) The graph is sketched in Figure 25.38.

EVALUATE: In Exercise 25.36 the current is 0.47 A . When the $5.0\ \Omega$ resistor is replaced by the $26.1\ \Omega$ resistor the current decreases to 0.21 A .

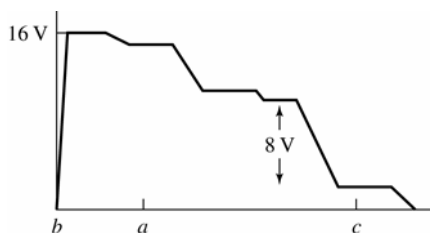


Figure 25.38

25.39. (a) IDENTIFY and SET UP: Assume that the current is clockwise. The circuit is sketched in Figure 25.39a.

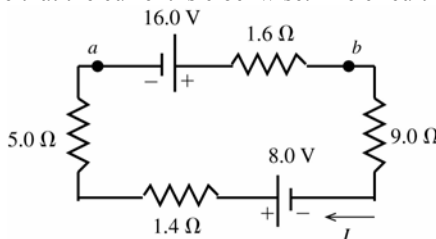


Figure 25.39a

Add up the potential rises and drops as travel clockwise around the circuit.

EXECUTE: $16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) + 8.0\text{ V} - I(1.4\ \Omega) - I(5.0\ \Omega) = 0$

$$I = \frac{16.0\text{ V} + 8.0\text{ V}}{9.0\ \Omega + 1.4\ \Omega + 5.0\ \Omega + 1.6\ \Omega} = \frac{24.0\text{ V}}{17.0\ \Omega} = 1.41\text{ A, clockwise}$$

EVALUATE: The 16.0 V battery drives the current clockwise more strongly than the 8.0 V battery does in the opposite direction.

(b) **IDENTIFY and SET UP:** Start at point a and travel through the battery to point b , keeping track of the potential changes. At point b the potential is V_b .

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) = V_b$

$$V_a - V_b = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega)$$

$$V_{ab} = -16.0\text{ V} + 2.3\text{ V} = -13.7\text{ V} \text{ (point } a \text{ is at lower potential; it is the negative terminal)}$$

EVALUATE: Could also go counterclockwise from a to b :

$$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} + (1.41\text{ A})(9.0\ \Omega) = V_b$$

$$V_{ab} = -13.7\text{ V, which checks.}$$

(c) **IDENTIFY and SET UP:** State at point a and travel through the battery to point c , keeping track of the potential changes.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) = V_c$

$$V_a - V_c = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega + 9.0\ \Omega)$$

$$V_{ac} = -16.0\text{ V} + 15.0\text{ V} = -1.0\text{ V} \text{ (point } a \text{ is at lower potential than point } c)$$

EVALUATE: Could also go counterclockwise from a to c :

$$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} = V_c$$

$$V_{ac} = -1.0\text{ V, which checks.}$$

(d) Call the potential zero at point a . Travel clockwise around the circuit. The graph is sketched in Figure 25.39b.

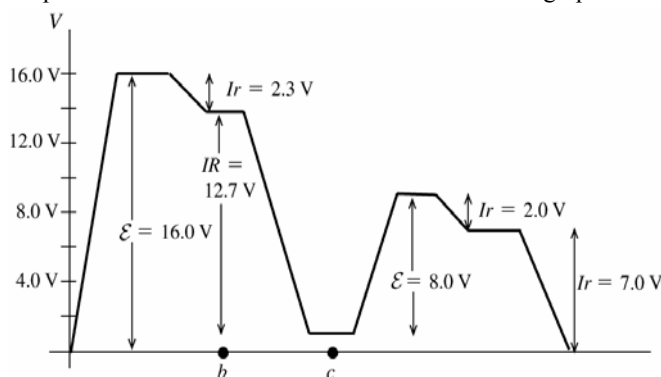


Figure 25.39b

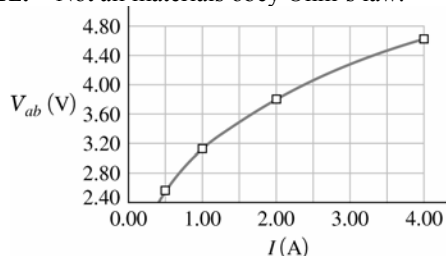
25.40. **IDENTIFY:** Ohm's law says $R = \frac{V_{ab}}{I}$ is a constant.

SET UP: (a) The graph is given in Figure 25.40a.

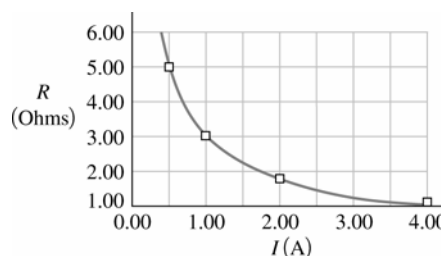
EXECUTE: (b) No. The graph of V_{ab} versus I is not a straight line so Thyrite does not obey Ohm's law.

(c) The graph of R versus I is given in Figure 25.40b. R is not constant; it decreases as I increases.

EVALUATE: Not all materials obey Ohm's law.



(a)



(b)

Figure 25.40

25.41. **IDENTIFY:** Ohm's law says $R = \frac{V_{ab}}{I}$ is a constant.

SET UP: (a) The graph is given in Figure 25.41.

EXECUTE: (b) The graph of V_{ab} versus I is a straight line so Nichrome obeys Ohm's law.

(c) R is the slope of the graph in part (a). $R = \frac{15.52 \text{ V} - 1.94 \text{ V}}{4.00 \text{ A} - 0.50 \text{ A}} = 3.88 \Omega$.

EVALUATE: V_{ab}/I for every I gives the same result for R , $R = 3.88 \Omega$.

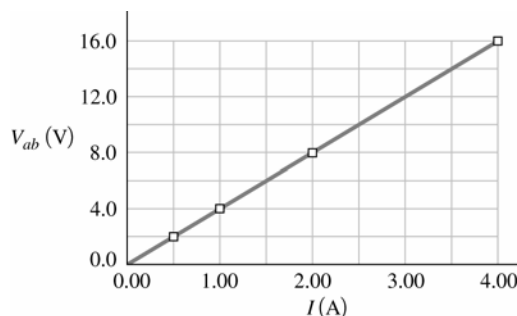


Figure 25.41

25.42. **IDENTIFY and SET UP:** For a resistor, $P = VI = V^2/R$ and $V = IR$.

EXECUTE: (a) $R = \frac{V^2}{P} = \frac{(15.0 \text{ V})^2}{327 \text{ W}} = 0.688 \Omega$

(b) $I = \frac{V}{R} = \frac{15.0 \text{ V}}{0.688 \Omega} = 21.8 \text{ A}$

EVALUATE: We could also write $P = VI$ to calculate $I = \frac{P}{V} = \frac{327 \text{ W}}{15.0 \text{ V}} = 21.8 \text{ A}$.

- 25.43. IDENTIFY:** The bulbs are each connected across a 120-V potential difference.
SET UP: Use $P = V^2/R$ to solve for R and Ohm's law ($I = V/R$) to find the current.
EXECUTE: (a) $R = V^2/P = (120 \text{ V})^2/(100 \text{ W}) = 144 \Omega$.
 (b) $R = V^2/P = (120 \text{ V})^2/(60 \text{ W}) = 240 \Omega$
 (c) For the 100-W bulb: $I = V/R = (120 \text{ V})/(144 \Omega) = 0.833 \text{ A}$
 For the 60-W bulb: $I = (120 \text{ V})/(240 \Omega) = 0.500 \text{ A}$
EVALUATE: The 60-W bulb has *more* resistance than the 100-W bulb, so it draws less current.
- 25.44. IDENTIFY:** Across 120 V, a 75-W bulb dissipates 75 W. Use this fact to find its resistance, and then find the power the bulb dissipates across 220 V.
SET UP: $P = V^2/R$, so $R = V^2/P$
EXECUTE: Across 120 V: $R = (120 \text{ V})^2/(75 \text{ W}) = 192 \Omega$. Across a 220-V line, its power will be $P = V^2/R = (220 \text{ V})^2/(192 \Omega) = 252 \text{ W}$.
EVALUATE: The bulb dissipates much more power across 220 V, so it would likely blow out at the higher voltage. An alternative solution to the problem is to take the ratio of the powers.

$$\frac{P_{220}}{P_{120}} = \frac{V_{220}^2/R}{V_{120}^2/R} = \left(\frac{V_{220}}{V_{120}}\right)^2 = \left(\frac{220}{120}\right)^2$$
 This gives $P_{220} = (75 \text{ W})\left(\frac{220}{120}\right)^2 = 252 \text{ W}$.
- 25.45. IDENTIFY:** A "100-W" European bulb dissipates 100 W when used across 220 V.
(a) SET UP: Take the ratio of the power in the US to the power in Europe, as in the alternative method for problem 25.44, using $P = V^2/R$.
EXECUTE: $\frac{P_{\text{US}}}{P_{\text{E}}} = \frac{V_{\text{US}}^2/R}{V_{\text{E}}^2/R} = \left(\frac{V_{\text{US}}}{V_{\text{E}}}\right)^2 = \left(\frac{120 \text{ V}}{220 \text{ V}}\right)^2$. This gives $P_{\text{US}} = (100 \text{ W})\left(\frac{120 \text{ V}}{220 \text{ V}}\right)^2 = 29.8 \text{ W}$.
(b) SET UP: Use $P = IV$ to find the current.
EXECUTE: $I = P/V = (29.8 \text{ W})/(120 \text{ V}) = 0.248 \text{ A}$
EVALUATE: The bulb draws considerably less power in the U.S., so it would be much dimmer than in Europe.
- 25.46. IDENTIFY:** $P = VI$. Energy = Pt .
SET UP: $P = (9.0 \text{ V})(0.13 \text{ A}) = 1.17 \text{ W}$
EXECUTE: Energy = $(1.17 \text{ W})(1.5 \text{ h})(3600 \text{ s/h}) = 6320 \text{ J}$
EVALUATE: The energy consumed is proportional to the voltage, to the current and to the time.
- 25.47. IDENTIFY and SET UP:** By definition $p = \frac{P}{LA}$. Use $P = VI$, $E = VL$ and $I = JA$ to rewrite this expression in terms of the specified variables.
EXECUTE: (a) E is related to V and J is related to I , so use $P = VI$. This gives $p = \frac{VI}{LA}$

$$\frac{V}{L} = E \text{ and } \frac{I}{A} = J \text{ so } p = EJ$$

 (b) J is related to I and ρ is related to R , so use $P = IR^2$. This gives $p = \frac{I^2 R}{LA}$

$$I = JA \text{ and } R = \frac{\rho L}{A} \text{ so } p = \frac{J^2 A^2 \rho L}{LA^2} \rho J^2$$

 (c) E is related to V and ρ is related to R , so use $P = V^2/R$. This gives $p = \frac{V^2}{RLA}$

$$V = EL \text{ and } R = \frac{\rho L}{A} \text{ so } p = \frac{E^2 L^2}{LA} \left(\frac{A}{\rho L}\right) = \frac{E^2}{\rho}$$

EVALUATE: For a given material (ρ constant), p is proportional to J^2 or to E^2 .
- 25.48. IDENTIFY:** Calculate the current in the circuit. The power output of a battery is its terminal voltage times the current through it. The power dissipated in a resistor is $I^2 R$.
SET UP: The sum of the potential changes around the circuit is zero.
EXECUTE: (a) $I = \frac{8.0 \text{ V}}{17 \Omega} = 0.47 \text{ A}$. Then $P_{9\Omega} = I^2 R = (0.47 \text{ A})^2 (9.0 \Omega) = 1.1 \text{ W}$ and
 $P_{9\Omega} = I^2 R = (0.47 \text{ A})^2 (9.0 \Omega) = 2.0 \text{ W}$.
 (b) $P_{16\text{V}} = \mathcal{E}I - I^2 r = (16 \text{ V})(0.47 \text{ A}) - (0.47 \text{ A})^2 (1.6 \Omega) = 7.2 \text{ W}$.
 (c) $P_{8\text{V}} = \mathcal{E}I + Ir^2 = (8.0 \text{ V})(0.47 \text{ A}) + (0.47 \text{ A})^2 (1.4 \Omega) = 4.1 \text{ W}$.
EVALUATE: (d) (b) = (a) + (c). The rate at which the 16.0 V battery delivers electrical energy to the circuit equals the rate at which it is consumed in the 8.0 V battery and the 5.0Ω and 9.0Ω resistors.

- 25.49. (a) IDENTIFY and SET UP:** $P = VI$ and energy = (power) $\times$ (time).

EXECUTE: $P = VI = (12 \text{ V})(60 \text{ A}) = 720 \text{ W}$

The battery can provide this for 1.0 h, so the energy the battery has stored is

$$U = Pt = (720 \text{ W})(3600 \text{ s}) = 2.6 \times 10^6 \text{ J}$$

(b) IDENTIFY and SET UP: For gasoline the heat of combustion is $L_c = 46 \times 10^6 \text{ J/kg}$. Solve for the mass m required to supply the energy calculated in part (a) and use density $\rho = m/V$ to calculate V .

EXECUTE: The mass of gasoline that supplies $2.6 \times 10^6 \text{ J}$ is $m = \frac{2.6 \times 10^6 \text{ J}}{46 \times 10^6 \text{ J/kg}} = 0.0565 \text{ kg}$.

The volume of this mass of gasoline is

$$V = \frac{m}{\rho} = \frac{0.0565 \text{ kg}}{900 \text{ kg/m}^3} = 6.3 \times 10^{-5} \text{ m}^3 \left(\frac{1000 \text{ L}}{1 \text{ m}^3} \right) = 0.063 \text{ L}$$

(c) IDENTIFY and SET UP: Energy = (power) $\times$ (time); the energy is that calculated in part (a).

EXECUTE: $U = Pt$, $t = \frac{U}{P} = \frac{2.6 \times 10^6 \text{ J}}{450 \text{ W}} = 5800 \text{ s} = 97 \text{ min} = 1.6 \text{ h}$.

EVALUATE: The battery discharges at a rate of 720 W (for 60 A) and is charged at a rate of 450 W, so it takes longer to charge than to discharge.

- 25.50. IDENTIFY:** The rate of conversion of chemical to electrical energy in an emf is $\mathcal{E}I$. The rate of dissipation of electrical energy in a resistor R is I^2R .

SET UP: Example 25.10 finds that $I = 1.2 \text{ A}$ for this circuit. In Example 25.9, $\mathcal{E}I = 24 \text{ W}$ and $I^2r = 8 \text{ W}$. In Example 25.10, $I^2R = 12 \text{ W}$, or 11.5 W if expressed to three significant figures.

EXECUTE: (a) $P = \mathcal{E}I = (12 \text{ V})(1.2 \text{ A}) = 14.4 \text{ W}$. This is less than the previous value of 24 W.

(b) The energy dissipated in the battery is $P = I^2r = (1.2 \text{ A})^2(2.0 \Omega) = 2.9 \text{ W}$. This is less than 8 W, the amount found in Example (25.9).

(c) The net power output of the battery is $14.4 \text{ W} - 2.9 \text{ W} = 11.5 \text{ W}$. This is the same as the power dissipated in the 8.0Ω resistor.

EVALUATE: With the larger circuit resistance the current is less and the power input and power consumption are less.

- 25.51. IDENTIFY:** Some of the power generated by the internal emf of the battery is dissipated across the battery's internal resistance, so it is not available to the bulb.

SET UP: Use $P = I^2R$ and take the ratio of the power dissipated in the internal resistance r to the total power.

EXECUTE: $\frac{P_r}{P_{\text{Total}}} = \frac{I^2r}{I^2(r+R)} = \frac{r}{r+R} = \frac{3.5 \Omega}{28.5 \Omega} = 0.123 = 12.3\%$

EVALUATE: About 88% of the power of the battery goes to the bulb. The rest appears as heat in the internal resistance.

- 25.52. IDENTIFY:** The voltmeter reads the terminal voltage of the battery, which is the potential difference across the appliance. The terminal voltage is less than 15.0 V because some potential is lost across the internal resistance of the battery.

(a) SET UP: $P = V^2/R$ gives the power dissipated by the appliance.

EXECUTE: $P = (11.3 \text{ V})^2/(75.0 \Omega) = 1.70 \text{ W}$

(b) SET UP: The drop in terminal voltage ($\mathcal{E} - V_{ab}$) is due to the potential drop across the internal resistance r . Use $Ir = \mathcal{E} - V_{ab}$ to find the internal resistance r , but first find the current using $P = IV$.

EXECUTE: $I = P/V = (1.70 \text{ W})/(11.3 \text{ V}) = 0.151 \text{ A}$. Then $Ir = \mathcal{E} - V_{ab}$ gives $(0.151 \text{ A})r = 15.0 \text{ V} - 11.3 \text{ V}$ and $r = 24.6 \Omega$.

EVALUATE: The full 15.0 V of the battery would be available only when no current (or a very small current) is flowing in the circuit. This would be the case if the appliance had a resistance much greater than 24.6Ω .

- 25.53. IDENTIFY:** Solve for the current I in the circuit. Apply Eq. (25.17) to the specified circuit elements to find the rates of energy conversion.

SET UP: The circuit is sketched in Figure 25.53.

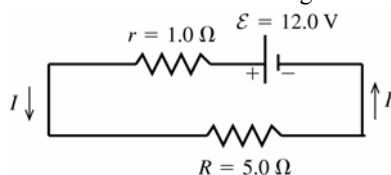


Figure 25.53

EXECUTE: Compute I :

$$\mathcal{E} - Ir - IR = 0$$

$$I = \frac{\mathcal{E}}{r + R} = \frac{12.0 \text{ V}}{1.0 \Omega + 5.0 \Omega} = 2.00 \text{ A}$$

(a) The rate of conversion of chemical energy to electrical energy in the emf of the battery is

$$P = \mathcal{E}I = (12.0 \text{ V})(2.00 \text{ A}) = 24.0 \text{ W}.$$

(b) The rate of dissipation of electrical energy in the internal resistance of the battery is

$$P = I^2 r = (2.00 \text{ A})^2 (1.0 \Omega) = 4.0 \text{ W}.$$

(c) The rate of dissipation of electrical energy in the external resistor R is $P = I^2 R = (2.00 \text{ A})^2 (5.0 \Omega) = 20.0 \text{ W}$.

EVALUATE: The rate of production of electrical energy in the circuit is 24.0 W. The total rate of consumption of electrical energy in the circuit is 4.00 W + 20.0 W = 24.0 W. Equal rate of production and consumption of electrical energy are required by energy conservation.

25.54. **IDENTIFY:** The power delivered to the bulb is $I^2 R$. Energy = Pt .

SET UP: The circuit is sketched in Figure 25.54. r_{total} is the combined internal resistance of both batteries.

EXECUTE: (a) $r_{\text{total}} = 0$. The sum of the potential changes around the circuit is zero, so

$$1.5 \text{ V} + 1.5 \text{ V} - I(17 \Omega) = 0. \quad I = 0.1765 \text{ A}. \quad P = I^2 R = (0.1765 \text{ A})^2 (17 \Omega) = 0.530 \text{ W}. \quad \text{This is also } (3.0 \text{ V})(0.1765 \text{ A}).$$

(b) Energy = $(0.530 \text{ W})(5.0 \text{ h})(3600 \text{ s/h}) = 9540 \text{ J}$

$$(c) \quad P = \frac{0.530 \text{ W}}{2} = 0.265 \text{ W}. \quad P = I^2 R \quad \text{so} \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.265 \text{ W}}{17 \Omega}} = 0.125 \text{ A}.$$

The sum of the potential changes around the circuit is zero, so $1.5 \text{ V} + 1.5 \text{ V} - IR - Ir_{\text{total}} = 0$.

$$r_{\text{total}} = \frac{3.0 \text{ V} - (0.125 \text{ A})(17 \Omega)}{0.125 \text{ A}} = 7.0 \Omega.$$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

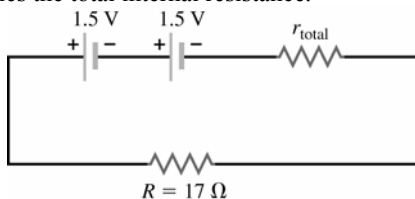


Figure 25.54

25.55. **IDENTIFY:** $P = I^2 R = \frac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120 \text{ V}$. Energy = Pt .

$$\text{EXECUTE: (a)} \quad P = \frac{V^2}{R} \quad \text{so} \quad R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{540 \text{ W}} = 26.7 \Omega$$

$$(b) \quad P = VI \quad \text{so} \quad I = \frac{P}{V} = \frac{540 \text{ W}}{120 \text{ V}} = 4.50 \text{ A}$$

$$(c) \quad \text{Assuming that } R \text{ remains } 26.7 \Omega, \quad P = \frac{V^2}{R} = \frac{(110 \text{ V})^2}{26.7 \Omega} = 453 \text{ W}. \quad P \text{ is smaller by a factor of } (110/120)^2.$$

EVALUATE: (d) With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than 26.7Ω and the power consumed will be greater than the value calculated in part (c).

25.56. **IDENTIFY:** From Eq. (25.24), $\rho = \frac{m}{ne^2 \tau}$.

SET UP: For silicon, $\rho = 2300 \Omega \cdot \text{m}$.

$$\text{EXECUTE: (a)} \quad \tau = \frac{m}{ne^2 \rho} = \frac{9.11 \times 10^{-31} \text{ kg}}{(1.0 \times 10^{16} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})^2 (2300 \Omega \cdot \text{m})} = 1.55 \times 10^{-12} \text{ s}.$$

EVALUATE: (b) The number of free electrons in copper ($8.5 \times 10^{28} \text{ m}^{-3}$) is much larger than in pure silicon ($1.0 \times 10^{16} \text{ m}^{-3}$). A smaller density of current carriers means a higher resistivity.

25.57. (a) **IDENTIFY and SET UP:** Use $R = \frac{\rho L}{A}$.

$$\text{EXECUTE: } \rho = \frac{RA}{L} = \frac{(0.104 \Omega) \pi (1.25 \times 10^{-3} \text{ m})^2}{14.0 \text{ m}} = 3.65 \times 10^{-8} \Omega \cdot \text{m}$$

EVALUATE: This value is similar to that for good metallic conductors in Table 25.1.

(b) IDENTIFY and SET UP: Use $V = EL$ to calculate E and then Ohm's law gives I .

EXECUTE: $V = EL = (1.28 \text{ V/m})(14.0 \text{ m}) = 17.9 \text{ V}$

$$I = \frac{V}{R} = \frac{17.9 \text{ V}}{0.104 \Omega} = 172 \text{ A}$$

EVALUATE: We could do the calculation another way:

$$E = \rho J \text{ so } J = \frac{E}{\rho} = \frac{1.28 \text{ V/m}}{3.65 \times 10^{-8} \Omega \cdot \text{m}} = 3.51 \times 10^7 \text{ A/m}^2$$

$$I = JA = (3.51 \times 10^7 \text{ A/m}^2) \pi (1.25 \times 10^{-3} \text{ m})^2 = 172 \text{ A, which checks}$$

(c) IDENTIFY and SET UP: Calculate $J = I/A$ or $J = E/\rho$ and then use Eq. (25.3) for the target variable v_d .

EXECUTE: $J = n|q|v_d = nev_d$

$$v_d = \frac{J}{ne} = \frac{3.51 \times 10^7 \text{ A/m}^2}{(8.5 \times 10^{28} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})} = 2.58 \times 10^{-3} \text{ m/s} = 2.58 \text{ mm/s}$$

EVALUATE: Even for this very large current the drift speed is small.

25.58. IDENTIFY: Use $R = \frac{\rho L}{A}$ to calculate the resistance of the silver tube. Then $I = V/R$.

SET UP: For silver, $\rho = 1.47 \times 10^{-8} \Omega \cdot \text{m}$. The silver tube is sketched in Figure 25.58. Since the thickness $T = 0.100 \text{ mm}$ is much smaller than the radius, $r = 2.00 \text{ cm}$, the cross section area of the silver is $2\pi rT$. The length of the tube is $l = 25.0 \text{ m}$.

$$\text{EXECUTE: } I = \frac{V}{R} = \frac{V}{\rho l / A} = \frac{VA}{\rho l} = \frac{V(2\pi rT)}{\rho l} = \frac{(12 \text{ V})(2\pi)(2.00 \times 10^{-2} \text{ m})(0.100 \times 10^{-3} \text{ m})}{(1.47 \times 10^{-8} \Omega \cdot \text{m})(25.0 \text{ m})} = 410 \text{ A}$$

EVALUATE: The resistance is small, $R = 0.0292 \Omega$, so 12.0 V produces a large current.

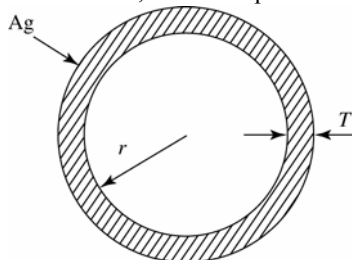


Figure 25.58

25.59. IDENTIFY and SET UP: With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\mathcal{E} = 12.6 \text{ V}$. With a wire of resistance R connected to the battery current I flows and $\mathcal{E} - Ir - IR = 0$, where r is the internal resistance of the battery. Apply this equation to each piece of wire to get two equations in the two unknowns.

EXECUTE: Call the resistance of the 20.0-m piece R_1 ; then the resistance of the 40.0-m piece is $R_2 = 2R_1$.

$$\mathcal{E} - I_1 r - I_1 R_1 = 0; \quad 12.6 \text{ V} - (7.00 \text{ A})r - (7.00 \text{ A})R_1 = 0$$

$$\mathcal{E} - I_2 r - I_2 (2R_1) = 0; \quad 12.6 \text{ V} - (4.20 \text{ A})r - (4.20 \text{ A})(2R_1) = 0$$

Solving these two equations in two unknowns gives $R_1 = 1.20 \Omega$. This is the resistance of 20.0 m, so the resistance of one meter is $[1.20 \Omega / (20.0 \text{ m})](1.00 \text{ m}) = 0.060 \Omega$

EVALUATE: We can also solve for r and we get $r = 0.600 \Omega$. When measuring small resistances, the internal resistance of the battery has a large effect.

25.60. IDENTIFY: Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R = R_{\text{Cu}} + R_{\text{Ag}}$. The total resistance is the sum of the resistances of each section.

$$E = \rho J = \frac{\rho I}{A}, \text{ so } E = \frac{IR}{L}, \text{ where } R \text{ is the resistance of a section and } L \text{ is its length.}$$

SET UP: For copper, $\rho_{\text{Cu}} = 1.72 \times 10^{-8} \Omega \cdot \text{m}$. For silver, $\rho_{\text{Ag}} = 1.47 \times 10^{-8} \Omega \cdot \text{m}$.

EXECUTE: (a) $I = \frac{V}{R} = \frac{V}{R_{\text{Cu}} + R_{\text{Ag}}}$. $R_{\text{Cu}} = \frac{\rho_{\text{Cu}} L_{\text{Cu}}}{A_{\text{Cu}}} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(0.8 \text{ m})}{(\pi/4)(6.0 \times 10^{-4} \text{ m})^2} = 0.049 \Omega$ and

$$R_{\text{Ag}} = \frac{\rho_{\text{Ag}} L_{\text{Ag}}}{A_{\text{Ag}}} = \frac{(1.47 \times 10^{-8} \Omega \cdot \text{m})(1.2 \text{ m})}{(\pi/4)(6.0 \times 10^{-4} \text{ m})^2} = 0.062 \Omega. \text{ This gives } I = \frac{5.0 \text{ V}}{0.049 \Omega + 0.062 \Omega} = 45 \text{ A}.$$

The current in the copper wire is 45 A.

(b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{\text{Cu}} = J \rho_{\text{Cu}} = \frac{IR_{\text{Cu}}}{L_{\text{Cu}}} = \frac{(45 \text{ A})(0.049 \Omega)}{0.8 \text{ m}} = 2.76 \text{ V/m}.$

(d) $E_{\text{Ag}} = J \rho_{\text{Ag}} = \frac{IR_{\text{Ag}}}{L_{\text{Ag}}} = \frac{(45 \text{ A})(0.062 \Omega)}{1.2 \text{ m}} = 2.33 \text{ V/m}.$

(e) $V_{\text{Ag}} = IR_{\text{Ag}} = (45 \text{ A})(0.062 \Omega) = 2.79 \text{ V}.$

EVALUATE: For the copper section, $V_{\text{Cu}} = IR_{\text{Cu}} = 2.21 \text{ V}$. Note that $V_{\text{Cu}} + V_{\text{Ag}} = 5.0 \text{ V}$, the voltage applied across the ends of the composite wire.

25.61. IDENTIFY: Conservation of charge requires that the current be the same in both sections of the wire.

$$E = \rho J = \frac{\rho I}{A}. \text{ For each section, } V = IR = JAR = \left(\frac{EA}{\rho} \right) \left(\frac{\rho I}{A} \right) = EL. \text{ The voltages across each section add.}$$

SET UP: $A = (\pi/4)D^2$, where D is the diameter.

EXECUTE: (a) The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

(b) $E_{1.6\text{mm}} = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(2.5 \times 10^{-3} \text{ A})}{(\pi/4)(1.6 \times 10^{-3} \text{ m})^2} = 2.14 \times 10^{-5} \text{ V/m}.$

(c) $E_{0.8\text{mm}} = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(2.5 \times 10^{-3} \text{ A})}{(\pi/4)(0.80 \times 10^{-3} \text{ m})^2} = 8.55 \times 10^{-5} \text{ V/m}.$ This is $4E_{1.6\text{mm}}.$

(d) $V = E_{1.6\text{mm}} L_{1.6\text{mm}} + E_{0.8\text{mm}} L_{0.8\text{mm}} = (2.14 \times 10^{-5} \text{ V/m})(1.20 \text{ m}) + (8.55 \times 10^{-5} \text{ V/m})(1.80 \text{ m}) = 1.80 \times 10^{-4} \text{ V}.$

EVALUATE: The currents are the same but the current density is larger in the thinner section and the electric field is larger there.

25.62. IDENTIFY: $I = JA$.

SET UP: From Example 25.1, an 18-gauge wire has $A = 8.17 \times 10^{-3} \text{ cm}^2$.

EXECUTE: (a) $I = JA = (1.0 \times 10^5 \text{ A/cm}^2)(8.17 \times 10^{-3} \text{ cm}^2) = 820 \text{ A}$

(b) $A = I/J = (1000 \text{ A})/(1.0 \times 10^6 \text{ A/cm}^2) = 1.0 \times 10^{-3} \text{ cm}^2.$ $A = \pi r^2$ so

$$r = \sqrt{A/\pi} = \sqrt{(1.0 \times 10^{-3} \text{ cm}^2)/\pi} = 0.0178 \text{ cm} \text{ and } d = 2r = 0.36 \text{ mm}.$$

EVALUATE: These wires can carry very large currents.

25.63. (a) IDENTIFY: Apply Eq. (25.10) to calculate the resistance of each thin disk and then integrate over the truncated cone to find the total resistance.

SET UP:

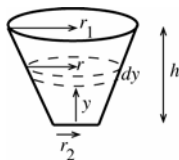


Figure 25.63

EXECUTE: The radius of a truncated cone a distance y above the bottom is given by $r = r_2 + (y/h)(r_1 - r_2) = r_2 + y\beta$ with $\beta = (r_1 - r_2)/h$

Consider a thin slice a distance y above the bottom. The slice has thickness dy and radius r . The resistance of the slice is

$$dR = \frac{\rho dy}{A} = \frac{\rho dy}{\pi r^2} = \frac{\rho dy}{\pi (r_2 + \beta y)^2}$$

The total resistance of the cone is obtained by integrating over these thin slices:

$$R = \int dR = \frac{\rho}{\pi} \int_0^h \frac{dy}{(r_2 + \beta y)^2} = \frac{\rho}{\pi} \left[-\frac{1}{\beta} (r_2 + \beta y)^{-1} \right]_0^h = -\frac{\rho}{\pi\beta} \left[\frac{1}{r_2 + h\beta} - \frac{1}{r_2} \right]$$

But $r_2 + h\beta = r_1$

$$R = \frac{\rho}{\pi\beta} \left[\frac{1}{r_2} - \frac{1}{r_1} \right] = \frac{\rho}{\pi} \left(\frac{h}{r_1 - r_2} \right) \left(\frac{r_1 - r_2}{r_1 r_2} \right) = \frac{\rho h}{\pi r_1 r_2}$$

(b) EVALUATE: Let $r_1 = r_2 = r$. Then $R = \rho h / \pi r^2 = \rho L / A$ where $A = \pi r^2$ and $L = h$. This agrees with Eq. (25.10).

- 25.64. IDENTIFY:** Divide the region into thin spherical shells of radius r and thickness dr . The total resistance is the sum of the resistances of the thin shells and can be obtained by integration.

SET UP: $I = V/R$ and $J = I/4\pi r^2$, where $4\pi r^2$ is the surface area of a shell of radius r .

EXECUTE: (a) $dR = \frac{\rho dr}{4\pi r^2} \Rightarrow R = \frac{\rho}{4\pi} \int_a^b \frac{dr}{r^2} = -\frac{\rho}{4\pi} \frac{1}{r} \Big|_a^b = \frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{\rho}{4\pi} \left(\frac{b-a}{ab} \right)$.

(b) $I = \frac{V_{ab}}{R} = \frac{V_{ab} 4\pi ab}{\rho(b-a)}$ and $J = \frac{I}{A} = \frac{V_{ab} 4\pi ab}{\rho(b-a) 4\pi r^2} = \frac{V_{ab} ab}{\rho(b-a) r^2}$.

(c) If the thickness of the shells is small, then $4\pi ab \approx 4\pi a^2$ is the surface area of the conducting material.

$$R = \frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{\rho(b-a)}{4\pi ab} \approx \frac{\rho L}{4\pi a^2} = \frac{\rho L}{A}, \text{ where } L = b-a.$$

EVALUATE: The current density in the material is proportional to $1/r^2$.

- 25.65. IDENTIFY and SET UP:** Use $E = \rho J$ to calculate the current density between the plates. Let A be the area of each plate; then $I = JA$.

EXECUTE: $J = \frac{E}{\rho}$ and $E = \frac{\sigma}{K\epsilon_0} = \frac{Q}{KA\epsilon_0}$

Thus $J = \frac{Q}{KA\epsilon_0\rho}$ and $I = JA = \frac{Q}{K\epsilon_0\rho}$, as was to be shown.

EVALUATE: $C = K\epsilon_0 A/d$ and $V = Q/C = Qd/K\epsilon_0 A$ so the result can also be written as $I = VA/d\rho$. The resistance of the dielectric is $R = V/I = d\rho/A$, which agrees with Eq. (25.10).

- 25.66. IDENTIFY:** As the resistance R varies, the current in the circuit also varies, which causes the potential drop across the internal resistance of the battery to vary.

SET UP: The largest current will occur when $R = 0$, and the smallest current will occur when $R \rightarrow \infty$. The largest terminal voltage will occur when the current is zero ($R \rightarrow \infty$) and the smallest terminal voltage will be when the current is a maximum ($R = 0$).

EXECUTE: (a) As $R \rightarrow \infty$, $I \rightarrow 0$, so $V_{ab} \rightarrow \mathcal{E} = 15.0 \text{ V}$, which is the largest reading of the voltmeter. When $R = 0$, the current is largest at $(15.0 \text{ V})/(4.00 \Omega) = 3.75 \text{ A}$, so the smallest terminal voltage is $V_{ab} = \mathcal{E} - rI = 15.0 \text{ V} - (4.00 \Omega)(3.75 \text{ A}) = 0$.

(b) From part (a), the maximum current is 3.75 A when $R = 0$, and the minimum current is 0.00 A when $R \rightarrow \infty$.

(c) The graphs are sketched in Figure 25.66.

EVALUATE: Increasing the resistance R increases the terminal voltage, but at the same time it decreases the current in the circuit.

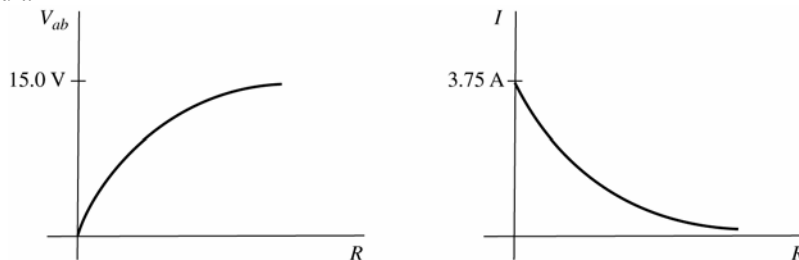


Figure 25.66

- 25.67. IDENTIFY:** Apply $R = \frac{\rho L}{A}$.

SET UP: For mercury at 20°C , $\rho = 9.5 \times 10^{-7} \Omega \cdot \text{m}$, $\alpha = 0.00088 (\text{C}^\circ)^{-1}$ and $\beta = 18 \times 10^{-5} (\text{C}^\circ)^{-1}$.

EXECUTE: (a) $R = \frac{\rho L}{A} = \frac{(9.5 \times 10^{-7} \Omega \cdot \text{m})(0.12 \text{ m})}{(\pi/4)(0.0016 \text{ m})^2} = 0.057 \Omega$.

(b) $\rho(T) = \rho_0(1 + \alpha\Delta T)$ gives $\rho(60^\circ\text{C}) = (9.5 \times 10^{-7} \Omega \cdot \text{m})(1 + (0.00088 \text{ (}^\circ\text{C)}^{-1})(40^\circ\text{C})) = 9.83 \times 10^{-7} \Omega \cdot \text{m}$, so $\Delta\rho = 3.34 \times 10^{-8} \Omega \cdot \text{m}$.

(c) $\Delta V = \beta V_0 \Delta T$ gives $\Delta L = A(\beta L_0 \Delta T)$. Therefore

$\Delta L = \beta L_0 \Delta T = (18 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1})(0.12 \text{ m})(40^\circ\text{C}) = 8.64 \times 10^{-4} \text{ m} = 0.86 \text{ mm}$. The cross sectional area of the mercury remains constant because the diameter of the glass tube doesn't change. All of the change in volume of the mercury must be accommodated by a change in length of the mercury column.

(d) $R = \frac{\rho L}{A}$ gives $\Delta R = \frac{L \Delta \rho}{A} + \frac{\rho \Delta L}{A}$.

$$\Delta R = \frac{(3.34 \times 10^{-8} \Omega \cdot \text{m})(0.12 \text{ m})}{(\pi/4)(0.0016 \text{ m})^2} + \frac{(9.5 \times 10^{-8} \Omega \cdot \text{m})(0.86 \times 10^{-3} \text{ m})}{(\pi/4)(0.0016 \text{ m})^2} = 2.40 \times 10^{-3} \Omega.$$

EVALUATE: (e) From Equation (25.12),

$$\alpha = \frac{1}{\Delta T} \left(\frac{R}{R_0} - 1 \right) = \frac{1}{40^\circ\text{C}} \left(\frac{(0.057 \Omega + 2.40 \times 10^{-3} \Omega)}{0.057 \Omega} - 1 \right) = 1.1 \times 10^{-3} \text{ (}^\circ\text{C)}^{-1}.$$

This value is 25% greater than the temperature coefficient of resistivity and the length increase is important.

25.68. IDENTIFY: Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: (a) $I = \frac{8.0 \text{ V} - 4.0 \text{ V}}{24.0 \Omega} = 0.167 \text{ A}$. $V_d + 8.00 \text{ V} - I(0.50 \Omega + 8.00 \Omega) = V_a$, so

$$V_{ad} = 8.00 \text{ V} - (0.167 \text{ A})(8.50 \Omega) = 6.58 \text{ V}.$$

(b) The terminal voltage is $V_{bc} = V_b - V_c$. $V_c + 4.00 \text{ V} + I(0.50 \Omega) = V_b$ and

$$V_{bc} = +4.00 \text{ V} + (0.167 \text{ A})(0.50 \Omega) = +4.08 \text{ V}.$$

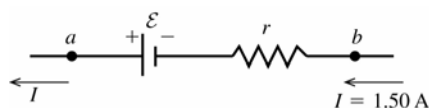
(c) Adding another battery at point d in the opposite sense to the 8.0 V battery produces a counterclockwise current with magnitude $I = \frac{10.3 \text{ V} - 8.0 \text{ V} + 4.0 \text{ V}}{24.5 \Omega} = 0.257 \text{ A}$. Then $V_c + 4.00 \text{ V} - I(0.50 \Omega) = V_b$ and

$$V_{bc} = 4.00 \text{ V} - (0.257 \text{ A})(0.50 \Omega) = 3.87 \text{ V}.$$

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.69. IDENTIFY: In each case write the terminal voltage in terms of $\mathcal{E}$, I , and r . Since I is known, this gives two equations in the two unknowns $\mathcal{E}$ and r .

SET UP: The battery with the 1.50 A current is sketched in Figure 25.69a.



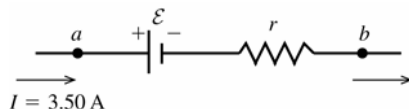
$$V_{ab} = 8.4 \text{ V}$$

$$V_{ab} = \mathcal{E} - Ir$$

$$\mathcal{E} - (1.50 \text{ A})r = 8.4 \text{ V}$$

Figure 25.69a

The battery with the 3.50 A current is sketched in Figure 25.69b.



$$V_{ab} = 9.4 \text{ V}$$

$$V_{ab} = \mathcal{E} + Ir$$

$$\mathcal{E} + (3.5 \text{ A})r = 9.4 \text{ V}$$

Figure 25.69b

EXECUTE: (a) Solve the first equation for $\mathcal{E}$ and use that result in the second equation:

$$\mathcal{E} = 8.4 \text{ V} + (1.50 \text{ A})r$$

$$8.4 \text{ V} + (1.50 \text{ A})r + (3.50 \text{ A})r = 9.4 \text{ V}$$

$$(5.00 \text{ A})r = 1.0 \text{ V so } r = \frac{1.0 \text{ V}}{5.00 \text{ A}} = 0.20 \Omega$$

(b) Then $\mathcal{E} = 8.4 \text{ V} + (1.50 \text{ A})r = 8.4 \text{ V} + (1.50 \text{ A})(0.20 \Omega) = 8.7 \text{ V}$

EVALUATE: When the current passes through the emf in the direction from $-$ to $+$, the terminal voltage is less than the emf and when it passes through from $+$ to $-$, the terminal voltage is greater than the emf.

25.70. **IDENTIFY:** $V = IR$. $P = I^2 R$.

SET UP: The total resistance is the resistance of the person plus the internal resistance of the power supply.

EXECUTE: (a) $I = \frac{V}{R_{\text{tot}}} = \frac{14 \times 10^3 \text{ V}}{10 \times 10^3 \Omega + 2000 \Omega} = 1.17 \text{ A}$

(b) $P = I^2 R = (1.17 \text{ A})^2 (10 \times 10^3 \Omega) = 1.37 \times 10^4 \text{ J} = 13.7 \text{ kJ}$

(c) $R_{\text{tot}} = \frac{V}{I} = \frac{14 \times 10^3 \text{ V}}{1.00 \times 10^{-3} \text{ A}} = 14 \times 10^6 \Omega$. The resistance of the power supply would need to be

$14 \times 10^6 \Omega - 10 \times 10^3 \Omega = 14 \times 10^6 \Omega = 14 \text{ M}\Omega$.

EVALUATE: The current through the body in part (a) is large enough to be fatal.

25.71. **IDENTIFY:** $R = \frac{\rho L}{A}$. $V = IR$. $P = I^2 R$.

SET UP: The area of the end of a cylinder of radius r is πr^2 .

EXECUTE: (a) $R = \frac{(5.0 \Omega \cdot \text{m})(1.6 \text{ m})}{\pi (0.050 \text{ m})^2} = 1.0 \times 10^3 \Omega$

(b) $V = IR = (100 \times 10^{-3} \text{ A})(1.0 \times 10^3 \Omega) = 100 \text{ V}$

(c) $P = I^2 R = (100 \times 10^{-3} \text{ A})^2 (1.0 \times 10^3 \Omega) = 10 \text{ W}$

EVALUATE: The resistance between the hands when the skin is wet is about a factor of ten less than when the skin is dry (Problem 25.70).

25.72. **IDENTIFY:** The cost of operating an appliance is proportional to the amount of energy consumed. The energy depends on the power the item consumes and the length of time for which it is operated.

SET UP: At a constant power, the energy is equal to Pt , and the total cost is the cost per kilowatt-hour (kWh) times the time the energy (in kWh).

EXECUTE: (a) Use the fact that $1.00 \text{ kWh} = (1000 \text{ J/s})(3600 \text{ s}) = 3.60 \times 10^6 \text{ J}$, and one year contains $3.156 \times 10^7 \text{ s}$.

$$(75 \text{ J/s}) \left(\frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}} \right) \left(\frac{\$0.120}{3.60 \times 10^6 \text{ J}} \right) = \$78.90$$

(b) At 8 h/day, the refrigerator runs for $1/3$ of a year. Using the same procedure as above gives

$$(400 \text{ J/s}) \left(\frac{1}{3} \right) \left(\frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}} \right) \left(\frac{\$0.120}{3.60 \times 10^6 \text{ J}} \right) = \$140.27$$

EVALUATE: Electric lights can be a substantial part of the cost of electricity in the home if they are left on for a long time!

25.73. **IDENTIFY:** Set the sum of the potential rises and drops around the circuit equal to zero and solve for I .

SET UP: The circuit is sketched in Figure 25.73.

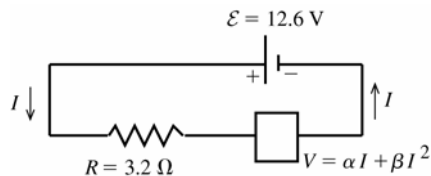


Figure 25.73

EXECUTE:

$$\mathcal{E} - IR - V = 0$$

$$\mathcal{E} - IR - \alpha I - \beta I^2 = 0$$

$$\beta I^2 + (R + \alpha)I - \mathcal{E} = 0$$

The quadratic formula gives $I = (1/2\beta) \left[-(R + \alpha) \pm \sqrt{(R + \alpha)^2 + 4\beta\mathcal{E}} \right]$

I must be positive, so take the $+$ sign

$$I = (1/2\beta) \left[-(R + \alpha) + \sqrt{(R + \alpha)^2 + 4\beta\mathcal{E}} \right]$$

$$I = -2.692 \text{ A} + 4.116 \text{ A} = 1.42 \text{ A}$$

EVALUATE: For this I the voltage across the thermistor is 8.0 V . The voltage across the resistor must then be $12.6 \text{ V} - 8.0 \text{ V} = 4.6 \text{ V}$, and this agrees with Ohm's law for the resistor.

- 25.74. (a) IDENTIFY:** The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A$. The diameter D of the cable is twice its radius.

$$P = \frac{V^2}{R} = \frac{V^2}{(\rho L/A)} = \frac{AV^2}{\rho L} = \frac{\pi r^2 V^2}{\rho L}. \text{ The electric field in the cable is equal to the potential difference across its}$$

ends divided by the length of the cable: $E = V/L$.

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(50.0 \text{ W})(1.72 \times 10^{-8} \Omega \cdot \text{m})(1500 \text{ m})}{\pi (220.0 \text{ V})^2}} = 9.21 \times 10^{-5} \text{ m}. \quad D = 2r = 0.184 \text{ mm}$$

(b) SET UP: $E = V/L$

EXECUTE: $E = (220 \text{ V})/(1500 \text{ m}) = 0.147 \text{ V/m}$

EVALUATE: This would be an extremely thin (and hence fragile) cable.

- 25.75. IDENTIFY:** The ammeter acts as a resistance in the circuit loop. Set the sum of the potential rises and drops around the circuit equal to zero.

(a) SET UP: The circuit with the ammeter is sketched in Figure 25.75a.

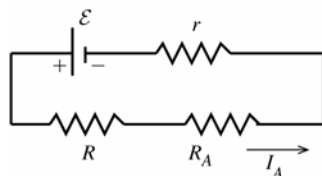


Figure 25.75a

EXECUTE:

$$I_A = \frac{\mathcal{E}}{r + R + R_A}$$

$$\mathcal{E} = I_A(r + R + R_A)$$

SET UP: The circuit with the ammeter removed is sketched in Figure 25.75b.

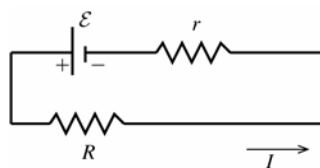


Figure 25.75b

EXECUTE:

$$I = \frac{\mathcal{E}}{R + r}$$

Combining the two equations gives

$$I = \left(\frac{1}{R + r} \right) I_A (r + R + R_A) = I_A \left(1 + \frac{R_A}{r + R} \right)$$

(b) Want $I_A = 0.990I$. Use this in the result for part (a).

$$I = 0.990I \left(1 + \frac{R_A}{r + R} \right)$$

$$0.010 = 0.990 \left(\frac{R_A}{r + R} \right)$$

$$R_A = (r + R)(0.010/0.990) = (0.45 \Omega + 3.80 \Omega)(0.010/0.990) = 0.0429 \Omega$$

$$\text{(c) } I - I_A = \frac{\mathcal{E}}{r + R} - \frac{\mathcal{E}}{r + R + R_A}$$

$$I - I_A = \mathcal{E} \left(\frac{r + R + R_A - r - R}{(r + R)(r + R + R_A)} \right) = \frac{\mathcal{E} R_A}{(r + R)(r + R + R_A)}.$$

EVALUATE: The difference between I and I_A increases as R_A increases. If R_A is larger than the value calculated in part (b) then I_A differs from I by more than 1.0%.

- 25.76. IDENTIFY:** Since the resistivity is a function of the position along the length of the cylinder, we must integrate to find the resistance.

(a) SET UP: The resistance of a cross-section of thickness dx is $dR = \rho dx/A$.

EXECUTE: Using the given function for the resistivity and integrating gives

$$R = \int \frac{\rho dx}{A} = \int_0^L \frac{(a + bx^2) dx}{\pi r^2} = \frac{aL + bL^3/3}{\pi r^2}.$$

Now get the constants a and b : $\rho(0) = a = 2.25 \times 10^{-8} \Omega \cdot \text{m}$ and

$$\rho(L) = a + bL^2 \text{ gives } 8.50 \times 10^{-8} \Omega \cdot \text{m} = 2.25 \times 10^{-8} \Omega \cdot \text{m} + b(1.50 \text{ m})^2$$

which gives $b = 2.78 \times 10^{-8} \Omega/\text{m}$. Now use the above result to find R .

$$R = \frac{(2.25 \times 10^{-8} \Omega \cdot \text{m})(1.50 \text{ m}) + (2.78 \times 10^{-8} \Omega/\text{m})(1.50 \text{ m})^3/3}{\pi(0.0110 \text{ m})^2} = 1.71 \times 10^{-4} \Omega = 171 \mu\Omega$$

(b) IDENTIFY: Use the definition of resistivity to find the electric field at the midpoint of the cylinder, where $x = L/2$.

SET UP: $E = \rho J$. Evaluate the resistivity, using the given formula, for $x = L/2$.

$$\text{EXECUTE: At the midpoint, } x = L/2, \text{ giving } E = \frac{\rho I}{\pi r^2} = \frac{[a + b(L/2)^2]I}{\pi r^2}.$$

$$E = \frac{[2.25 \times 10^{-8} \Omega \cdot \text{m} + (2.78 \times 10^{-8} \Omega/\text{m})(0.750 \text{ m})^2](1.75 \text{ A})}{\pi(0.0110 \text{ m})^2} = 1.76 \times 10^{-4} \text{ V/m}$$

(c) IDENTIFY: For the first segment, the result is the same as in part (a) except that the upper limit of the integral is $L/2$ instead of L .

$$\text{SET UP: Integrating using the upper limit of } L/2 \text{ gives } R_1 = \frac{a(L/2) + (b/3)(L^3/8)}{\pi r^2}.$$

EXECUTE: Substituting the numbers gives

$$R_1 = \frac{(2.25 \times 10^{-8} \Omega \cdot \text{m})(0.750 \text{ m}) + (2.78 \times 10^{-8} \Omega/\text{m})/3((1.50 \text{ m})^3/8)}{\pi(0.0110 \text{ m})^2} = 5.47 \times 10^{-5} \Omega$$

The resistance R_2 of the second half is equal to the total resistance minus the resistance of the first half.

$$R_2 = R - R_1 = 1.71 \times 10^{-4} \Omega - 5.47 \times 10^{-5} \Omega = 1.16 \times 10^{-4} \Omega$$

EVALUATE: The second half has a greater resistance than the first half because the resistance increases with distance along the cylinder.

- 25.77. **IDENTIFY:** The power supplied to the house is $P = VI$. The rate at which electrical energy is dissipated in the wires is $I^2 R$, where $R = \frac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$.

EXECUTE: (a) The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

$$\text{(b) } P = VI \text{ gives } I = \frac{P}{V} = \frac{4200 \text{ W}}{120 \text{ V}} = 35 \text{ A, so the 8-gauge wire is necessary, since it can carry up to 40 A.}$$

$$\text{(c) } P = I^2 R = \frac{I^2 \rho L}{A} = \frac{(35 \text{ A})^2 (1.72 \times 10^{-8} \Omega \cdot \text{m})(42.0 \text{ m})}{(\pi/4)(0.00326 \text{ m})^2} = 106 \text{ W.}$$

$$\text{(d) If 6-gauge wire is used, } P = \frac{I^2 \rho L}{A} = \frac{(35 \text{ A})^2 (1.72 \times 10^{-8} \Omega \cdot \text{m})(42 \text{ m})}{(\pi/4)(0.00412 \text{ m})^2} = 66 \text{ W. The decrease in energy}$$

consumption is $\Delta E = \Delta Pt = (40 \text{ W})(365 \text{ days/yr})(12 \text{ h/day}) = 175 \text{ kWh/yr}$ and the savings is $(175 \text{ kWh/yr})(\$0.11/\text{kWh}) = \19.25 per year.

EVALUATE: The cost of the 4200 W used by the appliances is \$2020. The savings is about 1%.

- 25.78. **IDENTIFY:** $R_T = R_0(1 + \alpha[T - T_0])$. $R = \frac{V}{I}$. $P = VI$.

SET UP: When the temperature increases the resistance increases and the current decreases.

$$\text{EXECUTE: (a) } \frac{V}{I_T} = \frac{V}{I_0}(1 + \alpha[T - T_0]). \quad I_0 = I_T(1 + \alpha[T - T_0]).$$

$$T - T_0 = \frac{I_0 - I_T}{\alpha I_T} = \frac{1.35 \text{ A} - 1.23 \text{ A}}{(1.23 \text{ A})(4.5 \times 10^{-4} (\text{C}^\circ)^{-1})} = 217 \text{ C}^\circ. \quad T = 20^\circ\text{C} + 217^\circ\text{C} = 237^\circ\text{C}$$

$$\text{(b) (i) } P = VI = (120 \text{ V})(1.35 \text{ A}) = 162 \text{ W} \quad \text{(ii) } P = (120 \text{ V})(1.23 \text{ A}) = 148 \text{ W}$$

EVALUATE: $P = V^2/R$ shows that the power dissipated decreases when the resistance increases.

- 25.79. **(a) IDENTIFY:** Set the sum of the potential rises and drops around the circuit equal to zero and solve for the resulting equation for the current I . Apply Eq. (25.17) to each circuit element to find the power associated with it.

SET UP: The circuit is sketched in Figure 25.79.

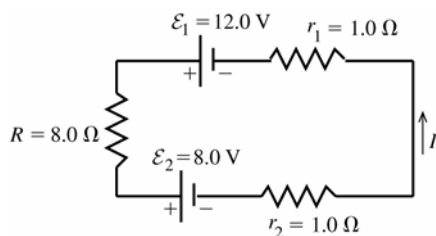


Figure 25.79

EXECUTE:

$$\mathcal{E}_1 - \mathcal{E}_2 - I(r_1 + r_2 + R) = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R}$$

$$I = \frac{12.0 \text{ V} - 8.0 \text{ V}}{1.0 \Omega + 1.0 \Omega + 8.0 \Omega}$$

$$I = 0.40 \text{ A}$$

(b) $P = I^2 R + I^2 r_1 + I^2 r_2 = I^2 (R + r_1 + r_2) = (0.40 \text{ A})^2 (8.0 \Omega + 1.0 \Omega + 1.0 \Omega)$

$$P = 1.6 \text{ W}$$

(c) Chemical energy is converted to electrical energy in a battery when the current goes through the battery from the negative to the positive terminal, so the electrical energy of the charges increases as the current passes through. This happens in the 12.0 V battery, and the rate of production of electrical energy is

$$P = \mathcal{E}_1 I = (12.0 \text{ V})(0.40 \text{ A}) = 4.8 \text{ W}.$$

(d) Electrical energy is converted to chemical energy in a battery when the current goes through the battery from the positive to the negative terminal, so the electrical energy of the charges decreases as the current passes through. This happens in the 8.0 V battery, and the rate of consumption of electrical energy is

$$P = \mathcal{E}_2 I = (8.0 \text{ V})(0.40 \text{ A}) = 3.2 \text{ W}.$$

(e) EVALUATE: Total rate of production of electrical energy = 4.8 W. Total rate of consumption of electrical energy = 1.6 W + 3.2 W = 4.8 W, which equals the rate of production, as it must.

25.80. IDENTIFY: Apply $R = \frac{\rho L}{A}$ for each material. The total resistance is the sum of the resistances of the rod and the wire. The rate at which energy is dissipated is $I^2 R$.

SET UP: For steel, $\rho = 2.0 \times 10^{-7} \Omega \cdot \text{m}$. For copper, $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$.

EXECUTE: **(a)** $R_{\text{steel}} = \frac{\rho L}{A} = \frac{(2.0 \times 10^{-7} \Omega \cdot \text{m})(2.0 \text{ m})}{(\pi/4)(0.018 \text{ m})^2} = 1.57 \times 10^{-3} \Omega$ and

$$R_{\text{Cu}} = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(35 \text{ m})}{(\pi/4)(0.008 \text{ m})^2} = 0.012 \Omega. \text{ This gives}$$

$$V = IR = I(R_{\text{steel}} + R_{\text{Cu}}) = (15000 \text{ A})(1.57 \times 10^{-3} \Omega + 0.012 \Omega) = 204 \text{ V}.$$

(b) $E = Pt = I^2 R t = (15000 \text{ A})^2 (0.0136 \Omega) (65 \times 10^{-6} \text{ s}) = 199 \text{ J}.$

EVALUATE: $I^2 R$ is large but t is very small, so the energy deposited is small. The wire and rod each have a mass of about 1 kg, so their temperature rise due to the deposited energy will be small.

25.81. IDENTIFY and SET UP: The terminal voltage is $V_{ab} = \mathcal{E} - Ir = IR$, where R is the resistance connected to the battery. During the charging the terminal voltage is $V_{ab} = \mathcal{E} + Ir$. $P = VI$ and energy is $E = Pt$. $I^2 r$ is the rate at which energy is dissipated in the internal resistance of the battery.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir = 12.0 \text{ V} + (10.0 \text{ A})(0.24 \Omega) = 14.4 \text{ V}.$

(b) $E = Pt = IVt = (10 \text{ A})(14.4 \text{ V})(5)(3600 \text{ s}) = 2.59 \times 10^6 \text{ J}.$

(c) $E_{\text{diss}} = P_{\text{diss}} t = I^2 r t = (10 \text{ A})^2 (0.24 \Omega) (5)(3600 \text{ s}) = 4.32 \times 10^5 \text{ J}.$

(d) Discharged at 10 A: $I = \frac{\mathcal{E}}{r + R} \Rightarrow R = \frac{\mathcal{E} - Ir}{I} = \frac{12.0 \text{ V} - (10 \text{ A})(0.24 \Omega)}{10 \text{ A}} = 0.96 \Omega.$

(e) $E = Pt = IVt = (10 \text{ A})(9.6 \text{ V})(5)(3600 \text{ s}) = 1.73 \times 10^6 \text{ J}.$

(f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in

(c): $E_{\text{diss}} = 4.32 \times 10^5 \text{ J}.$

(g) Part of the energy originally supplied was stored in the battery and part was lost in the internal resistance. So the stored energy was less than what was supplied during charging. Then when discharging, even more energy is lost in the internal resistance, and only what is left is dissipated by the external resistor.

- 25.82. IDENTIFY and SET UP:** The terminal voltage is $V_{ab} = \mathcal{E} - Ir = IR$, where R is the resistance connected to the battery. During the charging the terminal voltage is $V_{ab} = \mathcal{E} + Ir$. $P = VI$ and energy is $E = Pt$. $I^2 r$ is the rate at which energy is dissipated in the internal resistance of the battery.

EXECUTE: (a) $V_{ab} = \mathcal{E} + Ir = 12.0 \text{ V} + (30 \text{ A})(0.24 \Omega) = 19.2 \text{ V}$.

(b) $E = Pt = IVt = (30 \text{ A})(19.2 \text{ V})(1.7)(3600 \text{ s}) = 3.53 \times 10^6 \text{ J}$.

(c) $E_{\text{diss}} = P_{\text{diss}} t = I^2 R t = (30 \text{ A})^2 (0.24 \Omega)(1.7)(3600 \text{ s}) = 1.32 \times 10^6 \text{ J}$.

(d) Discharged at 30 A: $I = \frac{\mathcal{E}}{r + R}$ gives $R = \frac{\mathcal{E} - Ir}{I} = \frac{12.0 \text{ V} - (30 \text{ A})(0.24 \Omega)}{30 \text{ A}} = 0.16 \Omega$.

(e) $E = Pt = I^2 R t = (30 \text{ A})^2 (0.16 \Omega)(1.7)(3600 \text{ s}) = 8.81 \times 10^5 \text{ J}$.

(f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{\text{diss}} = 1.32 \times 10^6 \text{ J}$.

(g) Again, part of the energy originally supplied was stored in the battery and part was lost in the internal resistance. So the stored energy was less than what was supplied during charging. Then when discharging, even more energy is lost in the internal resistance, and what is left is dissipated over the external resistor. This time, at a higher current, much more energy is lost in the internal resistance. Slow charging and discharging is more energy efficient.

- 25.83. IDENTIFY and SET UP:** Follow the steps specified in the problem.

EXECUTE: (a) $\Sigma F = ma = |q|E$ gives $\frac{|q|}{m} = \frac{a}{E}$.

(b) If the electric field is constant, $V_{bc} = EL$ and $\frac{|q|}{m} = \frac{aL}{V_{bc}}$.

(c) The free charges are “left behind” so the left end of the rod is negatively charged, while the right end is positively charged. Thus the right end, point c , is at the higher potential.

(d) $a = \frac{V_{bc} |q|}{mL} = \frac{(1.0 \times 10^{-3} \text{ V})(1.6 \times 10^{-19} \text{ C})}{(9.11 \times 10^{-31} \text{ kg})(0.50 \text{ m})} = 3.5 \times 10^8 \text{ m/s}^2$.

EVALUATE: (e) Performing the experiment in a rotational way enables one to keep the experimental apparatus in a localized area—whereas an acceleration like that obtained in (d), if linear, would quickly have the apparatus moving at high speeds and large distances. Also, the rotating spool of thin wire can have many turns of wire and the total potential is the sum of the potentials in each turn, the potential in each turn times the number of turns.

- 25.84. IDENTIFY:** $\mathcal{E} - IR - V = 0$

SET UP: With $T = 293 \text{ K}$, $\frac{e}{kT} = 39.6 \text{ V}^{-1}$.

EXECUTE: (a) $\mathcal{E} = IR + V$ gives $2.00 \text{ V} = I(1.0 \Omega) + V$. Dropping units and using the expression given in the problem for I , this becomes $2.00 = I_s[\exp(eV/kT) - 1] + V$.

(b) For $I_s = 1.50 \times 10^{-3} \text{ A}$ and $T = 293 \text{ K}$, $1333 = \exp[39.6 V] - 1 + 667V$. Trial and error shows that the right-hand side (rhs) above, for specific V values, equals 1333 V when $V = 0.179 \text{ V}$. The current then is

just $I = I_s[\exp(39.6 V) - 1] = (1.5 \times 10^{-3} \text{ A})(\exp([39.6][0.179]) - 1) = 1.80 \text{ A}$.

EVALUATE: The voltage across the resistor R is 1.80 V. The diode does not obey Ohm’s law.

- 25.85. IDENTIFY:** Apply $R = \frac{\rho L}{A}$ to find the resistance of a thin slice of the rod and integrate to find the total R .

$V = IR$. Also find $R(x)$, the resistance of a length x of the rod.

SET UP: $E(x) = \rho(x)J$

EXECUTE: (a) $dR = \frac{\rho dx}{A} = \frac{\rho_0 \exp[-x/L] dx}{A}$ so

$R = \frac{\rho_0}{A} \int_0^L \exp[-x/L] dx = \frac{\rho_0}{A} [-L \exp[-x/L]]_0^L = \frac{\rho_0 L}{A} (1 - e^{-1})$ and $I = \frac{V_0}{R} = \frac{V_0 A}{\rho_0 L (1 - e^{-1})}$. With an upper limit of x

rather than L in the integration, $R(x) = \frac{\rho_0 L}{A} (1 - e^{-x/L})$.

(b) $E(x) = \rho(x)J = \frac{I \rho_0 e^{-x/L}}{A} = \frac{V_0 e^{-x/L}}{L(1 - e^{-1})}$.

$$(c) V = V_0 - IR(x). \quad V = V_0 - \left(\frac{V_0 A}{\rho_0 L [1 - e^{-1}]} \right) \left(\frac{\rho_0 L}{A} \right) (1 - e^{-x/L}) = V_0 \frac{(e^{-x/L} - e^{-1})}{(1 - e^{-1})}$$

(d) Graphs of resistivity, electric field and potential from $x = 0$ to L are given in Figure 25.85. Each quantity is given in terms of the indicated unit.

EVALUATE: The current is the same at all points in the rod. Where the resistivity is larger the electric field must be larger, in order to produce the same current density.

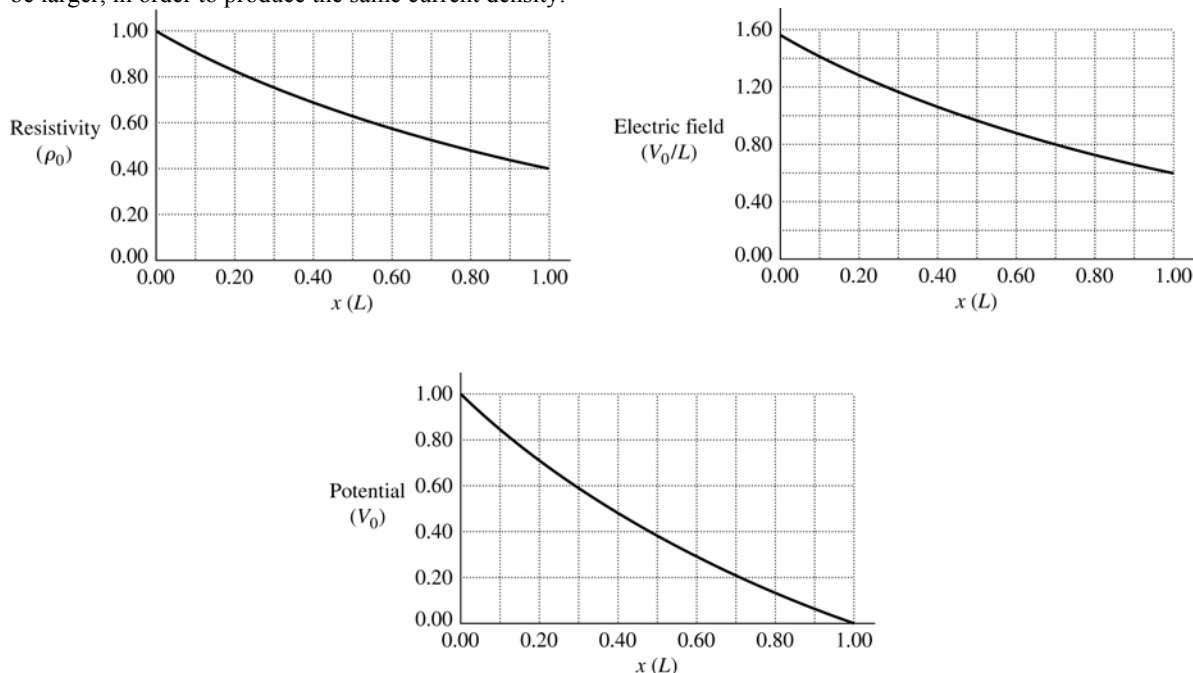


Figure 25.85

25.86. **IDENTIFY:** The power output of the source is $VI = (\mathcal{E} - Ir)I$.

SET UP: The short-circuit current is $I_{\text{short circuit}} = \mathcal{E}/r$.

EXECUTE: (a) $P = \mathcal{E}I - I^2r$, so $\frac{dP}{dI} = \mathcal{E} - 2Ir = 0$ for maximum power output and $I_{P \text{ max}} = \frac{1}{2} \frac{\mathcal{E}}{r} = \frac{1}{2} I_{\text{short circuit}}$.

(b) For the maximum power output of part (a), $I = \frac{\mathcal{E}}{r + R} = \frac{1}{2} \frac{\mathcal{E}}{r}$. $r + R = 2r$ and $R = r$.

$$\text{Then, } P = I^2 R = \left(\frac{\mathcal{E}}{2r} \right)^2 r = \frac{\mathcal{E}^2}{4r}.$$

EVALUATE: When R is smaller than r , I is large and the I^2r losses in the battery are large. When R is larger than r , I is small and the power output $\mathcal{E}I$ of the battery emf is small.

25.87. **IDENTIFY:** Use $\alpha = -n/T$ in $\alpha = \frac{1}{\rho} \frac{d\rho}{dT}$ to get a separable differential equation that can be integrated.

SET UP: For carbon, $\rho = 3.5 \times 10^{-5} \Omega \cdot \text{m}$ and $\alpha = -5 \times 10^{-4} (\text{K})^{-1}$.

EXECUTE: (a) $\alpha = \frac{1}{\rho} \left(\frac{d\rho}{dT} \right) = -\frac{n}{T} \Rightarrow \frac{ndT}{T} = \frac{d\rho}{\rho} \Rightarrow \ln(T^{-n}) = \ln(\rho) \Rightarrow \rho = \frac{a}{T^n}$.

(b) $n = -\alpha T = -(-5 \times 10^{-4} (\text{K})^{-1})(293 \text{ K}) = 0.15$.

$$\rho = \frac{a}{T^n} \Rightarrow a = \rho T^n = (3.5 \times 10^{-5} \Omega \cdot \text{m})(293 \text{ K})^{0.15} = 8.0 \times 10^{-5} \Omega \cdot \text{m} \cdot \text{K}^{0.15}.$$

(c) $T = -196^\circ\text{C} = 77 \text{ K}$: $\rho = \frac{8.0 \times 10^{-5}}{(77 \text{ K})^{0.15}} = 4.3 \times 10^{-5} \Omega \cdot \text{m}$.

$T = -300^\circ\text{C} = 573 \text{ K}$: $\rho = \frac{8.0 \times 10^{-5}}{(573 \text{ K})^{0.15}} = 3.2 \times 10^{-5} \Omega \cdot \text{m}$.

EVALUATE: α is negative and decreases as T decreases, so ρ changes more rapidly with temperature at lower temperatures.

DIRECT-CURRENT CIRCUITS

- 26.1. IDENTIFY:** The newly-formed wire is a combination of series and parallel resistors.
SET UP: Each of the three linear segments has resistance $R/3$. The circle is two $R/6$ resistors in parallel.
EXECUTE: The resistance of the circle is $R/12$ since it consists of two $R/6$ resistors in parallel. The equivalent resistance is two $R/3$ resistors in series with an $R/6$ resistor, giving $R_{\text{equiv}} = R/3 + R/3 + R/12 = 3R/4$.
EVALUATE: The equivalent resistance of the original wire has been reduced because the circle's resistance is less than it was as a linear wire.
- 26.2. IDENTIFY:** It may appear that the meter measures X directly. But note that X is in parallel with three other resistors, so the meter measures the equivalent parallel resistance between ab .
SET UP: We use the formula for resistors in parallel.
EXECUTE: $1/(2.00 \, \Omega) = 1/X + 1/(15.0 \, \Omega) + 1/(5.0 \, \Omega) + 1/(10.0 \, \Omega)$, so $X = 7.5 \, \Omega$.
EVALUATE: X is *greater* than the equivalent parallel resistance of $2.00 \, \Omega$.
- 26.3. (a) IDENTIFY:** Suppose we have two resistors in parallel, with $R_1 < R_2$.
SET UP: The equivalent resistance is $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$.
EXECUTE: It is always true that $\frac{1}{R_1} + \frac{1}{R_2} > \frac{1}{R_1}$. Therefore $\frac{1}{R_{\text{eq}}} > \frac{1}{R_1}$ and $R_{\text{eq}} < R_1$.
EVALUATE: The equivalent resistance is always less than that of the smallest resistor.
(b) IDENTIFY: Suppose we have N resistors in parallel, with $R_1 < R_2 < \dots < R_N$.
SET UP: The equivalent resistance is $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$.
EXECUTE: It is always true that $\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} > \frac{1}{R_1}$. Therefore $\frac{1}{R_{\text{eq}}} > \frac{1}{R_1}$ and $R_{\text{eq}} < R_1$.
EVALUATE: The equivalent resistance is always less than that of the smallest resistor.
- 26.4. IDENTIFY:** For resistors in parallel the voltages are the same and equal to the voltage across the equivalent resistance.
SET UP: $V = IR$. $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$.
EXECUTE: (a) $R_{\text{eq}} = \left(\frac{1}{32 \, \Omega} + \frac{1}{20 \, \Omega} \right)^{-1} = 12.3 \, \Omega$.
(b) $I = \frac{V}{R_{\text{eq}}} = \frac{240 \, \text{V}}{12.3 \, \Omega} = 19.5 \, \text{A}$.
(c) $I_{32\Omega} = \frac{V}{R} = \frac{240 \, \text{V}}{32 \, \Omega} = 7.5 \, \text{A}$; $I_{20\Omega} = \frac{V}{R} = \frac{240 \, \text{V}}{20 \, \Omega} = 12 \, \text{A}$.
EVALUATE: More current flows through the resistor that has the smaller R .
- 26.5. IDENTIFY:** The equivalent resistance will vary for the different connections because the series-parallel combinations vary, and hence the current will vary.
SET UP: First calculate the equivalent resistance using the series-parallel formulas, then use Ohm's law ($V = RI$) to find the current.
EXECUTE: (a) $1/R = 1/(15.0 \, \Omega) + 1/(30.0 \, \Omega)$ gives $R = 10.0 \, \Omega$. $I = V/R = (35.0 \, \text{V})/(10.0 \, \Omega) = 3.50 \, \text{A}$.
(b) $1/R = 1/(10.0 \, \Omega) + 1/(35.0 \, \Omega)$ gives $R = 7.78 \, \Omega$. $I = (35.0 \, \text{V})/(7.78 \, \Omega) = 4.50 \, \text{A}$.
(c) $1/R = 1/(20.0 \, \Omega) + 1/(25.0 \, \Omega)$ gives $R = 11.11 \, \Omega$, so $I = (35.0 \, \text{V})/(11.11 \, \Omega) = 3.15 \, \text{A}$.

(d) From part (b), the resistance of the triangle alone is $7.78\ \Omega$. Adding the $3.00\text{-}\Omega$ internal resistance of the battery gives an equivalent resistance for the circuit of $10.78\ \Omega$. Therefore the current is $I = (35.0\text{ V})/(10.78\ \Omega) = 3.25\text{ A}$

EVALUATE: It makes a big difference how the triangle is connected to the battery.

- 26.6. IDENTIFY:** The potential drop is the same across the resistors in parallel, and the current into the parallel combination is the same as the current through the $45.0\text{-}\Omega$ resistor.

(a) SET UP: Apply Ohm's law in the parallel branch to find the current through the $45.0\text{-}\Omega$ resistor. Then apply Ohm's law to the $45.0\text{-}\Omega$ resistor to find the potential drop across it.

EXECUTE: The potential drop across the $25.0\text{-}\Omega$ resistor is $V_{25} = (25.0\ \Omega)(1.25\text{ A}) = 31.25\text{ V}$. The potential drop across each of the parallel branches is 31.25 V . For the $15.0\text{-}\Omega$ resistor: $I_{15} = (31.25\text{ V})/(15.0\ \Omega) = 2.083\text{ A}$. The resistance of the $10.0\text{-}\Omega + 15.0\ \Omega$ combination is $25.0\ \Omega$, so the current through it must be the same as the current through the upper $25.0\ \Omega$ resistor: $I_{10+15} = 1.25\text{ A}$. The sum of currents in the parallel branch will be the current through the $45.0\text{-}\Omega$ resistor.

$$I_{\text{Total}} = 1.25\text{ A} + 2.083\text{ A} + 1.25\text{ A} = 4.58\text{ A}$$

Apply Ohm's law to the $45.0\ \Omega$ resistor: $V_{45} = (4.58\text{ A})(45.0\ \Omega) = 206\text{ V}$

(b) SET UP: First find the equivalent resistance of the circuit and then apply Ohm's law to it.

EXECUTE: The resistance of the parallel branch is $1/R = 1/(25.0\ \Omega) + 1/(15.0\ \Omega) + 1/(25.0\ \Omega)$, so $R = 6.82\ \Omega$. The equivalent resistance of the circuit is $6.82\ \Omega + 45.0\ \Omega + 35.00\ \Omega = 86.82\ \Omega$. Ohm's law gives $V_{\text{Bat}} = (86.82\ \Omega)(4.58\text{ A}) = 398\text{ V}$.

EVALUATE: The emf of the battery is the sum of the potential drops across each of the three segments (parallel branch and two series resistors).

- 26.7. IDENTIFY:** First do as much series-parallel reduction as possible.

SET UP: The $45.0\text{-}\Omega$ and $15.0\text{-}\Omega$ resistors are in parallel, so first reduce them to a single equivalent resistance. Then find the equivalent series resistance of the circuit.

EXECUTE: $1/R_p = 1/(45.0\ \Omega) + 1/(15.0\ \Omega)$ and $R_p = 11.25\ \Omega$. The total equivalent resistance is $18.0\ \Omega + 11.25\ \Omega + 3.26\ \Omega = 32.5\ \Omega$. Ohm's law gives $I = (25.0\text{ V})/(32.5\ \Omega) = 0.769\text{ A}$.

EVALUATE: The circuit appears complicated until we realize that the $45.0\text{-}\Omega$ and $15.0\text{-}\Omega$ resistors are in parallel.

- 26.8. IDENTIFY:** Eq. (26.2) gives the equivalent resistance of the three resistors in parallel. For resistors in parallel, the voltages are the same and the currents add.

(a) SET UP: The circuit is sketched in Figure 26.8a.

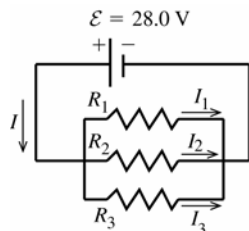


Figure 26.8a

EXECUTE: parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{1.60\ \Omega} + \frac{1}{2.40\ \Omega} + \frac{1}{4.80\ \Omega}$$

$$R_{\text{eq}} = 0.800\ \Omega$$

(b) For resistors in parallel the voltage is the same across each and equal to the applied voltage;

$$V_1 = V_2 = V_3 = \mathcal{E} = 28.0\text{ V}$$

$$V = IR \text{ so } I_1 = \frac{V_1}{R_1} = \frac{28.0\text{ V}}{1.60\ \Omega} = 17.5\text{ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{28.0\text{ V}}{2.40\ \Omega} = 11.7\text{ A} \text{ and } I_3 = \frac{V_3}{R_3} = \frac{28.0\text{ V}}{4.8\ \Omega} = 5.8\text{ A}$$

(c) The currents through the resistors add to give the current through the battery:

$$I = I_1 + I_2 + I_3 = 17.5\text{ A} + 11.7\text{ A} + 5.8\text{ A} = 35.0\text{ A}$$

EVALUATE: Alternatively, we can use the equivalent resistance R_{eq} as shown in Figure 26.8b.

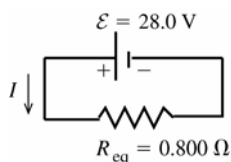


Figure 26.8b

$$\mathcal{E} - IR_{\text{eq}} = 0$$

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{28.0\text{ V}}{0.800\ \Omega} = 35.0\text{ A},$$

which checks

(d) As shown in part (b), the voltage across each resistor is 28.0 V .

(e) IDENTIFY and SET UP: We can use any of the three expressions for P : $P = VI = I^2R = V^2/R$. They will all give the same results, if we keep enough significant figures in intermediate calculations.

EXECUTE: Using $P = V^2 / R$, $P_1 = V_1^2 / R_1 = \frac{(28.0 \text{ V})^2}{1.60 \Omega} = 490 \text{ W}$, $P_2 = V_2^2 / R_2 = \frac{(28.0 \text{ V})^2}{2.40 \Omega} = 327 \text{ W}$, and

$$P_3 = V_3^2 / R_3 = \frac{(28.0 \text{ V})^2}{4.80 \Omega} = 163 \text{ W}$$

EVALUATE: The total power dissipated is $P_{\text{out}} = P_1 + P_2 + P_3 = 980 \text{ W}$. This is the same as the power

$$P_{\text{in}} = \mathcal{E}I = (28.0 \text{ V})(35.0 \text{ A}) = 980 \text{ W} \text{ delivered by the battery.}$$

(f) $P = V^2 / R$. The resistors in parallel each have the same voltage, so the power P is largest for the one with the least resistance.

26.9. IDENTIFY: For a series network, the current is the same in each resistor and the sum of voltages for each resistor equals the battery voltage. The equivalent resistance is $R_{\text{eq}} = R_1 + R_2 + R_3$. $P = I^2 R$.

SET UP: Let $R_1 = 1.60 \Omega$, $R_2 = 2.40 \Omega$, $R_3 = 4.80 \Omega$.

EXECUTE: **(a)** $R_{\text{eq}} = 1.60 \Omega + 2.40 \Omega + 4.80 \Omega = 8.80 \Omega$

$$\textbf{(b)} \quad I = \frac{V}{R_{\text{eq}}} = \frac{28.0 \text{ V}}{8.80 \Omega} = 3.18 \text{ A}$$

(c) $I = 3.18 \text{ A}$, the same as for each resistor.

$$\textbf{(d)} \quad V_1 = IR_1 = (3.18 \text{ A})(1.60 \Omega) = 5.09 \text{ V} \quad V_2 = IR_2 = (3.18 \text{ A})(2.40 \Omega) = 7.63 \text{ V}.$$

$$V_3 = IR_3 = (3.18 \text{ A})(4.80 \Omega) = 15.3 \text{ V}. \text{ Note that } V_1 + V_2 + V_3 = 28.0 \text{ V}.$$

$$\textbf{(e)} \quad P_1 = I^2 R_1 = (3.18 \text{ A})^2 (1.60 \Omega) = 16.2 \text{ W} \quad P_2 = I^2 R_2 = (3.18 \text{ A})^2 (2.40 \Omega) = 24.3 \text{ W}.$$

$$P_3 = I^2 R_3 = (3.18 \text{ A})^2 (4.80 \Omega) = 48.5 \text{ W}.$$

(f) Since $P = I^2 R$ and the current is the same for each resistor, the resistor with the greatest R dissipates the greatest power.

EVALUATE: When resistors are connected in parallel, the resistor with the smallest R dissipates the greatest power.

26.10. (a) IDENTIFY: The current, and hence the power, depends on the potential difference across the resistor.

SET UP: $P = V^2 / R$

$$\textbf{EXECUTE: (a)} \quad V = \sqrt{PR} = \sqrt{(5.0 \text{ W})(15,000 \Omega)} = 274 \text{ V}$$

$$\textbf{(b)} \quad P = V^2 / R = (120 \text{ V})^2 / (9,000 \Omega) = 1.6 \text{ W}$$

SET UP: (c) If the larger resistor generates 2.00 W, the smaller one will generate less and hence will be safe.

Therefore the maximum power in the larger resistor must be 2.00 W. Use $P = I^2 R$ to find the maximum current through the series combination and use Ohm's law to find the potential difference across the combination.

EXECUTE: $P = I^2 R$ gives $I = P/R = (2.00 \text{ W})/(150 \Omega) = 0.0133 \text{ A}$. The same current flows through both resistors, and their equivalent resistance is 250 Ω . Ohm's law gives $V = IR = (0.0133 \text{ A})(250 \Omega) = 3.33 \text{ V}$.

Therefore $P_{150} = 2.00 \text{ W}$ and $P_{100} = I^2 R = (0.0133 \text{ A})^2 (100 \Omega) = 0.0177 \text{ W}$.

EVALUATE: If the resistors in a series combination all have the same power rating, it is the *largest* resistance that limits the amount of current.

26.11. IDENTIFY: For resistors in parallel, the voltages are the same and the currents add. $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$ so $R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$,

For resistors in series, the currents are the same and the voltages add. $R_{\text{eq}} = R_1 + R_2$.

SET UP: The rules for combining resistors in series and parallel lead to the sequences of equivalent circuits shown in Figure 26.11.

EXECUTE: $R_{\text{eq}} = 5.00 \Omega$. In Figure 26.11c, $I = \frac{60.0 \text{ V}}{5.00 \Omega} = 12.0 \text{ A}$. This is the current through each of the

resistors in Figure 26.11b. $V_{12} = IR_{12} = (12.0 \text{ A})(2.00 \Omega) = 24.0 \text{ V}$. $V_{34} = IR_{34} = (12.0 \text{ A})(3.00 \Omega) = 36.0 \text{ V}$. Note

that $V_{12} + V_{34} = 60.0 \text{ V}$. V_{12} is the voltage across R_1 and across R_2 , so $I_1 = \frac{V_{12}}{R_1} = \frac{24.0 \text{ V}}{3.00 \Omega} = 8.00 \text{ A}$ and

$$I_2 = \frac{V_{12}}{R_2} = \frac{24.0 \text{ V}}{6.00 \Omega} = 4.00 \text{ A}. \quad V_{34} \text{ is the voltage across } R_3 \text{ and across } R_4, \text{ so } I_3 = \frac{V_{34}}{R_3} = \frac{36.0 \text{ V}}{12.0 \Omega} = 3.00 \text{ A and}$$

$$I_4 = \frac{V_{34}}{R_4} = \frac{36.0 \text{ V}}{4.00 \Omega} = 9.00 \text{ A}.$$

EVALUATE: Note that $I_1 + I_2 = I_3 + I_4$.

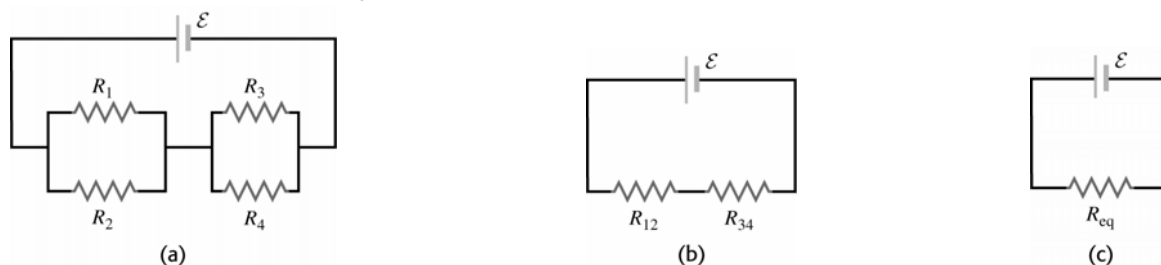


Figure 26.11

- 26.12. IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.12a.

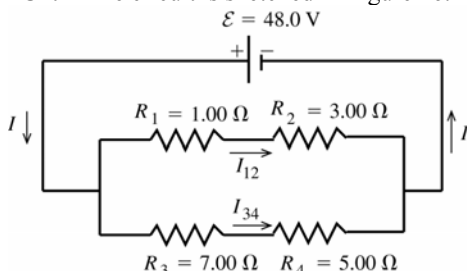


Figure 26.12a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00 \, \Omega$

R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0 \, \Omega$

The circuit is equivalent to the circuit sketched in Figure 26.12b.

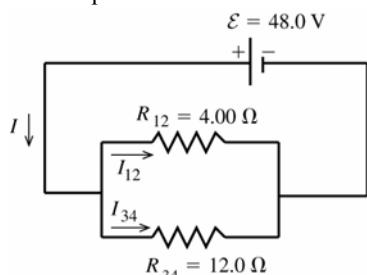


Figure 26.12b

R_{12} and R_{34} in parallel are equivalent to

$$R_{\text{eq}} \text{ given by } \frac{1}{R_{\text{eq}}} = \frac{1}{R_{12}} + \frac{1}{R_{34}} = \frac{R_{12} + R_{34}}{R_{12}R_{34}}$$

$$R_{\text{eq}} = \frac{R_{12}R_{34}}{R_{12} + R_{34}}$$

$$R_{\text{eq}} = \frac{(4.00 \, \Omega)(12.0 \, \Omega)}{4.00 \, \Omega + 12.0 \, \Omega} = 3.00 \, \Omega$$

The voltage across each branch of the parallel combination is $\mathcal{E}$, so $\mathcal{E} - I_{12}R_{12} = 0$.

$$I_{12} = \frac{\mathcal{E}}{R_{12}} = \frac{48.0 \, \text{V}}{4.00 \, \Omega} = 12.0 \, \text{A}$$

$$\mathcal{E} - I_{34}R_{34} = 0 \text{ so } I_{34} = \frac{\mathcal{E}}{R_{34}} = \frac{48.0 \, \text{V}}{12.0 \, \Omega} = 4.0 \, \text{A}$$

The current is 12.0 A through the 1.00 Ω and 3.00 Ω resistors, and it is 4.0 A through the 7.00 Ω and 5.00 Ω resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0 \, \text{A} + 4.0 \, \text{A} = 16.0 \, \text{A}$, and this is equal to

$$\mathcal{E}/R_{\text{eq}} = 48.0 \, \text{V}/3.00 \, \Omega = 16.0 \, \text{A}.$$

- 26.13. IDENTIFY:** In both circuits, with and without R_4 , replace series and parallel combinations of resistors by their equivalents. Calculate the currents and voltages in the equivalent circuit and infer from this the currents and voltages in the original circuit. Use $P = I^2R$ to calculate the power dissipated in each bulb.

(a) SET UP: The circuit is sketched in Figure 26.13a.

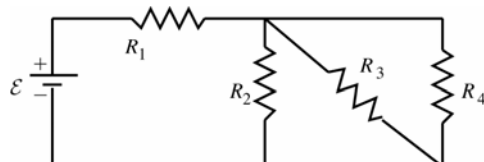


Figure 26.13a

EXECUTE: R_2 , R_3 , and R_4 are in parallel, so their equivalent resistance

$$R_{\text{eq}} \text{ is given by } \frac{1}{R_{\text{eq}}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_{\text{eq}}} = \frac{3}{4.50 \, \Omega} \text{ and } R_{\text{eq}} = 1.50 \, \Omega.$$

The equivalent circuit is drawn in Figure 26.13b.

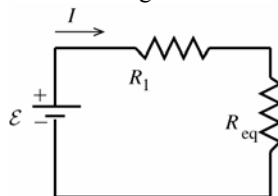


Figure 26.13b

$$\mathcal{E} - I(R_1 + R_{eq}) = 0$$

$$I = \frac{\mathcal{E}}{R_1 + R_{eq}}$$

$$I = \frac{9.00 \text{ V}}{4.50 \Omega + 1.50 \Omega} = 1.50 \text{ A and } I_1 = 1.50 \text{ A}$$

$$\text{Then } V_1 = I_1 R_1 = (1.50 \text{ A})(4.50 \Omega) = 6.75 \text{ V}$$

$$I_{eq} = 1.50 \text{ A, } V_{eq} = I_{eq} R_{eq} = (1.50 \text{ A})(1.50 \Omega) = 2.25 \text{ V}$$

For resistors in parallel the voltages are equal and are the same as the voltage across the equivalent resistor, so $V_2 = V_3 = V_4 = 2.25 \text{ V}$.

$$I_2 = \frac{V_2}{R_2} = \frac{2.25 \text{ V}}{4.50 \Omega} = 0.500 \text{ A, } I_3 = \frac{V_3}{R_3} = 0.500 \text{ A, } I_4 = \frac{V_4}{R_4} = 0.500 \text{ A}$$

EVALUATE: Note that $I_2 + I_3 + I_4 = 1.50 \text{ A}$, which is I_{eq} . For resistors in parallel the currents add and their sum is the current through the equivalent resistor.

(b) SET UP: $P = I^2 R$

$$\text{EXECUTE: } P_1 = (1.50 \text{ A})^2 (4.50 \Omega) = 10.1 \text{ W}$$

$$P_2 = P_3 = P_4 = (0.500 \text{ A})^2 (4.50 \Omega) = 1.125 \text{ W, which rounds to } 1.12 \text{ W. } R_1 \text{ glows brightest.}$$

EVALUATE: Note that $P_2 + P_3 + P_4 = 3.37 \text{ W}$. This equals $P_{eq} = I_{eq}^2 R_{eq} = (1.50 \text{ A})^2 (1.50 \Omega) = 3.37 \text{ W}$, the power dissipated in the equivalent resistor.

(c) SET UP: With R_4 removed the circuit becomes the circuit in Figure 26.13c.

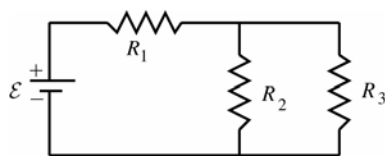


Figure 26.13c

EXECUTE: R_2 and R_3 are in parallel and their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{2}{4.50 \Omega} \text{ and } R_{eq} = 2.25 \Omega$$

The equivalent circuit is shown in Figure 26.13d.

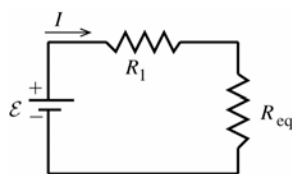


Figure 26.13d

$$\mathcal{E} - I(R_1 + R_{eq}) = 0$$

$$I = \frac{\mathcal{E}}{R_1 + R_{eq}}$$

$$I = \frac{9.00 \text{ V}}{4.50 \Omega + 2.25 \Omega} = 1.333 \text{ A}$$

$$I_1 = 1.33 \text{ A, } V_1 = I_1 R_1 = (1.333 \text{ A})(4.50 \Omega) = 6.00 \text{ V}$$

$$I_{eq} = 1.33 \text{ A, } V_{eq} = I_{eq} R_{eq} = (1.333 \text{ A})(2.25 \Omega) = 3.00 \text{ V and } V_2 = V_3 = 3.00 \text{ V.}$$

$$I_2 = \frac{V_2}{R_2} = \frac{3.00 \text{ V}}{4.50 \Omega} = 0.667 \text{ A, } I_3 = \frac{V_3}{R_3} = 0.667 \text{ A}$$

(d) SET UP: $P = I^2 R$

$$\text{EXECUTE: } P_1 = (1.333 \text{ A})^2 (4.50 \Omega) = 8.00 \text{ W}$$

$$P_2 = P_3 = (0.667 \text{ A})^2 (4.50 \Omega) = 2.00 \text{ W.}$$

(e) EVALUATE: When R_4 is removed, P_1 decreases and P_2 and P_3 increase. Bulb R_1 glows less brightly and bulbs R_2 and R_3 glow more brightly. When R_4 is removed the equivalent resistance of the circuit increases and the current through R_1 decreases. But in the parallel combination this current divides into two equal currents rather than three, so the currents through R_2 and R_3 increase. Can also see this by noting that with R_4 removed and less current through R_1 the voltage drop across R_1 is less so the voltage drop across R_2 and across R_3 must become larger.

26.14. IDENTIFY: Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: From Ohm's law, the voltage drop across the $6.00\ \Omega$ resistor is $V = IR = (4.00\ \text{A})(6.00\ \Omega) = 24.0\ \text{V}$. The voltage drop across the $8.00\ \Omega$ resistor is the same, since these two resistors are wired in parallel. The current through the $8.00\ \Omega$ resistor is then $I = V/R = 24.0\ \text{V}/8.00\ \Omega = 3.00\ \text{A}$. The current through the $25.0\ \Omega$ resistor is the sum of these two currents: $7.00\ \text{A}$. The voltage drop across the $25.0\ \Omega$ resistor is $V = IR = (7.00\ \text{A})(25.0\ \Omega) = 175\ \text{V}$, and total voltage drop across the top branch of the circuit is $175\ \text{V} + 24.0\ \text{V} = 199\ \text{V}$, which is also the voltage drop across the $20.0\ \Omega$ resistor. The current through the $20.0\ \Omega$ resistor is then $I = V/R = 199\ \text{V}/20\ \Omega = 9.95\ \text{A}$.

EVALUATE: The total current through the battery is $7.00\ \text{A} + 9.95\ \text{A} = 16.95\ \text{A}$. Note that we did not need to calculate the emf of the battery.

26.15. IDENTIFY: Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: The current through $2.00\text{-}\Omega$ resistor is $6.00\ \text{A}$. Current through $1.00\text{-}\Omega$ resistor also is $6.00\ \text{A}$ and the voltage is $6.00\ \text{V}$. Voltage across the $6.00\text{-}\Omega$ resistor is $12.0\ \text{V} + 6.0\ \text{V} = 18.0\ \text{V}$. Current through the $6.00\text{-}\Omega$ resistor is $(18.0\ \text{V})/(6.00\ \Omega) = 3.00\ \text{A}$. The battery emf is $18.0\ \text{V}$.

EVALUATE: The current through the battery is $6.00\ \text{A} + 3.00\ \text{A} = 9.00\ \text{A}$. The equivalent resistor of the resistor network is $2.00\ \Omega$, and this equals $(18.0\ \text{V})/(9.00\ \text{A})$.

26.16. IDENTIFY: The filaments must be connected such that the current can flow through each separately, and also through both in parallel, yielding three possible current flows. The parallel situation always has less resistance than any of the individual members, so it will give the highest power output of $180\ \text{W}$, while the other two must give power outputs of $60\ \text{W}$ and $120\ \text{W}$.

SET UP: $P = V^2/R$, where R is the equivalent resistance.

EXECUTE: (a) $60\ \text{W} = \frac{V^2}{R_1}$ gives $R_1 = \frac{(120\ \text{V})^2}{60\ \text{W}} = 240\ \Omega$. $120\ \text{W} = \frac{V^2}{R_2}$ gives $R_2 = \frac{(120\ \text{V})^2}{120\ \text{W}} = 120\ \Omega$. For these

two resistors in parallel, $R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = 80\ \Omega$ and $P = \frac{V^2}{R_{\text{eq}}} = \frac{(120\ \text{V})^2}{80\ \Omega} = 180\ \text{W}$, which is the desired value.

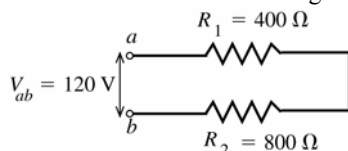
(b) If R_1 burns out, the $120\ \text{W}$ setting stays the same, the $60\ \text{W}$ setting does not work and the $180\ \text{W}$ setting goes to $120\ \text{W}$: brightnesses of zero, medium and medium.

(c) If R_2 burns out, the $60\ \text{W}$ setting stays the same, the $120\ \text{W}$ setting does not work, and the $180\ \text{W}$ setting is now $60\ \text{W}$: brightnesses of low, zero and low.

EVALUATE: Since in each case $120\ \text{V}$ is supplied to each filament network, the lowest resistance dissipates the greatest power.

26.17. IDENTIFY: For resistors in series, the voltages add and the current is the same. For resistors in parallel, the voltages are the same and the currents add. $P = I^2 R$.

(a) **SET UP:** The circuit is sketched in Figure 26.17a.



For resistors in series the current is the same through each.

Figure 26.17a

EXECUTE: $R_{\text{eq}} = R_1 + R_2 = 1200\ \Omega$. $I = \frac{V}{R_{\text{eq}}} = \frac{120\ \text{V}}{1200\ \Omega} = 0.100\ \text{A}$. This is the current drawn from the line.

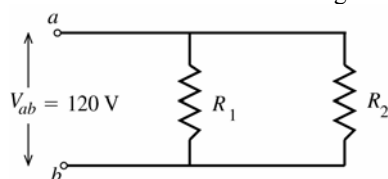
(b) $P_1 = I^2 R_1 = (0.100\ \text{A})^2 (400\ \Omega) = 4.0\ \text{W}$

$P_2 = I^2 R_2 = (0.100\ \text{A})^2 (800\ \Omega) = 8.0\ \text{W}$

(c) $P_{\text{out}} = P_1 + P_2 = 12.0\ \text{W}$, the total power dissipated in both bulbs. Note that

$P_{\text{in}} = V_{ab} I = (120\ \text{V})(0.100\ \text{A}) = 12.0\ \text{W}$, the power delivered by the potential source, equals P_{out} .

(d) **SET UP:** The circuit is sketched in Figure 26.17b.



For resistors in parallel the voltage across each resistor is the same.

Figure 26.17b

EXECUTE: $I_1 = \frac{V_1}{R_1} = \frac{120 \text{ V}}{400 \Omega} = 0.300 \text{ A}$, $I_2 = \frac{V_2}{R_2} = \frac{120 \text{ V}}{800 \Omega} = 0.150 \text{ A}$

EVALUATE: Note that each current is larger than the current when the resistors are connected in series.

(e) EXECUTE: $P_1 = I_1^2 R_1 = (0.300 \text{ A})^2 (400 \Omega) = 36.0 \text{ W}$

$P_2 = I_2^2 R_2 = (0.150 \text{ A})^2 (800 \Omega) = 18.0 \text{ W}$

(f) $P_{\text{out}} = P_1 + P_2 = 54.0 \text{ W}$

EVALUATE: Note that the total current drawn from the line is $I = I_1 + I_2 = 0.450 \text{ A}$. The power input from the line is $P_{\text{in}} = V_{ab} I = (120 \text{ V})(0.450 \text{ A}) = 54.0 \text{ W}$, which equals the total power dissipated by the bulbs.

(g) The bulb that is dissipating the most power glows most brightly. For the series connection the currents are the same and by $P = I^2 R$ the bulb with the larger R has the larger P ; the 800Ω bulb glows more brightly. For the parallel combination the voltages are the same and by $P = V^2 / R$ the bulb with the smaller R has the larger P ; the 400Ω bulb glows more brightly.

(h) The total power output P_{out} equals $P_{\text{in}} = V_{ab} I$, so P_{out} is larger for the parallel connection where the current drawn from the line is larger (because the equivalent resistance is smaller.)

- 26.18. IDENTIFY:** Use $P = V^2 / R$ with $V = 120 \text{ V}$ and the wattage for each bulb to calculate the resistance of each bulb. When connected in series the voltage across each bulb will not be 120 V and the power for each bulb will be different.

SET UP: For resistors in series the currents are the same and $R_{\text{eq}} = R_1 + R_2$.

EXECUTE: **(a)** $R_{60\text{W}} = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{60 \text{ W}} = 240 \Omega$; $R_{200\text{W}} = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{200 \text{ W}} = 72 \Omega$.

Therefore, $I_{60\text{W}} = I_{200\text{W}} = \frac{\mathcal{E}}{R} = \frac{240 \text{ V}}{(240 \Omega + 72 \Omega)} = 0.769 \text{ A}$.

(b) $P_{60\text{W}} = I^2 R = (0.769 \text{ A})^2 (240 \Omega) = 142 \text{ W}$; $P_{200\text{W}} = I^2 R = (0.769 \text{ A})^2 (72 \Omega) = 42.6 \text{ W}$.

(c) The 60 W bulb burns out quickly because the power it delivers (142 W) is 2.4 times its rated value.

EVALUATE: In series the largest resistance dissipates the greatest power.

- 26.19. IDENTIFY and SET UP:** Replace series and parallel combinations of resistors by their equivalents until the circuit is reduced to a single loop. Use the loop equation to find the current through the 20.0Ω resistor. Set $P = I^2 R$ for the 20.0Ω resistor equal to the rate Q/t at which heat goes into the water and set $Q = mc\Delta T$.

EXECUTE: Replace the network by the equivalent resistor, as shown in Figure 26.19.

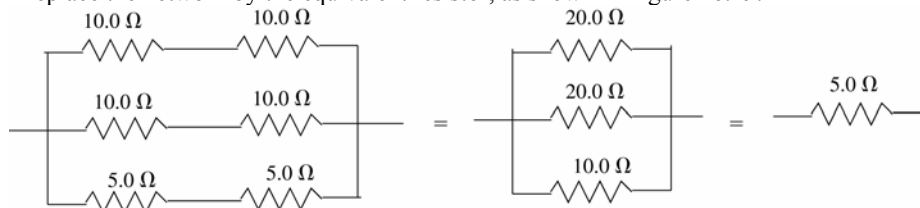


Figure 26.19

$30.0 \text{ V} - I(20.0 \Omega + 5.0 \Omega + 5.0 \Omega) = 0$; $I = 1.00 \text{ A}$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2 R = 20.0 \text{ W}$. $Q = Pt$ and $Q = mc\Delta T$ gives

$t = \frac{mc\Delta T}{P} = \frac{(0.100 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(48.0 \text{ C}^\circ)}{20.0 \text{ W}} = 1.01 \times 10^3 \text{ s}$

EVALUATE: The battery is supplying heat at the rate $P = \mathcal{E}I = 30.0 \text{ W}$. In the series circuit, more energy is dissipated in the larger resistor (20.0Ω) than in the smaller ones (5.00Ω).

- 26.20. IDENTIFY:** $P = I^2 R$ determines R_1 . R_1 , R_2 and the 10.0Ω resistor are all in parallel so have the same voltage. Apply the junction rule to find the current through R_2 .

SET UP: $P = I^2 R$ for a resistor and $P = \mathcal{E}I$ for an emf. The emf inputs electrical energy into the circuit and electrical energy is removed in the resistors.

EXECUTE: **(a)** $P_1 = I_1^2 R_1$. $20 \text{ W} = (2 \text{ A})^2 R_1$ and $R_1 = 5.00 \Omega$. R_1 and 10Ω are in parallel, so

$(10 \Omega)I_{10} = (5 \Omega)(2 \text{ A})$ and $I_{10} = 1 \text{ A}$. So $I_2 = 3.50 \text{ A} - I_1 - I_{10} = 0.50 \text{ A}$. R_1 and R_2 are in parallel, so

$(0.50 \text{ A})R_2 = (2 \text{ A})(5 \Omega)$ and $R_2 = 20.0 \Omega$.

(b) $\mathcal{E} = V_1 = (2.00 \text{ A})(5.00 \Omega) = 10.0 \text{ V}$

(c) From part (a), $I_2 = 0.500 \text{ A}$, $I_{10} = 1.00 \text{ A}$

(d) $P_1 = 20.0 \text{ W}$ (given). $P_2 = I_2^2 R_2 = (0.50 \text{ A})^2 (20 \Omega) = 5.00 \text{ W}$. $P_{10} = I_{10}^2 R_{10} = (1.0 \text{ A})^2 (10 \Omega) = 10.0 \text{ W}$. The total rate at which the resistors remove electrical energy is $P_{\text{Resist}} = 20 \text{ W} + 5 \text{ W} + 10 \text{ W} = 35.0 \text{ W}$. The total rate at which the battery inputs electrical energy is $P_{\text{Battery}} = I \mathcal{E} = (3.50 \text{ A})(10.0 \text{ V}) = 35.0 \text{ W}$. $P_{\text{Resist}} = P_{\text{Battery}}$, which agrees with conservation of energy.

EVALUATE: The three resistors are in parallel, so the voltage for each is the battery voltage, 10.0 V. The currents in the three resistors add to give the current in the battery.

- 26.21. **IDENTIFY:** Apply Kirchhoff's point rule at point a to find the current through R . Apply Kirchhoff's loop rule to loops (1) and (2) shown in Figure 26.21a to calculate R and $\mathcal{E}$. Travel around each loop in the direction shown.

(a) **SET UP:**

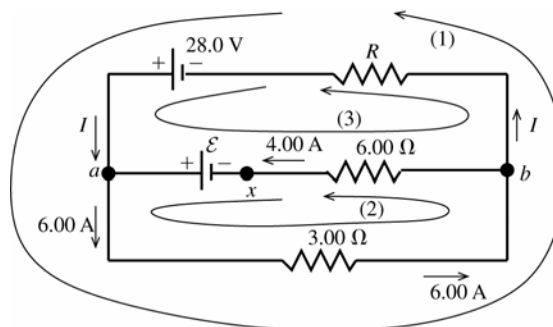


Figure 26.21a

EXECUTE: Apply Kirchhoff's point rule to point a : $\sum I = 0$ so $I + 4.00 \text{ A} - 6.00 \text{ A} = 0$
 $I = 2.00 \text{ A}$ (in the direction shown in the diagram).

(b) Apply Kirchhoff's loop rule to loop (1): $-(6.00 \text{ A})(3.00 \Omega) - (2.00 \text{ A})R + 28.0 \text{ V} = 0$
 $-18.0 \text{ V} - (2.00 \Omega)R + 28.0 \text{ V} = 0$

$$R = \frac{28.0 \text{ V} - 18.0 \text{ V}}{2.00 \text{ A}} = 5.00 \Omega$$

(c) Apply Kirchhoff's loop rule to loop (2): $-(6.00 \text{ A})(3.00 \Omega) - (4.00 \text{ A})(6.00 \Omega) + \mathcal{E} = 0$
 $\mathcal{E} = 18.0 \text{ V} + 24.0 \text{ V} = 42.0 \text{ V}$

EVALUATE: Can check that the loop rule is satisfied for loop (3), as a check of our work:

$$28.0 \text{ V} - \mathcal{E} + (4.00 \text{ A})(6.00 \Omega) - (2.00 \text{ A})R = 0$$

$$28.0 \text{ V} - 42.0 \text{ V} + 24.0 \text{ V} - (2.00 \text{ A})(5.00 \Omega) = 0$$

$$52.0 \text{ V} = 42.0 \text{ V} + 10.0 \text{ V}$$

$$52.0 \text{ V} = 52.0 \text{ V}, \text{ so the loop rule is satisfied for this loop.}$$

(d) **IDENTIFY:** If the circuit is broken at point x there can be no current in the 6.00Ω resistor. There is now only a single current path and we can apply the loop rule to this path.

SET UP: The circuit is sketched in Figure 26.21b.

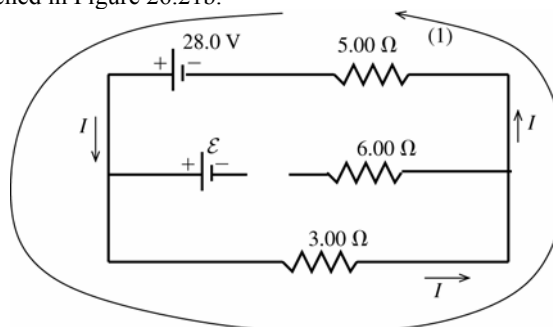


Figure 26.21b

EXECUTE: $+28.0 \text{ V} - (3.00 \Omega)I - (5.00 \Omega)I = 0$

$$I = \frac{28.0 \text{ V}}{8.00 \Omega} = 3.50 \text{ A}$$

EVALUATE: Breaking the circuit at x removes the 42.0 V emf from the circuit and the current through the 3.00Ω resistor is reduced.

26.22. IDENTIFY: Apply the loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.22. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$$+20.0 \text{ V} - (1.00 \text{ A})(1.00 \Omega) + (1.00 \text{ A})(4.00 \Omega) + (1.00 \text{ A})(1.00 \Omega) - \mathcal{E}_1 - (1.00 \text{ A})(6.00 \Omega) = 0$$

$\mathcal{E}_1 = 20.0 \text{ V} - 1.00 \text{ V} + 4.00 \text{ V} + 1.00 \text{ V} - 6.00 \text{ V} = 18.0 \text{ V}$. The loop rule applied to loop (2) gives:

$$+20.0 \text{ V} - (1.00 \text{ A})(1.00 \Omega) - (2.00 \text{ A})(1.00 \Omega) - \mathcal{E}_2 - (2.00 \text{ A})(2.00 \Omega) - (1.00 \text{ A})(6.00 \Omega) = 0$$

$\mathcal{E}_2 = 20.0 \text{ V} - 1.00 \text{ V} - 2.00 \text{ V} - 4.00 \text{ V} - 6.00 \text{ V} = 7.0 \text{ V}$. Going from b to a along the lower branch,

$V_b + (2.00 \text{ A})(2.00 \Omega) + 7.0 \text{ V} + (2.00 \text{ A})(1.00 \Omega) = V_a$. $V_b - V_a = -13.0 \text{ V}$; point b is at 13.0 V lower potential than point a .

EVALUATE: We can also calculate $V_b - V_a$ by going from b to a along the upper branch of the circuit.

$V_b - (1.00 \text{ A})(6.00 \Omega) + 20.0 \text{ V} - (1.00 \text{ A})(1.00 \Omega) = V_a$ and $V_b - V_a = -13.0 \text{ V}$. This agrees with $V_b - V_a$ calculated along a different path between b and a .

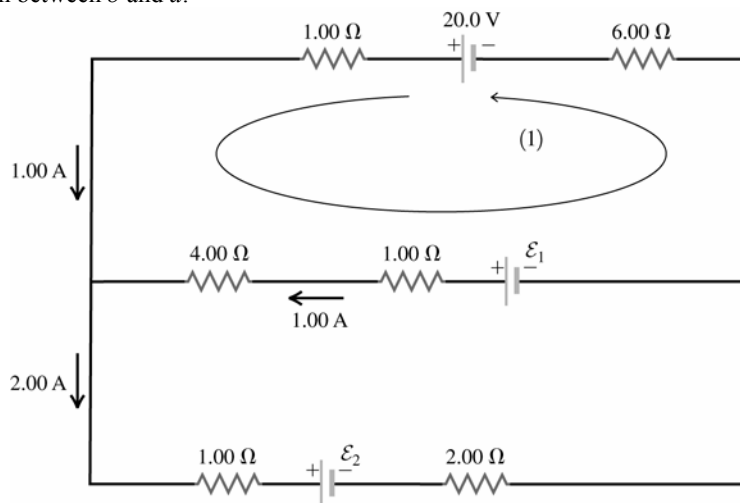


Figure 26.22

26.23. IDENTIFY: Apply the junction rule at points a , b , c and d to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1$, $\mathcal{E}_2$ and R .

(a) SET UP: The circuit is sketched in Figure 26.23.

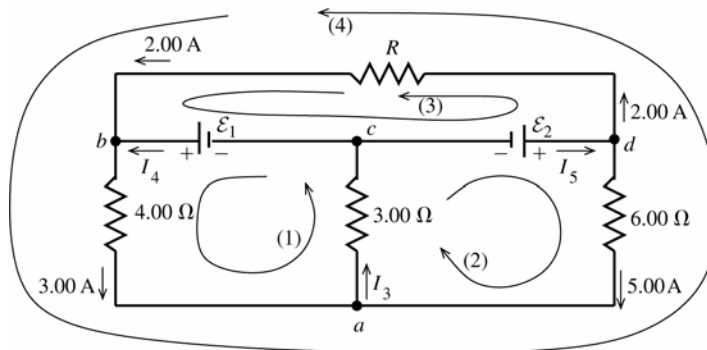


Figure 26.23

EXECUTE: Apply the junction rule to point a : $3.00 \text{ A} + 5.00 \text{ A} - I_3 = 0$

$$I_3 = 8.00 \text{ A}$$

Apply the junction rule to point b : $2.00 \text{ A} + I_4 - 3.00 \text{ A} = 0$

$$I_4 = 1.00 \text{ A}$$

Apply the junction rule to point c : $I_3 - I_4 - I_5 = 0$

$$I_5 = I_3 - I_4 = 8.00 \text{ A} - 1.00 \text{ A} = 7.00 \text{ A}$$

EVALUATE: As a check, apply the junction rule to point d : $I_5 - 2.00 \text{ A} - 5.00 \text{ A} = 0$

$$I_5 = 7.00 \text{ A}$$

(b) EXECUTE: Apply the loop rule to loop (1): $\mathcal{E}_1 - (3.00 \text{ A})(4.00 \Omega) - I_3(3.00 \Omega) = 0$

$$\mathcal{E}_1 = 12.0 \text{ V} + (8.00 \text{ A})(3.00 \Omega) = 36.0 \text{ V}$$

Apply the loop rule to loop (2): $\mathcal{E}_2 - (5.00 \text{ A})(6.00 \Omega) - I_3(3.00 \Omega) = 0$

$$\mathcal{E}_2 = 30.0 \text{ V} + (8.00 \text{ A})(3.00 \Omega) = 54.0 \text{ V}$$

(c) Apply the loop rule to loop (3): $-(2.00 \text{ A})R - \mathcal{E}_1 + \mathcal{E}_2 = 0$

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{2.00 \text{ A}} = \frac{54.0 \text{ V} - 36.0 \text{ V}}{2.00 \text{ A}} = 9.00 \Omega$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:

$$-(2.00 \text{ A})R - (3.00 \text{ A})(4.00 \Omega) + (5.00 \text{ A})(6.00 \Omega) = 0$$

$$-(2.00 \text{ A})(9.00 \Omega) - 12.0 \text{ V} + 30.0 \text{ V} = 0$$

$$-18.0 \text{ V} + 18.0 \text{ V} = 0$$

26.24. IDENTIFY: Use Kirchhoff's Rules to find the currents.

SET UP: Since the 1.0 V battery has the larger voltage, assume I_1 is to the left through the 10 V battery, I_2 is to the right through the 5 V battery, and I_3 is to the right through the 10Ω resistor. Go around each loop in the counterclockwise direction.

EXECUTE: Upper loop: $10.0 \text{ V} - (2.00 \Omega + 3.00 \Omega)I_1 - (1.00 \Omega + 4.00 \Omega)I_2 - 5.00 \text{ V} = 0$. This gives

$$5.0 \text{ V} - (5.00 \Omega)I_1 - (5.00 \Omega)I_2 = 0, \text{ and } \Rightarrow I_1 + I_2 = 1.00 \text{ A}.$$

Lower loop: $5.00 \text{ V} + (1.00 \Omega + 4.00 \Omega)I_2 - (10.0 \Omega)I_3 = 0$. This gives $5.00 \text{ V} + (5.00 \Omega)I_2 - (10.0 \Omega)I_3 = 0$, and $I_2 - 2I_3 = -1.00 \text{ A}$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = 0.800 \text{ A}, I_2 = 0.200 \text{ A}, I_3 = 0.600 \text{ A}.$$

(b) $V_{ab} = -(0.200 \text{ A})(4.00 \Omega) - (0.800 \text{ A})(3.00 \Omega) = -3.20 \text{ V}$.

EVALUATE: Traveling from b to a through the 4.00Ω and 3.00Ω resistors you pass through the resistors in the direction of the current and the potential decreases; point b is at higher potential than point a .

26.25. IDENTIFY: Apply the junction rule to reduce the number of unknown currents. Apply the loop rule to two loops to obtain two equations for the unknown currents I_1 and I_2

(a) SET UP: The circuit is sketched in Figure 26.25.

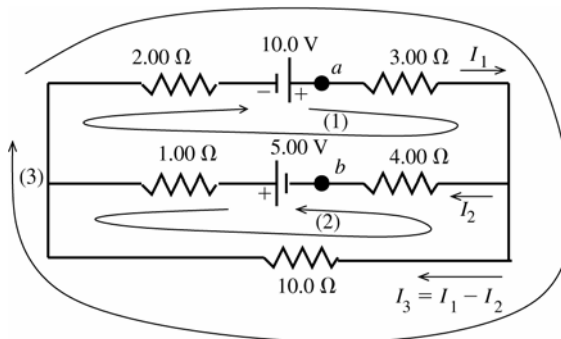


Figure 26.25

Let I_1 be the current in the 3.00Ω resistor and I_2 be the current in the 4.00Ω resistor and assume that these currents are in the directions shown. Then the current in the 10.0Ω resistor is $I_3 = I_1 - I_2$, in the direction shown, where we have used Kirchhoff's point rule to relate I_3 to I_1 and I_2 . If we get a negative answer for any of these currents we know the current is actually in the opposite direction to what we have assumed. Three loops and directions to travel around the loops are shown in the circuit diagram. Apply Kirchhoff's loop rule to each loop.

EXECUTE: loop (1)

$$+10.0 \text{ V} - I_1(3.00 \Omega) - I_2(4.00 \Omega) + 5.00 \text{ V} - I_2(1.00 \Omega) - I_1(2.00 \Omega) = 0$$

$$15.0 \text{ V} - (5.00 \Omega)I_1 - (5.00 \Omega)I_2 = 0$$

$$3.00 \text{ A} - I_1 - I_2 = 0$$

loop (2)

$$+5.00 \text{ V} - I_2(1.00 \, \Omega) + (I_1 - I_2)10.0 \, \Omega - I_2(4.00 \, \Omega) = 0$$

$$5.00 \text{ V} + (10.0 \, \Omega)I_1 - (15.0 \, \Omega)I_2 = 0$$

$$1.00 \text{ A} + 2.00I_1 - 3.00I_2 = 0$$

The first equation says $I_2 = 3.00 \text{ A} - I_1$.

Use this in the second equation: $1.00 \text{ A} + 2.00I_1 - 9.00 \text{ A} + 3.00I_1 = 0$

$$5.00I_1 = 8.00 \text{ A}, I_1 = 1.60 \text{ A}$$

Then $I_2 = 3.00 \text{ A} - I_1 = 3.00 \text{ A} - 1.60 \text{ A} = 1.40 \text{ A}$.

$$I_3 = I_1 - I_2 = 1.60 \text{ A} - 1.40 \text{ A} = 0.20 \text{ A}$$

EVALUATE: Loop (3) can be used as a check.

$$+10.0 \text{ V} - (1.60 \text{ A})(3.00 \, \Omega) - (0.20 \text{ A})(10.00 \, \Omega) - (1.60 \text{ A})(2.00 \, \Omega) = 0$$

$$10.0 \text{ V} = 4.8 \text{ V} + 2.0 \text{ V} + 3.2 \text{ V}$$

$$10.0 \text{ V} = 10.0 \text{ V}$$

We find that with our calculated currents the loop rule is satisfied for loop (3). Also, all the currents came out to be positive, so the current directions in the circuit diagram are correct.

(b) IDENTIFY and SET UP: To find $V_{ab} = V_a - V_b$ start at point b and travel to point a . Many different routes can be taken from b to a and all must yield the same result for V_{ab} .

EXECUTE: Travel through the $4.00 \, \Omega$ resistor and then through the $3.00 \, \Omega$ resistor:

$$V_b + I_2(4.00 \, \Omega) + I_1(3.00 \, \Omega) = V_a$$

$$V_a - V_b = (1.40 \text{ A})(4.00 \, \Omega) + (1.60 \text{ A})(3.00 \, \Omega) = 5.60 \text{ V} + 4.8 \text{ V} = 10.4 \text{ V} \text{ (point } a \text{ is at higher potential than point } b)$$

EVALUATE: Alternatively, travel through the 5.00 V emf, the $1.00 \, \Omega$ resistor, the $2.00 \, \Omega$ resistor, and the 10.0 V emf.

$$V_b + 5.00 \text{ V} - I_2(1.00 \, \Omega) - I_1(2.00 \, \Omega) + 10.0 \text{ V} = V_a$$

$$V_a - V_b = 15.0 \text{ V} - (1.40 \text{ A})(1.00 \, \Omega) - (1.60 \text{ A})(2.00 \, \Omega) = 15.0 \text{ V} - 1.40 \text{ V} - 3.20 \text{ V} = 10.4 \text{ V}, \text{ the same as before.}$$

26.26. IDENTIFY: Use Kirchhoff's rules to find the currents

SET UP: Since the 20.0 V battery has the largest voltage, assume I_1 is to the right through the 10.0 V battery, I_2 is to the left through the 20.0 V battery, and I_3 is to the right through the $10 \, \Omega$ resistor. Go around each loop in the counterclockwise direction.

EXECUTE: Upper loop: $10.0 \text{ V} + (2.00 \, \Omega + 3.00 \, \Omega)I_1 + (1.00 \, \Omega + 4.00 \, \Omega)I_2 - 20.00 \text{ V} = 0$.

$$-10.0 \text{ V} + (5.00 \, \Omega)I_1 + (5.00 \, \Omega)I_2 = 0, \text{ so } I_1 + I_2 = +2.00 \text{ A}.$$

Lower loop: $20.00 \text{ V} - (1.00 \, \Omega + 4.00 \, \Omega)I_2 - (10.0 \, \Omega)I_3 = 0$.

$$20.00 \text{ V} - (5.00 \, \Omega)I_2 - (10.0 \, \Omega)I_3 = 0, \text{ so } I_2 + 2I_3 = 4.00 \text{ A}.$$

Along with $I_2 = I_1 + I_3$, we can solve for the three currents and find $I_1 = +0.4 \text{ A}$, $I_2 = +1.6 \text{ A}$, $I_3 = +1.2 \text{ A}$.

$$\text{(b) } V_{ab} = I_2(4 \, \Omega) + I_1(3 \, \Omega) = (1.6 \text{ A})(4 \, \Omega) + (0.4 \text{ A})(3 \, \Omega) = 7.6 \text{ V}$$

EVALUATE: Traveling from b to a through the $4.00 \, \Omega$ and $3.00 \, \Omega$ resistors you pass through each resistor opposite to the direction of the current and the potential increases; point a is at higher potential than point b .

26.27. (a) IDENTIFY: With the switch open, the circuit can be solved using series-parallel reduction.

SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.

EXECUTE: The $30.0\text{-}\Omega$ and $50.0\text{-}\Omega$ resistors are in series, and hence have the same current. Using Ohm's law $I_{50} = (15.0 \text{ V})/(50.0 \, \Omega) = 0.300 \text{ A} = I_{30}$. The potential drop across the $75.0\text{-}\Omega$ resistor is the same as the potential drop across the $80.0\text{-}\Omega$ series combination. We can use this fact to find the current through the $75.0\text{-}\Omega$ resistor using Ohm's law: $V_{75} = V_{80} = (0.300 \text{ A})(80.0 \, \Omega) = 24.0 \text{ V}$ and $I_{75} = (24.0 \text{ V})/(75.0 \, \Omega) = 0.320 \text{ A}$.

The current through the unknown battery is the sum of the two currents we just found:

$$I_{\text{Total}} = 0.300 \text{ A} + 0.320 \text{ A} = 0.620 \text{ A}$$

The equivalent resistance of the resistors in parallel is $1/R_p = 1/(75.0 \, \Omega) + 1/(80.0 \, \Omega)$. This gives $R_p = 38.7 \, \Omega$. The equivalent resistance "seen" by the battery is $R_{\text{equiv}} = 20.0 \, \Omega + 38.7 \, \Omega = 58.7 \, \Omega$.

Applying Ohm's law to the battery gives $\mathcal{E} = R_{\text{equiv}}I_{\text{Total}} = (58.7 \, \Omega)(0.620 \text{ A}) = 36.4 \text{ V}$

(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the $50.0\text{-}\Omega$ resistor.

SET UP: Taking a loop around the right part of the circuit.

EXECUTE: Ohm's law gives $I = (25.0 \text{ V})/(50.0 \, \Omega) = 0.500 \text{ A}$

EVALUATE: The current through the $50.0\text{-}\Omega$ resistor, and the rest of the circuit, depends on whether or not the switch is open.

26.28. IDENTIFY: We need to use Kirchhoff's rules.

SET UP: Take a loop around the outside of the circuit, use the current at the upper junction, and then take a loop around the right side of the circuit.

EXECUTE: The outside loop gives $75.0 \text{ V} - (12.0 \Omega)(1.50 \text{ A}) - (48.0 \Omega)I_{48} = 0$, so $I_{48} = 1.188 \text{ A}$. At a junction we have $1.50 \text{ A} = I_{\varepsilon} + 1.188 \text{ A}$, and $I_{\varepsilon} = 0.313 \text{ A}$. A loop around the right part of the circuit gives $\mathcal{E} - (48 \Omega)(1.188 \text{ A}) + (15.0 \Omega)(0.313 \text{ A})$. $\mathcal{E} = 52.3 \text{ V}$, with the polarity shown in the figure in the problem.

EVALUATE: The unknown battery has a smaller emf than the known one, so the current through it goes against its polarity.

26.29. (a) IDENTIFY: With the switch open, we have a series circuit with two batteries.

SET UP: Take a loop to find the current, then use Ohm's law to find the potential difference between a and b .

EXECUTE: Taking the loop: $I = (40.0 \text{ V})/(175 \Omega) = 0.229 \text{ A}$. The potential difference between a and b is $V_b - V_a = +15.0 \text{ V} - (75.0 \Omega)(0.229 \text{ A}) = -2.14 \text{ V}$.

EVALUATE: The minus sign means that a is at a higher potential than b .

(b) IDENTIFY: With the switch closed, the ammeter part of the circuit divides the original circuit into two circuits. We can apply Kirchhoff's rules to both parts.

SET UP: Take loops around the left and right parts of the circuit, and then look at the current at the junction.

EXECUTE: The left-hand loop gives $I_{100} = (25.0 \text{ V})/(100.0 \Omega) = 0.250 \text{ A}$. The right-hand loop gives $I_{75} = (15.0 \text{ V})/(75.0 \Omega) = 0.200 \text{ A}$. At the junction just above the switch we have $I_{100} = 0.250 \text{ A}$ (in) and $I_{75} = 0.200 \text{ A}$ (out), so $I_A = 0.250 \text{ A} - 0.200 \text{ A} = 0.050 \text{ A}$, downward. The voltmeter reads zero because the potential difference across it is zero with the switch closed.

EVALUATE: The ideal ammeter acts like a short circuit, making a and b at the same potential. Hence the voltmeter reads zero.

26.30. IDENTIFY: The circuit is sketched in Figure 26.30a. Since all the external resistors are equal, the current must be symmetrical through them. That is, there can be no current through the resistor R for that would imply an imbalance in currents through the other resistors. With no current going through R , the circuit is like that shown in Figure 26.30b.

SET UP: For resistors in series, the equivalent resistance is $R_s = R_1 + R_2$. For resistors in parallel, the equivalent

resistance is $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

EXECUTE: The equivalent resistance of the circuit is $R_{\text{eq}} = \left(\frac{1}{2 \Omega} + \frac{1}{2 \Omega} \right)^{-1} = 1 \Omega$ and $I_{\text{total}} = \frac{13 \text{ V}}{1 \Omega} = 13 \text{ A}$. The two

parallel branches have the same resistance, so $I_{\text{each branch}} = \frac{1}{2} I_{\text{total}} = 6.5 \text{ A}$. The current through each 1Ω resistor is 6.5 A

and no current passes through R .

(b) As worked out above, $R_{\text{eq}} = 1 \Omega$.

(c) $V_{ab} = 0$, since no current flows through R .

EVALUATE: (d) R plays no role since no current flows through it and the voltage across it is zero.

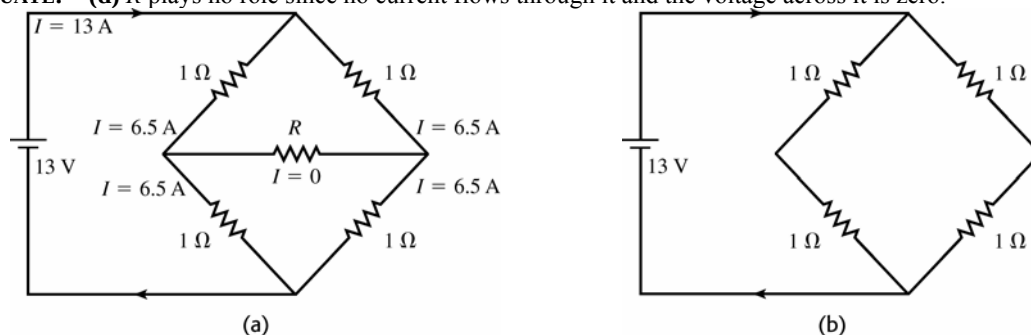


Figure 26.30

26.31. IDENTIFY: To construct an ammeter, add a shunt resistor in parallel with the galvanometer coil. To construct a voltmeter, add a resistor in series with the galvanometer coil.

SET UP: The full-scale deflection current is $500 \mu\text{A}$ and the coil resistance is 25.0Ω .

EXECUTE: (a) For a 20-mA ammeter, the two resistances are in parallel and the voltages across each are the same. $V_c = V_s$ gives $I_c R_c = I_s R_s$. $(500 \times 10^{-6} \text{ A})(25.0 \Omega) = (20 \times 10^{-3} \text{ A} - 500 \times 10^{-6} \text{ A})R_s$ and $R_s = 0.641 \Omega$.

(b) For a 500-mV voltmeter, the resistances are in series and the current is the same through each: $V_{ab} = I(R_c + R_s)$

$$\text{and } R_s = \frac{V_{ab}}{I} - R_c = \frac{500 \times 10^{-3} \text{ V}}{500 \times 10^{-6} \text{ A}} - 25.0 \Omega = 975 \Omega.$$

EVALUATE: The equivalent resistance of the voltmeter is $R_{eq} = R_s + R_c = 1000 \Omega$. The equivalent resistance of the ammeter is given by $\frac{1}{R_{eq}} = \frac{1}{R_{sh}} + \frac{1}{R_c}$ and $R_{eq} = 0.625 \Omega$. The voltmeter is a high-resistance device and the ammeter is a low-resistance device.

- 26.32. IDENTIFY:** The galvanometer is represented in the circuit as a resistance R_c . Use the junction rule to relate the current through the galvanometer and the current through the shunt resistor. The voltage drop across each parallel path is the same; use this to write an equation for the resistance R .

SET UP: The circuit is sketched in Figure 26.32.

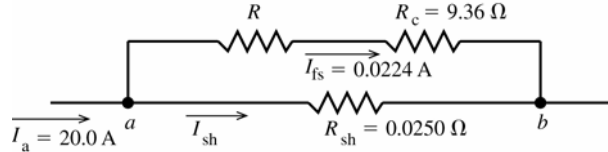


Figure 26.32

We want that $I_a = 20.0 \text{ A}$ in the external circuit to produce $I_{fs} = 0.0224 \text{ A}$ through the galvanometer coil.

EXECUTE: Applying the junction rule to point a gives $I_a - I_{fs} - I_{sh} = 0$

$$I_{sh} = I_a - I_{fs} = 20.0 \text{ A} - 0.0224 \text{ A} = 19.98 \text{ A}$$

The potential difference V_{ab} between points a and b must be the same for both paths between these two points:

$$I_{fs}(R + R_c) = I_{sh}R_{sh}$$

$$R = \frac{I_{sh}R_{sh}}{I_{fs}} - R_c = \frac{(19.98 \text{ A})(0.0250 \Omega)}{0.0224 \text{ A}} - 9.36 \Omega = 22.30 \Omega - 9.36 \Omega = 12.9 \Omega$$

EVALUATE: $R_{sh} \ll R + R_c$; most of the current goes through the shunt. Adding R decreases the fraction of the current that goes through R_c .

- 26.33. IDENTIFY:** The meter introduces resistance into the circuit, which affects the current through the 5.00-kΩ resistor and hence the potential drop across it.

SET UP: Use Ohm's law to find the current through the 5.00-kΩ resistor and then the potential drop across it.

EXECUTE: (a) The parallel resistance with the voltmeter is 3.33 kΩ, so the total equivalent resistance across the battery is 9.33 kΩ, giving $I = (50.0 \text{ V})/(9.33 \text{ k}\Omega) = 5.36 \text{ mA}$. Ohm's law gives the potential drop across the 5.00-kΩ resistor: $V_{5 \text{ k}\Omega} = (3.33 \text{ k}\Omega)(5.36 \text{ mA}) = 17.9 \text{ V}$

(b) The current in the circuit is now $I = (50.0 \text{ V})/(11.0 \text{ k}\Omega) = 4.55 \text{ mA}$. $V_{5 \text{ k}\Omega} = (5.00 \text{ k}\Omega)(4.55 \text{ mA}) = 22.7 \text{ V}$.

(c) % error = $(22.7 \text{ V} - 17.9 \text{ V})/(22.7 \text{ V}) = 0.214 = 21.4\%$. (We carried extra decimal places for accuracy since we had to subtract our answers.)

EVALUATE: The presence of the meter made a very large percent error in the reading of the "true" potential across the resistor.

- 26.34. IDENTIFY:** The resistance of the galvanometer can alter the resistance in a circuit.

SET UP: The shunt is in parallel with the galvanometer, so we find the parallel resistance of the ammeter. Then use Ohm's law to find the current in the circuit.

EXECUTE: (a) The resistance of the ammeter is given by $1/R_A = 1/(1.00 \Omega) + 1/(25.0 \Omega)$, so $R_A = 0.962 \Omega$. The current through the ammeter, and hence the current it measures, is $I = V/R = (25.0 \text{ V})/(15.96 \Omega) = 1.57 \text{ A}$.

(b) Now there is no meter in the circuit, so the total resistance is only 15.0 Ω. $I = (25.0 \text{ V})/(15.0 \Omega) = 1.67 \text{ A}$

(c) $(1.67 \text{ A} - 1.57 \text{ A})/(1.67 \text{ A}) = 0.060 = 6.0\%$

EVALUATE: A 1-Ω shunt can introduce noticeable error in the measurement of an ammeter.

- 26.35. IDENTIFY:** When the galvanometer reading is zero $\mathcal{E}_2 = IR_{cb}$ and $\mathcal{E}_1 = IR_{ab}$.

SET UP: R_{cb} is proportional to x and R_{ab} is proportional to l .

$$\text{EXECUTE: (a) } \mathcal{E}_2 = \mathcal{E}_1 \frac{R_{cb}}{R_{ab}} = \mathcal{E}_1 \frac{x}{l}.$$

(b) The value of the galvanometer's resistance is unimportant since no current flows through it.

$$\text{(c) } \mathcal{E}_2 = \mathcal{E}_1 \frac{x}{l} = (9.15 \text{ V}) \frac{0.365 \text{ m}}{1.000 \text{ m}} = 3.34 \text{ V}$$

EVALUATE: The potentiometer measures the emf $\mathcal{E}_2$ of the source directly, unaffected by the internal resistance of the source, since the measurement is made with no current through $\mathcal{E}_2$.

26.36. IDENTIFY: A half-scale reading occurs with $R = 600\ \Omega$, so the current through the galvanometer is half the full-scale current.

SET UP: The resistors R_s , R_c and R are in series, so the total resistance of the circuit is $R_{\text{total}} = R_s + R_c + R$.

EXECUTE: $\mathcal{E} = IR_{\text{total}}$. $1.50\text{ V} = \left(\frac{3.60 \times 10^{-3}\text{ A}}{2}\right)(15.0\ \Omega + 600\ \Omega + R_s)$ and $R_s = 218\ \Omega$.

EVALUATE: We have assumed that the device is linear, in the sense that the deflection is proportional to the current through the meter.

26.37. IDENTIFY: Apply $\mathcal{E} = IR_{\text{total}}$ to relate the resistance R_x to the current in the circuit.

SET UP: R , R_x and the meter are in series, so $R_{\text{total}} = R + R_x + R_M$, where $R_M = 65.0\ \Omega$ is the resistance of the meter. $I_{\text{fsd}} = 2.50\text{ mA}$ is the current required for full-scale deflection.

EXECUTE: (a) When the wires are shorted, the full-scale deflection current is obtained: $\mathcal{E} = IR_{\text{total}}$.

$1.52\text{ V} = (2.50 \times 10^{-3}\text{ A})(65.0\ \Omega + R)$ and $R = 543\ \Omega$.

(b) If the resistance $R_x = 200\ \Omega$: $I = \frac{V}{R_{\text{total}}} = \frac{1.52\text{ V}}{65.0\ \Omega + 543\ \Omega + R_x} = 1.88\text{ mA}$.

(c) $I_x = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{1.52\text{ V}}{65.0\ \Omega + 543\ \Omega + R_x}$ and $R_x = \frac{1.52\text{ V}}{I_x} - 608\ \Omega$. For each value of I_x we have:

For $I_x = \frac{1}{4}I_{\text{fsd}} = 6.25 \times 10^{-4}\text{ A}$, $R_x = \frac{1.52\text{ V}}{6.25 \times 10^{-4}\text{ A}} - 608\ \Omega = 1824\ \Omega$.

For $I_x = \frac{1}{2}I_{\text{fsd}} = 1.25 \times 10^{-3}\text{ A}$, $R_x = \frac{1.52\text{ V}}{1.25 \times 10^{-3}\text{ A}} - 608\ \Omega = 608\ \Omega$.

For $I_x = \frac{3}{4}I_{\text{fsd}} = 1.875 \times 10^{-3}\text{ A}$, $R_x = \frac{1.52\text{ V}}{1.875 \times 10^{-3}\text{ A}} - 608\ \Omega = 203\ \Omega$.

EVALUATE: The deflection of the meter increases when the resistance R_x decreases.

26.38. IDENTIFY: An uncharged capacitor is placed into a circuit. Apply the loop rule at each time.

SET UP: The voltage across a capacitor is $V_C = q/C$.

EXECUTE: (a) At the instant the circuit is completed, there is no voltage over the capacitor, since it has no charge stored.

(b) Since $V_C = 0$, the full battery voltage appears across the resistor $V_R = \mathcal{E} = 125\text{ V}$.

(c) There is no charge on the capacitor.

(d) The current through the resistor is $i = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{125\text{ V}}{7500\ \Omega} = 0.0167\text{ A}$.

(e) After a long time has passed the full battery voltage is across the capacitor and $i = 0$. The voltage across the capacitor balances the emf: $V_C = 125\text{ V}$. The voltage across the resistor is zero. The capacitor's charge is

$q = CV_C = (4.60 \times 10^{-6}\text{ F})(125\text{ V}) = 5.75 \times 10^{-4}\text{ C}$. The current in the circuit is zero.

EVALUATE: The current in the circuit starts at 0.0167 A and decays to zero. The charge on the capacitor starts at zero and rises to $q = 5.75 \times 10^{-4}\text{ C}$.

26.39. IDENTIFY: The capacitor discharges exponentially through the voltmeter. Since the potential difference across the capacitor is directly proportional to the charge on the plates, the voltage across the plates decreases exponentially with the same time constant as the charge.

SET UP: The reading of the voltmeter obeys the equation $V = V_0 e^{-t/RC}$, where RC is the time constant.

EXECUTE: (a) Solving for C and evaluating the result when $t = 4.00\text{ s}$ gives

$$C = \frac{t}{R \ln(V/V_0)} = \frac{4.00\text{ s}}{(3.40 \times 10^6\ \Omega) \ln\left(\frac{12.0\text{ V}}{3.00\text{ V}}\right)} = 8.49 \times 10^{-7}\text{ F}$$

(b) $\tau = RC = (3.40 \times 10^6\ \Omega)(8.49 \times 10^{-7}\text{ F}) = 2.89\text{ s}$

EVALUATE: In most laboratory circuits, time constants are much shorter than this one.

26.40. IDENTIFY: For a charging capacitor $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and $i(t) = \frac{\mathcal{E}}{R}e^{-t/\tau}$.

SET UP: The time constant is $RC = (0.895 \times 10^6\ \Omega)(12.4 \times 10^{-6}\text{ F}) = 11.1\text{ s}$.

EXECUTE: (a) At $t = 0\text{ s}$: $q = C\mathcal{E}(1 - e^{-t/RC}) = 0$.

$$\text{At } t = 5 \text{ s: } q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4 \times 10^{-6} \text{ F})(60.0 \text{ V})(1 - e^{-(5.0 \text{ s})/(11.1 \text{ s})}) = 2.70 \times 10^{-4} \text{ C.}$$

$$\text{At } t = 10 \text{ s: } q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4 \times 10^{-6} \text{ F})(60.0 \text{ V})(1 - e^{-(10.0 \text{ s})/(11.1 \text{ s})}) = 4.42 \times 10^{-4} \text{ C.}$$

$$\text{At } t = 20 \text{ s: } q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4 \times 10^{-6} \text{ F})(60.0 \text{ V})(1 - e^{-(20.0 \text{ s})/(11.1 \text{ s})}) = 6.21 \times 10^{-4} \text{ C.}$$

$$\text{At } t = 100 \text{ s: } q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4 \times 10^{-6} \text{ F})(60.0 \text{ V})(1 - e^{-(100 \text{ s})/(11.1 \text{ s})}) = 7.44 \times 10^{-4} \text{ C.}$$

(b) The current at time t is given by: $i = \frac{\mathcal{E}}{R} e^{-t/RC}$.

$$\text{At } t = 0 \text{ s: } i = \frac{60.0 \text{ V}}{8.95 \times 10^5 \Omega} e^{-0/11.1} = 6.70 \times 10^{-5} \text{ A.}$$

$$\text{At } t = 5 \text{ s: } i = \frac{60.0 \text{ V}}{8.95 \times 10^5 \Omega} e^{-5/11.1} = 4.27 \times 10^{-5} \text{ A.}$$

$$\text{At } t = 10 \text{ s: } i = \frac{60.0 \text{ V}}{8.95 \times 10^5 \Omega} e^{-10/11.1} = 2.27 \times 10^{-5} \text{ A.}$$

$$\text{At } t = 20 \text{ s: } i = \frac{60.0 \text{ V}}{8.95 \times 10^5 \Omega} e^{-20/11.1} = 1.11 \times 10^{-5} \text{ A.}$$

$$\text{At } t = 100 \text{ s: } i = \frac{60.0 \text{ V}}{8.95 \times 10^5 \Omega} e^{-100/11.1} = 8.20 \times 10^{-9} \text{ A.}$$

(c) The graphs of $q(t)$ and $i(t)$ are given in Figure 26.40a and 26.40b

EVALUATE: The charge on the capacitor increases in time as the current decreases.

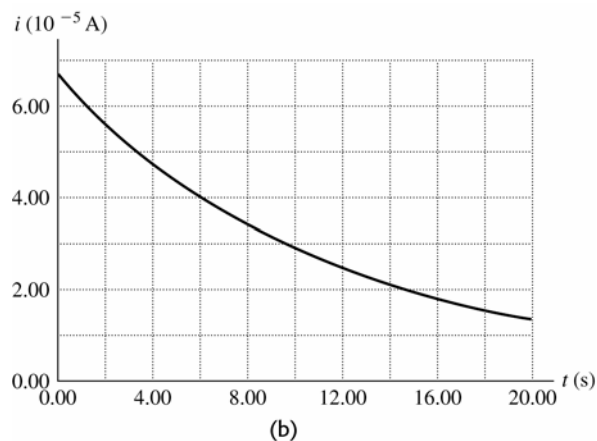
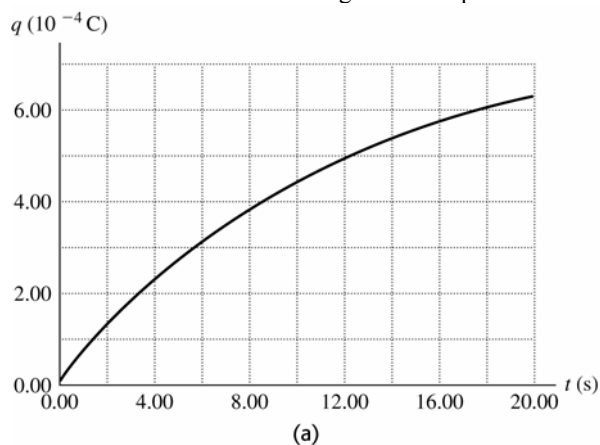


Figure 26.40

26.41. IDENTIFY: The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q , V must obey the same exponential equation as Q , $V = V_0 e^{-t/RC}$. The current is $I = (V_0/R) e^{-t/RC}$.

EXECUTE: (a) Solve for time when the potential across each capacitor is 10.0 V:

$$t = -RC \ln(V/V_0) = -(80.0 \Omega)(35.0 \mu\text{F}) \ln(10/45) = 4210 \mu\text{s} = 4.21 \text{ ms}$$

(b) $I = (V_0/R) e^{-t/RC}$. Using the above values, with $V_0 = 45.0 \text{ V}$, gives $I = 0.125 \text{ A}$.

EVALUATE: Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.42. IDENTIFY: In $\tau = RC$ use the equivalent capacitance of the two capacitors.

SET UP: For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$. For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$. Originally,

$$\tau = RC = 0.870 \text{ s.}$$

EXECUTE: (a) The combined capacitance of the two identical capacitors in series is given by $\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$,

so $C_{\text{eq}} = \frac{C}{2}$. The new time constant is thus $R(C/2) = \frac{0.870 \text{ s}}{2} = 0.435 \text{ s}$.

(b) With the two capacitors in parallel the new total capacitance is simply $2C$. Thus the time constant is $R(2C) = 2(0.870 \text{ s}) = 1.74 \text{ s}$.

EVALUATE: The time constant is proportional to C_{eq} . For capacitors in series the capacitance is decreased and for capacitors in parallel the capacitance is increased.

- 26.43. IDENTIFY and SET UP:** Apply the loop rule. The voltage across the resistor depends on the current through it and the voltage across the capacitor depends on the charge on its plates.

EXECUTE: $\mathcal{E} - V_R - V_C = 0$

$\mathcal{E} = 120 \text{ V}$, $V_R = IR = (0.900 \text{ A})(80.0 \Omega) = 72 \text{ V}$, so $V_C = 48 \text{ V}$

$Q = CV = (4.00 \times 10^{-6} \text{ F})(48 \text{ V}) = 192 \mu\text{C}$

EVALUATE: The initial charge is zero and the final charge is $C\mathcal{E} = 480 \mu\text{C}$. Since current is flowing at the instant considered in the problem the capacitor is still being charged and its charge has not reached its final value.

- 26.44. IDENTIFY:** The charge is increasing while the current is decreasing. Both obey exponential equations, but they are not the same equation.

SET UP: The charge obeys the equation $Q = Q_{\max}(1 - e^{-t/RC})$, but the equation for the current is $I = I_{\max}e^{-t/RC}$.

EXECUTE: When the charge has reached $\frac{1}{4}$ of its maximum value, we have $Q_{\max}/4 = Q_{\max}(1 - e^{-t/RC})$, which says that the exponential term has the value $e^{-t/RC} = \frac{3}{4}$. The current at this time is $I = I_{\max}e^{-t/RC} = I_{\max}(3/4) = (3/4)[(10.0 \text{ V})/(12.0 \Omega)] = 0.625 \text{ A}$

EVALUATE: Notice that the current will be $\frac{3}{4}$, not $\frac{1}{4}$, of its maximum value when the charge is $\frac{1}{4}$ of its maximum. Although current and charge both obey exponential equations, the equations have different forms for a charging capacitor.

- 26.45. IDENTIFY:** The stored energy is proportional to the square of the charge on the capacitor, so it will obey an exponential equation, but not the same equation as the charge.

SET UP: The energy stored in the capacitor is $U = Q^2/2C$ and the charge on the plates is $Q_0 e^{-t/RC}$. The current is $I = I_0 e^{-t/RC}$.

EXECUTE: $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2C = U_0 e^{-2t/RC}$

When the capacitor has lost 80% of its stored energy, the energy is 20% of the initial energy, which is $U_0/5$. $U_0/5 = U_0 e^{-2t/RC}$ gives $t = (RC/2) \ln 5 = (25.0 \Omega)(4.62 \text{ pF})(\ln 5)/2 = 92.9 \text{ ps}$.

At this time, the current is $I = I_0 e^{-t/RC} = (Q_0/RC) e^{-t/RC}$, so

$$I = (3.5 \text{ nC})/[(25.0 \Omega)(4.62 \text{ pF})] e^{-(92.9 \text{ ps})/[(25.0 \Omega)(4.62 \text{ pF})]} = 13.6 \text{ A}.$$

EVALUATE: When the energy reduced by 80%, neither the current nor the charge are reduced by that percent.

- 26.46. IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0 e^{-t/RC}$ but the energy obeys the equation

$U = Q^2/2C = (Q_0 e^{-t/RC})^2/2C = U_0 e^{-2t/RC}$.

EXECUTE: (a) The charge is reduced by half: $Q_0/2 = Q_0 e^{-t/RC}$. This gives

$$t = RC \ln 2 = (175 \Omega)(12.0 \mu\text{F})(\ln 2) = 1.456 \text{ ms} = 1.46 \text{ ms}.$$

(b) The energy is reduced by half: $U_0/2 = U_0 e^{-2t/RC}$. This gives

$$t = (RC \ln 2)/2 = (1.456 \text{ ms})/2 = 0.728 \text{ ms}.$$

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

- 26.47. IDENTIFY:** In both cases, simplify the complicated circuit by eliminating the appropriate circuit elements. The potential across an uncharged capacitor is initially zero, so it behaves like a short circuit. A fully charged capacitor allows no current to flow through it.

(a) **SET UP:** Just after closing the switch, the uncharged capacitors all behave like short circuits, so any resistors in parallel with them are eliminated from the circuit.

EXECUTE: The equivalent circuit consists of 50Ω and 25Ω in parallel, with this combination in series with 75Ω , 15Ω , and the 100-V battery. The equivalent resistance is $90 \Omega + 16.7 \Omega = 106.7 \Omega$, which gives $I = (100 \text{ V})/(106.7 \Omega) = 0.937 \text{ A}$.

(b) **SET UP:** Long after closing the switch, the capacitors are essentially charged up and behave like open circuits since no charge can flow through them. They effectively eliminate any resistors in series with them since no current can flow through these resistors.

EXECUTE: The equivalent circuit consists of resistances of 75Ω , 15Ω , and three $25\text{-}\Omega$ resistors, all in series with the 100-V battery, for a total resistance of 165Ω . Therefore $I = (100 \text{ V})/(165 \Omega) = 0.606 \text{ A}$

EVALUATE: The initial and final behavior of the circuit can be calculated quite easily using simple series-parallel circuit analysis. Intermediate times would require much more difficult calculations!

- 26.48. IDENTIFY:** When the capacitor is fully charged the voltage V across the capacitor equals the battery emf and

$Q = CV$. For a charging capacitor, $q = Q(1 - e^{-t/RC})$.

SET UP: $\ln e^x = x$

EXECUTE: (a) $Q = CV = (5.90 \times 10^{-6} \text{ F})(28.0 \text{ V}) = 1.65 \times 10^{-4} \text{ C}$.

(b) $q = Q(1 - e^{-t/RC})$, so $e^{-t/RC} = 1 - \frac{q}{Q}$ and $R = \frac{-t}{C \ln(1 - q/Q)}$. After

$$t = 3 \times 10^{-3} \text{ s: } R = \frac{-3 \times 10^{-3} \text{ s}}{(5.90 \times 10^{-6} \text{ F})(\ln(1 - 110/165))} = 463 \Omega.$$

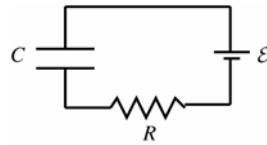
(c) If the charge is to be 99% of final value: $\frac{q}{Q} = (1 - e^{-t/RC})$ gives

$$t = -RC \ln(1 - q/Q) = -(463 \Omega)(5.90 \times 10^{-6} \text{ F}) \ln(0.01) = 0.0126 \text{ s}.$$

EVALUATE: The time constant is $\tau = RC = 2.73 \text{ ms}$. The time in part (b) is a bit more than one time constant and the time in part (c) is about 4.6 time constants.

26.49. IDENTIFY: For each circuit apply the loop rule to relate the voltages across the circuit elements.

(a) **SET UP:** With the switch in position 2 the circuit is the charging circuit shown in Figure 26.49a.



At $t = 0$, $q = 0$.

Figure 26.49a

EXECUTE: The charge q on the capacitor is given as a function of time by Eq.(26.12):

$$q = C\mathcal{E}(1 - e^{-t/RC})$$

$$Q_f = C\mathcal{E} = (1.50 \times 10^{-5} \text{ F})(18.0 \text{ V}) = 2.70 \times 10^{-4} \text{ C}.$$

$$RC = (980 \Omega)(1.50 \times 10^{-5} \text{ F}) = 0.0147 \text{ s}$$

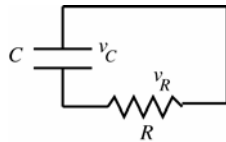
$$\text{Thus, at } t = 0.0100 \text{ s, } q = (2.70 \times 10^{-4} \text{ C})\left(1 - e^{-(0.0100 \text{ s})/(0.0147 \text{ s})}\right) = 133 \mu\text{C}.$$

$$(b) v_C = \frac{q}{C} = \frac{133 \mu\text{C}}{1.50 \times 10^{-5} \text{ F}} = 8.87 \text{ V}$$

The loop rule says $\mathcal{E} - v_C - v_R = 0$

$$v_R = \mathcal{E} - v_C = 18.0 \text{ V} - 8.87 \text{ V} = 9.13 \text{ V}$$

(c) **SET UP:** Throwing the switch back to position 1 produces the discharging circuit shown in Figure 26.49b.



The initial charge Q_0 is the charge calculated in part (b),
 $Q_0 = 133 \mu\text{C}$.

Figure 26.49b

EXECUTE: $v_C = \frac{q}{C} = \frac{133 \mu\text{C}}{1.50 \times 10^{-5} \text{ F}} = 8.87 \text{ V}$, the same as just before the switch is thrown. But now

$$v_C - v_R = 0, \text{ so } v_R = v_C = 8.87 \text{ V}.$$

(d) **SET UP:** In the discharging circuit the charge on the capacitor as a function of time is given by Eq.(26.16):

$$q = Q_0 e^{-t/RC}.$$

EXECUTE: $RC = 0.0147 \text{ s}$, the same as in part (a). Thus at $t = 0.0100 \text{ s}$, $q = (133 \mu\text{C})e^{-(0.0100 \text{ s})/(0.0147 \text{ s})} = 67.4 \mu\text{C}$.

EVALUATE: $t = 10.0 \text{ ms}$ is less than one time constant, so at the instant described in part (a) the capacitor is not fully charged; its voltage (8.87 V) is less than the emf. There is a charging current and a voltage drop across the resistor. In the discharging circuit the voltage across the capacitor starts at 8.87 V and decreases. After $t = 10.0 \text{ ms}$ it has decreased to $v_C = q/C = 4.49 \text{ V}$.

26.50. IDENTIFY: $P = VI = I^2 R$

SET UP: Problem 25.77 says that for 12-gauge wire the maximum safe current is 2.5 A.

EXECUTE: (a) $I = \frac{P}{V} = \frac{4100 \text{ W}}{240 \text{ V}} = 17.1 \text{ A}$. So we need at least 14-gauge wire (good up to 18 A). 12 gauge is also

ok (good up to 25 A).

$$(b) P = \frac{V^2}{R} \text{ and } R = \frac{V^2}{P} = \frac{(240 \text{ V})^2}{4100 \text{ W}} = 14 \Omega.$$

$$(c) \text{ At } 11\text{¢ per kWh, for 1 hour the cost is } (11\text{¢/kWh})(1 \text{ h})(4.1 \text{ kW}) = 45\text{¢}.$$

EVALUATE: The cost to operate the device is proportional to its power consumption.

- 26.51. IDENTIFY and SET UP:** The heater and hair dryer are in parallel so the voltage across each is 120 V and the current through the fuse is the sum of the currents through each appliance. As the power consumed by the dryer increases the current through it increases. The maximum power setting is the highest one for which the current through the fuse is less than 20 A.

EXECUTE: Find the current through the heater. $P = VI$ so $I = P/V = 1500 \text{ W}/120 \text{ V} = 12.5 \text{ A}$. The maximum total current allowed is 20 A, so the current through the dryer must be less than $20 \text{ A} - 12.5 \text{ A} = 7.5 \text{ A}$. The power dissipated by the dryer if the current has this value is $P = VI = (120 \text{ V})(7.5 \text{ A}) = 900 \text{ W}$. For P at this value or larger the circuit breaker trips.

EVALUATE: $P = V^2/R$ and for the dryer V is a constant 120 V. The higher power settings correspond to a smaller resistance R and larger current through the device.

- 26.52. IDENTIFY:** The current gets split evenly between all the parallel bulbs.

SET UP: A single bulb will draw $I = \frac{P}{V} = \frac{90 \text{ W}}{120 \text{ V}} = 0.75 \text{ A}$.

EXECUTE: Number of bulbs $\leq \frac{20 \text{ A}}{0.75 \text{ A}} = 26.7$. So you can attach 26 bulbs safely.

EVALUATE: In parallel the voltage across each bulb is the circuit voltage.

- 26.53. IDENTIFY and SET UP:** Ohm's law and Eq.(25.18) can be used to calculate I and P given V and R . Use Eq.(25.12) to calculate the resistance at the higher temperature.

(a) EXECUTE: When the heater element is first turned on it is at room temperature and has resistance $R = 20 \Omega$.

$$I = \frac{V}{R} = \frac{120 \text{ V}}{20 \Omega} = 6.0 \text{ A}$$

$$P = \frac{V^2}{R} = \frac{(120 \text{ V})^2}{20 \Omega} = 720 \text{ W}$$

(b) Find the resistance $R(T)$ of the element at the operating temperature of 280°C .

Take $T_0 = 23.0^\circ\text{C}$ and $R_0 = 20 \Omega$. Eq.(25.12) gives

$$R(T) = R_0(1 + \alpha(T - T_0)) = 20 \Omega \left(1 + (2.8 \times 10^{-3} (\text{C}^\circ)^{-1})(280^\circ\text{C} - 23.0^\circ\text{C}) \right) = 34.4 \Omega.$$

$$I = \frac{V}{R} = \frac{120 \text{ V}}{34.4 \Omega} = 3.5 \text{ A}$$

$$P = \frac{V^2}{R} = \frac{(120 \text{ V})^2}{34.4 \Omega} = 420 \text{ W}$$

EVALUATE: When the temperature increases, R increases and I and P decrease. The changes are substantial.

- 26.54. (a) IDENTIFY:** Two of the resistors in series would each dissipate one-half the total, or 1.2 W, which is ok. But the series combination would have an equivalent resistance of 800Ω , not the 400Ω that is required. Resistors in parallel have an equivalent resistance that is less than that of the individual resistors, so a solution is two in series in parallel with another two in series.

SET UP: The network can be simplified as shown in Figure 26.54a.

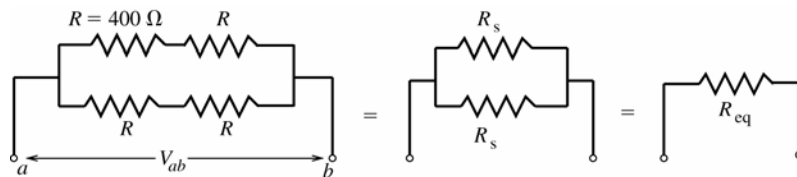


Figure 26.54a

EXECUTE: R_s is the resistance equivalent to two of the 400Ω resistors in series. $R_s = R + R = 800 \Omega$. R_{eq} is

the resistance equivalent to the two $R_s = 800 \Omega$ resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_s} + \frac{1}{R_s} = \frac{2}{R_s}$; $R_{eq} = \frac{800 \Omega}{2} = 400 \Omega$.

EVALUATE: This combination does have the required 400Ω equivalent resistance. It will be shown in part (b) that a total of 2.4 W can be dissipated without exceeding the power rating of each individual resistor.

IDENTIFY: Another solution is two resistors in parallel in series with two more in parallel.

SET UP: The network can be simplified as shown in Figure 26.54b.

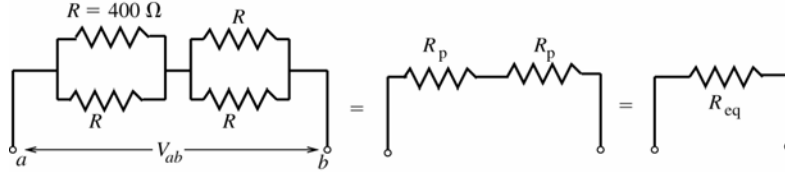


Figure 26.54b

EXECUTE: $\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} = \frac{2}{400 \Omega}$; $R_p = 200 \Omega$; $R_{eq} = R_p + R_p = 400 \Omega$

EVALUATE: This combination has the required 400Ω equivalent resistance. It will be shown in part (b) that a total of 2.4 W can be dissipated without exceeding the power rating of each individual resistor.

(b) IDENTIFY and SET UP: Find the applied voltage V_{ab} such that a total of 2.4 W is dissipated and then for this V_{ab} find the power dissipated by each resistor.

EXECUTE: For a combination with equivalent resistance $R_{eq} = 400 \Omega$ to dissipate 2.4 W the voltage V_{ab} applied to the network must be given by $P = V_{ab}^2 / R_{eq}$ so $V_{ab} = \sqrt{PR_{eq}} = \sqrt{(2.4 \text{ W})(400 \Omega)} = 31.0 \text{ V}$ and the current through the equivalent resistance is $I = V_{ab} / R = 31.0 \text{ V} / 400 \Omega = 0.0775 \text{ A}$. For the first combination this means 31.0 V across each parallel branch and $\frac{1}{2}(31.0 \text{ V}) = 15.5 \text{ V}$ across each 400Ω resistor. The power dissipated by each individual resistor is then $P = V^2 / R = (15.5 \text{ V})^2 / 400 \Omega = 0.60 \text{ W}$, which is less than the maximum allowed value of 1.20 W . For the second combination this means a voltage of $IR_p = (0.0775 \text{ A})(200 \Omega) = 15.5 \text{ V}$ across each parallel combination and hence across each separate resistor. The power dissipated by each resistor is again $P = V^2 / R = (15.5 \text{ V})^2 / 400 \Omega = 0.60 \text{ W}$, which is less than the maximum allowed value of 1.20 W .

EVALUATE: The symmetry of each network says that each resistor in the network dissipates the same power. So, for a total of 2.4 W dissipated by the network, each resistor dissipates $(2.4 \text{ W}) / 4 = 0.60 \text{ W}$, which agrees with the above analysis.

26.55. IDENTIFY: The Cu and Ni cables are in parallel. For each, $R = \frac{\rho L}{A}$.

SET UP: The composite cable is sketched in Figure 26.55. The cross sectional area of the nickel segment is πa^2 and the area of the copper portion is $\pi(b^2 - a^2)$. For nickel $\rho = 7.8 \times 10^{-8} \Omega \cdot \text{m}$ and for copper $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$.

EXECUTE: $\frac{1}{R_{\text{Cable}}} = \frac{1}{R_{\text{Ni}}} + \frac{1}{R_{\text{Cu}}}$. $R_{\text{Ni}} = \rho_{\text{Ni}} L / A = \rho_{\text{Ni}} \frac{L}{\pi a^2}$ and $R_{\text{Cu}} = \rho_{\text{Cu}} L / A = \rho_{\text{Cu}} \frac{L}{\pi(b^2 - a^2)}$. Therefore,

$$\frac{1}{R_{\text{Cable}}} = \frac{\pi a^2}{\rho_{\text{Ni}} L} + \frac{\pi(b^2 - a^2)}{\rho_{\text{Cu}} L}$$

$$\frac{1}{R_{\text{Cable}}} = \frac{\pi}{L} \left(\frac{a^2}{\rho_{\text{Ni}}} + \frac{b^2 - a^2}{\rho_{\text{Cu}}} \right) = \frac{\pi}{20 \text{ m}} \left[\frac{(0.050 \text{ m})^2}{7.8 \times 10^{-8} \Omega \cdot \text{m}} + \frac{(0.100 \text{ m})^2 - (0.050 \text{ m})^2}{1.72 \times 10^{-8} \Omega \cdot \text{m}} \right] \text{ and } R_{\text{Cable}} = 13.6 \times 10^{-6} \Omega = 13.6 \mu\Omega.$$

(b) $R = \rho_{\text{eff}} \frac{L}{A} = \rho_{\text{eff}} \frac{L}{\pi b^2}$. This gives $\rho_{\text{eff}} = \frac{\pi b^2 R}{L} = \frac{\pi(0.10 \text{ m})^2 (13.6 \times 10^{-6} \Omega)}{20 \text{ m}} = 2.14 \times 10^{-8} \Omega \cdot \text{m}$

EVALUATE: The effective resistivity of the cable is about 25% larger than the resistivity of copper. If nickel had infinite resistivity and only the copper portion conducted, the resistance of the cable would be $14.6 \mu\Omega$, which is not much larger than the resistance calculated in part (a).

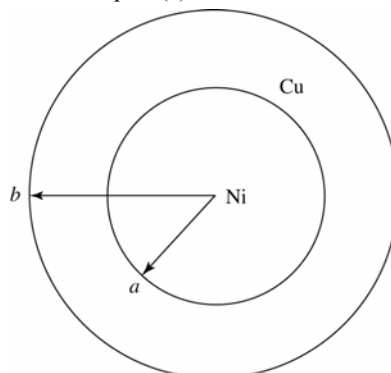


Figure 26.55

- 26.56. IDENTIFY and SET UP:** Let $R = 1.00\ \Omega$, the resistance of one wire. Each half of the wire has $R_h = R/2 = 0.500\ \Omega$. The combined wires are the same as a resistor network. Use the rules for equivalent resistance for resistors in series and parallel to find the resistance of the network, as shown in Figure 26.56.
- EXECUTE:**

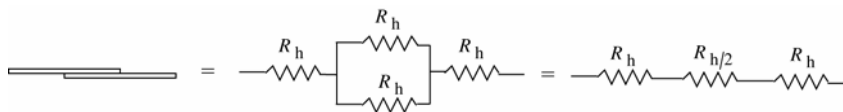


Figure 26.56

The equivalent resistance is $R_h + R_h/2 + R_h = 5R_h/2 = \frac{5}{2}(0.500\ \Omega) = 1.25\ \Omega$

EVALUATE: If the two wires were connected end-to-end, the total resistance would be $2.00\ \Omega$. If they were joined side-by-side, the total resistance would be $0.500\ \Omega$. Our answer is between these two limiting values.

- 26.57. IDENTIFY:** The terminal voltage of the battery depends on the current through it and therefore on the equivalent resistance connected to it. The power delivered to each bulb is $P = I^2 R$, where I is the current through it.

SET UP: The terminal voltage of the source is $\mathcal{E} - Ir$.

EXECUTE: (a) The equivalent resistance of the two bulbs is $1.0\ \Omega$. This equivalent resistance is in series with the

internal resistance of the source, so the current through the battery is $I = \frac{V}{R_{\text{total}}} = \frac{8.0\ \text{V}}{1.0\ \Omega + 0.80\ \Omega} = 4.4\ \text{A}$ and the

current through each bulb is $2.2\ \text{A}$. The voltage applied to each bulb is $\mathcal{E} - Ir = 8.0\ \text{V} - (4.4\ \text{A})(0.80\ \Omega) = 4.4\ \text{V}$.

Therefore, $P_{\text{bulb}} = I^2 R = (2.2\ \text{A})^2 (2.0\ \Omega) = 9.7\ \text{W}$.

(b) If one bulb burns out, then $I = \frac{V}{R_{\text{total}}} = \frac{8.0\ \text{V}}{2.0\ \Omega + 0.80\ \Omega} = 2.9\ \text{A}$. The current through the remaining bulb is

$2.9\ \text{A}$, and $P = I^2 R = (2.9\ \text{A})^2 (2.0\ \Omega) = 16.3\ \text{W}$. The remaining bulb is brighter than before, because it is consuming more power.

EVALUATE: In Example 26.2 the internal resistance of the source is negligible and the brightness of the remaining bulb doesn't change when one burns out.

- 26.58. IDENTIFY:** Half the current flows through each parallel resistor and the full current flows through the third resistor, that is in series with the parallel combination. Therefore, only the series resistor will be at its maximum power.

SET UP: $P = I^2 R$

EXECUTE: The maximum allowed power is when the total current is the maximum allowed value of

$I = \sqrt{P/R} = \sqrt{36\ \text{W}/2.4\ \Omega} = 3.9\ \text{A}$. Then half the current flows through the parallel resistors and the maximum power is $P_{\text{max}} = (I/2)^2 R + (I/2)^2 R + I^2 R = \frac{3}{2} I^2 R = \frac{3}{2} (3.9\ \text{A})^2 (2.4\ \Omega) = 54\ \text{W}$.

EVALUATE: If all three resistors were in series or all three were in parallel, then the maximum power would be $3(36\ \text{W}) = 108\ \text{W}$. For the network in this problem, the maximum power is half this value.

- 26.59. IDENTIFY:** The ohmmeter reads the equivalent resistance between points a and b . Replace series and parallel combinations by their equivalent.

SET UP: For resistors in parallel, $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$. For resistors in series, $R_{\text{eq}} = R_1 + R_2$

EXECUTE: Circuit (a): The $75.0\ \Omega$ and $40.0\ \Omega$ resistors are in parallel and have equivalent resistance $26.09\ \Omega$. The $25.0\ \Omega$ and $50.0\ \Omega$ resistors are in parallel and have an equivalent resistance of $16.67\ \Omega$. The equivalent

network is given in Figure 26.59a. $\frac{1}{R_{\text{eq}}} = \frac{1}{100.0\ \Omega} + \frac{1}{23.05\ \Omega}$, so $R_{\text{eq}} = 18.7\ \Omega$.

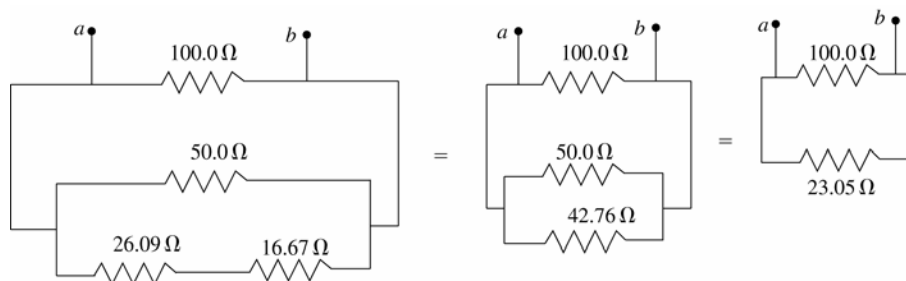


Figure 26.59a

Circuit (b): The $30.0\ \Omega$ and $45.0\ \Omega$ resistors are in parallel and have equivalent resistance $18.0\ \Omega$. The

equivalent network is given in Figure 26.59b. $\frac{1}{R_{\text{eq}}} = \frac{1}{10.0\ \Omega} + \frac{1}{30.3\ \Omega}$, so $R_{\text{eq}} = 7.5\ \Omega$.

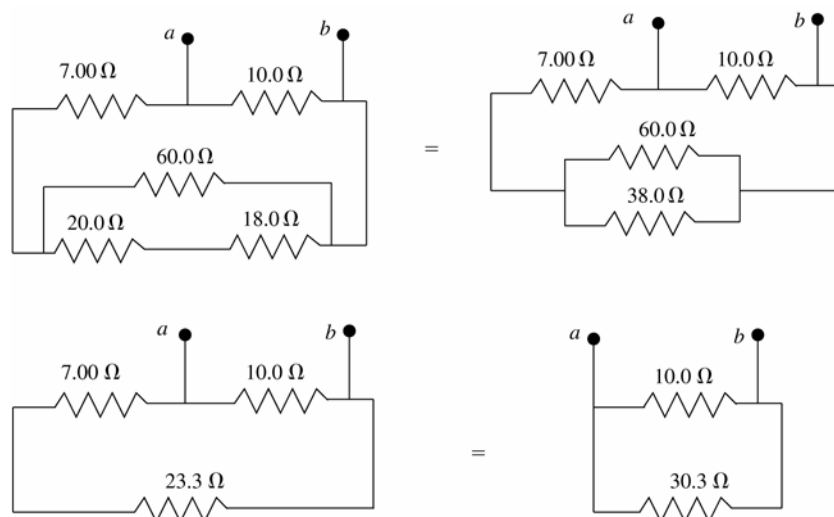


Figure 26.59b

EVALUATE: In circuit (a) the resistance along one path between a and b is $100.0\ \Omega$, but that is not the equivalent resistance between these points. A similar comment can be made about circuit (b).

26.60. IDENTIFY: Heat, which is generated in the resistor, melts the ice.

SET UP: Find the rate at which heat is generated in the $20.0\text{-}\Omega$ resistor using $P = V^2/R$. Then use the heat of fusion of ice to find the rate at which the ice melts. The heat dH to melt a mass of ice dm is $dH = L_F dm$, where L_F is the latent heat of fusion. The rate at which heat enters the ice, dH/dt , is the power P in the resistor, so $P = L_F dm/dt$. Therefore the rate of melting of the ice is $dm/dt = P/L_F$.

EXECUTE: The equivalent resistance of the parallel branch is $5.00\ \Omega$, so the total resistance in the circuit is $35.0\ \Omega$. Therefore the total current in the circuit is $I_{\text{Total}} = (45.0\ \text{V})/(35.0\ \Omega) = 1.286\ \text{A}$. The potential difference across the $20.0\text{-}\Omega$ resistor in the ice is the same as the potential difference across the parallel branch: $V_{\text{ice}} = I_{\text{Total}} R_p = (1.286\ \text{A})(5.00\ \Omega) = 6.429\ \text{V}$. The rate of heating of the ice is $P_{\text{ice}} = V_{\text{ice}}^2/R = (6.429\ \text{V})^2/(20.0\ \Omega) = 2.066\ \text{W}$. This power goes into to heat to melt the ice, so

$$dm/dt = P/L_F = (2.066\ \text{W})/(3.34 \times 10^5\ \text{J/kg}) = 6.19 \times 10^{-6}\ \text{kg/s} = 6.19 \times 10^{-3}\ \text{g/s}$$

EVALUATE: The melt rate is about $6\ \text{mg/s}$, which is not much. It would take $1000\ \text{s}$ to melt just $6\ \text{g}$ of ice.

26.61. IDENTIFY: Apply the junction rule to express the currents through the $5.00\ \Omega$ and $8.00\ \Omega$ resistors in terms of I_1 , I_2 and I_3 . Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.61.

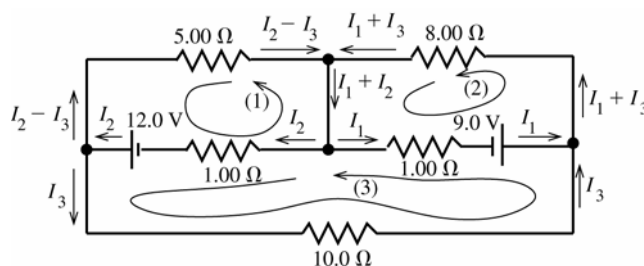


Figure 26.61

The current in each branch has been written in terms of I_1 , I_2 and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$$-12.0\ \text{V} + I_2(1.00\ \Omega) + (I_2 - I_3)(5.00\ \Omega) = 0$$

$$I_2(6.00\ \Omega) - I_3(5.00\ \Omega) = 12.0\ \text{V} \quad \text{eq.(1)}$$

Apply the loop rule to loop (2).

$$-I_1(1.00\ \Omega) + 9.00\ \text{V} - (I_1 + I_3)(8.00\ \Omega) = 0$$

$$I_1(9.00\ \Omega) + I_3(8.00\ \Omega) = 9.00\ \text{V} \quad \text{eq.(2)}$$

Apply the loop rule to loop (3).

$$-I_3(10.0\ \Omega) - 9.00\ \text{V} + I_1(1.00\ \Omega) - I_2(1.00\ \Omega) + 12.0\ \text{V} = 0$$

$$-I_1(1.00\ \Omega) + I_2(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\ \text{V} \quad \text{eq.(3)}$$

$$\text{Eq.(1) gives } I_2 = 2.00\ \text{A} + \frac{5}{6}I_3; \text{ eq.(2) gives } I_1 = 1.00\ \text{A} - \frac{8}{9}I_3$$

$$\text{Using these results in eq.(3) gives } -(1.00\ \text{A} - \frac{8}{9}I_3)(1.00\ \Omega) + (2.00\ \text{A} + \frac{5}{6}I_3)(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\ \text{V}$$

$$(\frac{16+15+180}{18})I_3 = 2.00\ \text{A}; I_3 = \frac{18}{211}(2.00\ \text{A}) = 0.171\ \text{A}$$

$$\text{Then } I_2 = 2.00\ \text{A} + \frac{5}{6}I_3 = 2.00\ \text{A} + \frac{5}{6}(0.171\ \text{A}) = 2.14\ \text{A} \text{ and } I_1 = 1.00\ \text{A} - \frac{8}{9}I_3 = 1.00\ \text{A} - \frac{8}{9}(0.171\ \text{A}) = 0.848\ \text{A}.$$

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\ \Omega$, $8.00\ \Omega$ and $10.0\ \Omega$ resistors. Going around the loop clockwise: $-(I_2 - I_3)(5.00\ \Omega) + (I_1 + I_3)(8.00\ \Omega) + I_3(10.0\ \Omega) = -9.85\ \text{V} + 8.15\ \text{V} + 1.71\ \text{V}$, which does equal zero, apart from rounding.

26.62. IDENTIFY: Apply the junction rule and the loop rule to the circuit.

SET UP: Because of the polarity of each emf, the current in the $7.00\ \Omega$ resistor must be in the direction shown in Figure 26.62a. Let I be the current in the $24.0\ \text{V}$ battery.

EXECUTE: The loop rule applied to loop (1) gives: $+24.0\ \text{V} - (1.80\ \text{A})(7.00\ \Omega) - I(3.00\ \Omega) = 0$. $I = 3.80\ \text{A}$. The junction rule then says that the current in the middle branch is $2.00\ \text{A}$, as shown in Figure 26.62b. The loop rule applied to loop (2) gives: $+\mathcal{E} - (1.80\ \text{A})(7.00\ \Omega) + (2.00\ \text{A})(2.00\ \Omega) = 0$ and $\mathcal{E} = 8.6\ \text{V}$.

EVALUATE: We can check our results by applying the loop rule to loop (3) in Figure 26.62b:

$$+24.0\ \text{V} - \mathcal{E} - (2.00\ \text{A})(2.00\ \Omega) - (3.80\ \text{A})(3.00\ \Omega) = 0 \text{ and } \mathcal{E} = 24.0\ \text{V} - 4.0\ \text{V} - 11.4\ \text{V} = 8.6\ \text{V}, \text{ which agrees with our result from loop (2).}$$

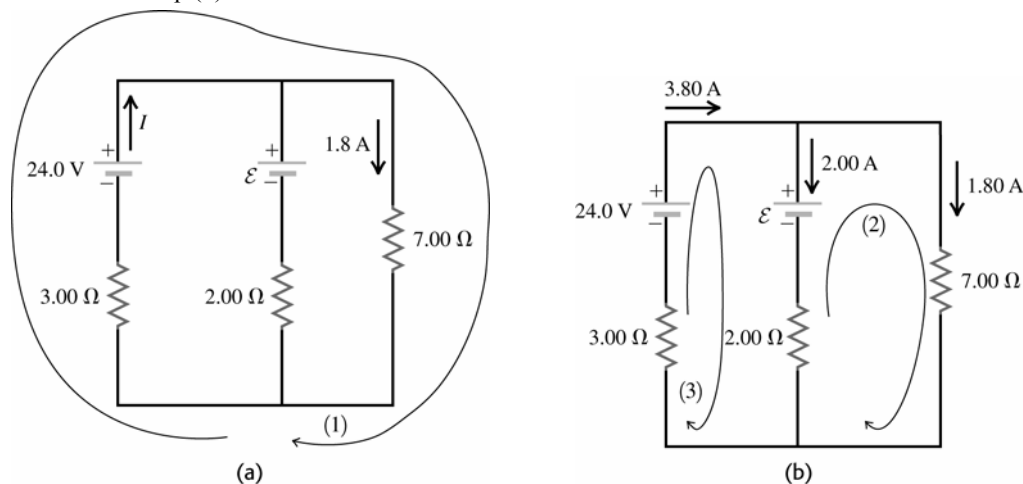


Figure 26.62

26.63. IDENTIFY and SET UP: The circuit is sketched in Figure 26.63.

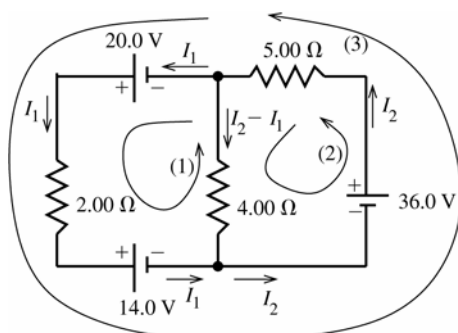


Figure 26.63

Two unknown currents I_1 (through the $2.00\ \Omega$ resistor) and I_2 (through the $5.00\ \Omega$ resistor) are labeled on the circuit diagram. The current through the $4.00\ \Omega$ resistor has been written as $I_2 - I_1$ using the junction rule.

Apply the loop rule to loops (1) and (2) to get two equations for the unknown currents, I_1 and I_2 . Loop (3) can then be used to check the results.

EXECUTE: loop (1): $+20.0 \text{ V} - I_1(2.00 \, \Omega) - 14.0 \text{ V} + (I_2 - I_1)(4.00 \, \Omega) = 0$

$$6.00I_1 - 4.00I_2 = 6.00 \text{ A}$$

$$3.00I_1 - 2.00I_2 = 3.00 \text{ A} \quad \text{eq.(1)}$$

loop (2): $+36.0 \text{ V} - I_2(5.00 \, \Omega) - (I_2 - I_1)(4.00 \, \Omega) = 0$

$$-4.00I_1 + 9.00I_2 = 36.0 \text{ A} \quad \text{eq.(2)}$$

Solving eq. (1) for I_1 gives $I_1 = 1.00 \text{ A} + \frac{2}{3}I_2$

Using this in eq.(2) gives $-4.00(1.00 \text{ A} + \frac{2}{3}I_2) + 9.00I_2 = 36.0 \text{ A}$

$$(-\frac{8}{3} + 9.00)I_2 = 40.0 \text{ A} \text{ and } I_2 = 6.32 \text{ A.}$$

Then $I_1 = 1.00 \text{ A} + \frac{2}{3}I_2 = 1.00 \text{ A} + \frac{2}{3}(6.32 \text{ A}) = 5.21 \text{ A.}$

In summary then

Current through the $2.00 \, \Omega$ resistor: $I_1 = 5.21 \text{ A.}$

Current through the $5.00 \, \Omega$ resistor: $I_2 = 6.32 \text{ A.}$

Current through the $4.00 \, \Omega$ resistor: $I_2 - I_1 = 6.32 \text{ A} - 5.21 \text{ A} = 1.11 \text{ A.}$

EVALUATE: Use loop (3) to check. $+20.0 \text{ V} - I_1(2.00 \, \Omega) - 14.0 \text{ V} + 36.0 \text{ V} - I_2(5.00 \, \Omega) = 0$

$$(5.21 \text{ A})(2.00 \, \Omega) + (6.32 \text{ A})(5.00 \, \Omega) = 42.0 \text{ V}$$

$10.4 \text{ V} + 31.6 \text{ V} = 42.0 \text{ V}$, so the loop rule is satisfied for this loop.

26.64. IDENTIFY: Apply the loop and junction rules.

SET UP: Use the currents as defined on the circuit diagram in Figure 26.64 and obtain three equations to solve for the currents.

EXECUTE: Left loop: $14 - I_1 - 2(I_1 - I_2) = 0$ and $3I_1 - 2I_2 = 14$.

Top loop: $-2(I - I_1) + I_2 + I_1 = 0$ and $-2I + 3I_1 + I_2 = 0$.

Bottom loop: $-(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0$ and $-I + 3I_1 - 4I_2 = 0$.

Solving these equations for the currents we find: $I = I_{\text{battery}} = 10.0 \text{ A}$; $I_1 = I_{R_1} = 6.0 \text{ A}$; $I_2 = I_{R_3} = 2.0 \text{ A}$.

So the other currents are: $I_{R_2} = I - I_1 = 4.0 \text{ A}$; $I_{R_4} = I_1 - I_2 = 4.0 \text{ A}$; $I_{R_5} = I - I_1 + I_2 = 6.0 \text{ A}$.

(b) $R_{\text{eq}} = \frac{V}{I} = \frac{14.0 \text{ V}}{10.0 \text{ A}} = 1.40 \, \Omega$.

EVALUATE: It isn't possible to simplify the resistor network using the rules for resistors in series and parallel. But the equivalent resistance is still defined by $V = IR_{\text{eq}}$.

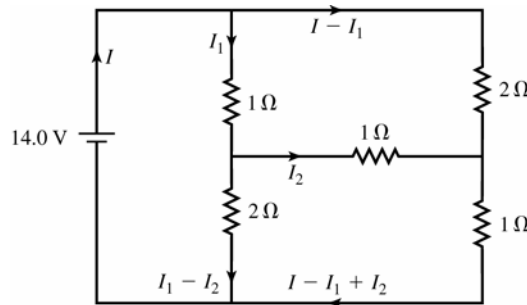


Figure 26.64

26.65. (a) IDENTIFY: Break the circuit between points a and b means no current in the middle branch that contains the $3.00 \, \Omega$ resistor and the 10.0 V battery. The circuit therefore has a single current path. Find the current, so that potential drops across the resistors can be calculated. Calculate V_{ab} by traveling from a to b , keeping track of the potential changes along the path taken.

SET UP: The circuit is sketched in Figure 26.65a.

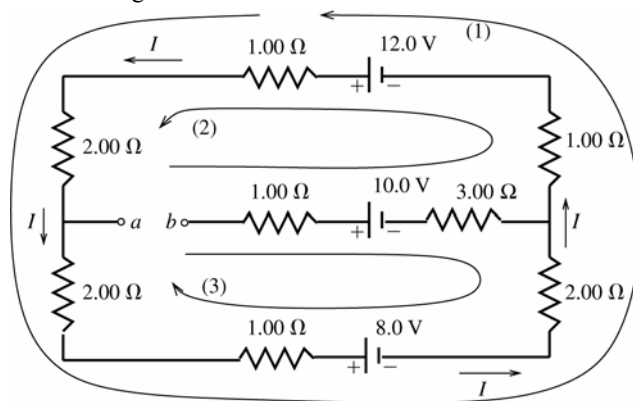


Figure 26.65a

EXECUTE: Apply the loop rule to loop (1).

$$+12.0 \text{ V} - I(1.00 \Omega + 2.00 \Omega + 2.00 \Omega + 1.00 \Omega) - 8.0 \text{ V} - I(2.00 \Omega + 1.00 \Omega) = 0$$

$$I = \frac{12.0 \text{ V} - 8.0 \text{ V}}{9.00 \Omega} = 0.4444 \text{ A.}$$

To find V_{ab} start at point b and travel to a , adding up the potential rises and drops. Travel on path (2) shown on the diagram. The 1.00Ω and 3.00Ω resistors in the middle branch have no current through them and hence no voltage across them. Therefore, $V_b - 10.0 \text{ V} + 12.0 \text{ V} - I(1.00 \Omega + 1.00 \Omega + 2.00 \Omega) = V_a$; thus

$$V_a - V_b = 2.0 \text{ V} - (0.4444 \text{ A})(4.00 \Omega) = +0.22 \text{ V} \quad (\text{point } a \text{ is at higher potential})$$

EVALUATE: As a check on this calculation we also compute V_{ab} by traveling from b to a on path (3).

$$V_b - 10.0 \text{ V} + 8.0 \text{ V} + I(2.00 \Omega + 1.00 \Omega + 2.00 \Omega) = V_a$$

$$V_{ab} = -2.00 \text{ V} + (0.4444 \text{ A})(5.00 \Omega) = +0.22 \text{ V, which checks.}$$

(b) IDENTIFY and SET UP: With points a and b connected by a wire there are three current branches, as shown in Figure 26.65b.

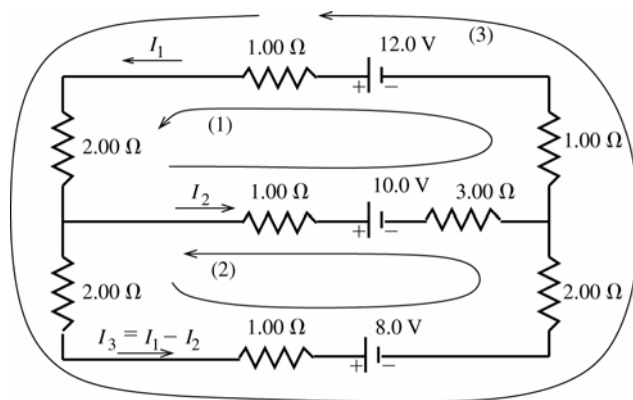


Figure 26.65b

The junction rule has been used to write the third current (in the 8.0 V battery) in terms of the other currents. Apply the loop rule to loops (1) and (2) to obtain two equations for the two unknowns I_1 and I_2 .

EXECUTE: Apply the loop rule to loop (1).

$$12.0 \text{ V} - I_1(1.00 \Omega) - I_1(2.00 \Omega) - I_2(1.00 \Omega) - 10.0 \text{ V} - I_2(3.00 \Omega) - I_1(1.00 \Omega) = 0$$

$$2.0 \text{ V} - I_1(4.00 \Omega) - I_2(4.00 \Omega) = 0$$

$$(2.00 \Omega)I_1 + (2.00 \Omega)I_2 = 1.0 \text{ V} \quad \text{eq.(1)}$$

Apply the loop rule to loop (2).

$$-(I_1 - I_2)(2.00 \Omega) - (I_1 - I_2)(1.00 \Omega) - 8.0 \text{ V} - (I_1 - I_2)(2.00 \Omega) + I_2(3.00 \Omega) + 10.0 \text{ V} + I_2(1.00 \Omega) = 0$$

$$2.0 \text{ V} - (5.00 \Omega)I_1 + (9.00 \Omega)I_2 = 0 \quad \text{eq.(2)}$$

Solve eq.(1) for I_2 and use this to replace I_2 in eq.(2).

$$I_2 = 0.50 \text{ A} - I_1$$

$$2.0 \text{ V} - (5.00 \Omega)I_1 + (9.00 \Omega)(0.50 \text{ A} - I_1) = 0$$

$$(14.0 \Omega)I_1 = 6.50 \text{ V} \text{ so } I_1 = (6.50 \text{ V})/(14.0 \Omega) = 0.464 \text{ A}$$

$$I_2 = 0.500 \text{ A} - 0.464 \text{ A} = 0.036 \text{ A}.$$

The current in the 12.0 V battery is $I_1 = 0.464 \text{ A}$.

EVALUATE: We can apply the loop rule to loop (3) as a check.

$$+12.0 \text{ V} - I_1(1.00 \Omega + 2.00 \Omega + 1.00 \Omega) - (I_1 - I_2)(2.00 \Omega + 1.00 \Omega + 2.00 \Omega) - 8.0 \text{ V} = 4.0 \text{ V} - 1.86 \text{ V} - 2.14 \text{ V} = 0,$$

as it should.

- 26.66. IDENTIFY:** Simplify the resistor networks as much as possible using the rule for series and parallel combinations of resistors. Then apply Kirchhoff's laws.

SET UP: First do the series/parallel reduction. This gives the circuit in Figure 26.66. The rate at which the 10.0Ω resistor generates thermal energy is $P = I^2 R$.

EXECUTE: Apply Kirchhoff's laws and solve for $\mathcal{E}$. $\Delta V_{\text{adef}} = 0: -(20 \Omega)(2 \text{ A}) - 5 \text{ V} - (20 \Omega)I_2 = 0$.

This gives $I_2 = -2.25 \text{ A}$. Then $I_1 + I_2 = 2 \text{ A}$ gives $I_1 = 2 \text{ A} - (-2.25 \text{ A}) = 4.25 \text{ A}$.

$\Delta V_{\text{abcedfa}} = 0: (15 \Omega)(4.25 \text{ A}) + \mathcal{E} - (20 \Omega)(-2.25 \text{ A}) = 0$. This gives $\mathcal{E} = -109 \text{ V}$. Since $\mathcal{E}$ is calculated to be negative, its polarity should be reversed.

(b) The parallel network that contains the 10.0Ω resistor in one branch has an equivalent resistance of 10Ω . The voltage across each branch of the parallel network is $V_{\text{par}} = RI = (10 \Omega)(2 \text{ A}) = 20 \text{ V}$. The current in the upper

branch is $I = \frac{V}{R} = \frac{20 \text{ V}}{30 \Omega} = \frac{2}{3} \text{ A}$. $Pt = E$, so $I^2 Rt = E$, where $E = 60.0 \text{ J}$. $(\frac{2}{3} \text{ A})^2 (10 \Omega)t = 60 \text{ J}$, and $t = 13.5 \text{ s}$.

EVALUATE: For the 10.0Ω resistor, $P = I^2 R = 4.44 \text{ W}$. The total rate at which electrical energy is input to the circuit in the emf is $(5.0 \text{ V})(2.0 \text{ A}) + (109 \text{ V})(4.25 \text{ A}) = 473 \text{ J}$. Only a small fraction of the energy is dissipated in the 10.0Ω resistor.

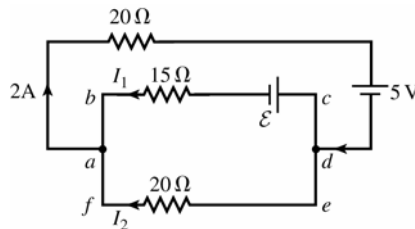


Figure 26.66

- 26.67. IDENTIFY:** In Figure 26.67, points a and c are at the same potential and points d and b are at the same potential, so we can calculate V_{ab} by calculating V_{cd} . We know the current through the resistor that is between points c and d . We thus can calculate the terminal voltage of the 24.0 V battery without calculating the current through it.

SET UP:

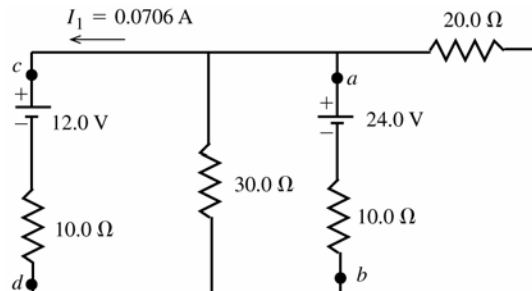


Figure 26.67

EXECUTE: $V_d + I_1(10.0 \Omega) + 12.0 \text{ V} = V_c$

$$V_c - V_d = 12.7 \text{ V}; V_a - V_b = V_c - V_d = 12.7 \text{ V}$$

EVALUATE: The voltage across each parallel branch must be the same. The current through the 24.0 V battery must be $(24.0 \text{ V} - 12.7 \text{ V})/(10.0 \Omega) = 1.13 \text{ A}$ in the direction b to a .

- 26.68. IDENTIFY:** The current through the $40.0\ \Omega$ resistor equals the current through the emf, and the current through each of the other resistors is less than or equal to this current. So, set $P_{40} = 1.00\ \text{W}$ and use this to solve for the current I through the emf. If $P_{40} = 1.00\ \text{W}$, then P for each of the other resistors is less than $1.00\ \text{W}$.
- SET UP:** Use the equivalent resistance for series and parallel combinations to simplify the circuit.
- EXECUTE:** $I^2 R = P$ gives $I^2(40\ \Omega) = 1\ \text{W}$, and $I = 0.158\ \text{A}$. Now use series / parallel reduction to simplify the circuit. The upper parallel branch is $6.38\ \Omega$ and the lower one is $25\ \Omega$. The series sum is now $126\ \Omega$. Ohm's law gives $\mathcal{E} = (126\ \Omega)(0.158\ \text{A}) = 19.9\ \text{V}$.
- EVALUATE:** The power input from the emf is $\mathcal{E}I = 3.14\ \text{W}$, so nearly one-third of the total power is dissipated in the $40.0\ \Omega$ resistor.
- 26.69. IDENTIFY and SET UP:** Simplify the circuit by replacing the parallel networks of resistors by their equivalents. In this simplified circuit apply the loop and junction rules to find the current in each branch.
- EXECUTE:** The $20.0\text{-}\Omega$ and $30.0\text{-}\Omega$ resistors are in parallel and have equivalent resistance $12.0\ \Omega$. The two resistors R are in parallel and have equivalent resistance $R/2$. The circuit is equivalent to the circuit sketched in Figure 26.69.

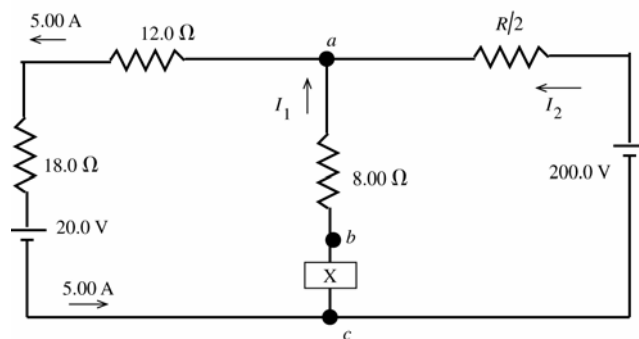


Figure 26.69

(a) Calculate V_{ca} by traveling along the branch that contains the $20.0\ \text{V}$ battery, since we know the current in that branch.

$$V_a - (5.00\ \text{A})(12.0\ \Omega) - (5.00\ \text{A})(18.0\ \Omega) - 20.0\ \text{V} = V_c$$

$$V_a - V_c = 20.0\ \text{V} + 90.0\ \text{V} + 60.0\ \text{V} = 170.0\ \text{V}$$

$$V_b - V_a = V_{ab} = 16.0\ \text{V}$$

$$X - V_{ba} = 170.0\ \text{V} \text{ so } X = 186.0\ \text{V}, \text{ with the upper terminal } +$$

(b) $I_1 = (16.0\ \text{V}) / (8.0\ \Omega) = 2.00\ \text{A}$

The junction rule applied to point a gives $I_2 + I_1 = 5.00\ \text{A}$, so $I_2 = 3.00\ \text{A}$. The current through the $200.0\ \text{V}$ battery is in the direction from the $-$ to the $+$ terminal, as shown in the diagram.

(c) $200.0\ \text{V} - I_2(R/2) = 170.0\ \text{V}$

$$(3.00\ \text{A})(R/2) = 30.0\ \text{V} \text{ so } R = 20.0\ \Omega$$

EVALUATE: We can check the loop rule by going clockwise around the outer circuit loop. This gives $+20.0\ \text{V} + (5.00\ \text{A})(18.0\ \Omega + 12.0\ \Omega) + (3.00\ \text{A})(10.0\ \Omega) - 200.0\ \text{V} = 20.0\ \text{V} + 150.0\ \text{V} + 30.0\ \text{V} - 200.0\ \text{V}$, which does equal zero.

26.70. IDENTIFY: $P_{\text{tot}} = \frac{V^2}{R_{\text{eq}}}$.

SET UP: Let R be the resistance of each resistor.

EXECUTE: When the resistors are in series, $R_{\text{eq}} = 3R$ and $P_s = \frac{V^2}{3R}$. When the resistors are in parallel, $R_{\text{eq}} = R/3$.

$$P_p = \frac{V^2}{R/3} = 3 \frac{V^2}{R} = 9P_s = 9(27\ \text{W}) = 243\ \text{W}.$$

EVALUATE: In parallel, the voltage across each resistor is the full applied voltage V . In series, the voltage across each resistor is $V/3$ and each resistor dissipates less power.

- 26.71. IDENTIFY and SET UP:** For part (a) use that the full emf is across each resistor. In part (b), calculate the power dissipated by the equivalent resistance, and in this expression express R_1 and R_2 in terms of P_1 , P_2 and $\mathcal{E}$.

EXECUTE: $P_1 = \mathcal{E}^2 / R_1$ so $R_1 = \mathcal{E}^2 / P_1$

$P_2 = \mathcal{E}^2 / R_2$ so $R_2 = \mathcal{E}^2 / P_2$

(a) When the resistors are connected in parallel to the emf, the voltage across each resistor is $\mathcal{E}$ and the power dissipated by each resistor is the same as if only the one resistor were connected. $P_{\text{tot}} = P_1 + P_2$

(b) When the resistors are connected in series the equivalent resistance is $R_{\text{eq}} = R_1 + R_2$

$$P_{\text{tot}} = \frac{\mathcal{E}^2}{R_1 + R_2} = \frac{\mathcal{E}^2}{\mathcal{E}^2 / P_1 + \mathcal{E}^2 / P_2} = \frac{P_1 P_2}{P_1 + P_2}$$

EVALUATE: The result in part (b) can be written as $\frac{1}{P_{\text{tot}}} = \frac{1}{P_1} + \frac{1}{P_2}$. Our results are that for parallel the powers add

and that for series the reciprocals of the power add. This is opposite the result for combining resistance. Since $P = \mathcal{E}^2 / R$ tells us that P is proportional to $1/R$, this makes sense.

- 26.72. IDENTIFY and SET UP:** Just after the switch is closed the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through R_3 is zero. After a long time the capacitor can be replaced by a break in the circuit.

EXECUTE: (a) Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{6.00 \, \Omega} + \frac{1}{3.00 \, \Omega} = \frac{3}{6.00 \, \Omega}; R_{\text{eq}} = 2.00 \, \Omega. \text{ In the absence of the capacitor, the total current in the circuit (the}$$

current through the $8.00 \, \Omega$ resistor) would be $i = \frac{\mathcal{E}}{R} = \frac{42.0 \, \text{V}}{8.00 \, \Omega + 2.00 \, \Omega} = 4.20 \, \text{A}$, of which $2/3$, or $2.80 \, \text{A}$, would

go through the $3.00 \, \Omega$ resistor and $1/3$, or $1.40 \, \text{A}$, would go through the $6.00 \, \Omega$ resistor. Since the current

through the capacitor is given by $i = \frac{V}{R} e^{-t/RC}$, at the instant $t = 0$ the circuit behaves as though the capacitor were

not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The $8.00 \, \Omega$ and the $6.00 \, \Omega$ resistors are now in series, and the current through them is $i = \mathcal{E} / R = (42.0 \, \text{V}) / (8.00 \, \Omega + 6.00 \, \Omega) = 3.00 \, \text{A}$.

The voltage drop across both the $6.00 \, \Omega$ resistor and the capacitor is thus $V = iR = (3.00 \, \text{A})(6.00 \, \Omega) = 18.0 \, \text{V}$.

(There is no current through the $3.00 \, \Omega$ resistor and so no voltage drop across it.) The charge on the capacitor is

$$Q = CV = (4.00 \times 10^{-6} \, \text{F})(18.0 \, \text{V}) = 7.2 \times 10^{-5} \, \text{C}.$$

EVALUATE: The equivalent resistance of R_2 and R_3 in parallel is less than R_3 , so initially the current through R_1 is larger than its value after a long time has elapsed.

- 26.73. (a) IDENTIFY and SET UP:** The circuit is sketched in Figure 26.73a.

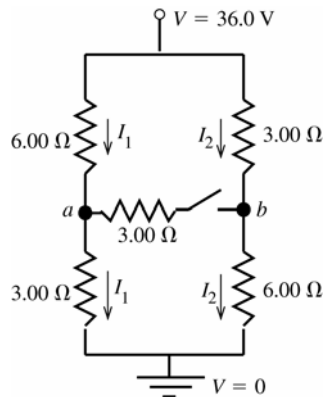


Figure 26.73a

With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

The potential drop across each parallel branch is $36.0 \, \text{V}$. Use this fact to calculate I_1 and I_2 . Then travel from point a to point b and keep track of the potential rises and drops in order to calculate V_{ab} .

EXECUTE: $-I_1(6.00\ \Omega + 3.00\ \Omega) + 36.0\ \text{V} = 0$

$$I_1 = \frac{36.0\ \text{V}}{6.00\ \Omega + 3.00\ \Omega} = 4.00\ \text{A}$$

$$-I_2(3.00\ \Omega + 6.00\ \Omega) + 36.0\ \text{V} = 0$$

$$I_2 = \frac{36.0\ \text{V}}{3.00\ \Omega + 6.00\ \Omega} = 4.00\ \text{A}$$

To calculate $V_{ab} = V_a - V_b$, start at point b and travel to point a , adding up all the potential rises and drops along the way. We can do this by going from b up through the $3.00\ \Omega$ resistor:

$$V_b + I_2(3.00\ \Omega) - I_1(6.00\ \Omega) = V_a$$

$$V_a - V_b = (4.00\ \text{A})(3.00\ \Omega) - (4.00\ \text{A})(6.00\ \Omega) = 12.0\ \text{V} - 24.0\ \text{V} = -12.0\ \text{V}$$

$$V_{ab} = -12.0\ \text{V} \text{ (point } a \text{ is } 12.0\ \text{V lower in potential than point } b)$$

EVALUATE: Alternatively, we can go from point b down through the $6.00\ \Omega$ resistor.

$$V_b - I_2(6.00\ \Omega) + I_1(3.00\ \Omega) = V_a$$

$$V_a - V_b = -(4.00\ \text{A})(6.00\ \Omega) + (4.00\ \text{A})(3.00\ \Omega) = -24.0\ \text{V} + 12.0\ \text{V} = -12.0\ \text{V}, \text{ which checks.}$$

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.73b. Use junction rule to write the current in each branch in terms of three unknown currents I_1 , I_2 , and I_3 . Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3 , the current through the switch. R_{eq} is calculated from $V = IR_{\text{eq}}$, where I is the total current that passes through the network.

SET UP:

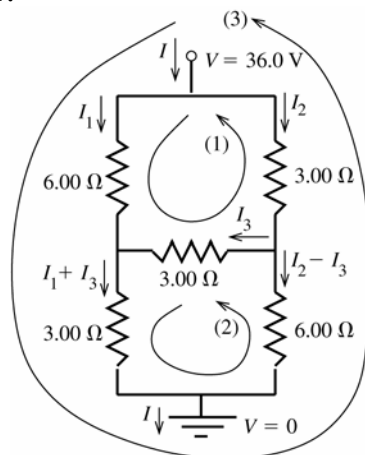


Figure 26.73b

The three unknown currents I_1 , I_2 , and I_3 are labeled on Figure 26.73b.

EXECUTE: Apply the loop rule to loops (1), (2), and (3).

loop (1): $-I_1(6.00\ \Omega) + I_3(3.00\ \Omega) + I_2(3.00\ \Omega) = 0$

$$I_2 = 2I_1 - I_3 \quad \text{eq.(1)}$$

loop (2): $-(I_1 + I_3)(3.00\ \Omega) + (I_2 - I_3)(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$$6I_2 - 12I_3 - 3I_1 = 0 \text{ so } 2I_2 - 4I_3 - I_1 = 0$$

Use eq.(1) to replace I_2 :

$$4I_1 - 2I_3 - 4I_3 - I_1 = 0$$

$$3I_1 = 6I_3 \text{ and } I_1 = 2I_3 \quad \text{eq.(2)}$$

loop (3) (This loop is completed through the battery [not shown], in the direction from the $-$ to the $+$ terminal.):

$$-I_1(6.00\ \Omega) - (I_1 + I_3)(3.00\ \Omega) + 36.0\ \text{V} = 0$$

$$9I_1 + 3I_3 = 36.0\ \text{A} \text{ and } 3I_1 + I_3 = 12.0\ \text{A} \quad \text{eq.(3)}$$

Use eq.(2) in eq.(3) to replace I_1 :

$$3(2I_3) + I_3 = 12.0\ \text{A}$$

$$I_3 = 12.0\ \text{A} / 7 = 1.71\ \text{A}$$

$$I_1 = 2I_3 = 3.42\ \text{A}$$

$$I_2 = 2I_1 - I_3 = 2(3.42\ \text{A}) - 1.71\ \text{A} = 5.13\ \text{A}$$

The current through the switch is $I_3 = 1.71\ \text{A}$.

(c) From the results in part (a) the current through the battery is $I = I_1 + I_2 = 3.42 \text{ A} + 5.13 \text{ A} = 8.55 \text{ A}$. The equivalent circuit is a single resistor that produces the same current through the 36.0 V battery, as shown in Figure 26.73c.

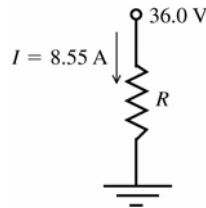


Figure 26.73c

$$-IR + 36.0 \text{ V} = 0$$

$$R = \frac{36.0 \text{ V}}{I} = \frac{36.0 \text{ V}}{8.55 \text{ A}} = 4.21 \Omega$$

EVALUATE: With the switch open (part a), point b is at higher potential than point a , so when the switch is closed the current flows in the direction from b to a . With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.

- 26.74. IDENTIFY:** With S open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When S is closed, current I flows through the 6.00Ω and 3.00Ω resistors.

SET UP: With the switch closed, a and b are at the same potential and the voltage across the 6.00Ω resistor equals the voltage across the $6.00 \mu\text{F}$ capacitor and the voltage is the same across the $3.00 \mu\text{F}$ capacitor and 3.00Ω resistor.

EXECUTE: (a) With an open switch: $V_{ab} = \mathcal{E} = 18.0 \text{ V}$.

(b) Point a is at a higher potential since it is directly connected to the positive terminal of the battery.

(c) When the switch is closed $18.0 \text{ V} = I(6.00 \Omega + 3.00 \Omega)$. $I = 2.00 \text{ A}$ and $V_b = (2.00 \text{ A})(3.00 \Omega) = 6.00 \text{ V}$.

(d) Initially the capacitor's charges were $Q_3 = CV = (3.00 \times 10^{-6} \text{ F})(18.0 \text{ V}) = 5.40 \times 10^{-5} \text{ C}$ and

$Q_6 = CV = (6.00 \times 10^{-6} \text{ F})(18.0 \text{ V}) = 1.08 \times 10^{-4} \text{ C}$. After the switch is closed

$Q_3 = CV = (3.00 \times 10^{-6} \text{ F})(18.0 \text{ V} - 12.0 \text{ V}) = 1.80 \times 10^{-5} \text{ C}$ and

$Q_6 = CV = (6.00 \times 10^{-6} \text{ F})(18.0 \text{ V} - 6.0 \text{ V}) = 7.20 \times 10^{-5} \text{ C}$. Both capacitors lose $3.60 \times 10^{-5} \text{ C}$.

EVALUATE: The voltage across each capacitor decreases when the switch is closed, because there is then current through each resistor and therefore a potential drop across each resistor.

- 26.75. IDENTIFY:** The current through the galvanometer for full-scale deflection is 0.0200 A. For each connection, there are two parallel branches and the voltage across each is the same.

SET UP: The sum of the two currents in the parallel branches for each connection equals the current into the meter for that connection.

EXECUTE: From the circuit we can derive three equations:

(i) $(R_1 + R_2 + R_3)(0.100 \text{ A} - 0.0200 \text{ A}) = (48.0 \Omega)(0.0200 \text{ A})$ and $R_1 + R_2 + R_3 = 12.0 \Omega$.

(ii) $(R_1 + R_2)(1.00 \text{ A} - 0.0200 \text{ A}) = (48.0 \Omega + R_3)(0.0200 \text{ A})$ and $R_1 + R_2 - 0.0204 R_3 = 0.980 \Omega$.

(iii) $R_1(10.0 \text{ A} - 0.0200 \text{ A}) = (48.0 \Omega + R_2 + R_3)(0.0200 \text{ A})$ and $R_1 - 0.002 R_2 - 0.002 R_3 = 0.096 \Omega$.

From (i) and (ii), $R_3 = 10.8 \Omega$. From (ii) and (iii), $R_2 = 1.08 \Omega$. Therefore, $R_1 = 0.12 \Omega$.

EVALUATE: For the 0.100 A setting the circuit consists of 48.0Ω and $R_1 + R_2 + R_3 = 12.0 \Omega$ in parallel and the equivalent resistance of the meter is 9.6Ω . For each of the other two settings the equivalent resistance of the meter is less than 9.6Ω .

- 26.76. IDENTIFY:** In each case the sum of the voltage drops across the resistors in the circuit must equal the full-scale voltage reading. The resistors are in series so the total resistance is the sum of the resistances in the circuit.

SET UP: For each range setting the circuit has the form shown in Figure 26.76.

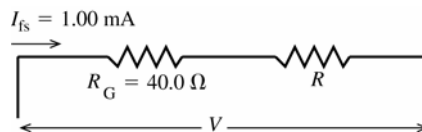


Figure 26.76

EXECUTE: 3.00 V

For $V = 3.00 \text{ V}$, $R = R_1$ and the total meter resistance R_m is $R_m = R_G + R_1$.

$$V = I_{fs} R_m \text{ so } R_m = \frac{V}{I_{fs}} = \frac{3.00 \text{ V}}{1.00 \times 10^{-3} \text{ A}} = 3.00 \times 10^3 \Omega.$$

$$R_m = R_G + R_1 \text{ so } R_1 = R_m - R_G = 3.00 \times 10^3 \Omega - 40.0 \Omega = 2960 \Omega$$

15.0 V

For $V = 15.0 \text{ V}$, $R = R_1 + R_2$ and the total meter resistance is $R_m = R_G + R_1 + R_2$.

$$V = I_{fs} R_m \text{ so } R_m = \frac{V}{I_{fs}} = \frac{15.0 \text{ V}}{1.00 \times 10^{-3} \text{ A}} = 1.50 \times 10^4 \Omega.$$

$$R_2 = R_m - R_G - R_1 = 1.50 \times 10^4 \Omega - 40.0 \Omega - 2960 \Omega = 1.20 \times 10^4 \Omega$$

150 V

For $V = 150 \text{ V}$, $R = R_1 + R_2 + R_3$ and the total meter resistance is $R_m = R_G + R_1 + R_2 + R_3$.

$$V = I_{fs} R_m \text{ so } R_m = \frac{V}{I_{fs}} = \frac{150 \text{ V}}{1.00 \times 10^{-3} \text{ A}} = 1.50 \times 10^5 \Omega.$$

$$R_3 = R_m - R_G - R_1 - R_2 = 1.50 \times 10^5 \Omega - 40.0 \Omega - 2960 \Omega - 1.20 \times 10^4 \Omega = 1.35 \times 10^5 \Omega.$$

EVALUATE: The greater the total resistance in series inside the meter the greater the potential difference between the two connections to the meter when the same 1.00 mA current flows through it.

- 26.77. IDENTIFY:** Connecting the voltmeter between point b and ground gives a resistor network and we can solve for the current through each resistor. The voltmeter reading equals the potential drop across the $200 \text{ k}\Omega$ resistor.

SET UP: For resistors in parallel, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$. For resistors in series, $R_{eq} = R_1 + R_2$.

EXECUTE: (a) $R_{eq} = 100 \text{ k}\Omega + \left(\frac{1}{200 \text{ k}\Omega} + \frac{1}{50 \text{ k}\Omega} \right)^{-1} = 140 \text{ k}\Omega$. The total current is $I = \frac{0.400 \text{ kV}}{140 \text{ k}\Omega} = 2.86 \times 10^{-3} \text{ A}$.

The voltage across the $200 \text{ k}\Omega$ resistor is $V_{200\text{k}\Omega} = IR = (2.86 \times 10^{-3} \text{ A}) \left(\frac{1}{200 \text{ k}\Omega} + \frac{1}{50 \text{ k}\Omega} \right)^{-1} = 114.4 \text{ V}$.

(b) If $V_R = 5.00 \times 10^6 \Omega$, then we carry out the same calculations as above to find $R_{eq} = 292 \text{ k}\Omega$,

$$I = 1.37 \times 10^{-3} \text{ A} \text{ and } V_{200\text{k}\Omega} = 263 \text{ V}.$$

(c) If $V_R = \infty$, then we find $R_{eq} = 300 \text{ k}\Omega$, $I = 1.33 \times 10^{-3} \text{ A}$ and $V_{200\text{k}\Omega} = 266 \text{ V}$.

EVALUATE: When a voltmeter of finite resistance is connected to a circuit, current flows through the voltmeter and the presence of the voltmeter alters the currents and voltages in the original circuit. The effect of the voltmeter on the circuit decreases as the resistance of the voltmeter increases.

- 26.78. IDENTIFY:** The circuit consists of two resistors in series with 110 V applied across the series combination.

SET UP: The circuit resistance is $30 \text{ k}\Omega + R$. The voltmeter reading of 68 V is the potential across the voltmeter terminals, equal to $I(30 \text{ k}\Omega)$.

EXECUTE: $I = \frac{110 \text{ V}}{(30 \text{ k}\Omega + R)}$. $I(30 \text{ k}\Omega) = 68 \text{ V}$ gives $(68 \text{ V})(30 \text{ k}\Omega + R) = (110 \text{ V})30 \text{ k}\Omega$ and $R = 18.5 \text{ k}\Omega$.

EVALUATE: This is a method for measuring large resistances.

- 26.79. IDENTIFY and SET UP:** Zero current through the galvanometer means the current I_1 through N is also the current through M and the current I_2 through P is the same as the current through X . And it means that points b and c are at the same potential, so $I_1 N = I_2 P$.

EXECUTE: (a) The voltage between points a and d is $\mathcal{E}$, so $I_1 = \frac{\mathcal{E}}{N + M}$ and $I_2 = \frac{\mathcal{E}}{P + X}$. Using these

expressions in $I_1 N = I_2 P$ gives $\frac{\mathcal{E}}{N + M} N = \frac{\mathcal{E}}{P + X} P$. $N(P + X) = P(N + M)$. $NX = PM$ and $X = MP/N$.

(b) $X = \frac{MP}{N} = \frac{(850.0 \Omega)(33.48 \Omega)}{15.00 \Omega} = 1897 \Omega$

EVALUATE: The measurement of X does not require that we know the value of the emf.

- 26.80. IDENTIFY:** Add resistors in series and parallel with the second galvanometer, so that the equivalent resistance is 65.0Ω and so that for a current of 1.50 mA into the device the current through the galvanometer is $3.60 \mu\text{A}$.

SET UP: In order for the second galvanometer to give the same full-scale deflection and to have the same resistance as the first, we need two additional resistances as shown in Figure 26.80.

EXECUTE: For $3.60 \mu\text{A}$ through R the current through R_1 is 1.496 mA. R and R_1 are in parallel so have equal voltages: $(3.6 \mu\text{A})(38.0 \Omega) = (1.496 \text{ mA})R_1$ and $R_1 = 91.4 \text{ m}\Omega$. And for the total resistance to be 65.0Ω :

$$65.0 \Omega = R_2 + \left(\frac{1}{38.0 \Omega} + \frac{1}{0.0914 \Omega} \right)^{-1} \text{ and } R_2 = 64.9 \Omega.$$

EVALUATE: Adding R_1 in parallel lowers the equivalent resistance so R_2 must be added in series to raise the equivalent resistance to 65.0Ω .

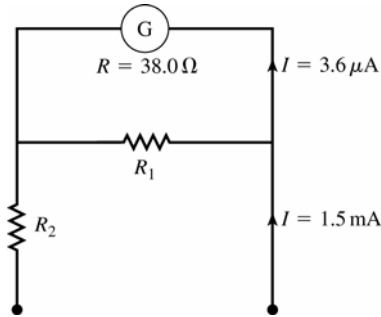


Figure 26.80

- 26.81. IDENTIFY and SET UP:** Without the meter, the circuit consists of the two resistors in series. When the meter is connected, its resistance is added to the circuit in parallel with the resistor it is connected across.

(a) EXECUTE: $I = I_1 = I_2$

$$I = \frac{90.0 \text{ V}}{R_1 + R_2} = \frac{90.0 \text{ V}}{224 \Omega + 589 \Omega} = 0.1107 \text{ A}$$

$$V_1 = I_1 R_1 = (0.1107 \text{ A})(224 \Omega) = 24.8 \text{ V}; V_2 = I_2 R_2 = (0.1107 \text{ A})(589 \Omega) = 65.2 \text{ V}$$

(b) SET UP: The resistor network is sketched in Figure 26.81a.

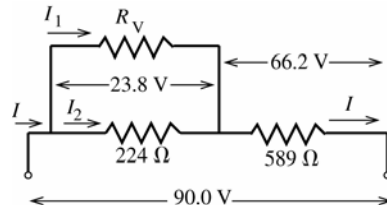


Figure 26.81c

The voltmeter reads the potential difference across its terminals, which is 23.8 V. If we can find the current I_1 through the voltmeter then we can use Ohm's law to find its resistance.

EXECUTE: The voltage drop across the 589Ω resistor is $90.0 \text{ V} - 23.8 \text{ V} = 66.2 \text{ V}$, so

$$I = \frac{V}{R} = \frac{66.2 \text{ V}}{589 \Omega} = 0.1124 \text{ A. The voltage drop across the } 224 \Omega \text{ resistor is } 23.8 \text{ V, so } I_2 = \frac{V}{R} = \frac{23.8 \text{ V}}{224 \Omega} = 0.1062 \text{ A.}$$

Then $I = I_1 + I_2$ gives $I_1 = I - I_2 = 0.1124 \text{ A} - 0.1062 \text{ A} = 0.0062 \text{ A}$. $R_v = \frac{V}{I_1} = \frac{23.8 \text{ V}}{0.0062 \text{ A}} = 3840 \Omega$

(c) SET UP: The circuit with the voltmeter connected is sketched in Figure 26.81b.

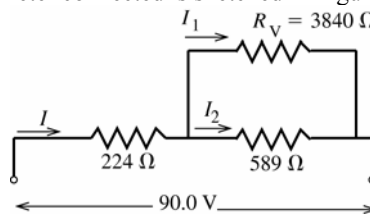


Figure 26.81b

EXECUTE: Replace the two resistors in parallel by their equivalent, as shown in Figure 26.81c.

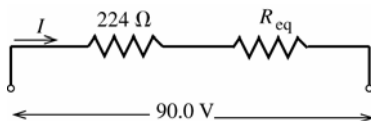


Figure 26.81c

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3840 \Omega} + \frac{1}{589 \Omega};$$

$$R_{\text{eq}} = \frac{(3840 \Omega)(589 \Omega)}{3840 \Omega + 589 \Omega} = 510.7 \Omega$$

$$I = \frac{90.0 \text{ V}}{224 \Omega + 510.7 \Omega} = 0.1225 \text{ A}$$

The potential drop across the 224Ω resistor then is $IR = (0.1225 \text{ A})(224 \Omega) = 27.4 \text{ V}$, so the potential drop across the 589Ω resistor and across the voltmeter (what the voltmeter reads) is $90.0 \text{ V} - 27.4 \text{ V} = 62.6 \text{ V}$.

(d) EVALUATE: No, any real voltmeter will draw some current and thereby reduce the current through the resistance whose voltage is being measured. Thus the presence of the voltmeter connected in parallel with the resistance lowers the voltage drop across that resistance. The resistance of the voltmeter is only about a factor of ten larger than the resistances in the circuit, so the voltmeter has a noticeable effect on the circuit.

26.82. IDENTIFY: Just after the connection is made, $q = 0$ and the voltage across the capacitor is zero. After a long time $i = 0$.

SET UP: The rate at which the resistor dissipates electrical energy is $P_R = V^2/R$, where V is the voltage across the resistor. The energy stored in the capacitor is $q^2/2C$. The power output of the source is $P_\epsilon = \mathcal{E}i$.

EXECUTE: (a) (i) $P_R = \frac{V^2}{R} = \frac{(120 \text{ V})^2}{4.26 \Omega} = 3380 \text{ W}$. (ii) $P_C = \frac{dU}{dt} = \frac{1}{2C} \frac{d(q^2)}{dt} = \frac{iq}{C} = 0$.

(iii) $P_\epsilon = \mathcal{E}I = (120 \text{ V}) \frac{120 \text{ V}}{4.26 \Omega} = 3380 \text{ W}$.

(b) After a long time, $i = 0$, so $P_R = 0$, $P_C = 0$, $P_\epsilon = 0$.

EVALUATE: Initially all the power output of the source is dissipated in the resistor. After a long time energy is stored in the capacitor but the amount stored isn't changing.

26.83. IDENTIFY: Apply the loop rule to the circuit. The initial current determines R . We can then use the time constant to calculate C .

SET UP: The circuit is sketched in Figure 26.83.

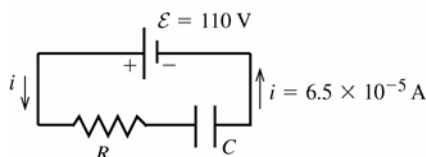


Figure 26.83

Initially, the charge of the capacitor is zero, so by $v = q/C$ the voltage across the capacitor is zero.

EXECUTE: The loop rule therefore gives $\mathcal{E} - iR = 0$ and $R = \frac{\mathcal{E}}{i} = \frac{110 \text{ V}}{6.5 \times 10^{-5} \text{ A}} = 1.7 \times 10^6 \Omega$

The time constant is given by $\tau = RC$ (Eq. 26.14), so $C = \frac{\tau}{R} = \frac{6.2 \text{ s}}{1.7 \times 10^6 \Omega} = 3.6 \mu\text{F}$.

EVALUATE: The resistance is large so the initial current is small and the time constant is large.

26.84. IDENTIFY: The energy stored in a capacitor is $U = q^2/2C$. The electrical power dissipated in the resistor is $P = i^2R$.

SET UP: For a discharging capacitor, $i = -\frac{q}{RC}$.

EXECUTE: (a) $U_0 = \frac{Q_0^2}{2C} = \frac{(0.0081 \text{ C})^2}{2(4.62 \times 10^{-6} \text{ F})} = 7.10 \text{ J}$.

(b) $P_0 = I_0^2 R = \left(\frac{Q_0}{RC}\right)^2 R = \frac{(0.0081 \text{ C})^2}{(850 \Omega)(4.62 \times 10^{-6} \text{ F})^2} = 3616 \text{ W}$

(c) When $U = \frac{1}{2}U_0 = \frac{1}{2} \frac{Q_0^2}{2C}$, $Q = \frac{Q_0}{\sqrt{2}}$. This gives $P = \left(\frac{Q}{RC}\right)^2 R = \frac{1}{2} \left(\frac{Q_0}{RC}\right)^2 R = \frac{1}{2}P_0 = 1808 \text{ W}$.

EVALUATE: All the energy originally stored in the capacitor is dissipated as current flow through the resistor.

26.85. IDENTIFY: $q = Q_0 e^{-t/RC}$. The time constant is $\tau = RC$.

SET UP: The charge of one electron has magnitude $e = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: (a) We will say that a capacitor is discharged if its charge is less than that of one electron. The time this takes is then given by $q = Q_0 e^{-t/RC}$, so $t = RC \ln(Q_0/e) = (6.7 \times 10^5 \Omega)(9.2 \times 10^{-7} \text{ F}) \ln(7.0 \times 10^{-6} \text{ C}/1.6 \times 10^{-19} \text{ C}) = 19.36 \text{ s}$, or 31.4 time constants.

EVALUATE: (b) As shown in part (a), $t = \tau \ln(Q_0/q)$ and so the number of time constants required to discharge the capacitor is independent of R and C , and depends only on the initial charge.

26.86. IDENTIFY: The energy changes exponentially, but it does not obey exactly the same equation as the charge since it is proportional to the square of the charge.

(a) **SET UP:** For charging, $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2C = U_0 e^{-2t/RC}$.

EXECUTE: To reduce the energy to $1/e$ of its initial value:

$$U_0/e = U_0 e^{-2t/RC}$$

$$t = RC/2$$

(b) SET UP: For discharging, $U = Q^2/2C = [Q_0(1 - e^{-t/RC})]^2/2C = U_{\max}(1 - e^{-t/RC})^2$

EXECUTE: To reach $1/e$ of the maximum energy, $U_{\max}/e = U_{\max}(1 - e^{-t/RC})^2$ and $t = -RC \ln\left(1 - \frac{1}{\sqrt{e}}\right)$.

EVALUATE: The time to reach $1/e$ of the maximum energy is not the same as the time to discharge to $1/e$ of the maximum energy.

- 26.87. IDENTIFY and SET UP:** For parts (a) and (b) evaluate the integrals as specified in the problem. The current as a function of time is given by Eq.(26.13) $i = \frac{\mathcal{E}}{R} e^{-t/RC}$. The energy stored in the capacitor is given by $Q^2/2C$.

EXECUTE: (a) $P = \mathcal{E}i$

The total energy supplied by the battery is $\int_0^\infty P dt = \int_0^\infty \mathcal{E}i dt = (\mathcal{E}^2/R) \int_0^\infty e^{-t/RC} dt = (\mathcal{E}^2/R) [-RC e^{-t/RC}]_0^\infty = C\mathcal{E}^2$.

(b) $P = i^2 R$

The total energy dissipated in the resistor is

$$\int_0^\infty P dt = \int_0^\infty i^2 R dt = (\mathcal{E}^2/R) \int_0^\infty e^{-2t/RC} dt = (\mathcal{E}^2/R) \left[-(RC/2) e^{-2t/RC} \right]_0^\infty = \frac{1}{2} C\mathcal{E}^2.$$

(c) The final charge on the capacitor is $Q = C\mathcal{E}$. The energy stored is $U = Q^2/(2C) = \frac{1}{2} C\mathcal{E}^2$. The final energy stored in the capacitor $(\frac{1}{2} C\mathcal{E}^2) =$ total energy supplied by the battery $(C\mathcal{E}^2) -$ energy dissipated in the resistor $(\frac{1}{2} C\mathcal{E}^2)$

(d) EVALUATE: $\frac{1}{2}$ of the energy supplied by the battery is stored in the capacitor. This fraction is independent of R . The other $\frac{1}{2}$ of the energy supplied by the battery is dissipated in the resistor. When R is small the current initially is large but dies away quickly. When R is large the current initially is small but lasts longer.

- 26.88. IDENTIFY:** $E = \int_0^\infty P dt$. The energy stored in a capacitor is $U = q^2/2C$.

SET UP: $i = -\frac{Q_0}{RC} e^{-t/RC}$

EXECUTE: $i = -\frac{Q_0}{RC} e^{-t/RC}$ gives $P = i^2 R = \frac{Q_0^2}{RC^2} e^{-2t/RC}$ and $E = \frac{Q_0^2}{RC^2} \int_0^\infty e^{-2t/RC} dt = \frac{Q_0^2}{RC^2} \frac{RC}{2} = \frac{Q_0^2}{2C} = U_0$.

EVALUATE: Increasing the energy stored in the capacitor increases current through the resistor as the capacitor discharges.

- 26.89. IDENTIFY and SET UP:**

EXECUTE: (a) Using Kirchhoff's Rules on the circuit we find:

Left loop: $92 - 140I_1 - 210I_2 + 55 = 0 \Rightarrow 147 - 140I_1 - 210I_2 = 0$.

Right loop: $57 - 35I_3 - 210I_2 + 55 = 0 \Rightarrow 112 - 210I_2 - 35I_3 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$.

Solving for the three currents we have: $I_1 = 0.300$ A, $I_2 = 0.500$ A, $I_3 = 0.200$ A.

(b) Leaving only the 92-V battery in the circuit:

Left loop: $92 - 140I_1 - 210I_2 = 0$. Right loop: $-35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = 0.541 \text{ A}, \quad I_2 = 0.077 \text{ A}, \quad I_3 = -0.464 \text{ A}.$$

(c) Leaving only the 57-V battery in the circuit:

Left loop: $140I_1 + 210I_2 = 0$. Right loop: $57 - 35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = -0.287 \text{ A}, \quad I_2 = 0.192 \text{ A}, \quad I_3 = 0.480 \text{ A}.$$

(d) Leaving only the 55-V battery in the circuit:

Left loop: $55 - 140I_1 - 210I_2 = 0$. Right loop: $55 - 35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = 0.046 \text{ A}, \quad I_2 = 0.231 \text{ A}, \quad I_3 = 0.185 \text{ A}.$$

(e) If we sum the currents from the previous three parts we find:

$$I_1 = 0.300 \text{ A}, \quad I_2 = 0.500 \text{ A}, \quad I_3 = 0.200 \text{ A}, \text{ just as in part (a).}$$

(f) Changing the 57-V battery for an 80-V battery just affects the calculation in part (c). It changes to: Left loop: $140I_1 + 210I_2 = 0$. Right loop: $80 - 35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = -0.403 \text{ A}, \quad I_2 = 0.269 \text{ A}, \quad I_3 = 0.672 \text{ A}.$$

The total current for the full circuit is the sum of (b), (d) and (f) above:

$$I_1 = 0.184 \text{ A}, \quad I_2 = 0.576 \text{ A}, \quad I_3 = 0.392 \text{ A}.$$

EVALUATE: This problem presents an alternative means of solving for currents in multiloop circuits.

26.90. IDENTIFY and SET UP: When C changes after the capacitor is charged, the voltage across the capacitor changes. Current flows through the resistor until the voltage across the capacitor again equals the emf.

EXECUTE: (a) Fully charged: $Q = CV = (10.0 \times 10^{-12} \text{ F})(1000 \text{ V}) = 1.00 \times 10^{-8} \text{ C}$.

(b) The initial current just after the capacitor is charged is $I_0 = \frac{\mathcal{E} - V_{C'}}{R} = \frac{\mathcal{E}}{R} - \frac{q}{RC'}$. This gives $i(t) = \left(\frac{\mathcal{E}}{R} - \frac{q}{RC'} \right) e^{-t/RC'}$,

where $C' = 1.1C$.

(c) We need a resistance such that the current will be greater than $1 \mu\text{A}$ for longer than $200 \mu\text{s}$. This requires that

at $t = 200 \mu\text{s}$, $i = 1.0 \times 10^{-6} \text{ A} = \frac{1}{R} \left(1000 \text{ V} - \frac{1.0 \times 10^{-8} \text{ C}}{1.1(1.0 \times 10^{-11} \text{ F})} \right) e^{-(2.0 \times 10^{-4} \text{ s})/R(1.1 \times 10^{-12} \text{ F})}$. This says

$1.0 \times 10^{-6} \text{ A} = \frac{1}{R} (90.9) e^{-(1.8 \times 10^7 \Omega)/R}$ and $18.3R - R \ln R - 1.8 \times 10^7 = 0$. Solving for R numerically we find

$$7.15 \times 10^6 \Omega \leq R \leq 7.01 \times 10^7 \Omega.$$

EVALUATE: If the resistance is too small, then the capacitor discharges too quickly, and if the resistance is too large, the current is not large enough.

26.91. IDENTIFY: Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.91.

EXECUTE: $R_T = 2R_1 + \left(\frac{1}{R_2} + \frac{1}{R_T} \right)^{-1} = 2R_1 + \frac{R_2 R_T}{R_2 + R_T}$. $R_T^2 - 2R_1 R_T - 2R_1 R_2 = 0$. $R_T = R_1 \pm \sqrt{R_1^2 + 2R_1 R_2}$. $R_T > 0$,

so $R_T = R_1 + \sqrt{R_1^2 + 2R_1 R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

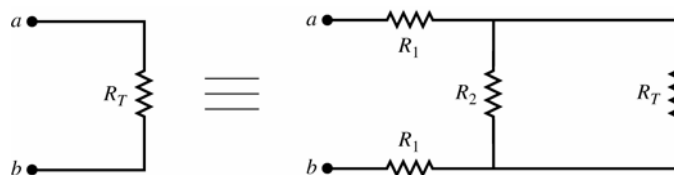


Figure 26.91

26.92. IDENTIFY: Assume a voltage V applied between points a and b and consider the currents that flow along each path between a and b .

SET UP: The currents are shown in Figure 26.92.

EXECUTE: Let current I enter at a and exit at b . At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is

$V = \left(\frac{I}{3} \right) R + \left(\frac{I}{6} \right) R + \left(\frac{I}{3} \right) R = \frac{5}{6} IR$. This must be the same as $V = IR_{\text{eq}}$, so $R_{\text{eq}} = \frac{5}{6} R$.

EVALUATE: The equivalent resistance is less than R , even though there are 12 resistors in the network.

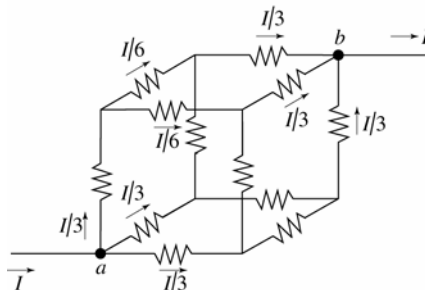


Figure 26.92

26.93. IDENTIFY: The network is the same as the one in Challenge Problem 26.91, and that problem shows that the equivalent resistance of the network is $R_T = \sqrt{R_1^2 + 2R_1R_2}$.

SET UP: The circuit can be redrawn as shown in Figure 26.93.

EXECUTE: (a) $V_{cd} = V_{ab} \frac{R_{eq}}{2R_1 + R_{eq}} = V_{ab} \frac{1}{2R_1/R_{eq} + 1}$ and $R_{eq} = \frac{R_2R_T}{R_2 + R_T}$. But $\beta = \frac{2R_1(R_T + R_2)}{R_T R_2} = \frac{2R_1}{R_{eq}}$, so

$$V_{cd} = V_{ab} \frac{1}{1 + \beta}.$$

$$(b) V_1 = \frac{V_0}{(1 + \beta)} \Rightarrow V_2 = \frac{V_1}{(1 + \beta)} = \frac{V_0}{(1 + \beta)^2} \Rightarrow V_n = \frac{V_{n-1}}{(1 + \beta)} = \frac{V_0}{(1 + \beta)^n}.$$

If $R_1 = R_2$, then $R_T = R_1 + \sqrt{R_1^2 + 2R_1R_1} = R_1(1 + \sqrt{3})$ and $\beta = \frac{2(2 + \sqrt{3})}{1 + \sqrt{3}} = 2.73$. So, for the n th segment to have 1%

of the original voltage, we need: $\frac{1}{(1 + \beta)^n} = \frac{1}{(1 + 2.73)^n} \leq 0.01$. This says $n = 4$, and then $V_4 = 0.005V_0$.

(c) $R_T = R_1 + \sqrt{R_1^2 + 2R_1R_2}$ gives $R_T = 6400 \Omega + \sqrt{(6400 \Omega)^2 + 2(6400 \Omega)(8.0 \times 10^8 \Omega)} = 3.2 \times 10^6 \Omega$ and $\beta = \frac{2(6400 \Omega)(3.2 \times 10^6 \Omega + 8.0 \times 10^8 \Omega)}{(3.2 \times 10^6 \Omega)(8.0 \times 10^8 \Omega)} = 4.0 \times 10^{-3}$.

(d) Along a length of 2.0 mm of axon, there are 2000 segments each $1.0 \mu\text{m}$ long. The voltage therefore

attenuates by $V_{2000} = \frac{V_0}{(1 + \beta)^{2000}}$, so $\frac{V_{2000}}{V_0} = \frac{1}{(1 + 4.0 \times 10^{-3})^{2000}} = 3.4 \times 10^{-4}$.

(e) If $R_2 = 3.3 \times 10^{12} \Omega$, then $R_T = 2.1 \times 10^8 \Omega$ and $\beta = 6.2 \times 10^{-5}$. This gives

$$\frac{V_{2000}}{V_0} = \frac{1}{(1 + 6.2 \times 10^{-5})^{2000}} = 0.88.$$

EVALUATE: As R_2 increases, β decreases and the potential difference decrease from one section to the next is less.

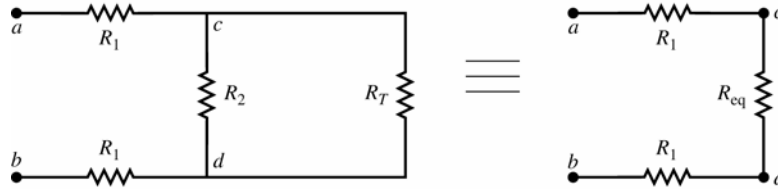


Figure 26.93

MAGNETIC FIELD AND MAGNETIC FORCES

27.1. IDENTIFY and SET UP: Apply Eq.(27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

EXECUTE: $\vec{v} = (+4.19 \times 10^4 \text{ m/s})\hat{i} + (-3.85 \times 10^4 \text{ m/s})\hat{j}$

(a) $\vec{B} = (1.40 \text{ T})\hat{i}$

$$\vec{F} = q\vec{v} \times \vec{B} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})[(4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{i} - (3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{i}]$$

$$\hat{i} \times \hat{i} = 0, \hat{j} \times \hat{i} = -\hat{k}$$

$$\vec{F} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})(-3.85 \times 10^4 \text{ m/s})(-\hat{k}) = (-6.68 \times 10^{-4} \text{ N})\hat{k}$$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.

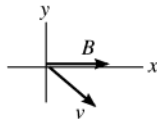


Figure 27.1a

The right-hand rule gives that $\vec{v} \times \vec{B}$ is directed out of the paper (+z-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v} \times \vec{B}$;

$\vec{F}$ is in the $-z$ -direction. This agrees with the direction calculated with unit vectors.

(b) **EXECUTE:** $\vec{B} = (1.40 \text{ T})\hat{k}$

$$\vec{F} = q\vec{v} \times \vec{B} = (-1.24 \times 10^{-8} \text{ C})(1.40 \text{ T})[(+4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{k} - (3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{k}]$$

$$\hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{k} = \hat{i}$$

$$\vec{F} = (-7.27 \times 10^{-4} \text{ N})(-\hat{j}) + (6.68 \times 10^{-4} \text{ N})\hat{i} = [(6.68 \times 10^{-4} \text{ N})\hat{i} + (7.27 \times 10^{-4} \text{ N})\hat{j}]$$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.

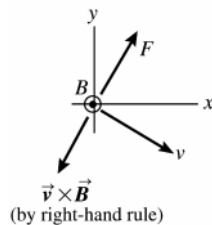


Figure 27.1b

The direction of $\vec{F}$ is opposite to $\vec{v} \times \vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

27.2. IDENTIFY: The net force must be zero, so the magnetic and gravity forces must be equal in magnitude and opposite in direction.

SET UP: The gravity force is downward so the force from the magnetic field must be upward. The charge's velocity and the forces are shown in Figure 27.2. Since the charge is negative, the magnetic force is opposite to the right-hand rule direction. The minimum magnetic field is when the field is perpendicular to $\vec{v}$. The force is also perpendicular to $\vec{B}$, so $\vec{B}$ is either eastward or westward.

EXECUTE: If $\vec{B}$ is eastward, the right-hand rule direction is into the page and $\vec{F}_B$ is out of the page, as required.

Therefore, $\vec{B}$ is eastward. $mg = |q|vB \sin \phi$. $\phi = 90^\circ$ and $B = \frac{mg}{v|q|} = \frac{(0.195 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{(4.00 \times 10^4 \text{ m/s})(2.50 \times 10^{-8} \text{ C})} = 1.91 \text{ T}$.

EVALUATE: The magnetic field could also have a component along the north-south direction, that would not contribute to the force, but then the field wouldn't have minimum magnitude.

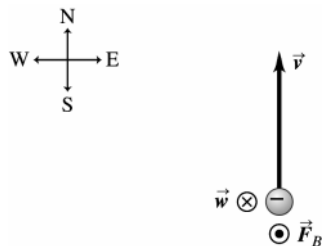


Figure 27.2

- 27.3. IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90^\circ$ to calculate F .

SET UP: The directions of $\vec{v}$, $\vec{B}$ and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: (a) When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50 \times 10^{-6} \text{ C})(4.75 \times 10^3 \text{ m/s})(1.25 \text{ T})\sin 90^\circ = 0.0505 \text{ N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

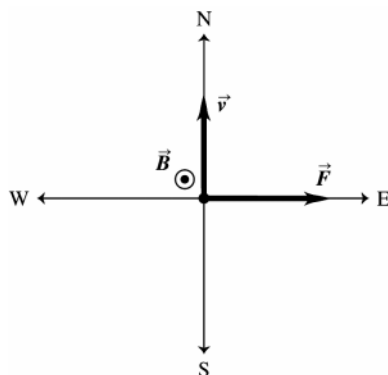


Figure 27.3

- 27.4. IDENTIFY:** Apply Newton's second law, with the force being the magnetic force.

SET UP: $\hat{j} \times \hat{i} = -\hat{k}$

EXECUTE: $\vec{F} = m\vec{a} = q\vec{v} \times \vec{B}$ gives $\vec{a} = \frac{q\vec{v} \times \vec{B}}{m}$ and

$$\vec{a} = \frac{(1.22 \times 10^{-8} \text{ C})(3.0 \times 10^4 \text{ m/s})(1.63 \text{ T})(\hat{j} \times \hat{i})}{1.81 \times 10^{-3} \text{ kg}} = -(0.330 \text{ m/s}^2)\hat{k}.$$

EVALUATE: The acceleration is in the $-z$ -direction and is perpendicular to both $\vec{v}$ and $\vec{B}$.

- 27.5. IDENTIFY:** Apply $F = |q|vB\sin\phi$ and solve for v .

SET UP: An electron has $q = -1.60 \times 10^{-19} \text{ C}$.

EXECUTE: $v = \frac{F}{|q|B\sin\phi} = \frac{4.60 \times 10^{-15} \text{ N}}{(1.6 \times 10^{-19} \text{ C})(3.5 \times 10^{-3} \text{ T})\sin 60^\circ} = 9.49 \times 10^6 \text{ m/s}$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

- 27.6. IDENTIFY:** Apply Newton's second law and $F = |q|vB\sin\phi$.

SET UP: ϕ is the angle between the direction of $\vec{v}$ and the direction of $\vec{B}$.

EXECUTE: (a) The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{qvB}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(2.50 \times 10^6 \text{ m/s})(7.4 \times 10^{-2} \text{ T})}{(9.11 \times 10^{-31} \text{ kg})} = 3.25 \times 10^{16} \text{ m/s}^2.$$

(b) If $a = \frac{1}{4}(3.25 \times 10^{16} \text{ m/s}^2) = \frac{qvB \sin \phi}{m}$, then $\sin \phi = 0.25$ and $\phi = 14.5^\circ$.

EVALUATE: The force and acceleration decrease as the angle ϕ approaches zero.

27.7. IDENTIFY: Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $\vec{v} = v_y \hat{j}$, with $v_y = -3.80 \times 10^3 \text{ m/s}$. $F_x = +7.60 \times 10^{-3} \text{ N}$, $F_y = 0$, and $F_z = -5.20 \times 10^{-3} \text{ N}$.

EXECUTE: (a) $F_x = q(v_y B_z - v_z B_y) = qv_y B_z$.

$$B_z = F_x / qv_y = (7.60 \times 10^{-3} \text{ N}) / [(7.80 \times 10^{-6} \text{ C})(-3.80 \times 10^3 \text{ m/s})] = -0.256 \text{ T}$$

$F_y = q(v_z B_x - v_x B_z) = 0$, which is consistent with $\vec{F}$ as given in the problem. There is no force component along the direction of the velocity.

$$F_z = q(v_x B_y - v_y B_x) = -qv_y B_x. \quad B_x = -F_z / qv_y = -0.175 \text{ T}.$$

(b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y .

$$(c) \quad \vec{B} \cdot \vec{F} = B_x F_x + B_y F_y + B_z F_z = (-0.175 \text{ T})(+7.60 \times 10^{-3} \text{ N}) + (-0.256 \text{ T})(-5.20 \times 10^{-3} \text{ N})$$

$$\vec{B} \cdot \vec{F} = 0. \quad \vec{B} \text{ and } \vec{F} \text{ are perpendicular (angle is } 90^\circ).$$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{v} \cdot \vec{F}$ is also zero.

27.8. IDENTIFY and SET UP: $\vec{F} = q\vec{v} \times \vec{B} = qB_z[v_x(\hat{i} \times \hat{k}) + v_y(\hat{j} \times \hat{k}) + v_z(\hat{k} \times \hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})]$.

EXECUTE: (a) Set the expression for $\vec{F}$ equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40 \times 10^{-7} \text{ N})}{-(-5.60 \times 10^{-9} \text{ C})(-1.25 \text{ T})} = -106 \text{ m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40 \times 10^{-7} \text{ N})}{(-5.60 \times 10^{-9} \text{ C})(-1.25 \text{ T})} = -48.6 \text{ m/s}.$$

(b) v_z does not contribute to the force, so is not determined by a measurement of $\vec{F}$.

$$(c) \quad \vec{v} \cdot \vec{F} = v_x F_x + v_y F_y + v_z F_z = \frac{F_y}{-qB_z} F_x + \frac{F_x}{qB_z} F_y = 0; \quad \theta = 90^\circ.$$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{B} \cdot \vec{F}$ is also zero.

27.9. IDENTIFY: Apply $\vec{F} = q\vec{v} \times \vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the $+y$ -direction, $B_y = 0$ and

$$\vec{B} = B_x \hat{i} + B_z \hat{k}. \quad \text{For the proton, } \vec{v} = (1.50 \text{ km/s}) \hat{i}.$$

EXECUTE: (a) For the proton, $\vec{F} = q(1.50 \times 10^3 \text{ m/s}) \hat{i} \times (B_x \hat{i} + B_z \hat{k}) = q(1.50 \times 10^3 \text{ m/s}) B_z (-\hat{j})$. $\vec{F} = (2.25 \times 10^{-16} \text{ N}) \hat{j}$,

$$\text{so } B_z = -\frac{2.25 \times 10^{-16} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(1.50 \times 10^3 \text{ m/s})} = -0.938 \text{ T}. \quad \text{The force on the proton is independent of } B_x. \quad \text{For the}$$

$$\text{electron, } \vec{v} = (4.75 \text{ km/s})(-\hat{k}). \quad \vec{F} = q\vec{v} \times \vec{B} = (-e)(4.75 \times 10^3 \text{ m/s})(-\hat{k}) \times (B_x \hat{i} + B_z \hat{k}) = +e(4.75 \times 10^3 \text{ m/s}) B_x \hat{j}.$$

The magnitude of the force is $F = e(4.75 \times 10^3 \text{ m/s}) |B_x|$. Since $F = 8.50 \times 10^{-16} \text{ N}$,

$$|B_x| = \frac{8.50 \times 10^{-16} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(4.75 \times 10^3 \text{ m/s})} = 1.12 \text{ T}. \quad B_x = \pm 1.12 \text{ T}. \quad \text{The sign of } B_x \text{ is not determined by measuring}$$

the magnitude of the force on the electron. $B = \sqrt{B_x^2 + B_z^2} = \sqrt{(\pm 1.12 \text{ T})^2 + (-0.938 \text{ T})^2} = 1.46 \text{ T}.$

$$\tan \theta = \frac{B_z}{B_x} = \frac{-0.938 \text{ T}}{\pm 1.12 \text{ T}}. \quad \theta = \pm 40^\circ. \quad \vec{B} \text{ is in the } xz\text{-plane and is either at } 40^\circ \text{ from the } +x\text{-direction toward the}$$

$-z$ -direction or 40° from the $-x$ -direction toward the $-z$ -direction.

(b) $\vec{B} = B_x \hat{i} + B_z \hat{k}$. $\vec{v} = (3.2 \text{ km/s})(-\hat{j})$.

$$\vec{F} = q\vec{v} \times \vec{B} = (-e)(3.2 \text{ km/s})(-\hat{j}) \times (B_x \hat{i} + B_z \hat{k}) = e(3.2 \times 10^3 \text{ m/s})(B_x(-\hat{k}) + B_z \hat{i}).$$

$$\vec{F} = e(3.2 \times 10^3 \text{ m/s})([\pm 1.12 \text{ T}]\hat{k} - [0.938 \text{ T}]\hat{i}) = -(4.80 \times 10^{-16} \text{ N})\hat{i} \pm (5.73 \times 10^{-16} \text{ N})\hat{k}$$

$$F = \sqrt{F_x^2 + F_z^2} = 7.47 \times 10^{-16} \text{ N}. \quad \tan \theta = \frac{F_z}{F_x} = \frac{\pm 5.73 \times 10^{-16} \text{ N}}{-4.80 \times 10^{-16} \text{ N}}. \quad \theta = \pm 50.0^\circ. \quad \text{The force is in the } xz\text{-plane and is}$$

directed at 50.0° from the $-x$ -axis toward either the $+z$ or $-z$ axis, depending on the sign of B_x .

EVALUATE: If the direction of the force on the first electron were measured, then the sign of B_x would be determined.

27.10. IDENTIFY: Magnetic field lines are closed loops, so the net flux through any closed surface is zero.

SET UP: Let magnetic field directed out of the enclosed volume correspond to positive flux and magnetic field directed into the volume correspond to negative flux.

EXECUTE: (a) The total flux must be zero, so the flux through the remaining surfaces must be -0.120 Wb .

(b) The shape of the surface is unimportant, just that it is closed.

(c) One possibility is sketched in Figure 27.10.

EVALUATE: In Figure 27.10 all the field lines that enter the cube also exit through the surface of the cube.

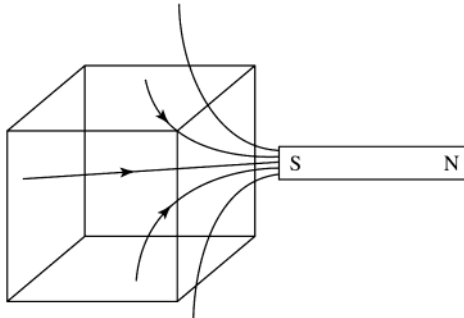


Figure 27.10

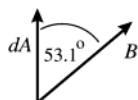
27.11. IDENTIFY and SET UP: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Circular area in the xy -plane, so $A = \pi r^2 = \pi(0.0650 \text{ m})^2 = 0.01327 \text{ m}^2$ and $d\vec{A}$ is in the z -direction. Use Eq.(1.18) to calculate the scalar product.

EXECUTE: (a) $\vec{B} = (0.230 \text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel ($\phi = 0^\circ$) so $\vec{B} \cdot d\vec{A} = B dA$.

B is constant over the circular area so $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA = B \int dA = BA = (0.230 \text{ T})(0.01327 \text{ m}^2) = 3.05 \times 10^{-3} \text{ Wb}$

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



$$\vec{B} \cdot d\vec{A} = B \cos \phi dA$$

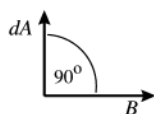
with $\phi = 53.1^\circ$

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos \phi dA = B \cos \phi \int dA = B \cos \phi A$

$$\Phi_B = (0.230 \text{ T}) \cos 53.1^\circ (0.01327 \text{ m}^2) = 1.83 \times 10^{-3} \text{ Wb}$$

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$$\vec{B} \cdot d\vec{A} = 0 \text{ since } d\vec{A} \text{ and } \vec{B} \text{ are perpendicular } (\phi = 90^\circ)$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = 0.$$

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.12. IDENTIFY: When $\vec{B}$ is uniform across the surface, $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi$.

SET UP: $\vec{A}$ is normal to the surface and is directed outward from the enclosed volume. For surface $abcd$, $\vec{A} = A\hat{i}$. For surface $befc$, $\vec{A} = -A\hat{k}$. For surface $ae fd$, $\cos \phi = 3/5$ and the flux is positive.

EXECUTE: (a) $\Phi_B(abcd) = \vec{B} \cdot \vec{A} = 0$.

(b) $\Phi_B(befc) = \vec{B} \cdot \vec{A} = -(0.128 \text{ T})(0.300 \text{ m})(0.300 \text{ m}) = -0.0115 \text{ Wb}$.

(c) $\Phi_B(aefd) = \vec{B} \cdot \vec{A} = BA \cos \phi = \frac{3}{5}(0.128 \text{ T})(0.500 \text{ m})(0.300 \text{ m}) = +0.0115 \text{ Wb}$.

(d) The net flux through the rest of the surfaces is zero since they are parallel to the x-axis. The total flux is the sum of all parts above, which is zero.

EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

27.13. IDENTIFY: The total flux through the bottle is zero because it is a closed surface.

SET UP: The total flux through the bottle is the flux through the plastic plus the flux through the open cap, so the sum of these must be zero. $\Phi_{\text{plastic}} + \Phi_{\text{cap}} = 0$.

$$\Phi_{\text{plastic}} = -\Phi_{\text{cap}} = -BA \cos \Phi = -B(\pi r^2) \cos \Phi$$

EXECUTE: Substituting the numbers gives $\Phi_{\text{plastic}} = -(1.75 \text{ T})\pi(0.0125 \text{ m})^2 \cos 25^\circ = -7.8 \times 10^{-4} \text{ Wb}$

EVALUATE: It would be impossible to calculate the flux through the plastic directly because of the complicated shape of the bottle, but with a little thought we can find this flux through a simple calculation.

27.14. IDENTIFY: $p = mv$ and $L = Rp$, since the velocity and linear momentum are tangent to the circular path.

SET UP: $|q|vB = mv^2/R$.

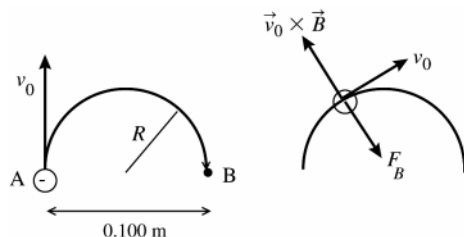
EXECUTE: (a) $p = mv = m\left(\frac{RqB}{m}\right) = RqB = (4.68 \times 10^{-3} \text{ m})(6.4 \times 10^{-19} \text{ C})(1.65 \text{ T}) = 4.94 \times 10^{-21} \text{ kg m/s}$.

(b) $L = Rp = R^2qB = (4.68 \times 10^{-3} \text{ m})^2(6.4 \times 10^{-19} \text{ C})(1.65 \text{ T}) = 2.31 \times 10^{-23} \text{ kg} \cdot \text{m}^2/\text{s}$.

EVALUATE: $\vec{p}$ is tangent to the orbit and $\vec{L}$ is perpendicular to the orbit plane.

27.15. (a) IDENTIFY: Apply Eq.(27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v} \times \vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0 \times \vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0 \times \vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's 2nd law to calculate the magnitude of $\vec{B}$: $\sum \vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma$

$$F_B = m(v^2/R)$$

$$F_B = |q|vB \sin \phi = |q|vB, \text{ so } |q|vB = m(v^2/R)$$

$$B = \frac{mv}{|q|R} = \frac{(9.109 \times 10^{-31} \text{ kg})(1.41 \times 10^6 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(0.050 \text{ m})} = 1.60 \times 10^{-4} \text{ T}$$

(b) **IDENTIFY and SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0 .

$$\text{EXECUTE: } \text{The distance along the semicircular path is } \pi R, \text{ so } t = \frac{\pi R}{v_0} = \frac{\pi(0.050 \text{ m})}{1.41 \times 10^6 \text{ m/s}} = 1.11 \times 10^{-7} \text{ s}$$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.16. IDENTIFY: Newton's second law gives $|q|vB = mv^2/R$. The speed v is constant and equals v_0 . The direction of the magnetic force must be in the direction of the acceleration and is toward the center of the semicircular path.

SET UP: A proton has $q = +1.60 \times 10^{-19} \text{ C}$ and $m = 1.67 \times 10^{-27} \text{ kg}$. The direction of the magnetic force is given by the right-hand rule.

EXECUTE: (a) $B = \frac{mv}{qR} = \frac{(1.67 \times 10^{-27} \text{ kg})(1.41 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.0500 \text{ m})} = 0.294 \text{ T}$

The direction of the magnetic field is out of the page (the charge is positive), in order for $\vec{F}$ to be directed to the right at point A.

(b) The time to complete half a circle is $t = \pi R / v_0 = 1.11 \times 10^{-7} \text{ s}$.

EVALUATE: The magnetic field required to produce this path for a proton has a different magnitude (because of the different mass) and opposite direction (because of opposite sign of the charge) than the field required to produce the path for an electron.

- 27.17. IDENTIFY and SET UP:** Use conservation of energy to find the speed of the ball when it reaches the bottom of the shaft. The right-hand rule gives the direction of $\vec{F}$ and Eq.(27.1) gives its magnitude. The number of excess electrons determines the charge of the ball.

EXECUTE: $q = (4.00 \times 10^8)(-1.602 \times 10^{-19} \text{ C}) = -6.408 \times 10^{-11} \text{ C}$

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy$; $v = \sqrt{2gy} = 49.5 \text{ m/s}$

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v} \times \vec{B}$ is north. Since $q < 0$, $\vec{F}$ is south.

$F = |q|vB \sin \theta = (6.408 \times 10^{-11} \text{ C})(49.5 \text{ m/s})(0.250 \text{ T}) \sin 90^\circ = 7.93 \times 10^{-10} \text{ N}$

EVALUATE: Both the charge and speed of the ball are relatively small so the magnetic force is small, much less than the gravity force of 1.5 N.

- 27.18. IDENTIFY:** Since the particle moves perpendicular to the uniform magnetic field, the radius of its path is

$R = \frac{mv}{|q|B}$. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

SET UP: The alpha particle has charge $q = +2e = 3.20 \times 10^{-19} \text{ C}$.

EXECUTE: (a) $R = \frac{(6.64 \times 10^{-27} \text{ kg})(35.6 \times 10^3 \text{ m/s})}{(3.20 \times 10^{-19} \text{ C})(1.10 \text{ T})} = 6.73 \times 10^{-4} \text{ m} = 0.673 \text{ mm}$. The alpha particle moves in a

circular arc of diameter $2R = 1.35 \text{ mm}$.

(b) For a very short time interval the displacement of the particle is in the direction of the velocity. The magnetic force is always perpendicular to this direction so it does no work. The work-energy theorem therefore says that the kinetic energy of the particle, and hence its speed, is constant.

(c) The acceleration is $a = \frac{F_B}{m} = \frac{|q|vB \sin \phi}{m} = \frac{(3.20 \times 10^{-19} \text{ C})(35.6 \times 10^3 \text{ m/s})(1.10 \text{ T}) \sin 90^\circ}{6.64 \times 10^{-27} \text{ kg}} = 1.88 \times 10^{12} \text{ m/s}^2$. We can

also use $a = \frac{v^2}{R}$ and the result of part (a) to calculate $a = \frac{(35.6 \times 10^3 \text{ m/s})^2}{6.73 \times 10^{-4} \text{ m}} = 1.88 \times 10^{12} \text{ m/s}^2$, the same result. The

acceleration is perpendicular to $\vec{v}$ and $\vec{B}$ and so is horizontal, toward the center of curvature of the particle's path.

EVALUATE: (d) The unbalanced force ($\vec{F}_B$) is perpendicular to $\vec{v}$, so it changes the direction of $\vec{v}$ but not its magnitude, which is the speed.

- 27.19. IDENTIFY:** In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $|q|vB = mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: (a) $K_1 + U_1 = K_2 + U_2$. $U_1 = K_2 = 0$, so $K_1 = U_2$ and $\frac{1}{2}mv^2 = ke^2/r$.

$$v = e\sqrt{\frac{2k}{mr}} = (1.602 \times 10^{-19} \text{ C})\sqrt{\frac{2k}{(3.34 \times 10^{-27} \text{ kg})(1.0 \times 10^{-15} \text{ m})}} = 1.2 \times 10^7 \text{ m/s}$$

(b) $\sum \vec{F} = m\vec{a}$ gives $qvB = mv^2/r$. $B = \frac{mv}{qr} = \frac{(3.34 \times 10^{-27} \text{ kg})(1.2 \times 10^7 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(2.50 \text{ m})} = 0.10 \text{ T}$.

EVALUATE: The speed calculated in part (a) is large, 4% of the speed of light.

- 27.20. IDENTIFY:** $F = |q|vB \sin \phi$. The direction of $\vec{F}$ is given by the right-hand rule.

SET UP: An electron has $q = -e$.

EXECUTE: (a) $F = |q|vB \sin \phi$. $B = \frac{F}{|q|v \sin \phi} = \frac{0.00320 \times 10^{-9} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(500,000 \text{ m/s}) \sin 90^\circ} = 5.00 \text{ T}$. If the angle ϕ is

less than 90° , a larger field is needed to produce the same force. The direction of the field must be toward the south so that $\vec{v} \times \vec{B}$ is downward.

$$(b) F = |q|vB \sin \phi. \quad v = \frac{F}{|q|B \sin \phi} = \frac{4.60 \times 10^{-12} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(2.10 \text{ T}) \sin 90^\circ} = 1.37 \times 10^7 \text{ m/s.}$$

If ϕ is less than 90° , the speed would have to be larger to have the same force. The force is upward, so $\vec{v} \times \vec{B}$ must be downward since the electron is negative, and the velocity must be toward the south.

EVALUATE: The component of $\vec{B}$ along the direction of $\vec{v}$ produces no force and the component of $\vec{v}$ along the direction of $\vec{B}$ produces no force.

27.21. (a) IDENTIFY and SET UP: Apply Newton's 2nd law, with $a = v^2/R$ since the path of the particle is circular.

$$\text{EXECUTE: } \sum \vec{F} = m\vec{a} \text{ says } |q|vB = m(v^2/R)$$

$$v = \frac{|q|BR}{m} = \frac{(1.602 \times 10^{-19} \text{ C})(2.50 \text{ T})(6.96 \times 10^{-3} \text{ m})}{3.34 \times 10^{-27} \text{ kg}} = 8.35 \times 10^5 \text{ m/s}$$

(b) IDENTIFY and SET UP: The speed is constant so $t = \text{distance}/v$.

$$\text{EXECUTE: } t = \frac{\pi R}{v} = \frac{\pi(6.96 \times 10^{-3} \text{ m})}{8.35 \times 10^5 \text{ m/s}} = 2.62 \times 10^{-8} \text{ s}$$

(c) IDENTIFY and SET UP: kinetic energy gained = electric potential energy lost

$$\text{EXECUTE: } \frac{1}{2}mv^2 = |q|V$$

$$V = \frac{mv^2}{2|q|} = \frac{(3.34 \times 10^{-27} \text{ kg})(8.35 \times 10^5 \text{ m/s})^2}{2(1.602 \times 10^{-19} \text{ C})} = 7.27 \times 10^3 \text{ V} = 7.27 \text{ kV}$$

EVALUATE: The deuteron has a much larger mass to charge ratio than an electron so a much larger B is required for the same v and R . The deuteron has positive charge so gains kinetic energy when it goes from high potential to low potential.

27.22. IDENTIFY: For motion in an arc of a circle, $a = \frac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is $1.67 \times 10^{-27} \text{ kg}$.

EXECUTE: (a) $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives

$$|q|vB \sin \phi = m \frac{v^2}{R}. \quad \phi = 90^\circ \text{ and } v = \frac{|q|BR}{m} = \frac{3(1.60 \times 10^{-19} \text{ C})(0.250 \text{ T})(0.475 \text{ m})}{12(1.67 \times 10^{-27} \text{ kg})} = 2.84 \times 10^6 \text{ m/s}.$$

$$(b) F_B = |q|vB \sin \phi = 3(1.60 \times 10^{-19} \text{ C})(2.84 \times 10^6 \text{ m/s})(0.250 \text{ T}) \sin 90^\circ = 3.41 \times 10^{-13} \text{ N}.$$

$w = mg = 12(1.67 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2) = 1.96 \times 10^{-25} \text{ N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: (c) The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

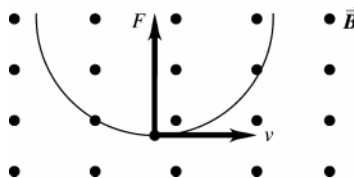


Figure 27.22

27.23. IDENTIFY: Example 27.3 shows that $B = \frac{m2\pi f}{|q|}$, where f is the frequency, in Hz, of the electromagnetic waves that are produced.

SET UP: An electron has charge $q = -e$ and mass $m = 9.11 \times 10^{-31} \text{ kg}$. A proton has charge $q = +e$ and mass $m = 1.67 \times 10^{-27} \text{ kg}$.

$$\text{EXECUTE: (a)} \quad B = \frac{m2\pi f}{|q|} = \frac{(9.11 \times 10^{-31} \text{ kg})2\pi(3.00 \times 10^{12} \text{ Hz})}{(1.60 \times 10^{-19} \text{ C})} = 107 \text{ T.}$$

This is about 2.4 times the greatest magnitude of magnetic field yet obtained on earth.

(b) Protons have a greater mass than the electrons, so a greater magnetic field would be required to accelerate them with the same frequency and there would be no advantage in using them.

EVALUATE: Electromagnetic waves with frequency $f = 3.0$ THz have a wavelength in air of

$$\lambda = \frac{v}{f} = 3.0 \times 10^{-4} \text{ m. The shorter the wavelength the greater the frequency and the greater the magnetic field that}$$

is required. B depends only on f and on the mass-to-charge ratio of the particle that moves in the circular path.

27.24. IDENTIFY: The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18 \text{ cm}/(\pi/2) = 0.751 \text{ cm}$. Solving for the magnetic field:

$$B = mv/qR = (1.67 \times 10^{-27} \text{ kg})(1200 \text{ m/s})/[(1.60 \times 10^{-19} \text{ C})(0.00751 \text{ m})] = 1.67 \times 10^{-3} \text{ T}$$

EVALUATE: This field is about 10 times stronger than the Earth's magnetic field, but much weaker than many laboratory fields.

27.25. IDENTIFY: When a particle of charge $-e$ is accelerated through a potential difference of magnitude V , it gains kinetic energy eV . When it moves in a circular path of radius R , its acceleration is $\frac{v^2}{R}$.

SET UP: An electron has charge $q = -e = -1.60 \times 10^{-19} \text{ C}$ and mass $9.11 \times 10^{-31} \text{ kg}$.

$$\text{EXECUTE: } \frac{1}{2}mv^2 = eV \text{ and } v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^3 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 2.65 \times 10^7 \text{ m/s. } \vec{F} = m\vec{a} \text{ gives}$$

$$|q|vB \sin \phi = m \frac{v^2}{R}. \phi = 90^\circ \text{ and } B = \frac{mv}{|q|R} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.65 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.180 \text{ m})} = 8.38 \times 10^{-4} \text{ T.}$$

EVALUATE: The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

27.26. IDENTIFY: After being accelerated through a potential difference V the ion has kinetic energy qV . The acceleration in the circular path is v^2/R .

SET UP: The ion has charge $q = +e$.

$$\text{EXECUTE: } K = qV = +eV. \frac{1}{2}mv^2 = eV \text{ and } v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(220 \text{ V})}{1.16 \times 10^{-26} \text{ kg}}} = 7.79 \times 10^4 \text{ m/s. } F_B = |q|vB \sin \phi.$$

$$\phi = 90^\circ. \vec{F} = m\vec{a} \text{ gives } |q|vB = m \frac{v^2}{R}. R = \frac{mv}{|q|B} = \frac{(1.16 \times 10^{-26} \text{ kg})(7.79 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.723 \text{ T})} = 7.81 \times 10^{-3} \text{ m} = 7.81 \text{ mm.}$$

EVALUATE: The larger the accelerating voltage, the larger the speed of the particle and the larger the radius of its path in the magnetic field.

27.27. (a) IDENTIFY and SET UP: Eq.(27.4) gives the total force on the proton. At $t = 0$,

$$\vec{F} = q\vec{v} \times \vec{B} = q(v_x \hat{i} + v_z \hat{k}) \times B_x \hat{i} = qv_z B_x \hat{j}. \vec{F} = (1.60 \times 10^{-19} \text{ C})(2.00 \times 10^5 \text{ m/s})(0.500 \text{ T})\hat{j} = (1.60 \times 10^{-14} \text{ N})\hat{j}.$$

(b) Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive and there is a component of acceleration in this direction.

(c) EXECUTE: In the plane perpendicular to $\vec{B}$ (the yz -plane) the motion is circular. But there is a velocity component in the direction of $\vec{B}$, so the motion is a helix. The electric field in the $+\hat{i}$ direction exerts a force in the $+\hat{i}$ direction. This force produces an acceleration in the $+\hat{i}$ direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the yz -plane, so the electric field does not affect the radius of the helix.

(d) IDENTIFY and SET UP: Eq.(27.12) and $T = 2\pi/\omega$ to calculate the period of the motion. Calculate a_x produced by the electric force and use a constant acceleration equation to calculate the displacement in the x -direction in time $T/2$.

EXECUTE: Calculate the period T : $\omega = |q|B/m$

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\pi(1.67 \times 10^{-27} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 1.312 \times 10^{-7} \text{ s. Then } t = T/2 = 6.56 \times 10^{-8} \text{ s. } v_{0x} = 1.50 \times 10^5 \text{ m/s}$$

$$a_x = \frac{F_x}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ V/m})}{1.67 \times 10^{-27} \text{ kg}} = +1.916 \times 10^{12} \text{ m/s}^2$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (1.50 \times 10^5 \text{ m/s})(6.56 \times 10^{-8} \text{ s}) + \frac{1}{2}(1.916 \times 10^{12} \text{ m/s}^2)(6.56 \times 10^{-8} \text{ s})^2 = 1.40 \text{ cm}$$

EVALUATE: The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

27.28. IDENTIFY: For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection. With only the magnetic force, $|q|vB = mv^2/R$

EXECUTE: (a) $v = E/B = (1.56 \times 10^4 \text{ V/m}) / (4.62 \times 10^{-3} \text{ T}) = 3.38 \times 10^6 \text{ m/s}$.

(b) The directions of the three vectors $\vec{v}$, $\vec{E}$ and $\vec{B}$ are sketched in Figure 27.28.

(c) $R = \frac{mv}{|q|B} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.38 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(4.62 \times 10^{-3} \text{ T})} = 4.17 \times 10^{-3} \text{ m}$.

$T = \frac{2\pi m}{|q|B} = \frac{2\pi R}{v} = \frac{2\pi(4.17 \times 10^{-3} \text{ m})}{(3.38 \times 10^6 \text{ m/s})} = 7.74 \times 10^{-9} \text{ s}$.

EVALUATE: For the field directions shown in Figure 27.28, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.

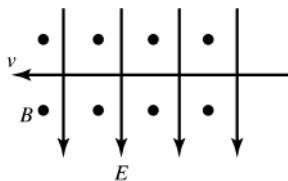


Figure 27.28

27.29. IDENTIFY: For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the plates: $qV = 1/2 mv^2$. The electric field between the plates due to the battery is $E = V_b/d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4(1.60 \times 10^{-19} \text{ C})(1750 \text{ V})}{6.64 \times 10^{-27} \text{ kg}}} = 4.11 \times 10^5 \text{ m/s}$$

The electric field between the plates, produced by the battery, is

$$E = V_b/d = (150 \text{ V})/(0.00820 \text{ m}) = 18,300 \text{ V/m}$$

The magnetic force must cancel the electric force:

$$B = E/v_\alpha = (18,300 \text{ V/m})/(4.11 \times 10^5 \text{ m/s}) = 0.0445 \text{ T}$$

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.30. IDENTIFY: For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85 \times 10^3 \text{ m/s})(1.35 \text{ T}) = 7898 \text{ N/C}$.

(a) If $q = 0.640 \times 10^{-9} \text{ C}$, the electric field direction is given by $-(\hat{j} \times (-\hat{k})) = \hat{i}$, since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320 \times 10^{-9} \text{ C}$, the electric field direction is given by $((-\hat{j}) \times (-\hat{k})) = \hat{i}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.31. IDENTIFY and SET UP: Use the fields in the velocity selector to find the speed v of the particles that pass through. Apply Newton's 2nd law with $a = v^2/R$ to the circular motion in the second region of the spectrometer. Solve for the mass m of the ion.

EXECUTE: In the velocity selector $|q|E = |q|vB$.

$$v = \frac{E}{B} = \frac{1.12 \times 10^5 \text{ V/m}}{0.540 \text{ T}} = 2.074 \times 10^5 \text{ m/s}$$

In the region of the circular path $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$ so $m = |q|RB/v$

Singly charged ion, so $|q| = +e = 1.602 \times 10^{-19} \text{ C}$

$$m = \frac{(1.602 \times 10^{-19} \text{ C})(0.310 \text{ m})(0.540 \text{ T})}{2.074 \times 10^5 \text{ m/s}} = 1.29 \times 10^{-25} \text{ kg}$$

Mass number = mass in atomic mass units, so is $\frac{1.29 \times 10^{-25} \text{ kg}}{1.66 \times 10^{-27} \text{ kg}} = 78$.

EVALUATE: Appendix D gives the average atomic mass of selenium to be 78.96. One of its isotopes has atomic mass 78.

27.32. IDENTIFY and SET UP: For a velocity selector, $E = vB$. For parallel plates with opposite charge, $V = Ed$.

EXECUTE: (a) $E = vB = (1.82 \times 10^6 \text{ m/s})(0.650 \text{ T}) = 1.18 \times 10^6 \text{ V/m}$.

(b) $V = Ed = (1.18 \times 10^6 \text{ V/m})(5.20 \times 10^{-3} \text{ m}) = 6.14 \text{ kV}$.

EVALUATE: Any charged particle with $v = 1.82 \times 10^6 \text{ m/s}$ will pass through undeflected, regardless of the sign and magnitude of its charge.

27.33. IDENTIFY: The magnetic force is $F = I\ell B \sin \phi$. For the wire to be completely supported by the field requires that $F = mg$ and that $\vec{F}$ and $\vec{w}$ are in opposite directions.

SET UP: The magnetic force is maximum when $\phi = 90^\circ$. The gravity force is downward.

EXECUTE: (a) $I\ell B = mg$. $I = \frac{mg}{\ell B} = \frac{(0.150 \text{ kg})(9.80 \text{ m/s}^2)}{(2.00 \text{ m})(0.55 \times 10^{-4} \text{ T})} = 1.34 \times 10^4 \text{ A}$. This is a very large current and ohmic

heating due to the resistance of the wire would be severe; such a current isn't feasible.

(b) The magnetic force must be upward. The directions of I , $\vec{B}$ and $\vec{F}$ are shown in Figure 27.33, where we have assumed that $\vec{B}$ is south to north. To produce an upward magnetic force, the current must be to the east. The wire must be horizontal and perpendicular to the earth's magnetic field.

EVALUATE: The magnetic force is perpendicular to both the direction of I and the direction of $\vec{B}$.

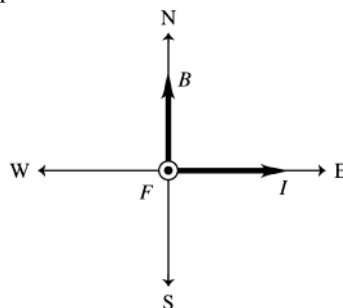


Figure 27.33

27.34. IDENTIFY: Apply $F = I\ell B \sin \phi$.

SET UP: $\ell = 0.0500 \text{ m}$ is the length of wire in the magnetic field. Since the wire is perpendicular to $\vec{B}$, $\phi = 90^\circ$.

EXECUTE: $F = I\ell B = (10.8 \text{ A})(0.0500 \text{ m})(0.550 \text{ T}) = 0.297 \text{ N}$.

EVALUATE: The force per unit length of wire is proportional to both B and I .

27.35. IDENTIFY: Apply $F = I\ell B \sin \phi$.

SET UP: Label the three segments in the field as a , b , and c . Let x be the length of segment a . Segment b has length 0.300 m and segment c has length $0.600 \text{ m} - x$. Figure 27.35a shows the direction of the forces on each segment. For each segment, $\phi = 90^\circ$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = I\ell B = (4.50 \text{ A})x(0.240 \text{ T})$. $F_c = (4.50 \text{ A})(0.600 \text{ m} - x)(0.240 \text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50 \text{ A})(0.600 \text{ m})(0.240 \text{ T}) = 0.648 \text{ N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50 \text{ A})(0.300 \text{ m})(0.240 \text{ T}) = 0.324 \text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.35b.

$$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648 \text{ N})^2 + (0.324 \text{ N})^2} = 0.724 \text{ N}. \quad \tan \theta = \frac{F_{ac}}{F_b} = \frac{0.648 \text{ N}}{0.324 \text{ N}} \quad \text{and} \quad \theta = 63.4^\circ. \quad \text{The net force has}$$

magnitude 0.724 N and its direction is specified by $\theta = 63.4^\circ$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90^\circ$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

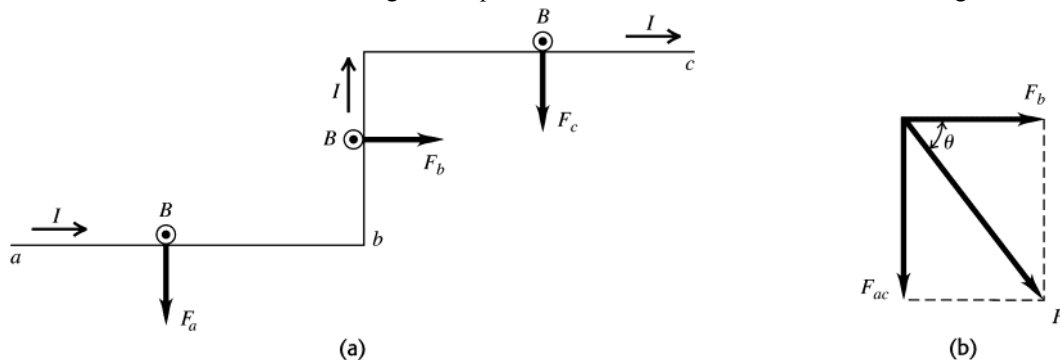


Figure 27.35

- 27.36. IDENTIFY and SET UP:** $F = I l B \sin \phi$. The direction of $\vec{F}$ is given by applying the right-hand rule to the directions of I and $\vec{B}$.
- EXECUTE:** (a) The current and field directions are shown in Figure 27.36a. The right-hand rule gives that $\vec{F}$ is directed to the south, as shown. $\phi = 90^\circ$ and $F = (1.20 \text{ A})(1.00 \times 10^{-2} \text{ m})(0.588 \text{ T}) = 7.06 \times 10^{-3} \text{ N}$.
- (b) The right-hand rule gives that $\vec{F}$ is directed to the west, as shown in Figure 27.36b. $\phi = 90^\circ$ and $F = 7.06 \times 10^{-3} \text{ N}$, the same as in part (a).
- (c) The current and field directions are shown in Figure 27.36c. The right-hand rule gives that $\vec{F}$ is 60.0° north of west. $\phi = 90^\circ$ so $F = 7.06 \times 10^{-3} \text{ N}$, the same as in part (a).
- EVALUATE:** In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

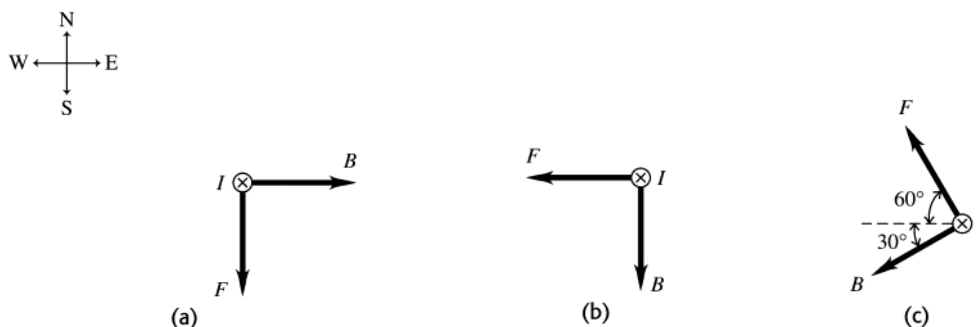


Figure 27.36

- 27.37. IDENTIFY:** $F = I l B \sin \phi$.
- SET UP:** Since the field is perpendicular to the rod it is perpendicular to the current and $\phi = 90^\circ$.
- EXECUTE:** $I = \frac{F}{lB} = \frac{0.13 \text{ N}}{(0.200 \text{ m})(0.067 \text{ T})} = 9.7 \text{ A}$
- EVALUATE:** The force and current are proportional. We have assumed that the entire 0.200 m length of the rod is in the magnetic field.
- 27.38. IDENTIFY:** Apply $\vec{F} = I \vec{l} \times \vec{B}$.
- SET UP:** The magnetic field of a bar magnet points away from the north pole and toward the south pole.
- EXECUTE:** Between the poles of the magnet, the magnetic field points to the right. Using the fingertips of your right hand, rotate the current vector by 90° into the direction of the magnetic field vector. Your thumb points downward—which is the direction of the magnetic force.
- EVALUATE:** If the two magnets had their poles interchanged, then the force would be upward.
- 27.39. IDENTIFY and SET UP:** The magnetic force is given by Eq.(27.19). $F_l = mg$ when the bar is just ready to levitate. When I becomes larger, $F_l > mg$ and $F_l - mg$ is the net force that accelerates the bar upward. Use Newton's 2nd law to find the acceleration.

(a) EXECUTE: $IlB = mg$, $I = \frac{mg}{lB} = \frac{(0.750 \text{ kg})(9.80 \text{ m/s}^2)}{(0.500 \text{ m})(0.450 \text{ T})} = 32.67 \text{ A}$

$\mathcal{E} = IR = (32.67 \text{ A})(25.0 \Omega) = 817 \text{ V}$

(b) $R = 2.0 \Omega$, $I = \mathcal{E}/R = (816.7 \text{ V})/(2.0 \Omega) = 408 \text{ A}$

$F_l = IlB = 92 \text{ N}$

$a = (F_l - mg)/m = 113 \text{ m/s}^2$

EVALUATE: I increases by over an order of magnitude when R changes to $F_l \gg mg$ and a is an order of magnitude larger than g .

- 27.40. IDENTIFY: The magnetic force $\vec{F}_B$ must be upward and equal to mg . The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB \sin \phi$, with $\phi = 90^\circ$. $I = \frac{V}{R}$, where V is the battery voltage.

EXECUTE: (a) The forces are shown in Figure 27.40. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \frac{IlB}{g} = \frac{VlB}{Rg} = \frac{(175 \text{ V})(0.600 \text{ m})(1.50 \text{ T})}{(5.00 \Omega)(9.80 \text{ m/s}^2)} = 3.21 \text{ kg}$.

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

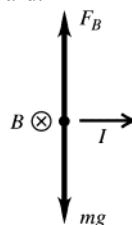


Figure 27.40

- 27.41. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$ to each segment of the conductor: the straight section parallel to the x axis, the semicircular section and the straight section that is perpendicular to the plane of the figure in Example 27.8.

SET UP: $\vec{B} = B_x \hat{i}$. The force is zero when the current is along the direction of $\vec{B}$.

EXECUTE: (a) The force on the straight section along the $-x$ -axis is zero. For the half of the semicircle at negative x the force is out of the page. For the half of the semicircle at positive x the force is into the page. The net force on the semicircular section is zero. The force on the straight section that is perpendicular to the plane of the figure is in the $-y$ -direction and has magnitude $F = IlB$. The total magnetic force on the conductor is IlB , in the $-y$ -direction.

EVALUATE: (b) If the semicircular section is replaced by a straight section along the x -axis, then the magnetic force on that straight section would be zero, the same as it is for the semicircle.

- 27.42. IDENTIFY: $\tau = IAB \sin \phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90^\circ$.

EXECUTE: (a) $\tau = IAB = (6.2 \text{ A})(0.050 \text{ m})(0.080 \text{ m})(0.19 \text{ T}) = 4.7 \times 10^{-3} \text{ N} \cdot \text{m}$

(b) $\mu = IA = (6.2 \text{ A})(0.050 \text{ m})(0.080 \text{ m}) = 0.025 \text{ A} \cdot \text{m}^2$

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90^\circ$.

- 27.43. IDENTIFY: The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2 .

EXECUTE: (a) $T = 2\pi r/v = 1.5 \times 10^{-16} \text{ s}$

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1 \text{ mA}$.

(c) $\mu = IA = I\pi r^2 = 9.3 \times 10^{-24} \text{ A} \cdot \text{m}^2$

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.44. IDENTIFY: $\tau = IAB \sin \phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated 30.0° .

EXECUTE: (a) The forces on each side of the coil are shown in Figure 27.44a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0^\circ$ and $\sin \phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0^\circ$ and the net torque is

$\tau = (1)(1.40 \text{ A})(0.220 \text{ m})(0.350 \text{ m})(1.50 \text{ T})\sin 30.0^\circ = 0.0808 \text{ N} \cdot \text{m}$. The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle ϕ .

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

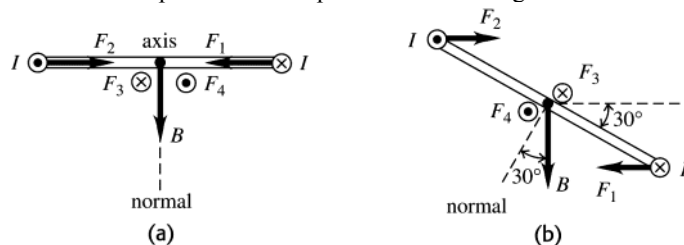


Figure 27.44

27.45. IDENTIFY: The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

SET UP: The magnetic torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, and the rotational form of Newton's second law is

$\sum \tau = I\alpha$. The magnetic field is parallel to the plane of the loop.

EXECUTE: (a) The coil rotates about axis A_2 because the only torque is along top and bottom sides of the coil.

(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is $1/3$ of the total perimeter, so its mass is $(1/3)(210 \text{ g}) = 70 \text{ g} = 0.070 \text{ kg}$. The mass of each 0.500-m segment is half this amount, or 0.035 kg . The result is

$$I = 2(0.070 \text{ kg})(0.250 \text{ m})^2 + 2\left(\frac{1}{12}\right)(0.035 \text{ kg})(0.500 \text{ m})^2 = 0.0102 \text{ kg} \cdot \text{m}^2$$

The torque is

$$|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = IAB \sin 90^\circ = (2.00 \text{ A})(0.500 \text{ m})(1.00 \text{ m})(3.00 \text{ T}) = 3.00 \text{ N} \cdot \text{m}$$

Using the above values, the rotational form of Newton's second law gives

$$\alpha = \frac{\tau}{I} = 290 \text{ rad/s}^2$$

EVALUATE: This angular acceleration will not continue because the torque changes as the coil turns.

27.46. IDENTIFY: $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\mu B \cos \phi$, where $\mu = NIB$. $\tau = \mu B \sin \phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: (a) $\phi = 90^\circ$. $\tau = NIAB \sin(90^\circ) = NIAB$, direction $\hat{k} \times \hat{j} = -\hat{i}$. $U = -\mu B \cos \phi = 0$.

(b) $\phi = 0$. $\tau = NIAB \sin(0) = 0$, no direction. $U = -\mu B \cos \phi = -NIAB$.

(c) $\phi = 90^\circ$. $\tau = NIAB \sin(90^\circ) = NIAB$, direction $-\hat{k} \times \hat{j} = \hat{i}$. $U = -\mu B \cos \phi = 0$.

(d) $\phi = 180^\circ$: $\tau = NIAB \sin(180^\circ) = 0$, no direction, $U = -\mu B \cos(180^\circ) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.47. IDENTIFY and SET UP: The potential energy is given by Eq.(27.27): $U = \vec{\mu} \cdot \vec{B}$. The scalar product depends on the angle between $\vec{\mu}$ and $\vec{B}$.

EXECUTE: For $\vec{\mu}$ and $\vec{B}$ parallel, $\phi = 0^\circ$ and $\vec{\mu} \cdot \vec{B} = \mu B \cos \phi = \mu B$. For $\vec{\mu}$ and $\vec{B}$ antiparallel,

$\phi = 180^\circ$ and $\vec{\mu} \cdot \vec{B} = \mu B \cos \phi = -\mu B$.

$U_1 = +\mu B$, $U_2 = -\mu B$

$\Delta U = U_2 - U_1 = -2\mu B = -2(1.45 \text{ A} \cdot \text{m}^2)(0.835 \text{ T}) = -2.42 \text{ J}$

EVALUATE: U is maximum when $\vec{\mu}$ and $\vec{B}$ are antiparallel and minimum when they are parallel. When the coil is rotated as specified its magnetic potential energy decreases.

- 27.48. IDENTIFY:** Apply Eq.(27.29) in order to calculate I . The power drawn from the line is $P_{\text{supplied}} = IV_{ab}$. The mechanical power is the power supplied minus the I^2r electrical power loss in the internal resistance of the motor.
SET UP: $V_{ab} = 120\text{ V}$, $\mathcal{E} = 105\text{ V}$, and $r = 3.2\ \Omega$.

EXECUTE: (a) $V_{ab} = \mathcal{E} + Ir \Rightarrow I = \frac{V_{ab} - \mathcal{E}}{r} = \frac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}$.

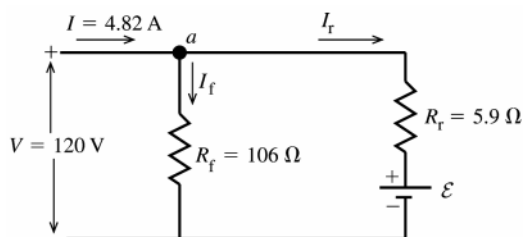
(b) $P_{\text{supplied}} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}$.

(c) $P_{\text{mech}} = IV_{ab} - I^2r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}$.

EVALUATE: If the rotor isn't turning, when the motor is first turned on or if the rotor bearings fail, then $\mathcal{E} = 0$ and $I = \frac{120\text{ V}}{3.2\ \Omega} = 37.5\text{ A}$. This large current causes large I^2r heating and can trip the circuit breaker.

- 27.49. IDENTIFY:** The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_r and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.49.



$\mathcal{E}$ is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.49

EXECUTE: (a) The field coils and the rotor are in parallel with the applied potential difference V , so $V = I_f R_f$.

$$I_f = \frac{V}{R_f} = \frac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$$

(b) Applying the junction rule to point a in the circuit diagram gives $I - I_f - I_r = 0$.

$$I_r = I - I_f = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}.$$

(c) The potential drop across the rotor, $I_r R_r + \mathcal{E}$, must equal the applied potential difference V : $V = I_r R_r + \mathcal{E}$

$$\mathcal{E} = V - I_r R_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$$P_{\text{in}} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}$$

electrical power loss in the two resistances

$$P_{\text{loss}} = I_f^2 R_f + I_r^2 R_r = (1.13\text{ A})^2 (106\ \Omega) + (3.69\text{ A})^2 (5.9\ \Omega) = 216\text{ W}$$

mechanical power output

$$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 578\text{ W} - 216\text{ W} = 362\text{ W}$$

The mechanical power output is the power associated with the induced emf $\mathcal{E}$

$$P_{\text{out}} = P_{\mathcal{E}} = \mathcal{E} I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}, \text{ which agrees with the above calculation.}$$

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.12. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

- 27.50. IDENTIFY:** The field and rotor coils are in parallel, so $V_{ab} = I_f R_f = \mathcal{E} + I_r R_r$ and $I = I_f + I_r$, where I is the current drawn from the line. The power input to the motor is $P = V_{ab} I$. The power output of the motor is the power input minus the electrical power losses in the resistances and friction losses.

SET UP: $V_{ab} = 120\text{ V}$. $I = 4.82\text{ A}$.

EXECUTE: (a) Field current $I_f = \frac{120\text{ V}}{218\ \Omega} = 0.550\text{ A}$.

(b) Rotor current $I_r = I_{\text{total}} - I_f = 4.82\text{ A} - 0.550\text{ A} = 4.27\text{ A}$.

(c) $V = \mathcal{E} + I_r R_r$ and $\mathcal{E} = V - I_r R_r = 120 \text{ V} - (4.27 \text{ A})(5.9 \Omega) = 94.8 \text{ V}$.

(d) $P_r = I_r^2 R_r = (0.550 \text{ A})^2 (218 \Omega) = 65.9 \text{ W}$.

(e) $P_r = I_r^2 R_r = (4.27 \text{ A})^2 (5.9 \Omega) = 108 \text{ W}$.

(f) Power input $= (120 \text{ V})(4.82 \text{ A}) = 578 \text{ W}$.

(g) Efficiency $= \frac{P_{\text{output}}}{P_{\text{input}}} = \frac{(578 \text{ W} - 65.9 \text{ W} - 108 \text{ W} - 45 \text{ W})}{578 \text{ W}} = \frac{359 \text{ W}}{578 \text{ W}} = 0.621$.

EVALUATE: $I^2 R$ losses in the resistance of the rotor and field coils are larger than the friction losses for this motor.

- 27.51. IDENTIFY:** The drift velocity is related to the current density by Eq.(25.4). The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

(a) **SET UP:** The section of the silver ribbon is sketched in Figure 27.51a.

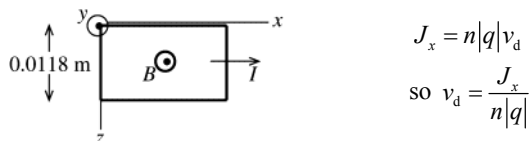


Figure 27.51a

EXECUTE: $J_x = \frac{I}{A} = \frac{I}{y_1 z_1} = \frac{120 \text{ A}}{(0.23 \times 10^{-3} \text{ m})(0.0118 \text{ m})} = 4.42 \times 10^7 \text{ A/m}^2$

$v_d = \frac{J_x}{n|q|} = \frac{4.42 \times 10^7 \text{ A/m}^2}{(5.85 \times 10^{28} / \text{m}^3)(1.602 \times 10^{-19} \text{ C})} = 4.7 \times 10^{-3} \text{ m/s} = 4.7 \text{ mm/s}$

(b) magnitude of $\vec{E}$

$|q|E_z = |q|v_d B_y$

$E_z = v_d B_y = (4.7 \times 10^{-3} \text{ m/s})(0.95 \text{ T}) = 4.5 \times 10^{-3} \text{ V/m}$

direction of $\vec{E}$

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.51b.

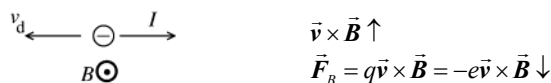


Figure 27.51b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.51c.

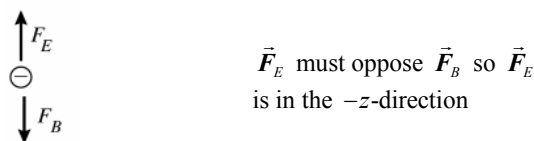


Figure 27.51c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$ -direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\mathcal{E}_{\text{Hall}} = E z_1 = (4.5 \times 10^{-3} \text{ V/m})(0.0118 \text{ m}) = 53 \mu\text{V}$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.13. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

- 27.52. IDENTIFY:** Apply Eq.(27.30).

SET UP: $A = y_1 z_1$. $E = \mathcal{E}/z_1$. $|q| = e$.

EXECUTE: $n = \frac{J_x B_y}{|q| E_z} = \frac{I B_y}{A |q| E_z} = \frac{I B_y z_1}{A |q| \mathcal{E}} = \frac{I B_y}{y_1 |q| \mathcal{E}}$

$n = \frac{(78.0 \text{ A})(2.29 \text{ T})}{(2.3 \times 10^{-4} \text{ m})(1.6 \times 10^{-19} \text{ C})(1.31 \times 10^{-4} \text{ V})} = 3.7 \times 10^{28} \text{ electrons/m}^3$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.53. (a) IDENTIFY: Use Eq.(27.2) to relate $\vec{v}$, $\vec{B}$, and $\vec{F}$.

SET UP: The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in Figure 27.53a.

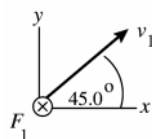


Figure 27.53a

$\vec{F} = q\vec{v} \times \vec{B}$ says that $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z -component.

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in Figure 27.53b.

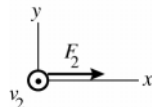


Figure 27.53b

$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x -component.

Both pieces of information taken together say that $\vec{B}$ is in the y -direction; $\vec{B} = B_y \hat{j}$.

EXECUTE: Use the information given about $\vec{F}_2$ to calculate F_y : $\vec{F}_2 = F_2 \hat{i}$, $\vec{v}_2 = v_2 \hat{k}$, $\vec{B} = B_y \hat{j}$.

$\vec{F}_2 = q\vec{v}_2 \times \vec{B}$ says $F_2 \hat{i} = qv_2 B_y \hat{k} \times \hat{j} = qv_2 B_y (-\hat{i})$ and $F_2 = -qv_2 B_y$

$B_y = -F_2/(qv_2) = -F_2/(qv_1)$. $\vec{B}$ has the magnitude $F_2/(qv_1)$ and is in the $-y$ -direction.

(b) $F_1 = qvB \sin \phi = qv_1 |B_y|/\sqrt{2} = F_2/\sqrt{2}$

EVALUATE: $v_1 = v_2$. $\vec{v}_2$ is perpendicular to $\vec{B}$ whereas only the component of $\vec{v}_1$ perpendicular to $\vec{B}$ contributes to the force, so it is expected that $F_2 > F_1$, as we found.

27.54. IDENTIFY: Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $B_x = 0.450$ T, $B_y = 0$ and $B_z = 0$.

EXECUTE: $F_x = q(v_y B_z - v_z B_y) = 0$.

$F_y = q(v_z B_x - v_x B_z) = (9.45 \times 10^{-8} \text{ C})(5.85 \times 10^4 \text{ m/s})(0.450 \text{ T}) = 2.49 \times 10^{-3} \text{ N}$.

$F_z = q(v_x B_y - v_y B_x) = -(9.45 \times 10^{-8} \text{ C})(-3.11 \times 10^4 \text{ m/s})(0.450 \text{ T}) = 1.32 \times 10^{-3} \text{ N}$.

EVALUATE: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. We can verify that $\vec{F} \cdot \vec{v} = 0$. Since $\vec{B}$ is along the x -axis, v_x does not affect the force components.

27.55. IDENTIFY: The sum of the magnetic, electrical, and gravitational forces must be zero to aim at and hit the target.

SET UP: The magnetic field must point to the left when viewed in the direction of the target for no net force. The net force is zero, so $\sum F = F_B - F_E - mg = 0$ and $qvB - qE - mg = 0$.

EXECUTE: Solving for B gives

$$B = \frac{qE + mg}{qv} = \frac{(2500 \times 10^{-6} \text{ C})(27.5 \text{ N/C}) + (0.0050 \text{ kg})(9.80 \text{ m/s}^2)}{(2500 \times 10^{-6} \text{ C})(12.8 \text{ m/s})} = 3.7 \text{ T}$$

The direction should be perpendicular to the initial velocity of the coin.

EVALUATE: This is a very strong magnetic field, but achievable in some labs.

27.56. IDENTIFY: Apply $R = mv/|q|B$. $\omega = v/R$

SET UP: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

EXECUTE: **(a)** $K = 2.7 \text{ MeV} = (2.7 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 4.32 \times 10^{-13} \text{ J}$.

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4.32 \times 10^{-13} \text{ J})}{1.67 \times 10^{-27} \text{ kg}}} = 2.27 \times 10^7 \text{ m/s}.$$

$$R = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg})(2.27 \times 10^7 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(3.5 \text{ T})} = 0.068 \text{ m. Also, } \omega = \frac{v}{R} = \frac{2.27 \times 10^7 \text{ m/s}}{0.068 \text{ m}} = 3.34 \times 10^8 \text{ rad/s.}$$

(b) If the energy reaches the final value of 5.4 MeV , the velocity increases by $\sqrt{2}$, as does the radius, to 0.096 m . The angular frequency is unchanged from part (a) so is $3.34 \times 10^8 \text{ rad/s}$.

EVALUATE: $\omega = |q|B/m$, so ω is independent of the energy of the protons. The orbit radius increases when the energy of the proton increases.

- 27.57. (a) IDENTIFY and SET UP:** The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's 2nd law and $a_c = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$

$$v = \frac{|q|BR}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.85 \text{ T})(0.40 \text{ m})}{1.67 \times 10^{-27} \text{ kg}} = 3.257 \times 10^7 \text{ m/s.}$$

The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(3.257 \times 10^7 \text{ m/s})^2 = 8.9 \times 10^{-13} \text{ J} = 5.6 \text{ MeV}$

(b) The time for one revolution is the period $T = \frac{2\pi R}{v} = \frac{2\pi(0.40 \text{ m})}{3.257 \times 10^7 \text{ m/s}} = 7.7 \times 10^{-8} \text{ s}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{|q|BR}{m}\right)^2 = \frac{1}{2}\frac{|q|^2 B^2 R^2}{m}$. Or, $B = \frac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a

factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85 \text{ T}) = 1.2 \text{ T}$.

(d) $v = \frac{|q|BR}{m} = \frac{(3.20 \times 10^{-19} \text{ C})(0.85 \text{ T})(0.40 \text{ m})}{6.65 \times 10^{-27} \text{ kg}} = 1.636 \times 10^7 \text{ m/s}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65 \times 10^{-27} \text{ kg})(1.636 \times 10^7 \text{ m/s})^2 = 8.9 \times 10^{-13} \text{ J} = 5.5 \text{ MeV}$, the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c),

$$K = \frac{1}{2}m\left(\frac{|q|BR}{m}\right)^2. \text{ For alpha particles } |q| \text{ is larger by a factor of 2 and } m \text{ is larger by a factor of 4 (approximately).}$$

Thus $|q|^2/m$ is unchanged and K is the same.

- 27.58. IDENTIFY:** Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $\vec{v} = -v\hat{j}$

EXECUTE: (a) $\vec{F} = -qv[B_x(\hat{j} \times \hat{i}) + B_y(\hat{j} \times \hat{j}) + B_z(\hat{j} \times \hat{k})] = qvB_x\hat{k} - qvB_z\hat{i}$

(b) $B_x > 0$, $B_z < 0$, sign of B_y doesn't matter.

(c) $\vec{F} = |q|vB_x\hat{i} - |q|vB_z\hat{k}$ and $|\vec{F}| = \sqrt{2}|q|vB_x$.

EVALUATE: $\vec{F}$ is perpendicular to $\vec{v}$, so $\vec{F}$ has no y -component.

- 27.59. IDENTIFY:** The contact at a will break if the bar rotates about b . The magnetic field is directed out of the page, so the magnetic torque is counterclockwise, whereas the gravity torque is clockwise in the figure in the problem. The maximum current corresponds to zero net torque, in which case the torque due to gravity is just equal to the torque due to the magnetic field.

SET UP: The magnetic force is perpendicular to the bar and has moment arm $l/2$, where $l = 0.750 \text{ m}$ is the

length of the bar. The gravity torque is $mg\left(\frac{l}{2}\cos 60.0^\circ\right)$

EXECUTE: $\tau_{\text{gravity}} = \tau_B$ and $mg\frac{l}{2}\cos 60.0^\circ = IlB\sin 90^\circ\frac{l}{2}$. This gives

$$I = \frac{mg\cos 60.0^\circ}{lB\sin 90^\circ} = \frac{(0.458 \text{ kg})(9.80 \text{ m/s}^2)(\cos 60.0^\circ)}{(0.750 \text{ m})(1.55 \text{ T})(1)} = 1.93 \text{ A}$$

EVALUATE: Once contact is broken, the magnetic torque ceases. The 90.0° angle in the expression for τ_B is the angle between the direction of I and the direction of $\vec{B}$.

- 27.60. IDENTIFY:** Apply $R = \frac{mv}{|q|B}$.

SET UP: Assume $D \ll R$

EXECUTE: (a) The path is sketched in Figure 27.60.

(b) Motion is circular: $x^2 + y^2 = R^2 \Rightarrow x = D \Rightarrow y_1 = \sqrt{R^2 - D^2}$ (path of deflected particle)

$y_2 = R$ (equation for tangent to the circle, path of undeflected particle).

$$d = y_2 - y_1 = R - \sqrt{R^2 - D^2} = R - R\sqrt{1 - \frac{D^2}{R^2}} = R\left[1 - \sqrt{1 - \frac{D^2}{R^2}}\right]. \text{ If } R \gg D, d \approx R\left[1 - \left(1 - \frac{1}{2}\frac{D^2}{R^2}\right)\right] = \frac{D^2}{2R}. \text{ For a}$$

particle moving in a magnetic field, $R = \frac{mv}{qB}$. But $\frac{1}{2}mv^2 = qV$, so $R = \frac{1}{B}\sqrt{\frac{2mV}{q}}$. Thus, the deflection

$$d \approx \frac{D^2 B}{2} \sqrt{\frac{q}{2mV}} = \frac{D^2 B}{2} \sqrt{\frac{e}{2mV}}.$$

(c) $d = \frac{(0.50 \text{ m})^2 (5.0 \times 10^{-5} \text{ T})}{2} \sqrt{\frac{(1.6 \times 10^{-19} \text{ C})}{2(9.11 \times 10^{-31} \text{ kg})(750 \text{ V})}} = 0.067 \text{ m} = 6.7 \text{ cm}$. $d \approx 13\%$ of D , which is fairly significant.

EVALUATE: In part (c), $R = \frac{1}{B}\sqrt{\frac{2mV}{e}} = \frac{D^2}{2d} = \left(\frac{D}{2d}\right)D = 3.7D$ and $\left(\frac{R}{D}\right)^2 = 14$, so the approximation made in part (b) is valid.

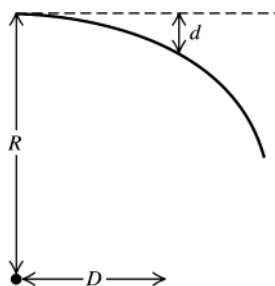


Figure 27.60

27.61. IDENTIFY and SET UP: Use Eq.(27.2) to relate $q, \vec{v}, \vec{B}$ and $\vec{F}$. The force $\vec{F}$ and $\vec{a}$ are related by Newton's 2nd law.

$$\vec{B} = -(0.120 \text{ T})\hat{k}, \vec{v} = (1.05 \times 10^6 \text{ m/s})(-3\hat{i} + 4\hat{j} + 12\hat{k}), F = 1.25 \text{ N}$$

(a) **EXECUTE:** $\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{F} = q(-0.120 \text{ T})(1.05 \times 10^6 \text{ m/s})(-3\hat{i} \times \hat{k} + 4\hat{j} \times \hat{k} + 12\hat{k} \times \hat{k})$$

$$\hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{k} = 0$$

$$\vec{F} = -q(1.26 \times 10^5 \text{ N/C})(+3\hat{j} + 4\hat{i}) = -q(1.26 \times 10^5 \text{ N/C})(+4\hat{i} + 3\hat{j})$$

The magnitude of the vector $+4\hat{i} + 3\hat{j}$ is $\sqrt{3^2 + 4^2} = 5$. Thus $F = -q(1.26 \times 10^5 \text{ N/C})(5)$.

$$q = -\frac{F}{5(1.26 \times 10^5 \text{ N/C})} = -\frac{1.25 \text{ N}}{5(1.26 \times 10^5 \text{ N/C})} = -1.98 \times 10^{-6} \text{ C}$$

(b) $\sum \vec{F} = m\vec{a}$ so $\vec{a} = \vec{F}/m$

$$\vec{F} = -q(1.26 \times 10^5 \text{ N/C})(+4\hat{i} + 3\hat{j}) = -(-1.98 \times 10^{-6} \text{ C})(1.26 \times 10^5 \text{ N/C})(+4\hat{i} + 3\hat{j}) = +0.250 \text{ N}(+4\hat{i} + 3\hat{j})$$

$$\text{Then } \vec{a} = \vec{F}/m = \left(\frac{0.250 \text{ N}}{2.58 \times 10^{-15} \text{ kg}}\right)(+4\hat{i} + 3\hat{j}) = (9.69 \times 10^{13} \text{ m/s}^2)(+4\hat{i} + 3\hat{j})$$

(c) **IDENTIFY and SET UP:** $\vec{F}$ is in the xy -plane, so in the z -direction the particle moves with constant speed $12.6 \times 10^6 \text{ m/s}$. In the xy -plane the force $\vec{F}$ causes the particle to move in a circle, with $\vec{F}$ directed in towards the center of the circle.

EXECUTE: $\sum \vec{F} = m\vec{a}$ gives $F = m(v^2/R)$ and $R = mv^2/F$

$$v^2 = v_x^2 + v_y^2 = (-3.15 \times 10^6 \text{ m/s})^2 + (+4.20 \times 10^6 \text{ m/s})^2 = 2.756 \times 10^{13} \text{ m}^2/\text{s}^2$$

$$F = \sqrt{F_x^2 + F_y^2} = (0.250 \text{ N})\sqrt{4^2 + 3^2} = 1.25 \text{ N}$$

$$R = \frac{mv^2}{F} = \frac{(2.58 \times 10^{-15} \text{ kg})(2.756 \times 10^{13} \text{ m}^2/\text{s}^2)}{1.25 \text{ N}} = 0.0569 \text{ m} = 5.69 \text{ cm}$$

(d) IDENTIFY and SET UP: By Eq.(27.12) the cyclotron frequency is $f = \omega/2\pi = v/2\pi R$.

EXECUTE: The circular motion is in the xy -plane, so $v = \sqrt{v_x^2 + v_y^2} = 5.25 \times 10^6$ m/s.

$$f = \frac{v}{2\pi R} = \frac{5.25 \times 10^6 \text{ m/s}}{2\pi(0.0569 \text{ m})} = 1.47 \times 10^7 \text{ Hz, and } \omega = 2\pi f = 9.23 \times 10^7 \text{ rad/s}$$

(e) IDENTIFY and SET UP Compare t to the period T of the circular motion in the xy -plane to find the x and y coordinates at this t . In the z -direction the particle moves with constant speed, so $z = z_0 + v_z t$.

EXECUTE: The period of the motion in the xy -plane is given by $T = \frac{1}{f} = \frac{1}{1.47 \times 10^7 \text{ Hz}} = 6.80 \times 10^{-8} \text{ s}$

In $t = 2T$ the particle has returned to the same x and y coordinates. The z -component of the motion is motion with a constant velocity of $v_z = +12.6 \times 10^6$ m/s. Thus $z = z_0 + v_z t = 0 + (12.6 \times 10^6 \text{ m/s})(2)(6.80 \times 10^{-8} \text{ s}) = +1.71 \text{ m}$.

The coordinates at $t = 2T$ are $x = R, y = 0, z = +1.71 \text{ m}$.

EVALUATE: The circular motion is in the plane perpendicular to $\vec{B}$. The radius of this motion gets smaller when B increases and it gets larger when v increases. There is no magnetic force in the direction of $\vec{B}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the xy -plane and constant speed motion in the z -direction is a helical path.

27.62. IDENTIFY: The net magnetic force on the wire is the vector sum of the force on the straight segment plus the force on the curved section. We must integrate to get the force on the curved section.

SET UP: $\sum F = F_{\text{straight, top}} + F_{\text{curved}} + F_{\text{straight, bottom}}$ and $F_{\text{straight, top}} = F_{\text{straight, bottom}} = iL_{\text{straight}}B$. $F_{\text{curved, x}} = \int_0^\pi iRB \sin \theta d\theta = 2iRB$

(the same as if it were a straight segment $2R$ long) and $F_y = 0$ due to symmetry. Therefore, $F = 2iL_{\text{straight}}B + 2iRB$

EXECUTE: Using $L_{\text{straight}} = 0.55 \text{ m}$, $R = 0.95 \text{ m}$, $i = 3.40 \text{ A}$, and $B = 2.20 \text{ T}$ gives $F = 22 \text{ N}$, to right.

EVALUATE: Notice that the curve has no effect on the force. In other words, the force is the same as if the wire were simply a straight wire 3.00 m long.

27.63. IDENTIFY: $\tau = NIAB \sin \phi$.

SET UP: The area A is related to the diameter D by $A = \frac{1}{4}\pi D^2$.

EXECUTE: $\tau = NI(\frac{1}{4}\pi D^2)B \sin \phi$. τ is proportional to D^2 . Increasing D by a factor of 3 increases τ by a factor of $3^2 = 9$.

EVALUATE: The larger diameter means larger length of wire in the loop and also larger moment arms because parts of the loop are farther from the axis.

27.64. IDENTIFY: Apply $\vec{F} = q\vec{v} \times \vec{B}$

SET UP: $\vec{v} = v\hat{k}$

EXECUTE: (a) $\vec{F} = -qvB_y\hat{i} + qvB_x\hat{j}$. But $\vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$, so $3F_0 = -qvB_y$ and $4F_0 = qvB_x$

Therefore, $B_y = -\frac{3F_0}{qv}$, $B_x = \frac{4F_0}{qv}$ and B_z is undetermined.

$$(b) B = \frac{6F_0}{qv} = \sqrt{B_x^2 + B_y^2 + B_z^2} = \frac{F_0}{qv} \sqrt{9 + 16 + \left(\frac{qv}{F_0}\right)^2 B_z^2} = \frac{F_0}{qv} \sqrt{25 + \left(\frac{qv}{F_0}\right)^2 B_z^2}, \text{ so } B_z = \pm \frac{11F_0}{qv}.$$

EVALUATE: The force doesn't depend on B_z , since $\vec{v}$ is along the z -direction.

27.65. IDENTIFY: For the velocity selector, $E = vB$. For the circular motion in the field B' , $R = \frac{mv}{|q|B'}$.

SET UP: $B = B' = 0.701 \text{ T}$.

EXECUTE: $v = \frac{E}{B} = \frac{1.88 \times 10^4 \text{ N/C}}{0.701 \text{ T}} = 2.68 \times 10^4 \text{ m/s}$. $R = \frac{mv}{qB'}$, so

$$R_{82} = \frac{82(1.66 \times 10^{-27} \text{ kg})(2.68 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.701 \text{ T})} = 0.0325 \text{ m}.$$

$$R_{84} = \frac{84(1.66 \times 10^{-27} \text{ kg})(2.68 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.701 \text{ T})} = 0.0333 \text{ m}.$$

$$R_{86} = \frac{86(1.66 \times 10^{-27} \text{ kg})(2.68 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.701 \text{ T})} = 0.0341 \text{ m}.$$

The distance between two adjacent lines is $\Delta R = 1.6 \text{ mm}$.

EVALUATE: The distance between the ^{82}Kr line and the ^{84}Kr line is 1.6 mm and the distance between the ^{84}Kr line and the ^{86}Kr line is 1.6 mm. Adjacent lines are equally spaced since the ^{82}Kr versus ^{84}Kr and ^{84}Kr versus ^{86}Kr mass differences are the same.

- 27.66. IDENTIFY:** Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$

EXECUTE: (a) During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

$$(b) V = \frac{qB^2R^2}{2m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.150 \text{ T})^2(0.500 \text{ m})^2}{2(12)(1.66 \times 10^{-27} \text{ kg})} = 2.26 \times 10^4 \text{ V}$$

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\frac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{12} = 2\sqrt{\frac{2V(1 \text{ u})}{qB^2}}(\sqrt{14} - \sqrt{12}).$$

$$\Delta D = 2\sqrt{\frac{2(2.26 \times 10^4 \text{ V})(1.66 \times 10^{-27} \text{ kg})}{(1.6 \times 10^{-19} \text{ C})(0.150 \text{ T})^2}}(\sqrt{14} - \sqrt{12}) = 8.01 \times 10^{-2} \text{ m. This is about 8 cm and is easily distinguishable.}$$

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(2.26 \times 10^4 \text{ V})}{12(1.66 \times 10^{-27} \text{ kg})}} = 6.0 \times 10^5 \text{ m/s}$. This is very fast, but

well below the speed of light, so relativistic mechanics is not needed.

- 27.67. IDENTIFY:** The force exerted by the magnetic field is given by Eq.(27.19). The net force on the wire must be zero.
SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure 27.61 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.67a.

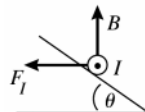


Figure 27.67a

The free-body diagram for the wire is given in Figure 27.67b.

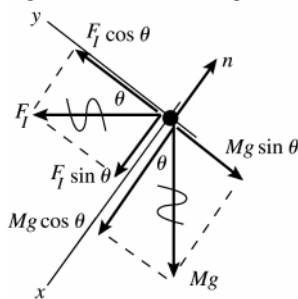


Figure 27.67b

$$\text{EXECUTE: } \sum F_y = 0$$

$$F_I \cos \theta - Mg \sin \theta = 0$$

$$F_I = ILB \sin \phi$$

$\phi = 90^\circ$ since $\vec{B}$ is perpendicular to the current direction.

$$\text{Thus } (ILB) \cos \theta - Mg \sin \theta = 0 \text{ and } I = \frac{Mg \tan \theta}{LB}$$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of Mg down the incline and the component of F_I up the incline is smaller; I must increase with θ to compensate. As

$\theta \rightarrow 0$, $I \rightarrow 0$ and as $\theta \rightarrow 90^\circ$, $I \rightarrow \infty$.

- 27.68. IDENTIFY:** The current in the bar is downward, so the magnetic force on it is vertically upwards. The net force on the bar is equal to the magnetic force minus the gravitational force, so Newton's second law gives the acceleration. The bar is in parallel with the $10.0\text{-}\Omega$ resistor, so we must use circuit analysis to find the initial current through it.

SET UP: First find the current. The equivalent resistance across the battery is $30.0 \, \Omega$, so the total current is $4.00 \, \text{A}$, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B - mg = iLB - mg$.

EXECUTE: Solving for the acceleration gives

$$a = \frac{iLB - mg}{m} = \frac{(2.0 \, \text{A})(1.50 \, \text{m})(1.60 \, \text{T}) - 3.00 \, \text{N}}{(3.00 \, \text{N}/9.80 \, \text{m/s}^2)} = 5.88 \, \text{m/s}^2.$$

The direction is upward.

EVALUATE: Once the bar is free of the conducting wires, its acceleration will become $9.8 \, \text{m/s}^2$ downward since only gravity will be acting on it.

- 27.69. IDENTIFY:** Calculate the acceleration of the ions when they first enter the field and assume this acceleration is constant. Apply conservation of energy to the acceleration of the ions by the potential difference.

SET UP: Assume $\vec{v} = v_x \hat{i}$ and neglect the y -component of $\vec{v}$ that is produced by the magnetic force.

EXECUTE: (a) $\frac{1}{2}mv_x^2 = qV$, so $v_x = \sqrt{\frac{2qV}{m}}$. Also, $a_y = \frac{qv_x B}{m}$ and $t = \frac{x}{v_x}$.

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}a_y \left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qv_x B}{m}\right)\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qBx^2}{m}\right)\left(\frac{m}{2qV}\right)^{1/2} = Bx^2\left(\frac{q}{8mV}\right)^{1/2}.$$

(b) This can be used for isotope separation since the mass in the denominator leads to different locations for different isotopes.

EVALUATE: For $B = 0.1 \, \text{T}$, $v = 1 \times 10^4 \, \text{m/s}$, $q = +e$ and $m = 12 \, \text{u} = 2.0 \times 10^{-26} \, \text{kg}$, $y = (1.0 \, \text{m}^{-2})x^2$. The approximation $y \ll x$ is valid as long as x is on the order of $10 \, \text{cm}$ or less.

- 27.70. IDENTIFY:** Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I = q/T = q/(1/f) = qf = q(\omega/2\pi) = q\omega/2\pi$. The torque is $\tau = \mu B \sin \phi$.

EXECUTE: In this case, $\phi = 90^\circ$ and $\mu = AB$, giving $\tau = IAB$. Combining the results for the torque and current

and using $A = \pi r^2$ gives $\tau = \left(\frac{q\omega}{2\pi}\right)\pi r^2 B = \frac{1}{2}q\omega r^2 B$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

- 27.71. IDENTIFY:** $R = \frac{mv}{|q|B}$.

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: (a) $B = \frac{mv}{|q|R} = \frac{(1.99 \times 10^{-26} \, \text{kg})(8.50 \times 10^3 \, \text{m/s})}{(1.60 \times 10^{-19} \, \text{C})(0.125 \, \text{m})} = 8.46 \times 10^{-3} \, \text{T}.$

(b) The only difference between the two isotopes is their masses. $\frac{R}{m} = \frac{v}{|q|B} = \text{constant}$ and $\frac{R_{12}}{m_{12}} = \frac{R_{13}}{m_{13}}.$

$$R_{13} = R_{12} \left(\frac{m_{13}}{m_{12}}\right) = (12.5 \, \text{cm}) \left(\frac{2.16 \times 10^{-26} \, \text{kg}}{1.99 \times 10^{-26} \, \text{kg}}\right) = 13.6 \, \text{cm}. \text{ The diameter is } 27.2 \, \text{cm}.$$

(c) The separation is $2(R_{13} - R_{12}) = 2(13.6 \, \text{cm} - 12.5 \, \text{cm}) = 2.2 \, \text{cm}$. This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

- 27.72. IDENTIFY:** The force exerted by the magnetic field is $F = ILB \sin \phi$. $a = F/m$ and is constant. Apply a constant acceleration equation to relate v and d .

SET UP: $\phi = 90^\circ$. The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: (a) $F = ILB$, to the right.

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v^2 = 2ad$ and $d = \frac{v^2}{2a} = \frac{v^2 m}{2ILB}.$

(c) $d = \frac{(1.12 \times 10^4 \, \text{m/s})^2 (25 \, \text{kg})}{2(2000 \, \text{A})(0.50 \, \text{m})(0.50 \, \text{T})} = 3.14 \times 10^6 \, \text{m} = 3140 \, \text{km}$

EVALUATE: $a = \frac{ILB}{m} = \frac{(20 \times 10^3 \, \text{A})(0.50 \, \text{m})(0.50 \, \text{T})}{25 \, \text{kg}} = 20 \, \text{m/s}^2$. The acceleration due to gravity is not negligible.

- 27.73. IDENTIFY:** Apply $F = ILB \sin \phi$ to calculate the force on each segment of the wire that is in the magnetic field. The net force is the vector sum of the forces on each segment.

SET UP: The direction of the magnetic force on each current segment in the field is shown in Figure 27.73. By symmetry, $F_a = F_b$. $\vec{F}_a$ and $\vec{F}_b$ are in opposite directions so their vector sum is zero. The net force equals F_c . For F_c , $\phi = 90^\circ$ and $l = 0.450$ m.

EXECUTE: $F_c = IlB = (6.00 \text{ A})(0.450 \text{ m})(0.666 \text{ T}) = 1.80 \text{ N}$. The net force is 1.80 N, directed to the left.

EVALUATE: The shape of the region of uniform field doesn't matter, as long as all of segment c is in the field and as long as the lengths of the portions of segments a and b that are in the field are the same.

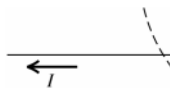


Figure 27.73

27.74. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$.

SET UP: $\vec{l} = l\hat{k}$

EXECUTE: (a) $\vec{F} = I(l\hat{k}) \times \vec{B} = Il[(-B_y)\hat{i} + (B_x)\hat{j}]$. This gives

$F_x = -IlB_y = -(9.00 \text{ A})(0.250 \text{ m})(-0.985 \text{ T}) = 2.22 \text{ N}$ and $F_y = IlB_x = (9.00 \text{ A})(0.250 \text{ m})(-0.242 \text{ T}) = -0.545 \text{ N}$.

$F_z = 0$, since the wire is in the z -direction.

(b) $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2.22 \text{ N})^2 + (0.545 \text{ N})^2} = 2.29 \text{ N}$.

EVALUATE: $\vec{F}$ must be perpendicular to the current direction, so $\vec{F}$ has no z component.

27.75. IDENTIFY: For the loop to be in equilibrium the net torque on it must be zero. Use Eq.(27.26) to calculate the torque due to the magnetic field and use Eq.(10.3) for the torque due to the gravity force.

SET UP: See Figure 27.75a.

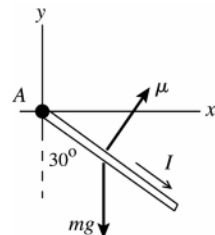


Figure 27.75a

Use $\sum \tau_A = 0$, where point A is at the origin.

EXECUTE: See Figure 27.75b.

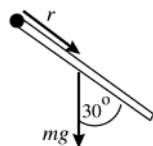


Figure 27.75b

$$\tau_{mg} = mgr \sin \phi = mg(0.400 \text{ m}) \sin 30.0^\circ$$

The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.75c.

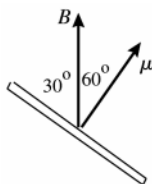


Figure 27.75c

$\vec{\tau}_B = \vec{\mu} \times \vec{B}$. For this torque to be counterclockwise ($\vec{\tau}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$ -direction.

$$\tau_B = \mu B \sin \phi = IAB \sin 60.0^\circ$$

$$\tau_B = \tau_{mg} \text{ gives } IAB \sin 60.0^\circ = mg(0.0400 \text{ m}) \sin 30.0^\circ$$

$$m = (0.15 \text{ g/cm})2(8.00 \text{ cm} + 6.00 \text{ cm}) = 4.2 \text{ g} = 4.2 \times 10^{-3} \text{ kg}$$

$$A = (0.800 \text{ m})(0.0600 \text{ m}) = 4.80 \times 10^{-3} \text{ m}^2$$

$$B = \frac{mg(0.0400 \text{ m})(\sin 30.0^\circ)}{IA \sin 60.0^\circ}$$

$$B = \frac{(4.2 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(0.0400 \text{ m}) \sin 30.0^\circ}{(8.2 \text{ A})(4.80 \times 10^{-3} \text{ m}^2) \sin 60.0^\circ} = 0.024 \text{ T}$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.76. IDENTIFY: The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB \sin \phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

EXECUTE: (a) $\tau = IAB \sin 60^\circ = (15.0 \text{ A})(0.060 \text{ m})(0.080 \text{ m})(0.48 \text{ T}) \sin 60^\circ = 0.030 \text{ N} \cdot \text{m}$, in the $-\hat{j}$ direction.

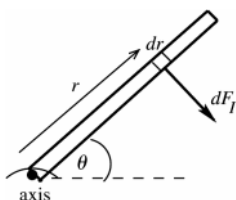
To keep the loop in place, you must provide a torque in the $+\hat{j}$ direction.

(b) $\tau = IAB \sin 30^\circ = (15.0 \text{ A})(0.60 \text{ m})(0.080 \text{ m})(0.48 \text{ T}) \sin 30^\circ = 0.017 \text{ N} \cdot \text{m}$, in the $+\hat{j}$ direction. You must provide a torque in the $-\hat{j}$ direction to keep the loop in place.

EVALUATE: (c) If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.77. IDENTIFY: Use Eq.(27.20) to calculate the force and then the torque on each small section of the rod and integrate to find the total magnetic torque. At equilibrium the torques from the spring force and from the magnetic force cancel. The spring force depends on the amount x the spring is stretched and then $U = \frac{1}{2}kx^2$ gives the energy stored in the spring.

(a) **SET UP:**



Divide the rod into infinitesimal sections of length dr , as shown in Figure 27.77.

Figure 27.77

EXECUTE: The magnetic force on this section is $dF_l = IBdr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = rdF_l = IBrdr$. The total torque is $\int d\tau = IB \int_0^l r dr = \frac{1}{2}Il^2B = 0.0442 \text{ N} \cdot \text{m}$, clockwise.

(b) **SET UP:** F_l produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left; the spring is stretched.

EXECUTE: Find x , the amount the spring is stretched:

$$\sum \tau = 0, \text{ axis at hinge, counterclockwise torques positive}$$

$$(kx)l \sin 53^\circ - \frac{1}{2}Il^2B = 0$$

$$x = \frac{IlB}{2k \sin 53.0^\circ} = \frac{(6.50 \text{ A})(0.200 \text{ m})(0.340 \text{ T})}{2(4.80 \text{ N/m}) \sin 53.0^\circ} = 0.05765 \text{ m}$$

$$U = \frac{1}{2}kx^2 = 7.98 \times 10^{-3} \text{ J}$$

EVALUATE: The magnetic torque calculated in part (a) is the same torque calculated from a force diagram in which the total magnetic force $F_l = IlB$ acts at the center of the rod. We didn't include a gravity torque since the problem said the rod had negligible mass.

27.78. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: (a) $F_{PQ} = (5.00 \text{ A})(0.600 \text{ m})(3.00 \text{ T})\sin(0^\circ) = 0 \text{ N}$.

$F_{RP} = (5.00 \text{ A})(0.800 \text{ m})(3.00 \text{ T})\sin(90^\circ) = 12.0 \text{ N}$, into the page.

$F_{QR} = (5.00 \text{ A})(1.00 \text{ m})(3.00 \text{ T})(0.800/1.00) = 12.0 \text{ N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the PR -axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the x -axis. $\tau = rF \sin \phi$ gives $\tau_{PQ} = r(0 \text{ N}) = 0$,

$\tau_{RP} = (0 \text{ m})F \sin \phi = 0$ and $\tau_{QR} = (0.300 \text{ m})(12.0 \text{ N})\sin(90^\circ) = 3.60 \text{ N} \cdot \text{m}$. The net torque is $3.60 \text{ N} \cdot \text{m}$.

(d) According to Eq.(27.28), $\tau = NIAB \sin \phi = (1)(5.00 \text{ A})(\frac{1}{2})(0.600 \text{ m})(0.800 \text{ m})(3.00 \text{ T})\sin(90^\circ) = 3.60 \text{ N} \cdot \text{m}$, which agrees with part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point Q will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB \sin \phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90^\circ$.

27.79. IDENTIFY: Use Eq.(27.20) to calculate the force on a short segment of the coil and integrate over the entire coil to find the total force.

SET UP: See Figure 27.79a.

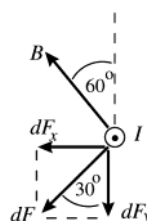


Figure 27.79a

See Figure 27.79b.

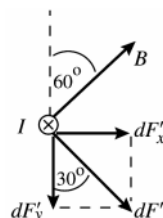


Figure 27.79b

Consider the force $d\vec{F}$ on a short segment dl at the left-hand side of the coil, as viewed in Figure 27.69 in the textbook. The current at this point is directed out of the page. $d\vec{F}$ is perpendicular both to $\vec{B}$ and to the direction of I .

Consider also the force $d\vec{F}'$ on a short segment on the opposite side of the coil, at the right-hand side of the coil in Figure 27.69 in the textbook. The current at this point is directed into the page.

The two sketches show that the x -components cancel and that the y -components add. This is true for all pairs of short segments on opposite sides of the coil. The net magnetic force on the coil is in the y -direction and its magnitude is given by $F = \int dF_y$.

EXECUTE: $dF = Idl B \sin \phi$. But $\vec{B}$ is perpendicular to the current direction so $\phi = 90^\circ$.

$$dF_y = dF \cos 30.0 = IB \cos 30.0^\circ dl$$

$$F = \int dF_y = IB \cos 30.0^\circ \int dl$$

But $\int dl = N(2\pi r)$, the total length of wire in the coil.

$$F = IB \cos 30.0^\circ N(2\pi r) = (0.950 \text{ A})(0.200 \text{ T})(\cos 30.0^\circ)(50)2\pi(0.0078 \text{ m}) = 0.444 \text{ N} \text{ and } \vec{F} = -(0.444 \text{ N})\hat{j}$$

EVALUATE: The magnetic field makes a constant angle with the plane of the coil but has a different direction at different points around the circumference of the coil so is not uniform. The net force is proportional to the magnitude of the current and reverses direction when the current reverses direction.

27.80. IDENTIFY: Conservation of energy relates the accelerating potential difference V to the final speed of the ions. In the magnetic field region the ions travel in an arc of a circle that has radius $R = \frac{mv}{|q|B}$.

SET UP: The quarter-circle paths of the two ions are shown in Figure 27.80. The separation at the detector is $\Delta r = R_{18} - R_{16}$. Each ion has charge $q = +e$.

EXECUTE: (a) Conservation of energy gives $|q|V = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2|q|V}{m}}$. $R = \frac{m}{|q|B} \sqrt{\frac{2|q|V}{m}} = \frac{\sqrt{2|q|mV}}{|q|B}$.

$$|q| = e \text{ for each ion. } \Delta r = R_{18} - R_{16} = \frac{\sqrt{2eV}}{eB} (\sqrt{m_{18}} - \sqrt{m_{16}}).$$

$$(b) V = \frac{(\Delta reB)^2}{2e(\sqrt{m_{18}} - \sqrt{m_{16}})^2} = \frac{e(\Delta r)^2 B^2}{2(\sqrt{m_{18}} - \sqrt{m_{16}})^2} = \frac{(1.60 \times 10^{-19} \text{ C})(4.00 \times 10^{-2} \text{ m})^2 (0.050 \text{ T})^2}{2(\sqrt{2.99 \times 10^{-26} \text{ kg}} - \sqrt{2.66 \times 10^{-26} \text{ kg}})^2}$$

$$V = 3.32 \times 10^3 \text{ V}.$$

EVALUATE: The speed of the ^{16}O ion after it has been accelerated through a potential difference of $V = 3.32 \times 10^3 \text{ V}$ is $2.00 \times 10^5 \text{ m/s}$. Increasing the accelerating voltage increases the separation of the two isotopes at the detector. But it does this by increasing the radius of the path for each ion, and this increases the required size of the magnetic field region.

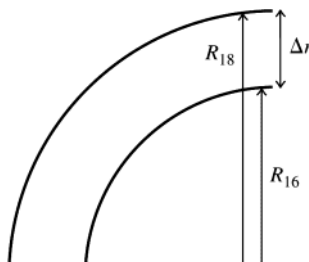


Figure 27.80

27.81. IDENTIFY: Apply $d\vec{F} = I d\vec{l} \times \vec{B}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I . Since the loop is in the xy -plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: (a) The magnetic field lines in the yz -plane are sketched in Figure 27.81.

$$(b) \text{ Side 1, that runs from } (0,0) \text{ to } (0,L): \vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 y}{L} dy \hat{i} = \frac{1}{2} B_0 L \hat{i}.$$

$$\text{Side 2, that runs from } (0,L) \text{ to } (L,L): \vec{F} = \int_{0,y=L}^L I d\vec{l} \times \vec{B} = I \int_{0,y=L}^L \frac{B_0 y}{L} dx \hat{j} = -IB_0 L \hat{j}.$$

$$\text{Side 3, that runs from } (L,L) \text{ to } (L,0): \vec{F} = \int_{L,x=L}^0 I d\vec{l} \times \vec{B} = I \int_{L,x=L}^0 \frac{B_0 y}{L} dy (-\hat{i}) = -\frac{1}{2} IB_0 L \hat{i}.$$

$$\text{Side 4, that runs from } (L,0) \text{ to } (0,0): \vec{F} = \int_{L,y=0}^0 I d\vec{l} \times \vec{B} = I \int_{L,y=0}^0 \frac{B_0 y}{L} dx \hat{j} = 0.$$

$$(c) \text{ The sum of all forces is } \vec{F}_{\text{total}} = -IB_0 L \hat{j}.$$

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

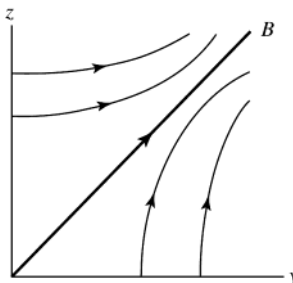


Figure 27.81

27.82. IDENTIFY: Apply $d\vec{F} = I d\vec{l} \times \vec{B}$ to each side of the loop. $\vec{\tau} = \vec{r} \times \vec{F}$.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I .

EXECUTE: (a) The magnetic field lines in the xy -plane are sketched in Figure 27.82.

(b) Side 1, that runs from $(0,0)$ to $(0,L)$: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 y}{L} dy (-\hat{k}) = -\frac{1}{2} B_0 L I \hat{k}$.

Side 2, that runs from $(0,L)$ to (L,L) : $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 x}{L} dx \hat{k} = \frac{1}{2} I B_0 L \hat{k}$.

Side 3, that runs from (L,L) to $(L,0)$: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 y}{L} dy \hat{k} = +\frac{1}{2} I B_0 L \hat{k}$.

Side 4, that runs from $(L,0)$ to $(0,0)$: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 x}{L} dx (-\hat{k}) = -\frac{1}{2} I B_0 L \hat{k}$.

(c) If free to rotate about the x -axis, the torques due to the forces on sides 1 and 3 cancel and the torque due to the forces on side 4 is zero. For side 2, $\vec{r} = L\hat{j}$. Therefore, $\vec{\tau} = \vec{r} \times \vec{F} = \frac{I B_0 L^2}{2} \hat{i} = \frac{1}{2} I A B_0 \hat{i}$.

(d) If free to rotate about the y -axis, the torques due to the forces on sides 2 and 4 cancel and the torque due to the forces on side 1 is zero. For side 3, $\vec{r} = L\hat{i}$. Therefore, $\vec{\tau} = \vec{r} \times \vec{F} = \frac{I B_0 L^2}{2} \hat{j} = -\frac{1}{2} I A B_0 \hat{j}$.

EVALUATE: (e) The equation for the torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ is not appropriate, since the magnetic field is not constant.

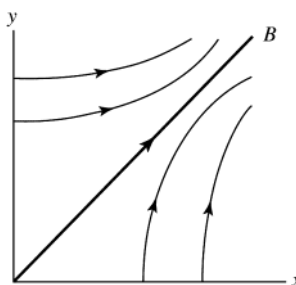


Figure 27.82

27.83. IDENTIFY: While the ends of the wire are in contact with the mercury and current flows in the wire, the magnetic field exerts an upward force and the wire has an upward acceleration. After the ends leave the mercury the electrical connection is broken and the wire is in free-fall.

(a) **SET UP:** After the wire leaves the mercury its acceleration is g , downward. The wire travels upward a total distance of 0.350 m from its initial position. Its ends lose contact with the mercury after the wire has traveled 0.025 m, so the wire travels upward 0.325 m after it leaves the mercury. Consider the motion of the wire after it leaves the mercury. Take $+y$ to be upward and take the origin at the position of the wire as it leaves the mercury.

$a_y = -9.80 \text{ m/s}^2$, $y - y_0 = +0.325 \text{ m}$, $v_y = 0$ (at maximum height), $v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(0.325 \text{ m})} = 2.52 \text{ m/s}$

(b) **SET UP:** Now consider the motion of the wire while it is in contact with the mercury. Take $+y$ to be upward and the origin at the initial position of the wire. Calculate the acceleration: $y - y_0 = +0.025 \text{ m}$, $v_{0y} = 0$ (starts from rest), $v_y = +2.52 \text{ m/s}$ (from part (a)), $a_y = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $a_y = \frac{v_y^2}{2(y - y_0)} = \frac{(2.52 \text{ m/s})^2}{2(0.025 \text{ m})} = 127 \text{ m/s}^2$

SET UP: The free-body diagram for the wire is given in Figure 27.83.

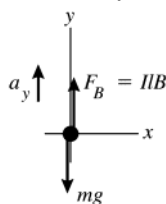


Figure 27.83

EXECUTE: $\sum F_y = ma_y$

$$F_B - mg = ma_y$$

$$IlB = m(g + a_y)$$

$$I = \frac{m(g + a_y)}{lB}$$

l is the length of the horizontal section of the wire; $l = 0.150$ m

$$I = \frac{(5.40 \times 10^{-5} \text{ kg})(9.80 \text{ m/s}^2 + 127 \text{ m/s}^2)}{(0.150 \text{ m})(0.00650 \text{ T})} = 7.58 \text{ A}$$

(c) IDENTIFY and SET UP: Use Ohm's law.

EXECUTE: $V = IR$ so $R = \frac{V}{I} = \frac{1.50 \text{ V}}{7.58 \text{ A}} = 0.198 \Omega$

EVALUATE: The current is large and the magnetic force provides a large upward acceleration. During this upward acceleration the wire moves a much shorter distance as it gains speed than the distance it moves while in free-fall with a much smaller acceleration, as it loses the speed it gained. The large current means the resistance of the wire must be small.

27.84. IDENTIFY and SET UP: Follow the procedures specified in the problem.

EXECUTE: (a) $d\vec{l} = d\hat{t}$, where $\hat{t}$ is a unit vector in the tangential direction. $d\vec{l} = R d\theta [-\sin\theta \hat{i} + \cos\theta \hat{j}]$. Note that this implies that when $\theta = 0$, the line element points in the $+y$ -direction, and when the angle is 90° , the line element points in the $-x$ -direction. This is in agreement with the diagram.

$$d\vec{F} = Id\vec{l} \times \vec{B} = IR d\theta [-\sin\theta \hat{i} + \cos\theta \hat{j}] \times (B_x \hat{i}) = IB_x R d\theta [-\cos\theta \hat{k}]$$

(b) $\vec{F} = \int_0^{2\pi} -\cos\theta IB_x R d\theta \hat{k} = -IB_x R \int_0^{2\pi} \cos\theta d\theta \hat{k} = 0$

(c) $d\vec{\tau} = \vec{r} \times d\vec{F} = R(\cos\theta \hat{i} + \sin\theta \hat{j}) \times (IB_x R d\theta [-\cos\theta \hat{k}]) = -R^2 IB_x d\theta (\sin\theta \cos\theta \hat{i} - \cos^2\theta \hat{j})$

(d) $\vec{\tau} = \int d\vec{\tau} = -R^2 IB_x \left(\int_0^{2\pi} \sin\theta \cos\theta d\theta \hat{i} - \int_0^{2\pi} \cos^2\theta d\theta \hat{j} \right) = IR^2 B_x \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right)_0^{2\pi} \hat{j} = IR^2 B_x \pi \hat{j} = I\pi R^2 B_x \hat{j} = IA \hat{k} \times B_x \hat{i}$

and $\vec{\tau} = \vec{\mu} \times \vec{B}$.

EVALUATE: Section 27.7 of the textbook derived $\vec{\tau} = \vec{\mu} \times \vec{B}$ for the case of a rectangular coil. This problem shows that the same result also applies to a circular coil.

27.85. (a) IDENTIFY: Use Eq.(27.27) to relate U , $\vec{\mu}$ and $\vec{B}$ and use Eq.(27.26) to relate $\vec{\tau}$, $\vec{\mu}$ and $\vec{B}$. We also know that $B_0^2 = B_x^2 + B_y^2 + B_z^2$. This gives three equations for the three components of $\vec{B}$.

SET UP: The loop and current are shown in Figure 27.85.

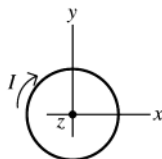


Figure 27.85

$\vec{\mu}$ is into the plane of the paper, in the $-z$ -direction

$$\vec{\mu} = -\mu \hat{k} = -IA \hat{k}$$

(b) **EXECUTE:** $\vec{\tau} = D(4\hat{i} - 3\hat{j})$, where $D > 0$.

$$\vec{\mu} = -IA \hat{k}, \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (-IA)(B_x \hat{k} \times \hat{i} + B_y \hat{k} \times \hat{j} + B_z \hat{k} \times \hat{k}) = IAB_y \hat{i} - IAB_x \hat{j}$$

Compare this to the expression given for $\vec{\tau}$: $IAB_y = 4D$ so $B_y = 4D/IA$ and $-IAB_x = -3D$ so $B_x = 3D/IA$

B_z doesn't contribute to the torque since $\vec{\mu}$ is along the z -direction. But $B = B_0$ and $B_x^2 + B_y^2 + B_z^2 = B_0^2$; with

$$B_0 = 13D/IA. \text{ Thus } B_z = \pm \sqrt{B_0^2 - B_x^2 - B_y^2} = \pm (D/IA) \sqrt{169 - 9 - 16} = \pm 12(D/IA)$$

That $U = -\vec{\mu} \cdot \vec{B}$ is negative determines the sign of B_z : $U = -(-IA \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) = +IAB_z$

So U negative says that B_z is negative, and thus $B_z = -12D/IA$.

EVALUATE: $\vec{\mu}$ is along the z -axis so only B_x and B_y contribute to the torque. B_x produces a y -component of $\vec{\tau}$ and B_y produces an x -component of $\vec{\tau}$. Only B_z affects U , and U is negative when $\vec{\mu}$ and $\vec{B}_z$ are parallel.

27.86. IDENTIFY: $I = \frac{\Delta q}{\Delta t}$ and $\mu = IA$.

SET UP: The direction of $\vec{\mu}$ is given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. I is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

EXECUTE: (a) $I_u = \frac{dq}{dt} = \frac{\Delta q}{\Delta t} = \frac{q_u v}{2\pi r} = \frac{ev}{3\pi r}$.

(b) $\mu_u = I_u A = \frac{ev}{3\pi r} \pi r^2 = \frac{evr}{3}$.

(c) Since there are two down quarks, each of half the charge of the up quark, $\mu_d = \mu_u = \frac{evr}{3}$. Therefore, $\mu_{\text{total}} = \frac{2evr}{3}$.

(d) $v = \frac{3\mu}{2er} = \frac{3(9.66 \times 10^{-27} \text{ A} \cdot \text{m}^2)}{2(1.60 \times 10^{-19} \text{ C})(1.20 \times 10^{-15} \text{ m})} = 7.55 \times 10^7 \text{ m/s}$.

EVALUATE: The speed calculated in part (d) is 25% of the speed of light.

27.87. IDENTIFY: Eq.(27.8) says that the magnetic field through any closed surface is zero.

SET UP: The cylindrical Gaussian surface has its top at $z = L$ and its bottom at $z = 0$. The rest of the surface is the curved portion of the cylinder and has radius r and length L . $B = 0$ at the bottom of the surface, since $z = 0$ there.

EXECUTE: (a) $\oint \vec{B} \cdot d\vec{A} = \int_{\text{top}} B_z dA + \int_{\text{curved}} B_r dA = \int_{\text{top}} (\beta L) dA + \int_{\text{curved}} B_r dA = 0$. This gives $0 = \beta L \pi r^2 + B_r 2\pi r L$, and

$$B_r(r) = -\frac{\beta r}{2}.$$

(b) The two diagrams in Figure 27.87 show views of the field lines from the top and side of the Gaussian surface.

EVALUATE: Only a portion of each field line is shown; the field lines are closed loops.

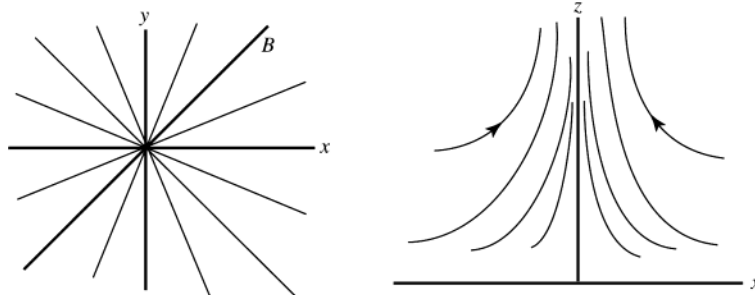


Figure 27.87

27.88. IDENTIFY: $U = -\vec{\mu} \cdot \vec{B}$. In part (b) apply conservation of energy.

SET UP: The kinetic energy of the rotating ring is $K = \frac{1}{2} I \omega^2$.

EXECUTE: (a) $\Delta U = -(\vec{\mu}_f \cdot \vec{B} - \vec{\mu}_i \cdot \vec{B}) = -(\vec{\mu}_f - \vec{\mu}_i) \cdot \vec{B} = \left[-\mu(-\hat{k} - (-0.8\hat{i} + 0.6\hat{j})) \right] \cdot \left[B_0(12\hat{i} + 3\hat{j} - 4\hat{k}) \right]$.

$$\Delta U = IAB_0[(-0.8)(+12) + (0.6)(+3) + (+1)(-4)] = (12.5 \text{ A})(4.45 \times 10^{-4} \text{ m}^2)(0.0115 \text{ T})(-11.8).$$

$$\Delta U = -7.55 \times 10^{-4} \text{ J}.$$

(b) $\Delta K = \frac{1}{2} I \omega^2$. $\omega = \sqrt{\frac{2\Delta K}{I}} = \sqrt{\frac{2(7.55 \times 10^{-4} \text{ J})}{8.50 \times 10^{-7} \text{ kg} \cdot \text{m}^2}} = 42.1 \text{ rad/s}$.

EVALUATE: The potential energy of the ring decreases and its kinetic energy increases.

27.89. IDENTIFY and SET UP: In the magnetic field, $R = \frac{mv}{qB}$. Once the particle exits the field it travels in a straight line.

Throughout the motion the speed of the particle is constant.

EXECUTE: (a) $R = \frac{mv}{qB} = \frac{(3.20 \times 10^{-11} \text{ kg})(1.45 \times 10^5 \text{ m/s})}{(2.15 \times 10^{-6} \text{ C})(0.420 \text{ T})} = 5.14 \text{ m}$.

(b) See Figure 27.89. The distance along the curve, d , is given by $d = R\theta$. $\sin\theta = \frac{0.35 \text{ m}}{5.14 \text{ m}}$, so $\theta = 2.78^\circ = 0.0486 \text{ rad}$. $d = R\theta = (5.14 \text{ m})(0.0486 \text{ rad}) = 0.25 \text{ m}$. And $t = \frac{d}{v} = \frac{0.25 \text{ m}}{1.45 \times 10^5 \text{ m/s}} = 1.72 \times 10^{-6} \text{ s}$.

(c) $\Delta x_1 = d \tan(\theta/2) = (0.25 \text{ m})\tan(2.79^\circ/2) = 6.08 \times 10^{-3} \text{ m}$.

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50 \text{ m})\tan 2.79^\circ = 0.0244 \text{ m}$. Therefore, $\Delta x = 6.08 \times 10^{-3} \text{ m} + 0.0244 \text{ m} = 0.0305 \text{ m}$.

EVALUATE: d is much less than R , so the horizontal deflection of the particle is much smaller than the distance it travels in the y -direction.

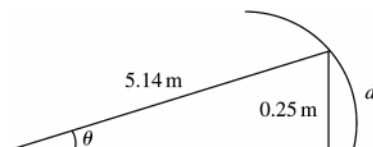


Figure 27.89

27.90. IDENTIFY: The current direction is perpendicular to $\vec{B}$, so $F = I\ell B$. If the liquid doesn't flow, a force $(\Delta p)A$ from the pressure difference must oppose F .

SET UP: $J = I/A$, where $A = hw$.

EXECUTE: (a) $\Delta p = F/A = I\ell B/A = J\ell B$.

(b) $J = \frac{\Delta p}{\ell B} = \frac{(1.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})}{(0.0350 \text{ m})(2.20 \text{ T})} = 1.32 \times 10^6 \text{ A/m}^2$.

EVALUATE: A current of 1 A in a wire with diameter 1 mm corresponds to a current density of $J = 1.36 \times 10^6 \text{ A/m}^2$, so the current density calculated in part (c) is a typical value for circuits.

27.91. IDENTIFY: The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \frac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y -direction is $W = Fy = qEy$.

At the top point, $\vec{F}_B$ is in the $-y$ -direction and $\vec{F}_E$ is in the $+y$ -direction.

EXECUTE: (a) The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y -direction of the particle, leading to the repeated motion.

(b) $W = qEy = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qEy}{m}}$.

(c) At the top, $F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y} \frac{2qEy}{m} = -qE$. $2qE = qvB$ and $v = \frac{2E}{B}$.

EVALUATE: The speed at the top depends on B because B determines the y -displacement and the work done by the electric force depends on the y -displacement.

SOURCES OF MAGNETIC FIELD

28.1. IDENTIFY and SET UP: Use Eq.(28.2) to calculate $\vec{B}$ at each point.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}, \text{ since } \hat{r} = \frac{\vec{r}}{r}.$$

$\vec{v} = (8.00 \times 10^6 \text{ m/s})\hat{j}$ and $\vec{r}$ is the vector from the charge to the point where the field is calculated.

EXECUTE: (a) $\vec{r} = (0.500 \text{ m})\hat{i}$, $r = 0.500 \text{ m}$

$$\vec{v} \times \vec{r} = v\hat{j} \times \hat{i} = -vr\hat{k}$$

$$\vec{B} = -\frac{\mu_0}{4\pi} \frac{qv}{r^2} \hat{k} = -(1 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(6.00 \times 10^{-6} \text{ C})(8.00 \times 10^6 \text{ m/s})}{(0.500 \text{ m})^2} \hat{k}$$

$$\vec{B} = -(1.92 \times 10^{-5} \text{ T})\hat{k}$$

(b) $\vec{r} = -(0.500 \text{ m})\hat{j}$, $r = 0.500 \text{ m}$

$$\vec{v} \times \vec{r} = -vr\hat{j} \times \hat{j} = 0 \text{ and } \vec{B} = 0.$$

(c) $\vec{r} = (0.500 \text{ m})\hat{k}$, $r = 0.500 \text{ m}$

$$\vec{v} \times \vec{r} = v\hat{j} \times \hat{k} = vr\hat{i}$$

$$\vec{B} = (1 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(6.00 \times 10^{-6} \text{ C})(8.00 \times 10^6 \text{ m/s})}{(0.500 \text{ m})^2} \hat{i} = +(1.92 \times 10^{-5} \text{ T})\hat{i}$$

(d) $\vec{r} = -(0.500 \text{ m})\hat{j} + (0.500 \text{ m})\hat{k}$, $r = \sqrt{(0.500 \text{ m})^2 + (0.500 \text{ m})^2} = 0.7071 \text{ m}$

$$\vec{v} \times \vec{r} = v(0.500 \text{ m})(-\hat{j} \times \hat{j} + \hat{j} \times \hat{k}) = (4.00 \times 10^6 \text{ m}^2/\text{s})\hat{i}$$

$$\vec{B} = (1 \times 10^{-7} \text{ T} \cdot \text{m/A}) \frac{(6.00 \times 10^{-6} \text{ C})(4.00 \times 10^6 \text{ m/s})}{(0.7071 \text{ m})^3} \hat{i} = +(6.79 \times 10^{-6} \text{ T})\hat{i}$$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{r}$. $B = 0$ along the direction of $\vec{v}$.

28.2. IDENTIFY: A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$, and its electric field is $E = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2}$

since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(1.60 \times 10^{-19} \text{ C})(2.2 \times 10^6 \text{ m/s}) \sin 90^\circ}{(5.3 \times 10^{-11} \text{ m})^2} = 13 \text{ T, out of the page.}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{(5.3 \times 10^{-11} \text{ m})^2} = 5.1 \times 10^{11} \text{ N/C, toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. IDENTIFY: A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

$$(a) B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(1.6 \times 10^{-19} \text{ C})(3.0 \times 10^7 \text{ m/s}) \sin 30^\circ}{(2.00 \times 10^{-6} \text{ m})^2}.$$

$B = 6.00 \times 10^{-8} \text{ T}$, out of the paper, and it is the same at point B .

$$(b) B = (1.00 \times 10^{-7} \text{ T} \cdot \text{m/A})(1.60 \times 10^{-19} \text{ C})(3.00 \times 10^7 \text{ m/s})/(2.00 \times 10^{-6} \text{ m})^2$$

$B = 1.20 \times 10^{-7} \text{ T}$, out of the page.

$$(c) B = 0 \text{ T since } \sin(180^\circ) = 0.$$

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the Earth's magnetic field.

28.4. IDENTIFY: Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2} (e \sin 40^\circ + 2e \sin 140^\circ)$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(1.60 \times 10^{-19} \text{ C})(2.50 \times 10^5 \text{ m/s})}{(1.75 \times 10^{-9} \text{ m})^2} (\sin 40^\circ + 2 \sin 140^\circ)$$

$$B = 2.52 \times 10^{-3} \text{ T} = 2.52 \text{ mT, out of the page.}$$

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$.

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: (a) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{i}$; $\vec{v} \times \vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v} \times \vec{r} = vr\hat{k}$, $r = 0.500 \text{ m}$.

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{|q|v}{r^2} = \frac{(1.0 \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)(4.80 \times 10^{-6} \text{ C})(6.80 \times 10^5 \text{ m/s})}{(0.500 \text{ m})^2} = 1.31 \times 10^{-6} \text{ T}.$$

q is negative, so $\vec{B} = -(1.31 \times 10^{-6} \text{ T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500 \text{ m})(\hat{i} + \hat{j})$; $\vec{v} \times \vec{r} = (0.500 \text{ m})v\hat{k}$, $r = 0.7071 \text{ m}$.

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{(|q||\vec{v} \times \vec{r}|/r^3)}{r^3} = \frac{(1.0 \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)(4.80 \times 10^{-6} \text{ C})(0.500 \text{ m})(6.80 \times 10^5 \text{ m/s})}{(0.7071 \text{ m})^3}.$$

$$B = 4.62 \times 10^{-7} \text{ T. } \vec{B} = -(4.62 \times 10^{-7} \text{ T})\hat{k}.$$

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v} \times \vec{r} = -vr\hat{j}$, $r = 0.500 \text{ m}$

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{|q|v}{r^2} = \frac{(1.0 \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)(4.80 \times 10^{-6} \text{ C})(6.80 \times 10^5 \text{ m/s})}{(0.500 \text{ m})^2} = 1.31 \times 10^{-6} \text{ T}.$$

$$\vec{B} = (1.31 \times 10^{-6} \text{ T})\hat{j}.$$

EVALUATE: In each case, $\vec{B}$ is perpendicular to both $\vec{r}$ and $\vec{v}$.

28.6. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and $\vec{r}$ is perpendicular to $\vec{v}$ in each case, so $\frac{|\vec{v} \times \vec{r}|}{r^3} = \frac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

$$\text{EXECUTE: (a) } B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi} \left(\frac{qv}{d^2} + \frac{q'v'}{d^2} \right).$$

$$B = \frac{\mu_0}{4\pi} \left(\frac{(8.0 \times 10^{-6} \text{ C})(4.5 \times 10^6 \text{ m/s})}{(0.120 \text{ m})^2} + \frac{(3.0 \times 10^{-6} \text{ C})(9.0 \times 10^6 \text{ m/s})}{(0.120 \text{ m})^2} \right) = 4.38 \times 10^{-4} \text{ T}.$$

The direction of $\vec{B}$ is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi} \frac{q q' v v'}{r^2} = (10^{-7} \text{ T} \cdot \text{m/A}) \frac{(8.00 \times 10^{-6} \text{ C})(3.00 \times 10^{-6} \text{ C})(4.50 \times 10^6 \text{ m/s})(9.00 \times 10^6 \text{ m/s})}{(0.240 \text{ m})^2}$$

$F_B = 1.69 \times 10^{-3} \text{ N}$. The force on the upper charge points up and the force on the lower charge points down. The

Coulomb force between the charges is $F_C = k \frac{q_1 q_2}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(8.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})}{(0.240 \text{ m})^2} = 3.75 \text{ N}$.

The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb

force to the magnetic force is $\frac{F_C}{F_B} = \frac{c^2}{v_1 v_2} = \frac{3.75 \text{ N}}{1.69 \times 10^{-3} \text{ N}} = 2.22 \times 10^3$; the Coulomb force is much larger.

(b) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

28.7. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \vec{r}}{r^3}$. For the magnetic force on q' , use $\vec{F}_B = q' \vec{v} \times \vec{B}_q$ and for the magnetic force on q use $\vec{F}_B = q \vec{v} \times \vec{B}_{q'}$.

SET UP: In part (a), $r = d$ and $\frac{|\vec{v} \times \vec{r}|}{r^3} = \frac{v}{d^2}$.

EXECUTE: (a) $q' = -q$; $B_q = \frac{\mu_0 q v}{4\pi d^2}$, into the page; $B_{q'} = \frac{\mu_0 q v'}{4\pi d^2}$, out of the page.

(i) $v' = \frac{v}{2}$ gives $B = \frac{\mu_0 q v}{4\pi d^2} (1 - \frac{1}{2}) = \frac{\mu_0 q v}{4\pi (2d^2)}$, into the page. (ii) $v' = v$ gives $B = 0$.

(iii) $v' = 2v$ gives $B = \frac{\mu_0 q v}{4\pi d^2}$, out of the page.

(b) The force that q exerts on q' is given by $\vec{F} = q' \vec{v}' \times \vec{B}_q$, so $F = \frac{\mu_0 q^2 v' v}{4\pi (2d)^2}$. $\vec{B}_q$ is into the page, so the force on q' is toward q . The force that q' exerts on q is toward q' . The force between the two charges is attractive.

(c) $F_B = \frac{\mu_0 q^2 v v'}{4\pi (2d)^2}$, $F_C = \frac{q^2}{4\pi \epsilon_0 (2d)^2}$ so $\frac{F_B}{F_C} = \mu_0 \epsilon_0 v v' = \mu_0 \epsilon_0 (3.00 \times 10^5 \text{ m/s})^2 = 1.00 \times 10^{-6}$.

EVALUATE: When charges of opposite sign move in opposite directions, the force between them is attractive. For the values specified in part (c), the magnetic force between the two charges is much smaller in magnitude than the Coulomb force between them.

28.8. IDENTIFY: Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB \sin \phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi} \frac{q v \sin \phi}{r^2}$.

EXECUTE: (a) Both fields are into the page, so their magnitudes add, giving

$$B = B_e + B_p = \frac{\mu_0}{4\pi} \left(\frac{ev}{r_e^2} + \frac{ev}{r_p^2} \right) \sin 90^\circ$$

$$B = \frac{\mu_0}{4\pi} (1.60 \times 10^{-19} \text{ C})(845,000 \text{ m/s}) \left[\frac{1}{(5.00 \times 10^{-9} \text{ m})^2} + \frac{1}{(4.00 \times 10^{-9} \text{ m})^2} \right]$$

$$B = 1.39 \times 10^{-3} \text{ T} = 1.39 \text{ mT, into the page.}$$

(b) Using $B = \frac{\mu_0}{4\pi} \frac{q v \sin \phi}{r^2}$, where $r = \sqrt{41} \text{ nm}$ and $\phi = 180^\circ - \arctan(5/4) = 128.7^\circ$, we get

$$B = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} (1.6 \times 10^{-19} \text{ C})(845,000 \text{ m/s}) \sin 128.7^\circ}{(\sqrt{41} \times 10^{-9} \text{ m})^2} = 2.58 \times 10^{-4} \text{ T, into the page.}$$

(c) $F_{\text{mag}} = qvB \sin 90^\circ = (1.60 \times 10^{-19} \text{ C})(845,000 \text{ m/s})(2.58 \times 10^{-4} \text{ T}) = 3.48 \times 10^{-17} \text{ N}$, in the $+x$ direction.

$F_{\text{elec}} = (1/4\pi \epsilon_0) e^2 / r^2 = \frac{(9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(\sqrt{41} \times 10^{-9} \text{ m})^2} = 5.62 \times 10^{-12} \text{ N}$, at 51.3° below the $+x$ -axis measured

clockwise.

EVALUATE: The electric force is much stronger than the magnetic force.

28.9. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

$$(a) \quad dB = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(10.0 \text{ A})(0.00110 \text{ m}) \sin 90^\circ}{(0.0500 \text{ m})^2} = 4.40 \times 10^{-7} \text{ T, out of the paper.}$$

(b) The same as above, except $r = \sqrt{(5.00 \text{ cm})^2 + (14.0 \text{ cm})^2}$ and $\phi = \arctan(5/14) = 19.65^\circ$, giving $dB = 1.67 \times 10^{-8} \text{ T}$, out of the page.

(c) $dB = 0$ since $\phi = 0^\circ$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3}$.

SET UP: The magnitude of the field due to the current element is $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$, where ϕ is the angle between $\vec{r}$ and the current direction.

EXECUTE: The magnetic field at the given points is:

$$dB_a = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2} = \frac{\mu_0}{4\pi} \frac{(200 \text{ A})(0.000100 \text{ m})}{(0.100 \text{ m})^2} = 2.00 \times 10^{-6} \text{ T.}$$

$$dB_b = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2} = \frac{\mu_0}{4\pi} \frac{(200 \text{ A})(0.000100 \text{ m}) \sin 45^\circ}{2(0.100 \text{ m})^2} = 0.705 \times 10^{-6} \text{ T.}$$

$$dB_c = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2} = \frac{\mu_0}{4\pi} \frac{(200 \text{ A})(0.000100 \text{ m})}{(0.100 \text{ m})^2} = 2.00 \times 10^{-6} \text{ T.}$$

$$dB_d = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl \sin(0^\circ)}{r^2} = 0.$$

$$dB_e = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2} = \frac{\mu_0}{4\pi} \frac{(200 \text{ A})(0.00100 \text{ m}) \frac{\sqrt{2}}{\sqrt{3}}}{3(0.100 \text{ m})^2} = 0.545 \times 10^{-6} \text{ T}$$

The field vectors at each point are shown in Figure 28.10.

EVALUATE: In each case $d\vec{B}$ is perpendicular to the current direction.

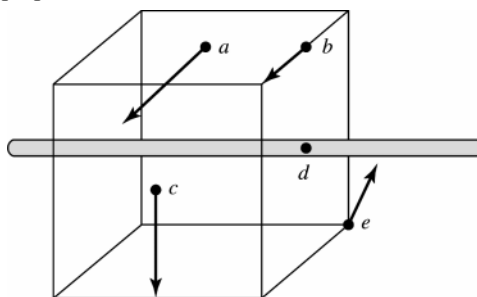


Figure 28.10

28.11. IDENTIFY and SET UP: The magnetic field produced by an infinitesimal current element is given by Eq.(28.6).

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\vec{l} \times \hat{r}}{r^2}$ As in Example 28.2 use this equation for the finite 0.500-mm segment of wire since the $\Delta l = 0.500 \text{ mm}$ length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I\Delta\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I\Delta\vec{l} \times \vec{r}}{r^3}$$

I is in the $+z$ -direction, so $\Delta\vec{l} = (0.500 \times 10^{-3} \text{ m})\hat{k}$

EXECUTE: (a) Field point is at $x = 2.00 \text{ m}$, $y = 0$, $z = 0$ so the vector $\vec{r}$ from the source point (at the origin) to the field point is $\vec{r} = (2.00 \text{ m})\hat{i}$.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times \hat{i} = +(1.00 \times 10^{-3} \text{ m}^2)\hat{j}$$

$$\vec{B} = \frac{(1 \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(1.00 \times 10^{-3} \text{ m}^2)}{(2.00 \text{ m})^3} \hat{j} = (5.00 \times 10^{-11} \text{ T})\hat{j}$$

(b) $\vec{r} = (2.00 \text{ m})\hat{j}$, $r = 2.00 \text{ m}$.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times \hat{j} = -(1.00 \times 10^{-3} \text{ m}^2)\hat{i}$$

$$\vec{B} = -\frac{(1 \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(1.00 \times 10^{-3} \text{ m}^2)}{(2.00 \text{ m})^3}\hat{i} = -(5.00 \times 10^{-11} \text{ T})\hat{i}$$

(c) $\vec{r} = (2.00 \text{ m})(\hat{i} + \hat{j})$, $r = \sqrt{2}(2.00 \text{ m})$.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times (\hat{i} + \hat{j}) = (1.00 \times 10^{-3} \text{ m}^2)(\hat{j} - \hat{i})$$

$$\vec{B} = \frac{(1 \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(1.00 \times 10^{-3} \text{ m}^2)}{[\sqrt{2}(2.00 \text{ m})]^3}(\hat{j} - \hat{i}) = (-1.77 \times 10^{-11} \text{ T})(\hat{i} - \hat{j})$$

(d) $\vec{r} = (2.00 \text{ m})\hat{k}$, $r = 2.00 \text{ m}$.

$$\Delta\vec{l} \times \vec{r} = (0.500 \times 10^{-3} \text{ m})(2.00 \text{ m})\hat{k} \times \hat{k} = 0; \vec{B} = 0.$$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{r}$ and $\Delta\vec{l}$. $B = 0$ along the length of the wire.

28.12. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current gives

$$dB = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(12.0 \text{ A})(0.00150 \text{ m})\left(\frac{2.50 \text{ cm}}{8.00 \text{ cm}}\right)}{(0.0800 \text{ m})^2} = 8.79 \times 10^{-8} \text{ T}$$

The field from the 24.0-A segment is twice this value, so the total field is $2.64 \times 10^{-7} \text{ T}$, into the page.

EVALUATE: The rest of each wire also produces field at P . We have calculated just the field from the two segments that are indicated in the problem.

28.13. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r = \frac{1}{2}\sqrt{(3.00 \text{ cm})^2 + (3.00 \text{ cm})^2} = 2.121 \text{ cm}$, we have

$$dB = 2 \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(28.0 \text{ A})(0.00200 \text{ m})\sin 45.0^\circ}{(0.02121 \text{ m})^2} = 1.76 \times 10^{-5} \text{ T, into the paper.}$$

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \phi}{r^2}$. All four fields are of equal magnitude and into the page, so their magnitudes add.

$$\text{EXECUTE: } dB = 4 \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{4\pi} \frac{(15.0 \text{ A})(0.00120 \text{ m})\sin 90^\circ}{(0.0500 \text{ m})^2} = 2.88 \times 10^{-6} \text{ T, into the page.}$$

EVALUATE: A small current element causes a small magnetic field.

28.15. IDENTIFY: We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

$$(a) B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(20,000 \text{ A})}{2\pi(5.0 \text{ m})} = 8 \times 10^{-4} \text{ T}$$

$$(b) B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(10 \text{ A})}{2\pi(0.050 \text{ m})} = 4.0 \times 10^{-5} \text{ T.}$$

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

28.16. IDENTIFY: The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (3.50 \times 10^{18} \text{ el/s})(1.60 \times 10^{-19} \text{ C/el}) = 0.560 \text{ A}$

Now find the magnetic field: $\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.560 \text{ A})}{2\pi(0.0400 \text{ m})} = 2.80 \times 10^{-6} \text{ T}$

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the Earth, so any experiments involving such a current would have to be shielded from the Earth's magnetic field, or at least would have to take it into consideration.

28.17. IDENTIFY: The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: First solve for the current, then substitute the numbers using the above equation.

(a) Solving for the current gives

$$I = 2\pi r B / \mu_0 = 2\pi(0.0200 \text{ m})(1.00 \times 10^{-4} \text{ T}) / (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) = 10.0 \text{ A}$$

(b) The earth's horizontal field points northward, so at all points directly above the wire the field of the wire would point northward.

(c) At all points directly east of the wire, its field would point northward.

EVALUATE: Even though the Earth's magnetic field is rather weak, it requires a fairly large current to cancel this field.

28.18. IDENTIFY: For each wire $B = \frac{\mu_0 I}{2\pi r}$ (Eq.28.9), and the direction of $\vec{B}$ is given by the right-hand rule (Fig. 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) **SET UP:** The two fields at this point have the directions shown in Figure 28.18a.

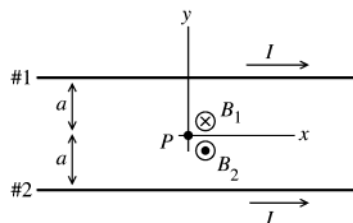


Figure 28.18a

EXECUTE: At point P midway between the two wires the fields $\vec{B}_1$ and $\vec{B}_2$ due to the two currents are in opposite directions, so $B = B_2 - B_1$.

But $B_1 = B_2 = \frac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) **SET UP:** The two fields at this point have the directions shown in Figure 28.18b.

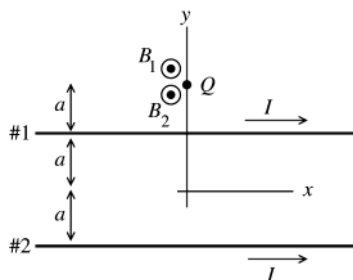


Figure 28.18b

EXECUTE: At point Q above the upper wire $\vec{B}_1$ and $\vec{B}_2$ are both directed out of the page ($+z$ -direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi a}, B_2 = \frac{\mu_0 I}{2\pi(3a)}$$

$$B = \frac{\mu_0 I}{2\pi a} \left(1 + \frac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a}; \vec{B} = \frac{2\mu_0 I}{3\pi a} \hat{k}$$

(c) **SET UP:** The two fields at this point have the directions shown in Figure 28.18c.

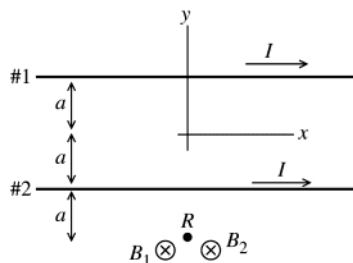


Figure 28.18c

EXECUTE: At point R below the lower wire $\vec{B}_1$ and $\vec{B}_2$ are both directed into the page ($-z$ -direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi(3a)}, B_2 = \frac{\mu_0 I}{2\pi a}$$

$$B_1 = \frac{\mu_0 I}{2\pi a} \left(1 + \frac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a}; \quad \vec{B} = -\frac{2\mu_0 I}{3\pi a} \hat{k}$$

EVALUATE: In the figures we have drawn, $\vec{B}$ due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

28.19. IDENTIFY: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is illustrated in

Figure 28.6 in the textbook. $\vec{B}_0 = (1.50 \times 10^{-6} \text{ T}) \hat{i}$.

EXECUTE: (a) At $(0, 0, 1 \text{ m})$, $\vec{B} = \vec{B}_0 - \frac{\mu_0 I}{2\pi r} \hat{i} = (1.50 \times 10^{-6} \text{ T}) \hat{i} - \frac{\mu_0 (8.00 \text{ A})}{2\pi (1.00 \text{ m})} \hat{i} = -(1.0 \times 10^{-7} \text{ T}) \hat{i}$.

(b) At $(1 \text{ m}, 0, 0)$, $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r} \hat{k} = (1.50 \times 10^{-6} \text{ T}) \hat{i} + \frac{\mu_0 (8.00 \text{ A})}{2\pi (1.00 \text{ m})} \hat{k}$.

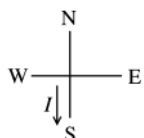
$\vec{B} = (1.50 \times 10^{-6} \text{ T}) \hat{i} + (1.6 \times 10^{-6} \text{ T}) \hat{k} = 2.19 \times 10^{-6} \text{ T}$, at $\theta = 46.8^\circ$ from x to z .

(c) At $(0, 0, -0.25 \text{ m})$, $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r} \hat{i} = (1.50 \times 10^{-6} \text{ T}) \hat{i} + \frac{\mu_0 (8.00 \text{ A})}{2\pi (0.25 \text{ m})} \hat{i} = (7.9 \times 10^{-6} \text{ T}) \hat{i}$.

EVALUATE: At point c the two fields are in the same direction and their magnitudes add. At point a they are in opposite directions and their magnitudes subtract. At point b the two fields are perpendicular.

28.20. IDENTIFY and SET UP: The magnitude of $\vec{B}$ is given by Eq.(28.9) and the direction is given by the right-hand rule.

(a) **EXECUTE:** Viewed from above, the current is in the direction shown in Figure 28.20.



Directly below the wire the direction of the magnetic field due to the current in the wire is east.

Figure 28.20

$$B = \frac{\mu_0 I}{2\pi r} = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{800 \text{ A}}{5.50 \text{ m}} \right) = 2.91 \times 10^{-5} \text{ T}$$

(b) **EVALUATE:** B from the current is nearly equal in magnitude to the earth's field, so, yes, the current really is a problem.

28.21. IDENTIFY: $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule in Section 20.7.

SET UP: Call the wires a and b , as indicated in Figure 28.21. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.21a. The fields at point 3 are shown in Figure 28.21b.

EXECUTE: (a) At P_1 , $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \frac{\mu_0 I}{2\pi r_a}$, $B_b = \frac{\mu_0 I}{2\pi r_b}$. $\vec{B}_a$ and $\vec{B}_b$ are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi} \left(\frac{1}{r_a} + \frac{1}{r_b} \right) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})}{2\pi} \left[\frac{1}{0.300 \text{ m}} + \frac{1}{0.200 \text{ m}} \right] = 6.67 \times 10^{-6} \text{ T}$$

$\vec{B}$ has magnitude $6.67 \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.21b, $\vec{B}_a$ is perpendicular to $\vec{r}_a$ and $\vec{B}_b$ is perpendicular to $\vec{r}_b$. $\tan \theta = \frac{5 \text{ cm}}{20 \text{ cm}}$ and $\theta = 14.04^\circ$.

$$r_a = r_b = \sqrt{(0.200 \text{ m})^2 + (0.050 \text{ m})^2} = 0.206 \text{ m} \text{ and } B_a = B_b.$$

$$B = B_a \cos \theta + B_b \cos \theta = 2B_a \cos \theta = 2 \left(\frac{\mu_0 I}{2\pi r_a} \right) \cos \theta = \frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.0 \text{ A}) \cos 14.04^\circ}{2\pi(0.206 \text{ m})} = 7.54 \mu\text{T}$$

B has magnitude $7.53 \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

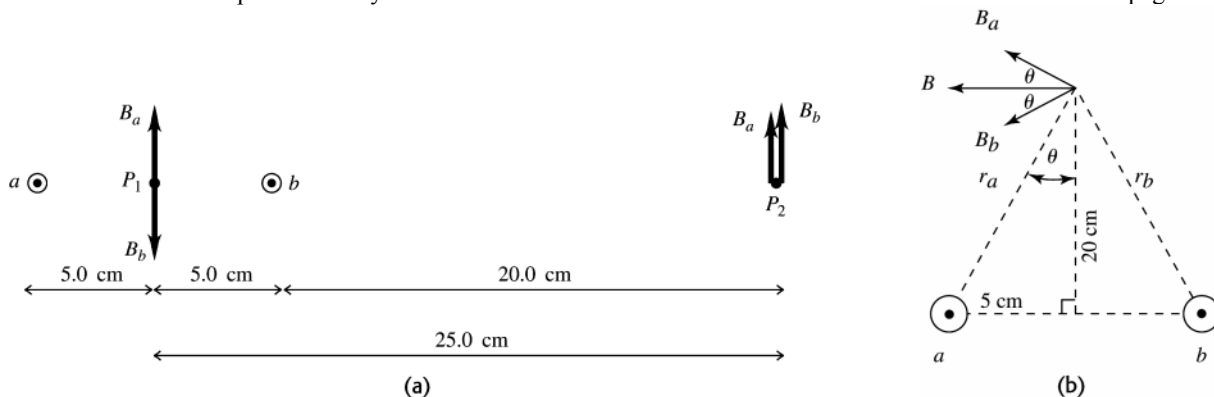


Figure 28.21

28.22. IDENTIFY: Use Eq.(28.9) and the right-hand rule to determine points where the fields of the two wires cancel.

(a) SET UP: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires. Consider a point a distance x from the wire carrying $I_2 = 75.0 \text{ A}$. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE:
$$\frac{\mu_0 I_1}{2\pi(0.400 \text{ m} - x)} = \frac{\mu_0 I_2}{2\pi x}$$

$$I_2(0.400 \text{ m} - x) = I_1 x; I_1 = 25.0 \text{ A}, I_2 = 75.0 \text{ A}$$

$x = 0.300 \text{ m}$; $B_{\text{tot}} = 0$ along a line 0.300 m from the wire carrying 75.0 A and 0.100 m from the wire carrying current 25.0 A .

(b) SET UP: Let the wire with $I_1 = 25.0 \text{ A}$ be 0.400 m above the wire with $I_2 = 75.0 \text{ A}$. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{\text{tot}} = 0$ only at points above both wires. Consider a point a distance x from the wire carrying $I_1 = 25.0 \text{ A}$. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE:
$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(0.400 \text{ m} + x)}$$

$$I_2 x = I_1(0.400 \text{ m} + x); x = 0.200 \text{ m}$$

$B_{\text{tot}} = 0$ along a line 0.200 m from the wire carrying current 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0 \text{ A}$.

EVALUATE: For parts (a) and (b) the locations of zero field are in different regions. In each case the points of zero field are closer to the wire that has the smaller current.

28.23. IDENTIFY: The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: (a) and (b) $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel. (c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a \cos 45^\circ + B_b \cos 45^\circ + B_c \cos 45^\circ + B_d \cos 45^\circ = 4B_a \cos 45^\circ = 4 \left(\frac{\mu_0 I}{2\pi r} \right) \cos 45^\circ.$$

$$r = \sqrt{(10 \text{ cm})^2 + (10 \text{ cm})^2} = 10\sqrt{2} \text{ cm} = 0.10\sqrt{2} \text{ m}, \text{ so}$$

$$B = 4 \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(100 \text{ A})}{2\pi(0.10\sqrt{2} \text{ m})} \cos 45^\circ = 4.0 \times 10^{-4} \text{ T}, \text{ to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

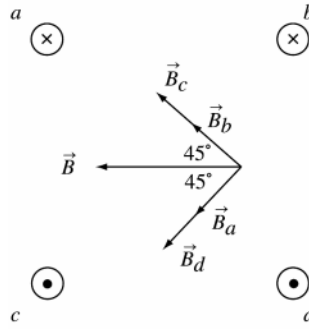


Figure 28.23

- 28.24. IDENTIFY:** Use Eq.(28.9) and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.
SET UP: The three known currents are shown in Figure 28.24.

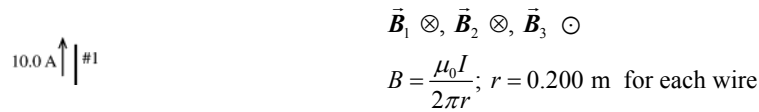


Figure 28.24

EXECUTE: Let $\odot$ be the positive z -direction. $I_1 = 10.0$ A, $I_2 = 8.0$ A, $I_3 = 20.0$ A. Then $B_1 = 1.00 \times 10^{-5}$ T, $B_2 = 0.80 \times 10^{-5}$ T, and $B_3 = 2.00 \times 10^{-5}$ T.

$$B_{1z} = -1.00 \times 10^{-5} \text{ T}, B_{2z} = -0.80 \times 10^{-5} \text{ T}, B_{3z} = +2.00 \times 10^{-5} \text{ T}$$

$$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$$

$$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0 \times 10^{-6} \text{ T}$$

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{r B_4}{(\mu_0 / 2\pi)} = \frac{(0.200 \text{ m})(2.0 \times 10^{-6} \text{ T})}{(2 \times 10^{-7} \text{ T} \cdot \text{m/A})} = 2.0 \text{ A}$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0 \text{ A} - 8.0 \text{ A} = 12.0 \text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0 \text{ A} + I_4 = 12.0 \text{ A}$.

- 28.25. IDENTIFY:** Apply Eq.(28.11).

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: (a) $F = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{\mu_0 (5.00 \text{ A})(2.00 \text{ A})(1.20 \text{ m})}{2\pi (0.400 \text{ m})} = 6.00 \times 10^{-6} \text{ N}$, and the force is repulsive since the

currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to $F = 2.40 \times 10^{-5} \text{ N}$.

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

- 28.26. IDENTIFY:** Apply Eq.(28.11).

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: (a) $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2 = \frac{F}{L} \frac{2\pi r}{\mu_0 I_1} = (4.0 \times 10^{-5} \text{ N/m}) \frac{2\pi (0.0250 \text{ m})}{\mu_0 (0.60 \text{ A})} = 8.33 \text{ A}$.

(b) The two wires repel so the currents are in opposite directions.

EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

- 28.27. IDENTIFY:** The lamp cord wires are two parallel current-carrying wires, so they must exert a magnetic force on each other.

SET UP: First find the current in the cord. Since it is connected to a light bulb, the power consumed by the bulb is $P = IV$. Then find the force per unit length using $F/L = \frac{\mu_0}{2\pi} \frac{II}{r}$.

EXECUTE: For the light bulb, $100 \text{ W} = I(120 \text{ V})$ gives $I = 0.833 \text{ A}$. The force per unit length is

$$F/L = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \frac{(0.833 \text{ A})^2}{0.003 \text{ m}} = 4.6 \times 10^{-5} \text{ N/m}$$

Since the currents are in opposite directions, the force is repulsive.

EVALUATE: This force is too small to have an appreciable effect for an ordinary cord.

- 28.28. IDENTIFY:** Apply Eq.(28.11) for the force from each wire.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi} \left(\frac{1}{d} - \frac{1}{2d} \right) = \frac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic forces cancel

so the net force is zero. On the bottom wire $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi} \left(-\frac{1}{d} + \frac{1}{2d} \right) = -\frac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

- 28.29. IDENTIFY:** The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.29.

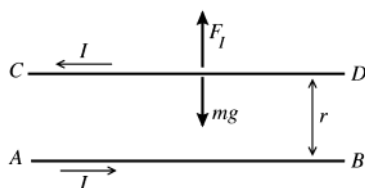


Figure 28.29

Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Eq.(28.11) says $F_I = \frac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda Lg$

Thus $F_I - mg = 0$ says $\frac{\mu_0 I^2 L}{2\pi h} = \lambda Lg$ and $h = \frac{\mu_0 I^2}{2\pi g \lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

- 28.30. IDENTIFY:** The magnetic field at the center of a circular loop is $B = \frac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

SET UP: Since the straight sections produce no field at P , the field at P is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: $B = \frac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \frac{\mu_0 I}{8R}$.

- 28.31. IDENTIFY:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.31a.

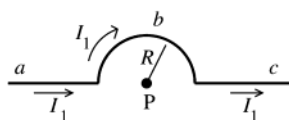


Figure 28.31a

Consider the three parts of this wire

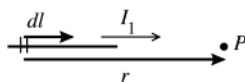
a : long straight section,

b : semicircle

c : long, straight section

Apply the Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}$ to each piece.

EXECUTE: part a See Figure 28.31b.

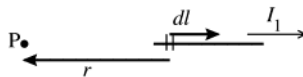


$$\begin{aligned} d\vec{l} \times \vec{r} &= 0, \\ \text{so } dB &= 0 \end{aligned}$$

Figure 28.31b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

part c See Figure 28.31c.



(c)

$$\begin{aligned} d\vec{l} \times \vec{r} &= 0, \\ \text{so } dB &= 0 \text{ and } B = 0 \text{ for this piece.} \end{aligned}$$

Figure 28.31c

part b See Figure 28.31d.

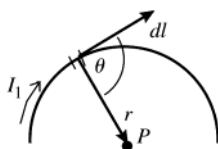


Figure 28.31d

$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

$|d\vec{l} \times \vec{r}| = r dl \sin \theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $|d\vec{l} \times \vec{r}| = R dl$

$$\begin{aligned} dB &= \frac{\mu_0}{4\pi} \frac{I |d\vec{l} \times \vec{r}|}{r^3} = \frac{\mu_0 I_1}{4\pi} \frac{R}{R^3} dl = \left(\frac{\mu_0 I_1}{4\pi R^2} \right) dl \\ B &= \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2} \right) \int dl = \left(\frac{\mu_0 I_1}{4\pi R^2} \right) (\pi R) = \frac{\mu_0 I_1}{4R} \end{aligned}$$

(We used that $\int dl$ is equal to πR , the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1 = \mu_0 I_1 / 4R$ and is directed into the page.



Figure 28.31e

For current in the direction shown in Figure 28.31e, a similar analysis gives $B_2 = \mu_0 I_2 / 4R$, out of the page

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \frac{\mu_0 |I_1 - I_2|}{4R}$.

EVALUATE: When $I_1 = I_2$, $B = 0$.

28.32. IDENTIFY: Apply Eq.(28.16).

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.0240 m.

EXECUTE: (a) $B_x = \mu_0 NI / 2a$, so $I = \frac{2aB_x}{\mu_0 N} = \frac{2(0.024 \text{ m})(0.0580 \text{ T})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(800)} = 2.77 \text{ A}$

(b) At the center, $B_c = \mu_0 NI / 2a$. At a distance x from the center,

$$B_x = \frac{\mu_0 NI a^2}{2(x^2 + a^2)^{3/2}} = \left(\frac{\mu_0 NI}{2a} \right) \left(\frac{a^3}{(x^2 + a^2)^{3/2}} \right) = B_c \left(\frac{a^3}{(x^2 + a^2)^{3/2}} \right). B_x = \frac{1}{2} B_c \text{ says } \frac{a^3}{(x^2 + a^2)^{3/2}} = \frac{1}{2}, \text{ and } (x^2 + a^2)^3 = 4a^6.$$

Since $a = 0.024 \text{ m}$, $x = 0.0184 \text{ m}$.

EVALUATE: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.33. IDENTIFY: Apply Eq.(28.16).

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.020 m.

EXECUTE: (a) $B_{\text{center}} = \frac{\mu_0 NI}{2a} = \frac{\mu_0 (600)(0.500 \text{ A})}{2(0.020 \text{ m})} = 9.42 \times 10^{-3} \text{ T}$.

(b) $B(x) = \frac{\mu_0 NI a^2}{2(x^2 + a^2)^{3/2}}$. $B(0.08 \text{ m}) = \frac{\mu_0 (600)(0.500 \text{ A})(0.020 \text{ m})^2}{2((0.080 \text{ m})^2 + (0.020 \text{ m})^2)^{3/2}} = 1.34 \times 10^{-4} \text{ T}$.

EVALUATE: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

- 28.34. IDENTIFY and SET UP:** The magnetic field at a point on the axis of N circular loops is given by

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}. \text{ Solve for } N \text{ and set } x = 0.0600 \text{ m.}$$

$$\text{EXECUTE: } N = \frac{2B_x(x^2 + a^2)^{3/2}}{\mu_0 I a^2} = \frac{2(6.39 \times 10^{-4} \text{ T})[(0.0600 \text{ m})^2 + (0.0600 \text{ m})^2]^{3/2}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.50 \text{ A})(0.0600 \text{ m})^2} = 69.$$

EVALUATE: At the center of the coil the field is $B_x = \frac{\mu_0 N I}{2a} = 1.8 \times 10^{-3} \text{ T}$. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

- 28.35. IDENTIFY:** Apply Ampere's law.

$$\text{SET UP: } \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

$$\text{EXECUTE: (a) } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83 \times 10^{-4} \text{ T} \cdot \text{m} \text{ and } I_{\text{encl}} = 305 \text{ A.}$$

(b) $-3.83 \times 10^{-4} \text{ T} \cdot \text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B} \cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

- 28.36. IDENTIFY:** Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

$$\text{EXECUTE: Path a: } I_{\text{encl}} = 0 \Rightarrow \oint \vec{B} \cdot d\vec{l} = 0.$$

$$\text{Path b: } I_{\text{encl}} = -I_1 = -4.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = -\mu_0(4.0 \text{ A}) = -5.03 \times 10^{-6} \text{ T} \cdot \text{m.}$$

$$\text{Path c: } I_{\text{encl}} = -I_1 + I_2 = -4.0 \text{ A} + 6.0 \text{ A} = 2.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0(2.0 \text{ A}) = 2.51 \times 10^{-6} \text{ T} \cdot \text{m}$$

$$\text{Path d: } I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = +\mu_0(4.0 \text{ A}) = 5.03 \times 10^{-6} \text{ T} \cdot \text{m.}$$

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

- 28.37. IDENTIFY:** Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r . By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

$$\text{EXECUTE: (a) For } a < r < b, I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}.$$

(b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

- 28.38. IDENTIFY:** Apply Ampere's law to calculate $\vec{B}$.

(a) **SET UP:** For $a < r < b$ the end view is shown in Figure 28.38a.

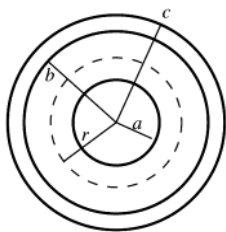


Figure 28.38a

Apply Ampere's law to a circle of radius r , where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

$$\text{EXECUTE: } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1$$

$$\text{Thus } B(2\pi r) = \mu_0 I_1 \text{ and } B = \frac{\mu_0 I_1}{2\pi r}$$

(b) **SET UP:** $r > c$: See Figure 28.38b.

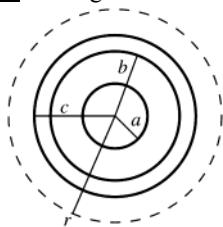


Figure 28.38b

Apply Ampere's law to a circle of radius r , where $r > c$. Both currents are in the positive direction.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$, $I_{\text{encl}} = I_1 + I_2$

Thus $B(2\pi r) = \mu_0(I_1 + I_2)$ and $B = \frac{\mu_0(I_1 + I_2)}{2\pi r}$

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

- 28.39. IDENTIFY:** The largest value of the field occurs at the surface of the cylinder. Inside the cylinder, the field increases linearly from zero at the center, and outside the field decreases inversely with distance from the central axis of the cylinder.

SET UP: At the surface of the cylinder, $B = \frac{\mu_0 I}{2\pi R}$, inside the cylinder, Eq. 28.21 gives $B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}$, and outside the field is $B = \frac{\mu_0 I}{2\pi r}$.

EXECUTE: For points inside the cylinder, the field is half its maximum value when $\frac{\mu_0 I}{2\pi} \frac{r}{R^2} = \frac{1}{2} \left(\frac{\mu_0 I}{2\pi R} \right)$, which gives $r = R/2$. Outside the cylinder, we have $\frac{\mu_0 I}{2\pi r} = \frac{1}{2} \left(\frac{\mu_0 I}{2\pi R} \right)$, which gives $r = 2R$.

EVALUATE: The field has half its maximum value at all points on cylinders coaxial with the wire but of radius $R/2$ and of radius $2R$.

28.40. IDENTIFY: $B = \mu_0 nI = \frac{\mu_0 NI}{L}$

SET UP: $L = 0.150 \text{ m}$

EXECUTE: $B = \frac{\mu_0 (600) (8.00 \text{ A})}{(0.150 \text{ m})} = 0.0402 \text{ T}$

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length.

- 28.41. (a) IDENTIFY and SET UP:** The magnetic field near the center of a long solenoid is given by Eq.(28.23), $B = \mu_0 nI$.

EXECUTE: Turns per unit length $n = \frac{B}{\mu_0 I} = \frac{0.0270 \text{ T}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(12.0 \text{ A})} = 1790 \text{ turns/m}$

(b) $N = nL = (1790 \text{ turns/m})(0.400 \text{ m}) = 716 \text{ turns}$

Each turn of radius R has a length $2\pi R$ of wire. The total length of wire required is

$N(2\pi R) = (716)(2\pi)(1.40 \times 10^{-2} \text{ m}) = 63.0 \text{ m}$.

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.42. IDENTIFY and SET UP: At the center of a long solenoid $B = \mu_0 nI = \mu_0 \frac{N}{L} I$.

EXECUTE: $I = \frac{BL}{\mu_0 N} = \frac{(0.150 \text{ T})(1.40 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4000)} = 41.8 \text{ A}$

EVALUATE: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

- 28.43. IDENTIFY and SET UP:** Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: (a) $B = \frac{\mu_0 I}{2\pi r}$ so $I = \frac{2\pi B}{\mu_0} = \frac{2\pi(2.00 \times 10^{-2} \text{ m})(37.2 \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 3.72 \times 10^6 \text{ A}.$

(b) $B = \frac{N\mu_0 I}{2R}$ so $I = \frac{2RB}{N\mu_0} = \frac{2(0.210 \text{ m})(37.2 \text{ T})}{(100)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 1.24 \times 10^5 \text{ A}.$

(c) $B = \mu_0 \frac{N}{L} I$ so $I = \frac{BL}{\mu_0 N} = \frac{(37.2 \text{ T})(0.320 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(40,000)} = 237 \text{ A}.$

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

- 28.44. IDENTIFY:** Example 28.10 shows that outside a toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \frac{\mu_0 NI}{2\pi r}.$

SET UP: The torus extends from $r_1 = 15.0 \text{ cm}$ to $r_2 = 18.0 \text{ cm}.$

EXECUTE: (a) $r = 0.12 \text{ m}$, which is outside the torus, so $B = 0.$

(b) $r = 0.16 \text{ m}$, so $B = \frac{\mu_0 NI}{2\pi r} = \frac{\mu_0(250)(8.50 \text{ A})}{2\pi(0.160 \text{ m})} = 2.66 \times 10^{-3} \text{ T}.$

(c) $r = 0.20 \text{ m}$, which is outside the torus, so $B = 0.$

EVALUATE: The magnetic field inside the torus is proportional to $1/r$, so it varies somewhat over the cross-section of the torus.

- 28.45. IDENTIFY:** Example 28.10 shows that inside a toroidal solenoid, $B = \frac{\mu_0 NI}{2\pi r}.$

SET UP: $r = 0.070 \text{ m}$

EXECUTE: $B = \frac{\mu_0 NI}{2\pi r} = \frac{\mu_0(600)(0.650 \text{ A})}{2\pi(0.070 \text{ m})} = 1.11 \times 10^{-3} \text{ T}.$

EVALUATE: If the radial thickness of the torus is small compared to its mean diameter, B is approximately uniform inside its windings.

- 28.46. IDENTIFY:** Use Eq.(28.24), with μ_0 replaced by $\mu = K_m \mu_0$, with $K_m = 80.$

SET UP: The contribution from atomic currents is the difference between B calculated with μ and B calculated with $\mu_0.$

EXECUTE: (a) $B = \frac{\mu NI}{2\pi r} = \frac{K_m \mu_0 NI}{2\pi r} = \frac{\mu_0(80)(400)(0.25 \text{ A})}{2\pi(0.060 \text{ m})} = 0.0267 \text{ T}.$

(b) The amount due to atomic currents is $B' = \frac{79}{80} B = \frac{79}{80}(0.0267 \text{ T}) = 0.0263 \text{ T}.$

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

- 28.47. IDENTIFY and SET UP:** $B = \frac{K_m \mu_0 NI}{2\pi r}$ (Eq.28.24, with μ_0 replaced by $K_m \mu_0$)

EXECUTE: (a) $K_m = 1400$

$I = \frac{2\pi r B}{\mu_0 K_m N} = \frac{(2.90 \times 10^{-2} \text{ m})(0.350 \text{ T})}{(2 \times 10^{-7} \text{ T} \cdot \text{m/A})(1400)(500)} = 0.0725 \text{ A}$

(b) $K_m = 5200$

$I = \frac{2\pi r B}{\mu_0 K_m N} = \frac{(2.90 \times 10^{-2} \text{ m})(0.350 \text{ T})}{(2 \times 10^{-7} \text{ T} \cdot \text{m/A})(5200)(500)} = 0.0195 \text{ A}$

EVALUATE: If the solenoid were air-filled instead, a much larger current would be required to produce the same magnetic field.

- 28.48. IDENTIFY:** Apply $B = \frac{K_m \mu_0 NI}{2\pi r}.$

SET UP: K_m is the relative permeability and $\chi_m = K_m - 1$ is the magnetic susceptibility.

EXECUTE: (a) $K_m = \frac{2\pi r B}{\mu_0 NI} = \frac{2\pi(0.2500 \text{ m})(1.940 \text{ T})}{\mu_0(500)(2.400 \text{ A})} = 2021.$

(b) $\chi_m = K_m - 1 = 2020.$

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021 = 9.6 \times 10^{-4} \text{ T}.$ The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.49. IDENTIFY: The magnetic field from the solenoid alone is $B_0 = \mu_0 nI$. The total magnetic field is $B = K_m B_0$. M is given by Eq.(28.29).

SET UP: $n = 6000$ turns/m

EXECUTE: (a) (i) $B_0 = \mu_0 nI = \mu_0 (6000 \text{ m}^{-1})(0.15 \text{ A}) = 1.13 \times 10^{-3} \text{ T}$.

(ii) $M = \frac{K_m - 1}{\mu_0} B_0 = \frac{5199}{\mu_0} (1.13 \times 10^{-3} \text{ T}) = 4.68 \times 10^6 \text{ A/m}$.

(iii) $B = K_m B_0 = (5200)(1.13 \times 10^{-3} \text{ T}) = 5.88 \text{ T}$.

(b) The directions of $\vec{B}$, $\vec{B}_0$ and $\vec{M}$ are shown in Figure 28.49. Silicon steel is paramagnetic and $\vec{B}_0$ and $\vec{M}$ are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

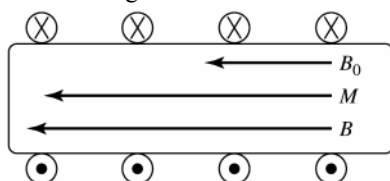


Figure 28.49

28.50. IDENTIFY: Curie's law (Eq.28.32) says that $1/M$ is proportional to T , so $1/\chi_m$ is proportional to T .

SET UP: The graph of $1/\chi_m$ versus the Kelvin temperature is given in Figure 28.50.

EXECUTE: The material does obey Curie's law because the graph in Figure 28.50 is a straight line. $M = C \frac{B}{T}$ and

$M = \frac{B - B_0}{\mu_0}$ says that $\chi_m = \frac{C\mu_0}{T}$. $1/\chi_m = \frac{T}{C\mu_0}$ and the slope of $1/\chi_m$ versus T is $1/(C\mu_0)$. Therefore, from the

graph the Curie constant is $C = \frac{1}{\mu_0(\text{slope})} = \frac{1}{\mu_0(5.13 \text{ K}^{-1})} = 1.55 \times 10^5 \text{ K} \cdot \text{A/T} \cdot \text{m}$.

EVALUATE: For this material Curie's law is valid over a wide temperature range.

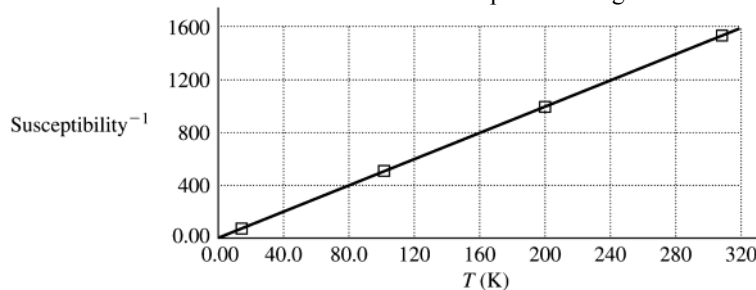


Figure 28.50

28.51. IDENTIFY: Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) **SET UP:** The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$. Both fields are into the paper, so

their magnitudes add, giving $B_{\text{net}} = B + B' = \frac{\mu_0}{4\pi} \left(\frac{qv \sin \phi}{r^2} + \frac{q'v' \sin \phi'}{r'^2} \right)$.

EXECUTE: Substituting numbers gives

$$B_{\text{net}} = \frac{\mu_0}{4\pi} \left[\frac{(8.00 \mu\text{C})(9.0 \times 10^4 \text{ m/s}) \sin 90^\circ}{(0.300 \text{ m})^2} + \frac{(5.00 \mu\text{C})(6.50 \times 10^4 \text{ m/s}) \sin 90^\circ}{(0.400 \text{ m})^2} \right]$$

$B_{\text{net}} = 1.00 \times 10^{-6} \text{ T} = 1.00 \mu\text{T}$, into the paper.

(b) **SET UP:** The magnetic force on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v} \times \vec{B}' = (qv)\hat{i} \times \frac{\mu_0}{4\pi} \frac{q\vec{v}' \times \hat{r}}{r^2} = (qv)\hat{i} \times \left(\frac{\mu_0}{4\pi} \frac{qv' \sin \phi}{r^2} \right) (-\hat{k}) = \left(\frac{\mu_0}{4\pi} \frac{qq'vv' \sin \phi}{r^2} \right) \hat{j}$$

where ϕ is the angle between $\vec{v}'$ and $\hat{r}'$.

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi} \left[\frac{(8.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})(9.00 \times 10^{-6} \text{ m/s})(6.50 \times 10^{-6} \text{ m/s})}{(0.500 \text{ m})^2} \right] \left(\frac{0.400}{0.500} \right) \hat{j}$$

$$\vec{F} = (7.49 \times 10^{-8} \text{ N}) \hat{j}.$$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.52. IDENTIFY: The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \frac{\mu_0 I}{2\pi r}$, and the force on a moving charge is $F = qvB \sin \phi$.

EXECUTE: $F = qvB \sin \phi = (ev\mu_0 I \sin \phi) / 2\pi r$. Substituting numbers gives

$$F = (1.60 \times 10^{-19} \text{ C})(6.00 \times 10^4 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.50 \text{ A})(\sin 90^\circ) / [2\pi(0.0450 \text{ m})]$$

$$F = 1.07 \times 10^{-19} \text{ N}$$

From the right hand rule for the cross product, the direction of $\vec{v} \times \vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of about 10^{11} m/s^2 .

28.53. IDENTIFY: Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q|vB \sin \phi$ and direction given by the right-hand rule.

For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: (a) $a = \frac{F}{m} = \frac{|q|vB \sin \phi}{m} = \frac{ev}{m} \left(\frac{\mu_0 I}{2\pi r} \right)$

$$a = \frac{(1.6 \times 10^{-17} \text{ C})(2.50 \times 10^5 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(25.0 \text{ A})}{(9.11 \times 10^{-31} \text{ kg})(2\pi)(0.0200 \text{ m})} = 1.1 \times 10^{13} \text{ m/s}^2,$$

away from the wire.

(b) The electric force must balance the magnetic force. $eE = evB$, and

$$E = vB = v \frac{\mu_0 I}{2\pi r} = \frac{(250,000 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(25.0 \text{ A})}{2\pi(0.0200 \text{ m})} = 62.5 \text{ N/C}.$$

The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: (c) $mg = (9.11 \times 10^{-31} \text{ kg})(9.8 \text{ m/s}^2) \approx 10^{-29} \text{ N}$. $F_{\text{el}} = eE = (1.6 \times 10^{-19} \text{ C})(62.5 \text{ N/C}) \approx 10^{-17} \text{ N}$.

$F_{\text{el}} \approx 10^{12} F_{\text{grav}}$, so we can neglect gravity.

28.54. IDENTIFY: Use Eq.(28.9) and the right-hand rule to calculate the magnetic field due to each wire. Add these field vectors to calculate the net field and then use Eq.(27.2) to calculate the force.

SET UP: Let the wire connected to the 25.0Ω resistor be #2 and the wire connected to the 10.0Ω resistor be #1.

Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton B_2 is $\otimes$ and B_1 is $\odot$.

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \text{ and } B_2 = \frac{\mu_0 I_2}{2\pi r}, \text{ with } r = 0.0250 \text{ m. } I_1 = (100.0 \text{ V}) / (10.0 \Omega) = 10.0 \text{ A and } I_2 = (100.0 \text{ V}) / (25.0 \Omega) = 4.00 \text{ A}$$

EXECUTE: $B_1 = 8.00 \times 10^{-5} \text{ T}$, $B_2 = 3.20 \times 10^{-5} \text{ T}$ and $B = B_1 - B_2 = 4.80 \times 10^{-5} \text{ T}$ and in the direction $\odot$.

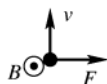


Figure 28.54

Force is to the right.

$$F = qvB = (1.602 \times 10^{-19} \text{ C})(650 \times 10^3 \text{ m/s})(4.80 \times 10^{-5} \text{ T}) = 5.00 \times 10^{-18} \text{ N}$$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnetic force is much larger than the gravity force on the proton.

28.55. IDENTIFY: Find the net magnetic field due to the two loops at the location of the proton and then find the force these fields exert on the proton.

SET UP: For a circular loop, the field on the axis, a distance x from the center of the loop is $B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$.

$R = 0.200 \text{ m}$ and $x = 0.125 \text{ m}$.

EXECUTE: The fields add, so $B = B_1 + B_2 = 2B_1 = 2 \left[\frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \right]$.

$$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.50 \text{ A})(0.200 \text{ m})^2}{[(0.200 \text{ m})^2 + (0.125 \text{ m})^2]^{3/2}} = 5.75 \times 10^{-6} \text{ T}.$$

$F = |q|vB \sin \phi = (1.6 \times 10^{-19} \text{ C})(2400 \text{ m/s})(5.75 \times 10^{-6} \text{ T}) \sin 90^\circ = 2.21 \times 10^{-21} \text{ N}$, perpendicular to the line ab and to the velocity.

EVALUATE: The weight of a proton is $w = mg = 1.6 \times 10^{-24} \text{ N}$, so the force from the loops is much greater than the gravity force on the proton.

28.56. IDENTIFY: The net magnetic field is the vector sum of the fields due to each wire.

SET UP: $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: (a) The currents are the same so points where the two fields are equal in magnitude are equidistant from the two wires. The net field is zero along the dashed line shown in Figure 28.56a.

(b) For the magnitudes of the two fields to be the same at a point, the point must be 3 times closer to the wire with the smaller current. The net field is zero along the dashed line shown in Figure 28.56b.

(c) As in (a), the points are equidistant from both wires. The net field is zero along the dashed line shown in Figure 28.56c.

EVALUATE: The lines of zero net field consist of points at which the fields of the two wires have opposite directions and equal magnitudes.

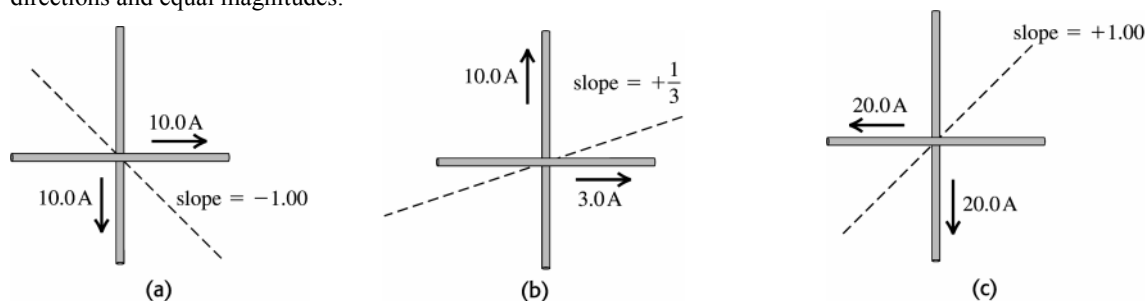


Figure 28.56

28.57. IDENTIFY: $\vec{B} = \frac{\mu_0 q \vec{v}_0 \times \hat{r}}{4\pi r^2}$

SET UP: $\hat{r} = \hat{i}$ and $r = 0.250 \text{ m}$, so $\vec{v}_0 \times \hat{r} = v_{0z} \hat{j} - v_{0y} \hat{k}$.

EXECUTE: $\vec{B} = \frac{\mu_0 q}{4\pi r^2} (v_{0z} \hat{j} - v_{0y} \hat{k}) = (6.00 \times 10^{-6} \text{ T}) \hat{j}$. $v_{0y} = 0$. $\frac{\mu_0 q}{4\pi r^2} v_{0z} = 6.00 \times 10^{-6} \text{ T}$ and

$$v_{0z} = \frac{4\pi (6.00 \times 10^{-6} \text{ T})(0.25 \text{ m})^2}{\mu_0 (-7.20 \times 10^{-3} \text{ C})} = -521 \text{ m/s}. \quad v_{0x} = \pm \sqrt{v_0^2 - v_{0y}^2 - v_{0z}^2} = \pm \sqrt{(800 \text{ m/s})^2 - (-521 \text{ m/s})^2} = \pm 607 \text{ m/s}.$$

The sign of v_{0x} isn't determined.

(b) Now $\vec{r} = \hat{j}$ and $r = 0.250 \text{ m}$. $\vec{B} = \frac{\mu_0 q \vec{v}_0 \times \hat{r}}{4\pi r^2} = \frac{\mu_0 q}{4\pi r^2} (v_{0x} \hat{k} - v_{0z} \hat{i})$.

$$B = \frac{\mu_0 |q|}{4\pi r^2} \sqrt{v_{0x}^2 + v_{0z}^2} = \frac{\mu_0 |q|}{4\pi r^2} v_0 = \frac{\mu_0 (7.20 \times 10^{-3} \text{ C})}{4\pi (0.250 \text{ m})^2} 800 \text{ m/s} = 9.20 \times 10^{-6} \text{ T}.$$

EVALUATE: The magnetic field in part (b) doesn't depend on the sign of v_{0x} .

28.58. IDENTIFY and SET UP: $\vec{B} = B_0(x/a)\hat{i}$

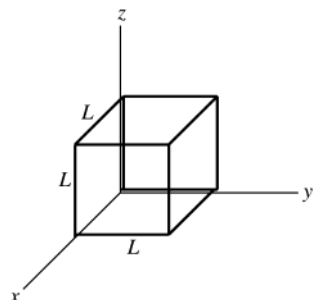


Figure 28.58

Apply Gauss's law for magnetic fields to a cube with side length L , one corner at the origin, and sides parallel to the x , y and z axes, as shown in Figure 28.58.

EXECUTE: Since $\vec{B}$ is parallel to the x -axis the only sides that have nonzero flux are the front side (parallel to the yz -plane at $x = L$) and the back side (parallel to the yz -plane at $x = 0$.)

front $\Phi_B = \int \vec{B} \cdot d\vec{A} = B_0(x/a) \int dA(\hat{i} \cdot \hat{i}) = B_0(x/a) \int dA$

$x = L$ on this face so $\vec{B} \cdot d\vec{A} = B_0(L/a) dA$

$\Phi_B = B_0(L/a) \int dA = B_0(L/a)L^2 = B_0(L^3/a)$

back On the back face $x = 0$ so $B = 0$ and $\Phi_B = 0$. The total flux through the cubical Gaussian surface is $\Phi_B = B_0(L^3/a)$.

EVALUATE: This violates Eq.(27.8), which says that $\Phi_B = 0$ for any closed surface. The claimed $\vec{B}$ is impossible because it has been shown to violate Gauss's law for magnetism.

- 28.59. IDENTIFY:** Use Eq.(28.9) and the right-hand rule to calculate the magnitude and direction of the magnetic field at P produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point P due to the two wires are sketched in Figure 28.59a.

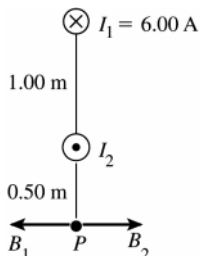


Figure 28.59a

EXECUTE: $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at P to be zero. $\vec{B}_2$ is to the right so I_2 is out of the page.

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi} \left(\frac{6.00 \text{ A}}{1.50 \text{ m}} \right) \quad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi} \left(\frac{I_2}{0.50 \text{ m}} \right)$$

$$B_1 = B_2 \text{ says } \frac{\mu_0}{2\pi} \left(\frac{6.00 \text{ A}}{1.50 \text{ m}} \right) = \frac{\mu_0}{2\pi} \left(\frac{I_2}{0.50 \text{ m}} \right)$$

$$I_2 = \left(\frac{0.50 \text{ m}}{1.50 \text{ m}} \right) (6.00 \text{ A}) = 2.00 \text{ A}$$

(b) SET UP: The directions of the fields at point Q are sketched in Figure 28.59b.

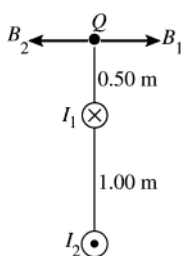


Figure 28.59b

EXECUTE: $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$$B_1 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{6.00 \text{ A}}{0.50 \text{ m}} \right) = 2.40 \times 10^{-6} \text{ T}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$$

$$B_2 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{2.00 \text{ A}}{1.50 \text{ m}} \right) = 2.67 \times 10^{-7} \text{ T}$$

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$ so

$$B = B_1 - B_2 = 2.40 \times 10^{-6} \text{ T} - 2.67 \times 10^{-7} \text{ T} = 2.13 \times 10^{-6} \text{ T}, \text{ and } \vec{B} \text{ is to the right.}$$

(c) SET UP: The directions of the fields at point S are sketched in Figure 28.59c.

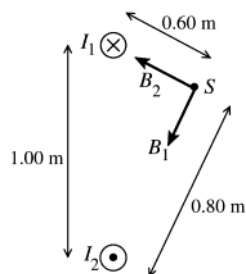


Figure 28.59c

EXECUTE: $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$$B_1 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{6.00 \text{ A}}{0.60 \text{ m}} \right) = 2.00 \times 10^{-6} \text{ T}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$$

$$B_2 = (2 \times 10^{-7} \text{ T} \cdot \text{m/A}) \left(\frac{2.00 \text{ A}}{0.80 \text{ m}} \right) = 5.00 \times 10^{-7} \text{ T}$$

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00 \times 10^{-6} \text{ T})^2 + (5.00 \times 10^{-7} \text{ T})^2} = 2.06 \times 10^{-6} \text{ T}$$

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so $\vec{B}$ is perpendicular to the line from the wire to the point where the field is calculated.

28.60. IDENTIFY: Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule and is perpendicular to the line from the wire to the point where field is calculated.

EXECUTE: (a) The magnetic field vectors are shown in Figure 28.60a.

(b) At a position on the x-axis $B_{\text{net}} = 2 \frac{\mu_0 I}{2\pi r} \sin \theta = \frac{\mu_0 I}{\pi \sqrt{x^2 + a^2}} \frac{a}{\sqrt{x^2 + a^2}} = \frac{\mu_0 I a}{\pi (x^2 + a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.60b.

EVALUATE: (d) The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x \gg a$, $B \approx \frac{\mu_0 I a}{\pi x^2}$.

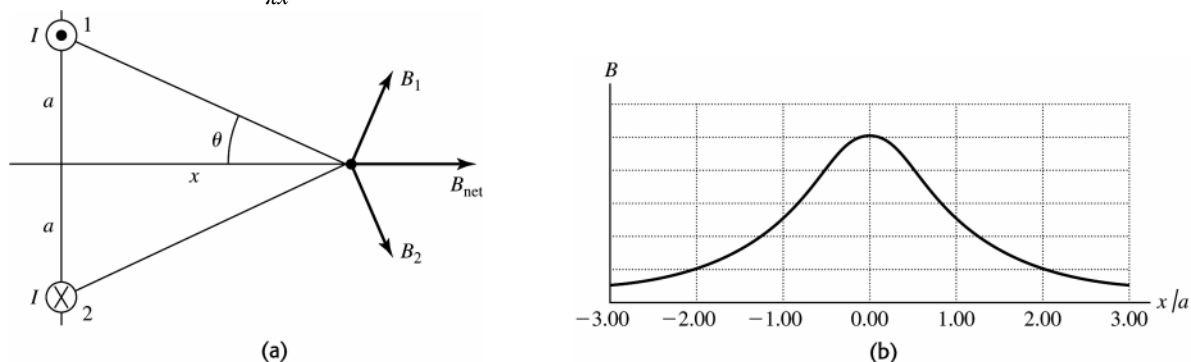


Figure 28.60

28.61. IDENTIFY: Apply $F = IlB \sin \phi$, with the magnetic field at point P that is calculated in problem 28.60.

SET UP: The net field of the first two wires at the location of the third wire is $B = \frac{\mu_0 I a}{\pi (x^2 + a^2)}$, in the $+x$ -direction.

EXECUTE: (a) Wire is carrying current into the page, so it feels a force in the $-y$ -direction.

$$\frac{F}{L} = IB = I \left(\frac{\mu_0 I a}{\pi (x^2 + a^2)} \right) = \frac{\mu_0 (6.00 \text{ A})^2 (0.400 \text{ m})}{\pi ((0.600 \text{ m})^2 + (0.400 \text{ m})^2)} = 1.11 \times 10^{-5} \text{ N/m}.$$

(b) If the wire carries current out of the page then the force felt will be in the opposite direction as in part (a). Thus the force will be $1.11 \times 10^{-5} \text{ N/m}$, in the $+y$ -direction.

EVALUATE: We could also calculate the force exerted by each of the first two wires and find the vector sum of the two forces.

28.62. IDENTIFY: The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring.

SET UP: The force of the spring is kx and the magnetic force on each wire is $F_{\text{mag}} = \frac{\mu_0 I^2 L}{2\pi x}$.

EXECUTE: Call x the distance the springs each stretch. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are two spring forces on each wire. Therefore $2kx = \frac{\mu_0 I^2 L}{2\pi x}$, which gives $x = \sqrt{\frac{\mu_0 I^2 L}{2\pi k}}$.

EVALUATE: Since $\mu_0/2\pi$ is small, x will likely be much less than the length of the wires.

28.63. IDENTIFY: Apply $\sum \vec{F} = \vec{0}$ to one of the wires. The force one wire exerts on the other depends on I so $\sum \vec{F} = \vec{0}$ gives two equations for the two unknowns T and I .

SET UP: The force diagram for one of the wires is given in Figure 28.63.

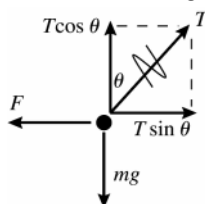


Figure 28.63

The force one wire exerts on the other is $F = \left(\frac{\mu_0 I^2}{2\pi r} \right) L$,

where $r = 2(0.040 \text{ m}) \sin \theta = 8.362 \times 10^{-3} \text{ m}$ is the distance between the two wires.

EXECUTE: $\sum F_y = 0$ gives $T \cos \theta = mg$ and $T = mg / \cos \theta$

$\sum F_x = 0$ gives $F = T \sin \theta = (mg / \cos \theta) \sin \theta = mg \tan \theta$

And $m = \lambda L$, so $F = \lambda L g \tan \theta$

$$\left(\frac{\mu_0 I^2}{2\pi r} \right) L = \lambda L g \tan \theta$$

$$I = \sqrt{\frac{\lambda g r \tan \theta}{(\mu_0 / 2\pi)}}$$

$$I = \sqrt{\frac{(0.0125 \text{ kg/m})(9.80 \text{ m/s}^2)(\tan 6.00^\circ)(8.362 \times 10^{-3} \text{ m})}{2 \times 10^{-7} \text{ T} \cdot \text{m/A}}} = 23.2 \text{ A}$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

28.64. IDENTIFY: Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

$$\text{EXECUTE: } F = F_t - F_b = \left(\frac{\mu_0 I_{\text{wire}}}{2\pi} \right) \left(\frac{Il}{r_t} - \frac{Il}{r_b} \right) = \frac{\mu_0 I I_{\text{wire}}}{2\pi} \left(\frac{1}{r_t} - \frac{1}{r_b} \right).$$

$$F = \frac{\mu_0 (5.00 \text{ A})(0.200 \text{ m})(14.0 \text{ A})}{2\pi} \left(\frac{1}{0.100 \text{ m}} - \frac{1}{0.026 \text{ m}} \right) = 7.97 \times 10^{-5} \text{ N. The force on the top segment is away}$$

from the wire, so the net force is away from the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform, it is stronger closer to the wire.

28.65. IDENTIFY: Find the magnetic field of the first loop at the location of the second loop and apply $\tau = |\vec{\mu} \times \vec{B}|$ and

$U = -\vec{\mu} \cdot \vec{B}$ to find μ and U .

SET UP: Since x is much larger than a' , assume B is uniform over the second loop and equal to its value on the axis of the first loop.

$$\text{EXECUTE: (a) } x \gg a \Rightarrow B = \frac{N\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \approx \frac{N\mu_0 I a^2}{2x^3}.$$

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B \sin \theta = (N'I'a') \left(\frac{N\mu_0 I a^2}{2x^3} \right) \sin \theta = \frac{NN'\mu_0 \pi I I' a^2 a' \sin \theta}{2x^3}$$

$$\text{(b) } U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta = -(N'I'a') \left(\frac{N\mu_0 I a^2}{2x^3} \right) \cos \theta = -\frac{NN'\mu_0 \pi I I' a^2 a' \cos \theta}{2x^3}.$$

EVALUATE: (c) Having $x \gg a$ allows us to simplify the form of the magnetic field, whereas assuming $x \gg a'$ means we can assume that the magnetic field from the first loop is constant over the second loop.

28.66. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P . The field at the center of a circular loop of radius R is

$$B = \frac{\mu_0 I}{2R}, \text{ so the field at the center of curvature of a semicircular loop is } B = \frac{\mu_0 I}{4R}.$$

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \frac{1}{2} \left(\frac{\mu_0 I}{2} \right) \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{\mu_0 I}{4a} \left(1 - \frac{a}{b} \right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.67. IDENTIFY: Find the vector sum of the fields due to each loop.

SET UP: For a single loop $B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field

along the x -axis from between them means that the “ x ” in the formula is different for each case:

EXECUTE:

$$\text{Left coil: } x \rightarrow x + \frac{a}{2} \Rightarrow B_l = \frac{\mu_0 N I a^2}{2((x + a/2)^2 + a^2)^{3/2}}.$$

$$\text{Right coil: } x \rightarrow x - \frac{a}{2} \Rightarrow B_r = \frac{\mu_0 N I a^2}{2((x - a/2)^2 + a^2)^{3/2}}.$$

So, the total field at a point a distance x from the point between them is

$$B = \frac{\mu_0 N I a^2}{2} \left(\frac{1}{((x + a/2)^2 + a^2)^{3/2}} + \frac{1}{((x - a/2)^2 + a^2)^{3/2}} \right).$$

(b) B versus x is graphed in Figure 28.67. Figure 28.67a is the total field and Figure 28.67b is the field from the right-hand coil.

(c) At point P , $x = 0$ and $B = \frac{\mu_0 N I a^2}{2} \left(\frac{1}{((a/2)^2 + a^2)^{3/2}} + \frac{1}{((-a/2)^2 + a^2)^{3/2}} \right) = \frac{\mu_0 N I a^2}{(5a^2/4)^{3/2}} = \left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 N I}{a}$

(d) $B = \left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 N I}{a} = \left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 (300)(6.00 \text{ A})}{(0.080 \text{ m})} = 0.0202 \text{ T}.$

(e) $\frac{dB}{dx} = \frac{\mu_0 N I a^2}{2} \left(\frac{-3(x + a/2)}{((x + a/2)^2 + a^2)^{5/2}} + \frac{-3(x - a/2)}{((x - a/2)^2 + a^2)^{5/2}} \right).$ At $x = 0$,

$$\left. \frac{dB}{dx} \right|_{x=0} = \frac{\mu_0 N I a^2}{2} \left(\frac{-3(a/2)}{((a/2)^2 + a^2)^{5/2}} + \frac{-3(-a/2)}{((-a/2)^2 + a^2)^{5/2}} \right) = 0.$$

$$\frac{d^2B}{dx^2} = \frac{\mu_0 N I a^2}{2} \left(\frac{-3}{((x + a/2)^2 + a^2)^{5/2}} + \frac{6(x + a/2)^2(5/2)}{((x + a/2)^2 + a^2)^{7/2}} + \frac{-3}{((x - a/2)^2 + a^2)^{5/2}} + \frac{6(x - a/2)^2(5/2)}{((x - a/2)^2 + a^2)^{7/2}} \right)$$

At $x = 0$, $\left. \frac{d^2B}{dx^2} \right|_{x=0} = \frac{\mu_0 N I a^2}{2} \left(\frac{-3}{((a/2)^2 + a^2)^{5/2}} + \frac{6(a/2)^2(5/2)}{((a/2)^2 + a^2)^{7/2}} + \frac{-3}{((a/2)^2 + a^2)^{5/2}} + \frac{6(-a/2)^2(5/2)}{((a/2)^2 + a^2)^{7/2}} \right) = 0.$

EVALUATE: Since both first and second derivatives are zero, the field can only be changing very slowly.

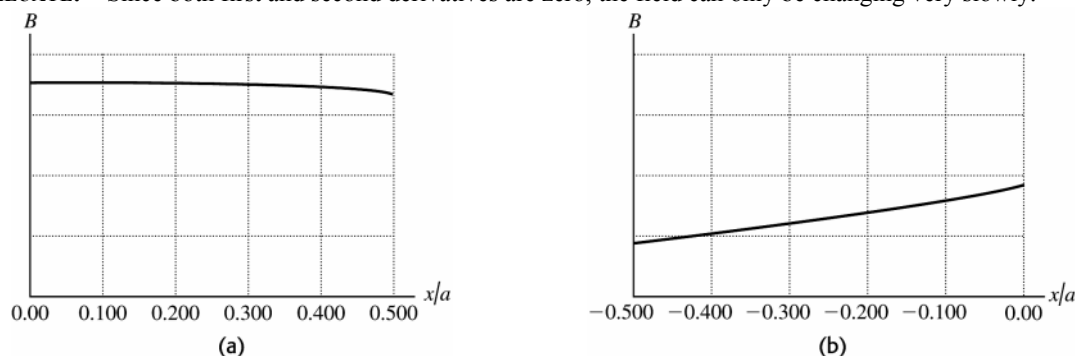


Figure 28.67

28.68. IDENTIFY: A current-carrying wire produces a magnetic field, but the strength of the field depends on the shape of the wire.

SET UP: The magnetic field at the center of a circular wire of radius a is $B = \mu_0 I / 2a$, and the field a distance x

from the center of a straight wire of length $2a$ is $B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}.$

EXECUTE: (a) Since the diameter $D = 2a$, we have $B = \mu_0 I / 2a = \mu_0 I / D.$

(b) In this case, the length of the wire is equal to the diameter of the circle, so $2a = \pi D$, giving $a = \pi D / 2$, and

$x = D/2.$ Therefore $B = \frac{\mu_0 I}{4\pi} \frac{2(\pi D/2)}{(D/2)\sqrt{D^2/4 + \pi^2 D^2/4}} = \frac{\mu_0 I}{D\sqrt{1 + \pi^2}}.$

EVALUATE: The field in part (a) is greater by a factor of $\sqrt{1 + \pi^2}$. It is reasonable that the field due to the circular wire is greater than the field due to the straight wire because more of the current is close to point A for the circular wire than it is for the straight wire.

28.69. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}.$

SET UP: The contribution from the straight segments is zero since $d\vec{l} \times \hat{r} = 0$. The magnetic field from the curved wire is just one quarter of a full loop.

EXECUTE: $B = \frac{1}{4} \left(\frac{\mu_0 I}{2R} \right) = \frac{\mu_0 I}{8R}$ and is directed out of the page.

EVALUATE: It is very simple to calculate B at point P but it would be much more difficult to calculate B at other points.

28.70. IDENTIFY: Apply $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The horizontal wire yields zero magnetic field since $d\vec{l} \times \vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2} \left(\frac{\mu_0 I}{2\pi R} \right) = \frac{\mu_0 I}{4\pi R}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq.(28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B .

28.71. (a) IDENTIFY: Consider current density J for a small concentric ring and integrate to find the total current in terms of α and R .

SET UP: We can't say $I = JA = J\pi R^2$, since J varies across the cross section.

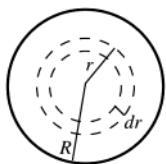


Figure 28.71

To integrate J over the cross section of the wire divide the wire cross section up into thin concentric rings of radius r and width dr , as shown in Figure 28.71.

EXECUTE: The area of such a ring is dA , and the current through it is $dI = J dA$; $dA = 2\pi r dr$ and $dI = J dA = \alpha r(2\pi r dr) = 2\pi\alpha r^2 dr$

$$I = \int dI = 2\pi\alpha \int_0^R r^2 dr = 2\pi\alpha(R^3/3) \text{ so } \alpha = \frac{3I}{2\pi R^3}$$

(b) IDENTIFY and SET UP: (i) $r \leq R$

Apply Ampere's law to a circle of radius $r < R$. Use the method of part (a) to find the current enclosed by the Ampere's law path.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through

the path is $I_{\text{encl}} = \int dI$, where the integration is from 0 to r . $I_{\text{encl}} = 2\pi\alpha \int_0^r r^2 dr = \frac{2\pi\alpha r^3}{3} - \frac{2\pi}{3} \left(\frac{3I}{2\pi R^3} \right) r^3 = \frac{Ir^3}{R^3}$. Thus

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \text{ gives } B(2\pi r) = \mu_0 \left(\frac{Ir^3}{R^3} \right) \text{ and } B = \frac{\mu_0 Ir^2}{2\pi R^3}$$

(ii) **IDENTIFY and SET UP:** $r \geq R$

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B(2\pi r)$

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0 I$ and $B = \frac{\mu_0 I}{2\pi r}$

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \leq R$) gives $B = \frac{\mu_0 I}{2\pi R}$. At $r = R$ the expression in (ii)

(for $r \geq R$) gives $B = \frac{\mu_0 I}{2\pi R}$, which is the same.

28.72. IDENTIFY: Apply Ampere's law to a circle of radius r in each case.

SET UP: Assume that the currents are uniform over the cross sections of the conductors.

EXECUTE: (a) $r < a \Rightarrow I_{\text{encl}} = I \left(\frac{A_r}{A_a} \right) = I \left(\frac{r^2}{a^2} \right)$. $\oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I \left(\frac{r^2}{a^2} \right)$ and $B = \frac{\mu_0 Ir}{2\pi a^2}$. When

$r = a$, $B = \frac{\mu_0 I}{2\pi a}$, which is just what was found in part (a) of Exercise 28.37.

$$(b) \quad b < r < c \Rightarrow I_{\text{encl}} = I - I \left(\frac{A_{b \rightarrow r}}{A_{b \rightarrow c}} \right) = I \left(1 - \frac{r^2 - b^2}{c^2 - b^2} \right). \oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I \left(1 - \frac{r^2 - b^2}{c^2 - b^2} \right) = \mu_0 I \left(\frac{c^2 - r^2}{c^2 - b^2} \right) \text{ and}$$

$$B = \frac{\mu_0 I}{2\pi r} \left(\frac{c^2 - r^2}{c^2 - b^2} \right). \text{ When } r = b, B = \frac{\mu_0 I}{2\pi b}, \text{ just as in part (a) of Exercise 28.37 and when } r = c, B = 0, \text{ just as in}$$

part (b) of Exercise 28.37.

EVALUATE: Unlike E , B is not zero within the conductors. B varies across the cross section of each conductor.

28.73. IDENTIFY: Apply $\oint \vec{B} \cdot d\vec{A} = 0$.

SET UP: Take the closed gaussian surface to be a cylinder whose axis coincides with the wire.

EXECUTE: If there is a magnetic field component in the z -direction, it must be constant because of the symmetry of the wire. Therefore the contribution to a surface integral over a closed cylinder, encompassing a long straight wire will be zero: no flux through the barrel of the cylinder, and equal but opposite flux through the ends. The radial field will have no contribution through the ends, but through the barrel:

$$0 = \oint \vec{B} \cdot d\vec{A} = \oint \vec{B}_r \cdot d\vec{A} = \int_{\text{barrel}} \vec{B}_r \cdot d\vec{A} = \int_{\text{barrel}} B_r dA = B_r A_{\text{barrel}} = 0. \text{ Therefore, } B_r = 0.$$

EVALUATE: The magnetic field of a long straight wire is everywhere tangent to a circular area whose plane is perpendicular to the wire, with the wire passing through the center of the circular area. This field produces zero flux through the cylindrical gaussian surface.

28.74. IDENTIFY: Apply Ampere's law to a circular path of radius r .

SET UP: Assume the current is uniform over the cross section of the conductor.

EXECUTE: (a) $r < a \Rightarrow I_{\text{encl}} = 0 \Rightarrow B = 0$.

$$(b) \quad a < r < b \Rightarrow I_{\text{encl}} = I \left(\frac{A_{a \rightarrow r}}{A_{a \rightarrow b}} \right) = I \left(\frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)} \right) = I \frac{(r^2 - a^2)}{(b^2 - a^2)}. \oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I \frac{(r^2 - a^2)}{(b^2 - a^2)} \text{ and } B = \frac{\mu_0 I}{2\pi r} \frac{(r^2 - a^2)}{(b^2 - a^2)}.$$

$$(c) \quad r > b \Rightarrow I_{\text{encl}} = I. \oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I \text{ and } B = \frac{\mu_0 I}{2\pi r}.$$

EVALUATE: The expression in part (b) gives $B = 0$ at $r = a$ and this agrees with the result of part (a). The

expression in part (b) gives $B = \frac{\mu_0 I}{2\pi b}$ at $r = b$ and this agrees with the result of part (c).

28.75. IDENTIFY: Use Ampere's law to find the magnetic field at $r = 2a$ from the axis. The analysis of Example 28.9 shows that the field outside the cylinder is the same as for a long, straight wire along the axis of the cylinder.

SET UP:

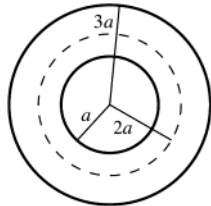


Figure 28.75

EXECUTE: Apply Ampere's law to a circular path of radius $2a$, as shown in Figure 28.75.

$$B(2\pi) = \mu_0 I_{\text{encl}}$$

$$I_{\text{encl}} = I \left(\frac{(2a)^2 - a^2}{(3a)^2 - a^2} \right) = 3I/8$$

$B = \frac{3}{16} \frac{\mu_0 I}{2\pi a}$; this is the magnetic field inside the metal at a distance of $2a$ from the cylinder axis. Outside the

cylinder, $B = \frac{\mu_0 I}{2\pi r}$. The value of r where these two fields are equal is given by $1/r = 3/(16a)$ and $r = 16a/3$.

EVALUATE: For $r < 3a$, as r increases the magnetic field increases from zero at $r = 0$ to $\mu_0 I / (2\pi(3a))$ at $r = 3a$. For $r > 3a$ the field decreases as r increases so it is reasonable for there to be a $r > 3a$ where the field is the same as at $r = 2a$.

28.76. IDENTIFY: The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

SET UP: For the long wire, $B = \frac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \frac{\mu_0 I_2}{2R}$.

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude:

$$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}. \text{ Thus, } I_1 = \frac{\pi D}{R} I_2.$$

EVALUATE: If I_1 is to the left the two fields add.

- 28.77. IDENTIFY:** Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find $\vec{B}$ at the specified distance from the center of the wire.

(a) SET UP:

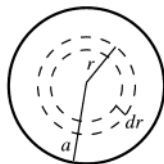


Figure 28.77a

Divide the cross section of the cylinder into thin concentric rings of radius r and width dr , as shown in Figure 28.77a. The current through each ring is $dI = J dA = J 2\pi r dr$.

EXECUTE: $dI = \frac{2I_0}{\pi a^2} \left[1 - (r/a)^2 \right] 2\pi r dr = \frac{4I_0}{a^2} \left[1 - (r/a)^2 \right] r dr$. The total current I is obtained by integrating dI

over the cross section $I = \int_0^a dI = \left(\frac{4I_0}{a^2} \right) \int_0^a (1 - r^2/a^2) r dr = \left(\frac{4I_0}{a^2} \right) \left[\frac{1}{2} r^2 - \frac{1}{4} r^4/a^2 \right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.77b.

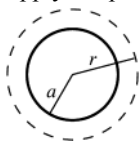


Figure 28.77b

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$$

$$I_{\text{encl}} = I_0 \text{ (the path encloses the entire cylinder)}$$

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \frac{\mu_0 I_0}{2\pi r}$.

(c) SET UP:

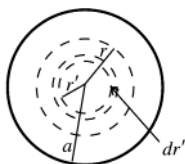


Figure 28.77c

Divide the cross section of the cylinder into concentric rings of radius r' and width dr' , as was done in part (a). See Figure 28.77c. The current dI through each ring

$$\text{is } dI = \frac{4I_0}{a^2} \left[1 - \left(\frac{r'}{a} \right)^2 \right] r' dr'$$

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2} \int_0^r \left[1 - \left(\frac{r'}{a} \right)^2 \right] r' dr' = \frac{4I_0}{a^2} \left[\frac{1}{2} (r')^2 - \frac{1}{4} (r')^4/a^2 \right]_0^r$$

$$I = \frac{4I_0}{a^2} (r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2} \left(2 - \frac{r^2}{a^2} \right)$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.77d.



Figure 28.77d

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$$

$$I_{\text{encl}} = \frac{I_0 r^2}{a^2} \left(2 - \frac{r^2}{a^2} \right) \text{ (from part (c))}$$

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 \frac{I_0 r^2}{a^2} (2 - r^2/a^2)$ and $B = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} (2 - r^2/a^2)$

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \frac{\mu_0 I_0}{2\pi a}$. Result in part (d) evaluated at

$r = a$: $B = \frac{\mu_0 I_0}{2\pi} \frac{a}{a^2} (2 - a^2/a^2) = \frac{\mu_0 I_0}{2\pi a}$. The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

- 28.78. IDENTIFY:** Apply Ampere's law to a circle of radius r .

SET UP: The current within a radius r is $I = \int \vec{J} \cdot d\vec{A}$, where the integration is over a disk of radius r .

EXECUTE: (a) $I_0 = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r} e^{(r-a)/\delta} \right) r dr d\theta = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b \delta \left. e^{(r-a)/\delta} \right|_0^a = 2\pi b \delta (1 - e^{-a/\delta}).$

$I_0 = 2\pi(600 \text{ A/m})(0.025 \text{ m})(1 - e^{(0.050/0.025)}) = 81.5 \text{ A}.$

(b) For $r \geq a$, $\oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \frac{\mu_0 I_0}{2\pi r}.$

(c) For $r \leq a$, $I(r) = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r'} e^{(r'-a)/\delta} \right) r' dr' d\theta = 2\pi b \int_0^r e^{(r'-a)/\delta} dr' = 2\pi b \delta \left. e^{(r'-a)/\delta} \right|_0^r.$

$I(r) = 2\pi b \delta (e^{(r-a)/\delta} - e^{-a/\delta}) = 2\pi b \delta e^{-a/\delta} (e^{r/\delta} - 1)$ and $I(r) = I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}.$

(d) For $r \leq a$, $\oint \vec{B} \cdot d\vec{l} = B(r) 2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}$ and $B = \frac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r (e^{a/\delta} - 1)}.$

(e) At $r = \delta = 0.025 \text{ m}$, $B = \frac{\mu_0 I_0 (e - 1)}{2\pi \delta (e^{a/\delta} - 1)} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.025 \text{ m}) (e^{0.050/0.025} - 1)} = 1.75 \times 10^{-4} \text{ T}.$

At $r = a = 0.050 \text{ m}$, $B = \frac{\mu_0 I_0 (e^{a/\delta} - 1)}{2\pi a (e^{a/\delta} - 1)} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.050 \text{ m})} = 3.26 \times 10^{-4} \text{ T}.$

At $r = 2a = 0.100 \text{ m}$, $B = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.100 \text{ m})} = 1.63 \times 10^{-4} \text{ T}.$

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

28.79. IDENTIFY: Evaluate the integral as specified in the problem.

SET UP: Eq.(28.15) says $B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}.$

EXECUTE: $\int_{-\infty}^{\infty} B_x dx = \int_{-\infty}^{\infty} \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} dx = \frac{\mu_0 I}{2} \int_{-\infty}^{\infty} \frac{1}{((x/a)^2 + 1)^{3/2}} d(x/a).$

$$B = \frac{\mu_0 I}{2} \int_{-\infty}^{\infty} \frac{dz}{(z^2 + 1)^{3/2}} \Rightarrow \int_{-\infty}^{\infty} B_x dx = \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{\mu_0 I}{2} (\sin \theta) \Big|_{-\pi/2}^{\pi/2} = \mu_0 I,$$

where we used the substitution $z = \tan \theta$ to go from the first to second line.

EVALUATE: This is just what Ampere's Law tells us to expect if we imagine the loop runs along the x -axis closing on itself at infinity: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$

28.80. IDENTIFY: Follow the procedure specified in the problem.

SET UP: The field and integration path are sketched in Figure 28.80.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $\vec{B} = B_0 \hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

Then $\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0$. $B_{cd} = 0$, so $B_{ab}L = 0$. But we have assumed that $B_{ab} \neq 0$. This is a contradiction

and violates Ampere's Law.

EVALUATE: It is often convenient to approximate B as confined to a particular region of space, but this result tells us that the boundary of such a region isn't sharp.

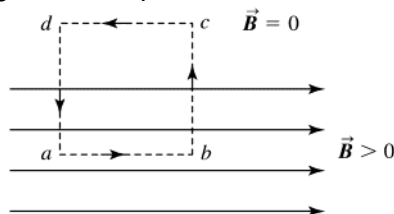


Figure 28.80

28.81. IDENTIFY: Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.

SET UP: Do parts (a) and (b) together.

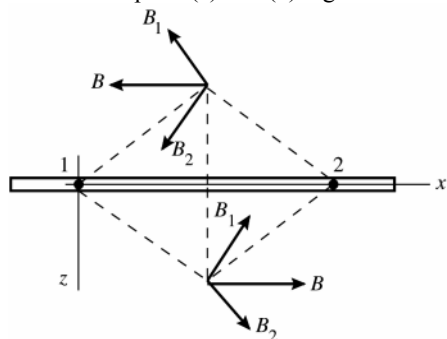


Figure 28.81a

Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.81a shows that for each pair the z -components cancel, and that above the sheet the field is in the $-x$ -direction and that below the sheet it is in the $+x$ -direction.

Also, by symmetry the magnitude of $\vec{B}$ a distance a above the sheet must equal the magnitude of $\vec{B}$ a distance a below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.81b.

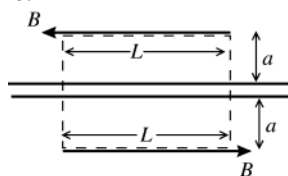


Figure 28.81b

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0$. On the long sides of length L , $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B , on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL$. n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL$. Ampere's law then gives $2BL = \mu_0 InL$ and $B = \frac{1}{2} \mu_0 In$.

EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge (Example 22.7).

28.82. IDENTIFY: Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.81 shows that for an infinite sheet $B = \frac{1}{2} \mu_0 In$. If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P , R and S are shown in Figure 28.82.

EXECUTE: (a) Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

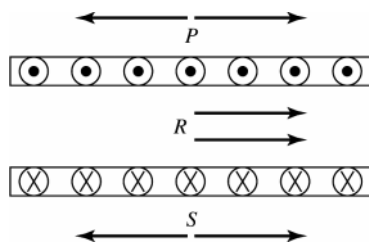


Figure 28.82

28.83. IDENTIFY and SET UP: Use Eq.(28.28) to calculate the total magnetic moment of a volume V of the iron. Use the density and atomic mass of iron to find the number of atoms in this volume and use that to find the magnetic dipole moment per atom.

EXECUTE: $M = \frac{\mu_{\text{total}}}{V}$, so $\mu_{\text{total}} = MV$. The average magnetic moment per atom is $\mu_{\text{atom}} = \mu_{\text{total}} / N = MV / N$,

where N is the number of atoms in volume V . The mass of volume V is $m = \rho V$, where ρ is the density.

($\rho_{\text{iron}} = 7.8 \times 10^3 \text{ kg/m}^3$). The number of moles of iron in volume V is

$$n = \frac{m}{55.847 \times 10^{-3} \text{ kg/mol}} = \frac{\rho V}{55.847 \times 10^{-3} \text{ kg/mol}}, \text{ where } 55.847 \times 10^{-3} \text{ kg/mol is the atomic mass}$$

of iron from appendix D. $N = nN_A$, where $N_A = 6.022 \times 10^{23}$ atoms/mol is Avogadro's number. Thus

$$N = nN_A = \frac{\rho V N_A}{55.847 \times 10^{-3} \text{ kg/mol}}.$$

$$\mu_{\text{atom}} = \frac{MV}{N} = MV \left(\frac{55.847 \times 10^{-3} \text{ kg/mol}}{\rho V N_A} \right) = \frac{M(55.847 \times 10^{-3} \text{ kg/mol})}{\rho N_A}.$$

$$\mu_{\text{atom}} = \frac{(6.50 \times 10^4 \text{ A/m})(55.847 \times 10^{-3} \text{ kg/mol})}{(7.8 \times 10^3 \text{ kg/m}^3)(6.022 \times 10^{23} \text{ atoms/mol})}$$

$$\mu_{\text{atom}} = 7.73 \times 10^{-25} \text{ A} \cdot \text{m}^2 = 7.73 \times 10^{-25} \text{ J/T}$$

$$\mu_B = 9.274 \times 10^{-24} \text{ A} \cdot \text{m}^2, \text{ so } \mu_{\text{atom}} = 0.0834 \mu_B.$$

EVALUATE: The magnetic moment per atom is much less than one Bohr magneton. The magnetic moments of each electron in the iron must be in different directions and mostly cancel each other.

28.84. IDENTIFY: The force on the cube of iron must equal the weight of the iron cube. The weight is proportional to the density and the magnetic force is proportional to μ , which is in turn proportional to K_m .

SET UP: The densities of iron, aluminum and silver are $\rho_{\text{Fe}} = 7.8 \times 10^3 \text{ kg/m}^3$, $\rho_{\text{Al}} = 2.7 \times 10^3 \text{ kg/m}^3$ and

$\rho_{\text{Ag}} = 10.5 \times 10^3 \text{ kg/m}^3$. The relative permeabilities of iron, aluminum and silver are $K_{\text{Fe}} = 1400$, $K_{\text{Al}} = 1.00022$ and

$K_{\text{Ag}} = 1.00 - 2.6 \times 10^{-5}$.

EXECUTE: (a) The microscopic magnetic moments of an initially unmagnetized ferromagnetic material experience torques from a magnet that aligns the magnetic domains with the external field, so they are attracted to the magnet. For a paramagnetic material, the same attraction occurs because the magnetic moments align themselves parallel to the external field. For a diamagnetic material, the magnetic moments align antiparallel to the external field so it is like two magnets repelling each other.

(b) The magnet can just pick up the iron cube so the force it exerts is

$$F_{\text{Fe}} = m_{\text{Fe}} g = \rho_{\text{Fe}} a^3 g = (7.8 \times 10^3 \text{ kg/m}^3)(0.020 \text{ m})^3 (9.8 \text{ m/s}^2) = 0.612 \text{ N. If the magnet tries to lift the aluminum cube of the same dimensions as the iron block, then the upward force felt by the cube is}$$

$$F_{\text{Al}} = \frac{K_{\text{Al}}}{K_{\text{Fe}}} (0.612 \text{ N}) = \frac{1.000022}{1400} (0.612 \text{ N}) = 4.37 \times 10^{-4} \text{ N. The weight of the aluminum cube is}$$

$$W_{\text{Al}} = m_{\text{Al}} g = \rho_{\text{Al}} a^3 g = (2.7 \times 10^3 \text{ kg/m}^3)(0.020 \text{ m})^3 (9.8 \text{ m/s}^2) = 0.212 \text{ N. Therefore, the ratio of the magnetic force}$$

$$\text{on the aluminum cube to the weight of the cube is } \frac{4.37 \times 10^{-4} \text{ N}}{0.212 \text{ N}} = 2.1 \times 10^{-3} \text{ and the magnet cannot lift it.}$$

(c) If the magnet tries to lift a silver cube of the same dimensions as the iron block, then the downward force felt

$$\text{by the cube is } F_{\text{Al}} = \frac{K_{\text{Ag}}}{K_{\text{Fe}}} (0.612 \text{ N}) = \frac{(1.00 - 2.6 \times 10^{-5})}{1400} (0.612 \text{ N}) = 4.37 \times 10^{-4} \text{ N. But the weight of the silver cube}$$

$$\text{is } W_{\text{Ag}} = m_{\text{Ag}} g = \rho_{\text{Ag}} a^3 g = (10.5 \times 10^3 \text{ kg/m}^3)(0.020 \text{ m})^3 (9.8 \text{ m/s}^2) = 0.823 \text{ N. So the ratio of the magnetic force on}$$

$$\text{the silver cube to the weight of the cube is } \frac{4.37 \times 10^{-4} \text{ N}}{0.823 \text{ N}} = 5.3 \times 10^{-4} \text{ and the magnet's effect would not be}$$

noticeable.

EVALUATE: Silver is diamagnetic and is repelled by the magnet. Aluminum is paramagnetic and is attracted by the magnet. But for both these materials the force is much less than the force on a similar cube of ferromagnetic iron.

28.85. IDENTIFY: The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

$$\text{SET UP: The magnetic force per unit length is } \frac{F}{L} = \frac{\mu_0 I^2}{2\pi d}, \text{ and the acceleration obeys the equation } F/L = m/L a.$$

The rms current over a short discharge time is $I_0 / \sqrt{2}$.

EXECUTE: (a) First get the force per unit length:

$$\frac{F}{L} = \frac{\mu_0}{2\pi} \frac{I^2}{d} = \frac{\mu_0}{2\pi d} \left(\frac{I_0}{\sqrt{2}} \right)^2 = \frac{\mu_0}{4\pi d} \left(\frac{V}{R} \right)^2 = \frac{\mu_0}{4\pi d} \left(\frac{Q_0}{RC} \right)^2$$

Now apply Newton's second law using the result above: $\frac{F}{L} = \frac{m}{L} a = \lambda a = \frac{\mu_0}{4\pi d} \left(\frac{Q_0}{RC} \right)^2$. Solving for a gives

$$a = \frac{\mu_0 Q_0^2}{4\pi \lambda R^2 C^2 d}. \text{ From the kinematics equation } v_x = v_{0x} + a_x t, \text{ we have } v_0 = at = aRC = \frac{\mu_0 Q_0^2}{4\pi \lambda RCd}$$

$$(b) \text{ Conservation of energy gives } \frac{1}{2} m v_0^2 = mgh \text{ and } h = \frac{v_0^2}{2g} = \frac{\left(\frac{\mu_0 Q_0^2}{4\pi \lambda RCd} \right)^2}{2g} = \frac{1}{2g} \left(\frac{\mu_0 Q_0^2}{4\pi \lambda RCd} \right)^2.$$

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.86. IDENTIFY: Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.81 shows that $B = \frac{1}{2} \mu_0 I n$ for an infinite current sheet. Let L be the width of the sheet, so $n = I/L$.

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q = L \Delta x \sigma$, so $I = \frac{\Delta Q}{\Delta t} = L \frac{\Delta x}{\Delta t} \sigma = L v \sigma$.

Approximating the belt as an infinite sheet $B = \frac{\mu_0 I}{2L} = \frac{\mu_0 v \sigma}{2}$. $\vec{B}$ is directed out of the page, as shown in Figure 28.86.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.



Figure 28.86

28.87. IDENTIFY: The rotating disk produces concentric rings of current. Calculate the field due to each ring and integrate over the surface of the disk to find the total field.

SET UP: At the center of a circular ring carrying current I , $B = \frac{\mu_0 I}{2r}$.

EXECUTE: The charge on a ring of radius r is $q = \sigma A = \sigma 2\pi r dr = \frac{2Qr dr}{a^2}$. If the disk rotates at n turns per

second, then the current from that ring is $dI = \frac{dq}{dt} = n dq = \frac{2Qn r dr}{a^2}$. Therefore, $dB = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r} \frac{2Qn r dr}{a^2} = \frac{\mu_0 n Q dr}{a^2}$.

We integrate out from the center to the edge of the disk and find $B = \int_0^a dB = \int_0^a \frac{\mu_0 n Q dr}{a^2} = \frac{\mu_0 n Q}{a}$.

EVALUATE: The magnetic field is proportional to the total charge on the disk and to its rotation rate.

28.88. IDENTIFY: There are two parts to the magnetic field: that from the half loop and that from the straight wire segment running from $-a$ to a .

SET UP: Apply Eq.(28.14). Let the ϕ be the angle that locates dl around the ring.

EXECUTE: $B_x(\text{ring}) = \frac{1}{2} B_{\text{loop}} = -\frac{\mu_0 I a^2}{4(x^2 + a^2)^{3/2}}$.

$$dB_y(\text{ring}) = dB \sin \theta \sin \phi = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}} \sin \phi = \frac{\mu_0 I a x \sin \phi d\phi}{4\pi (x^2 + a^2)^{3/2}} \text{ and}$$

$$B_y(\text{ring}) = \int_0^\pi dB_y(\text{ring}) = \int_0^\pi \frac{\mu_0 I a x \sin \phi d\phi}{4\pi (x^2 + a^2)^{3/2}} = \frac{\mu_0 I a x}{4\pi (x^2 + a^2)^{3/2}} \cos \phi \Big|_0^\pi = -\frac{\mu_0 I a x}{2\pi (x^2 + a^2)^{3/2}}.$$

$B_y(\text{rod}) = \frac{\mu_0 I a}{2\pi x (x^2 + a^2)^{1/2}}$, using Eq. (28.8). The total field components are:

$$B_x = -\frac{\mu_0 I a^2}{4(x^2 + a^2)^{3/2}} \text{ and } B_y = \frac{\mu_0 I a}{2\pi x (x^2 + a^2)^{1/2}} \left(1 - \frac{x^2}{x^2 + a^2} \right) = \frac{\mu_0 I a^3}{2\pi x (x^2 + a^2)^{3/2}}.$$

EVALUATE: $B_y = -\frac{2}{\pi} \frac{a}{x} B_x$. B_y decreases faster than B_x as x increases. For very small x , $B_x = -\frac{\mu_0 I}{4a}$ and $B_y = \frac{\mu_0 I}{2\pi a}$.

In this limit B_x is the field at the center of curvature of a semicircle and B_y is the field of a long straight wire.

ELECTROMAGNETIC INDUCTION

- 29.1. IDENTIFY:** Altering the orientation of a coil relative to a magnetic field changes the magnetic flux through the coil. This change then induces an emf in the coil.
SET UP: The flux through a coil of N turns is $\Phi = NBA \cos \phi$, and by Faraday's law the magnitude of the induced emf is $\mathcal{E} = d\Phi/dt$.
EXECUTE: (a) $\Delta\Phi = NBA = (50)(1.20 \text{ T})(0.250 \text{ m})(0.300 \text{ m}) = 4.50 \text{ Wb}$
 (b) $\mathcal{E} = d\Phi/dt = (4.50 \text{ Wb})/(0.222 \text{ s}) = 20.3 \text{ V}$
EVALUATE: This induced potential is certainly large enough to be easily detectable.
- 29.2. IDENTIFY:** $\mathcal{E} = \left| \frac{\Delta\Phi_B}{\Delta t} \right|$. $\Phi_B = BA \cos \phi$. Φ_B is the flux through each turn of the coil.
SET UP: $\phi_i = 0^\circ$. $\phi_f = 90^\circ$.
EXECUTE: (a) $\Phi_{B,i} = BA \cos 0^\circ = (6.0 \times 10^{-5} \text{ T})(12 \times 10^{-4} \text{ m}^2)(1) = 7.2 \times 10^{-8} \text{ Wb}$. The total flux through the coil is $N\Phi_{B,i} = (200)(7.2 \times 10^{-8} \text{ Wb}) = 1.44 \times 10^{-5} \text{ Wb}$. $\Phi_{B,f} = BA \cos 90^\circ = 0$.
 (b) $\mathcal{E} = \left| \frac{N\Phi_i - N\Phi_f}{\Delta t} \right| = \frac{1.44 \times 10^{-5} \text{ Wb}}{0.040 \text{ s}} = 3.6 \times 10^{-4} \text{ V} = 0.36 \text{ mV}$.
EVALUATE: The average induced emf depends on how rapidly the flux changes.
- 29.3. IDENTIFY and SET UP:** Use Faraday's law to calculate the average induced emf and apply Ohm's law to the coil to calculate the average induced current and charge that flows.
(a) EXECUTE: The magnitude of the average emf induced in the coil is $|\mathcal{E}_{av}| = N \left| \frac{\Delta\Phi_B}{\Delta t} \right|$. Initially,
 $\Phi_{B,i} = BA \cos \phi = BA$. The final flux is zero, so $|\mathcal{E}_{av}| = N \left| \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \right| = \frac{NBA}{\Delta t}$. The average induced current is
 $I = \frac{|\mathcal{E}_{av}|}{R} = \frac{NBA}{R\Delta t}$. The total charge that flows through the coil is $Q = I\Delta t = \left(\frac{NBA}{R\Delta t} \right) \Delta t = \frac{NBA}{R}$.
EVALUATE: The charge that flows is proportional to the magnetic field but does not depend on the time Δt .
 (b) The magnetic stripe consists of a pattern of magnetic fields. The pattern of charges that flow in the reader coil tell the card reader the magnetic field pattern and hence the digital information coded onto the card.
 (c) According to the result in part (a) the charge that flows depends only on the change in the magnetic flux and it does not depend on the rate at which this flux changes.
- 29.4. IDENTIFY and SET UP:** Apply the result derived in Exercise 29.3: $Q = NBA/R$. In the present exercise the flux changes from its maximum value of $\Phi_B = BA$ to zero, so this equation applies. R is the total resistance so here $R = 60.0 \Omega + 45.0 \Omega = 105.0 \Omega$.
EXECUTE: $Q = \frac{NBA}{R}$ says $B = \frac{QR}{NA} = \frac{(3.56 \times 10^{-5} \text{ C})(105.0 \Omega)}{120(3.20 \times 10^{-4} \text{ m}^2)} = 0.0973 \text{ T}$.
EVALUATE: A field of this magnitude is easily produced.
- 29.5. IDENTIFY:** Apply Faraday's law.
SET UP: Let $+z$ be the positive direction for $\vec{A}$. Therefore, the initial flux is positive and the final flux is zero.
EXECUTE: (a) and (b) $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{0 - (1.5 \text{ T})\pi(0.120 \text{ m})^2}{2.0 \times 10^{-3} \text{ s}} = +34 \text{ V}$. Since $\mathcal{E}$ is positive and $\vec{A}$ is toward us, the induced current is counterclockwise.
EVALUATE: The shorter the removal time, the larger the average induced emf.

29.6. IDENTIFY: Apply Eq.(29.4). $I = \mathcal{E}/R$.

SET UP: $d\Phi_B/dt = A dB/dt$.

EXECUTE: (a) $\mathcal{E} = \frac{Nd\Phi_B}{dt} = NA \frac{d}{dt}(B) = NA \frac{d}{dt}((0.012 \text{ T/s})t + (3.00 \times 10^{-5} \text{ T/s}^4)t^4)$

$$\mathcal{E} = NA((0.012 \text{ T/s}) + (1.2 \times 10^{-4} \text{ T/s}^4)t^3) = 0.0302 \text{ V} + (3.02 \times 10^{-4} \text{ V/s}^3)t^3.$$

(b) At $t = 5.00 \text{ s}$, $\mathcal{E} = 0.0302 \text{ V} + (3.02 \times 10^{-4} \text{ V/s}^3)(5.00 \text{ s})^3 = 0.0680 \text{ V}$. $I = \frac{\mathcal{E}}{R} = \frac{0.0680 \text{ V}}{600 \Omega} = 1.13 \times 10^{-4} \text{ A}$.

EVALUATE: The rate of change of the flux is increasing in time, so the induced current is not constant but rather increases in time.

29.7. IDENTIFY: Calculate the flux through the loop and apply Faraday's law.

SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance r from the wire, is $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: (a) When $B = \frac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \frac{\mu_0 i}{2\pi r} L dr$.

(c) $\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 i L}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i L}{2\pi} \ln(b/a)$.

(d) $\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{\mu_0 L}{2\pi} \ln(b/a) \frac{di}{dt}$.

(e) $\mathcal{E} = \frac{\mu_0 (0.240 \text{ m})}{2\pi} \ln(0.360/0.120)(9.60 \text{ A/s}) = 5.06 \times 10^{-7} \text{ V}$.

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.8. IDENTIFY: Apply Faraday's law.

SET UP: Let $\vec{A}$ be upward in Figure 29.28 in the textbook.

EXECUTE: (a) $|\mathcal{E}_{\text{ind}}| = \left| \frac{d\Phi_B}{dt} \right| = \left| \frac{d}{dt}(B_{\perp} A) \right|$

$$|\mathcal{E}_{\text{ind}}| = A \sin 60^\circ \left| \frac{dB}{dt} \right| = A \sin 60^\circ \left| \frac{d}{dt} \left((1.4 \text{ T}) e^{-(0.057 \text{ s}^{-1})t} \right) \right| = (\pi r^2)(\sin 60^\circ)(1.4 \text{ T})(0.057 \text{ s}^{-1}) e^{-(0.057 \text{ s}^{-1})t}$$

$$|\mathcal{E}_{\text{ind}}| = \pi (0.75 \text{ m})^2 (\sin 60^\circ)(1.4 \text{ T})(0.057 \text{ s}^{-1}) e^{-(0.057 \text{ s}^{-1})t} = (0.12 \text{ V}) e^{-(0.057 \text{ s}^{-1})t}.$$

(b) $\mathcal{E} = \frac{1}{10} \mathcal{E}_0 = \frac{1}{10} (0.12 \text{ V})$. $\frac{1}{10} (0.12 \text{ V}) = (0.12 \text{ V}) e^{-(0.057 \text{ s}^{-1})t}$. $\ln(1/10) = -(0.057 \text{ s}^{-1})t$ and $t = 40.4 \text{ s}$.

(c) $\vec{B}$ is in the direction of $\vec{A}$ so Φ_B is positive. B is getting weaker, so the magnitude of the flux is decreasing and $d\Phi_B/dt < 0$. Faraday's law therefore says $\mathcal{E} > 0$. Since $\mathcal{E} > 0$, the induced current must flow *counterclockwise* as viewed from above.

EVALUATE: The flux changes because the magnitude of the magnetic field is changing.

29.9. IDENTIFY and SET UP: Use Faraday's law to calculate the emf (magnitude and direction). The direction of the induced current is the same as the direction of the emf. The flux changes because the area of the loop is changing; relate dA/dt to dc/dt , where c is the circumference of the loop.

(a) **EXECUTE:** $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$

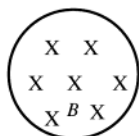
$$\Phi_B = BA = (B/4\pi)c^2$$

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = \left(\frac{B}{2\pi} \right) c \left| \frac{dc}{dt} \right|$$

At $t = 9.0 \text{ s}$, $c = 1.650 \text{ m} - (9.0 \text{ s})(0.120 \text{ m/s}) = 0.570 \text{ m}$

$$|\mathcal{E}| = (0.500 \text{ T})(1/2\pi)(0.570 \text{ m})(0.120 \text{ m/s}) = 5.44 \text{ mV}$$

(b) **SET UP:** The loop and magnetic field are sketched in Figure 29.9.



Take into the page to be the positive direction for $\vec{A}$. Then the magnetic flux is positive.

Figure 29.9

EXECUTE: The positive flux is decreasing in magnitude; $d\Phi_B/dt$ is negative and $\mathcal{E}$ is positive. By the right-hand rule, for $\vec{A}$ into the page, positive $\mathcal{E}$ is clockwise.

EVALUATE: Even though the circumference is changing at a constant rate, dA/dt is not constant and $|\mathcal{E}|$ is not constant. Flux $\otimes$ is decreasing so the flux of the induced current is $\otimes$ and this means that I is clockwise, which checks.

- 29.10. IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA \cos \phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: (a) and (c) The magnetic flux is constant, so the induced emf is zero.

(b) The area inside the field is changing. If we let x be the length (along the 30.0-cm side) in the field, then $A = (0.400 \text{ m})x$. $\Phi_B = BA = (0.400 \text{ m})x$

$$\mathcal{E} = d\Phi/dt = B d[(0.400 \text{ m})x]/dt = B(0.400 \text{ m})dx/dt = B(0.400 \text{ m})v$$

$$\mathcal{E} = (1.25 \text{ T})(0.400 \text{ m})(0.0200 \text{ m/s}) = 0.0100 \text{ V}$$

EVALUATE: It is not a large *flux* that induces an emf, but rather a large *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

- 29.11. IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA \cos \phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: (a) $\mathcal{E} = d\Phi/dt = d[A(B_0 + bx)]/dt = bA dx/dt = bAv$

(b) clockwise

(c) Same answers except the current is counterclockwise.

EVALUATE: Even though the coil remains within the magnetic field, the flux through it increases because the strength of the field is increasing.

- 29.12. IDENTIFY:** Use the results of Example 29.5.

SET UP: $\mathcal{E}_{\max} = NBA\omega$. $\mathcal{E}_{\text{av}} = \frac{2}{\pi} \mathcal{E}_{\max}$. $\omega = (440 \text{ rev/min}) \left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}} \right) = 46.1 \text{ rad/s}$.

EXECUTE: (a) $\mathcal{E}_{\max} = NBA\omega = (150)(0.060 \text{ T})\pi(0.025 \text{ m})^2(46.1 \text{ rad/s}) = 0.814 \text{ V}$

(b) $\mathcal{E}_{\text{av}} = \frac{2}{\pi} \mathcal{E}_{\max} = \frac{2}{\pi}(0.815 \text{ V}) = 0.519 \text{ V}$

EVALUATE: In $\mathcal{E}_{\max} = NBA\omega$, ω must be in rad/s.

- 29.13. IDENTIFY:** Apply the results of Example 29.5.

SET UP: $\mathcal{E}_{\max} = NBA\omega$

EXECUTE: $\omega = \frac{\mathcal{E}_{\max}}{NBA} = \frac{2.40 \times 10^{-2} \text{ V}}{(120)(0.0750 \text{ T})(0.016 \text{ m})^2} = 10.4 \text{ rad/s}$

EVALUATE: We may also express ω as 99.3 rev/min or 1.66 rev/s.

- 29.14. IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA \cos \phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: The flux is constant in each case, so the induced emf is zero in all cases.

EVALUATE: Even though the coil is moving within the magnetic field and has flux through it, this flux is not *changing*, so no emf is induced in the coil.

- 29.15. IDENTIFY and SET UP:** The field of the induced current is directed to oppose the change in flux.

EXECUTE: (a) The field is into the page and is increasing so the flux is increasing. The field of the induced current is out of the page. To produce field out of the page the induced current is counterclockwise.

(b) The field is into the page and is decreasing so the flux is decreasing. The field of the induced current is into the page. To produce field into the page the induced current is clockwise.

(c) The field is constant so the flux is constant and there is no induced emf and no induced current.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

- 29.16. IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.

SET UP and EXECUTE: The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.

EVALUATE: Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

- 29.17. IDENTIFY and SET UP:** Apply Lenz's law, in the form that states that the flux of the induced current tends to oppose the change in flux.

EXECUTE: (a) With the switch closed the magnetic field of coil A is to the right at the location of coil B. When the switch is opened the magnetic field of coil A goes away. Hence by Lenz's law the field of the current induced in coil B is to the right, to oppose the decrease in the flux in this direction. To produce magnetic field that is to the right the current in the circuit with coil B must flow through the resistor in the direction *a* to *b*.

(b) With the switch closed the magnetic field of coil A is to the right at the location of coil B. This field is stronger at points closer to coil A so when coil B is brought closer the flux through coil B increases. By Lenz's law the field of the induced current in coil B is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil B must flow through the resistor in the direction b to a .

(c) With the switch closed the magnetic field of coil A is to the right at the location of coil B. The current in the circuit that includes coil A increases when R is decreased and the magnetic field of coil A increases when the current through the coil increases. By Lenz's law the field of the induced current in coil B is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil B must flow through the resistor in the direction b to a .

EVALUATE: In parts (b) and (c) the change in the circuit causes the flux through circuit B to increase and in part (a) it causes the flux to decrease. Therefore, the direction of the induced current is the same in parts (b) and (c) and opposite in part (a).

29.18. IDENTIFY: Apply Lenz's law.

SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.

EXECUTE: (a) The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.

(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.

(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.19. IDENTIFY and SET UP: Lenz's law requires that the flux of the induced current opposes the change in flux.

EXECUTE: (a) Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is $\otimes$ and the induced current is clockwise.

(b) The current reaches a constant value so Φ_B is constant. $d\Phi_B/dt = 0$ and there is no induced current.

(c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.

EVALUATE: Only a change in flux produces an induced current. The induced current is in one direction when the current in the outer ring is increasing and is in the opposite direction when that current is decreasing.

29.20. IDENTIFY: Use the results of Example 29.6. Use the three approaches specified in the problem for determining the direction of the induced current. $I = \mathcal{E}/R$.

SET UP: Let $\vec{A}$ be directed into the figure, so a clockwise emf is positive.

EXECUTE: (a) $\mathcal{E} = vBl = (5.0 \text{ m/s})(0.750 \text{ T})(1.50 \text{ m}) = 5.6 \text{ V}$

(b) (i) Let q be a positive charge in the moving bar, as shown in Figure 29.20a. The magnetic force on this charge is $\vec{F} = q\vec{v} \times \vec{B}$, which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.

(ii) Φ_B is positive and is increasing in magnitude, so $d\Phi_B/dt > 0$. Then by Faraday's law $\mathcal{E} < 0$ and the emf and induced current are counterclockwise.

(iii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise* sense, as shown in Figure 29.20b.

(c) $\mathcal{E} = RI$. $I = \frac{\mathcal{E}}{R} = \frac{5.6 \text{ V}}{25 \Omega} = 0.22 \text{ A}$.

EVALUATE: All three methods agree on the direction of the induced current.

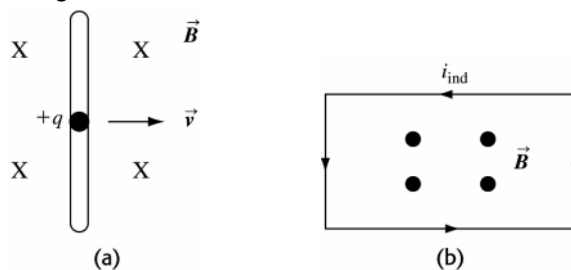


Figure 29.20

29.21. IDENTIFY: A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.

SET UP: The induced emf is $\mathcal{E} = vBl \sin \phi$, where ϕ is the angle between the velocity and the magnetic field.

EXECUTE: (a) $\mathcal{E} = vBL \sin \phi = (5.00 \text{ m/s})(0.450 \text{ T})(0.300 \text{ m})(\sin 90^\circ) = 0.675 \text{ V}$

(b) The positive charges are moved to end b , so b is at the higher potential.

(c) $E = V/L = (0.675 \text{ V})/(0.300 \text{ m}) = 2.25 \text{ V/m}$. The direction of $\vec{E}$ is from b to a .

(d) The positive charge are pushed to b , so b has an excess of positive charge.

(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.

EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.22. IDENTIFY: The moving bar has a motional emf induced across its ends, so it causes a current to flow.

SET UP: The induced potential is $\mathcal{E} = vBL$ and Ohm's law is $\mathcal{E} = IR$.

EXECUTE: (a) $\mathcal{E} = vBL = (5.0 \text{ m/s})(0.750 \text{ T})(1.50 \text{ m}) = 5.6 \text{ V}$

(b) $I = \mathcal{E}/R = (5.6 \text{ V})/(25 \Omega) = 0.23 \text{ A}$

EVALUATE: Both the induced potential and the current are large enough to have noticeable effects.

29.23. IDENTIFY: $\mathcal{E} = vBL$

SET UP: $L = 5.00 \times 10^{-2} \text{ m}$. $1 \text{ mph} = 0.4470 \text{ m/s}$.

EXECUTE: $v = \frac{\mathcal{E}}{BL} = \frac{1.50 \text{ V}}{(0.650 \text{ T})(5.00 \times 10^{-2} \text{ m})} = 46.2 \text{ m/s} = 103 \text{ mph}$.

EVALUATE: This is a large speed and not practical. It is also difficult to produce a 5.00 cm wide region of 0.650 T magnetic field.

29.24. IDENTIFY: $\mathcal{E} = vBL$.

SET UP: $1 \text{ mph} = 0.4470 \text{ m/s}$. $1 \text{ G} = 10^{-4} \text{ T}$.

EXECUTE: (a) $\mathcal{E} = (180 \text{ mph}) \left(\frac{0.4470 \text{ m/s}}{1 \text{ mph}} \right) (0.50 \times 10^{-4} \text{ T})(1.5 \text{ m}) = 6.0 \text{ mV}$. This is much too small to be noticeable.

(b) $\mathcal{E} = (565 \text{ mph}) \left(\frac{0.4470 \text{ m/s}}{1 \text{ mph}} \right) (0.50 \times 10^{-4} \text{ T})(64.4 \text{ m}) = 0.813 \text{ mV}$. This is too small to be noticeable.

EVALUATE: Even though the speeds and values of L are large, the earth's field is small and motional emfs due to the earth's field are not important in these situations.

29.25. IDENTIFY and SET UP: $\mathcal{E} = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_l that the magnetic field exerts on the rod because of the current in the rod.

EXECUTE: (a) $\mathcal{E} = vBL = (7.50 \text{ m/s})(0.800 \text{ T})(0.500 \text{ m}) = 3.00 \text{ V}$

(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from b to a in the rod.

(c) $I = \frac{\mathcal{E}}{R} = \frac{3.00 \text{ V}}{1.50 \Omega} = 2.00 \text{ A}$. $F_l = ILB \sin \phi = (2.00 \text{ A})(0.500 \text{ m})(0.800 \text{ T}) \sin 90^\circ = 0.800 \text{ N}$. $\vec{F}_l$ is to the left. To

keep the bar moving to the right at constant speed an external force with magnitude $F_{\text{ext}} = 0.800 \text{ N}$ and directed to the right must be applied to the bar.

(d) The rate at which work is done by the force F_{ext} is $F_{\text{ext}} v = (0.800 \text{ N})(7.50 \text{ m/s}) = 6.00 \text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2 R = (2.00 \text{ A})(1.50 \Omega) = 6.00 \text{ W}$. These two rates are equal, as is required by conservation of energy.

EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.26. IDENTIFY: Use Faraday's law to calculate the induced emf. Ohm's law applied to the loop gives I . Use Eq.(27.19) to calculate the force exerted on each side of the loop.

SET UP: The loop before it starts to enter the magnetic field region is sketched in Figure 29.26a.

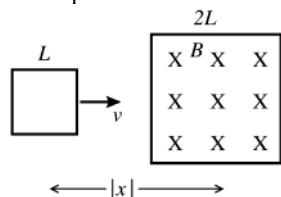


Figure 29.26a

EXECUTE: For $x < -3L/2$ or $x > 3L/2$ the loop is completely outside the field region. $\Phi_B = 0$, and $\frac{d\Phi_B}{dt} = 0$.

Thus $\mathcal{E} = 0$ and $I = 0$, so there is no force from the magnetic field and the external force F necessary to maintain constant velocity is zero.

SET UP: The loop when it is completely inside the field region is sketched in Figure 29.26b.

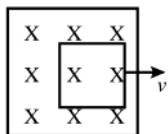


Figure 29.26b

EXECUTE: For $-L/2 < x < L/2$ the loop is completely inside the field region and $\Phi_B = BL^2$.

But $\frac{d\Phi_B}{dt} = 0$ so $\mathcal{E} = 0$ and $I = 0$. There is no force $\vec{F} = I\vec{l} \times \vec{B}$ from the magnetic field and the external force F necessary to maintain constant velocity is zero.

SET UP: The loop as it enters the magnetic field region is sketched in Figure 29.26c.

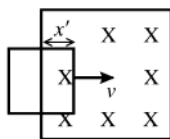


Figure 29.26c

EXECUTE: For $-3L/2 < x < -L/2$ the loop is entering the field region. Let x' be the length of the loop that is within the field.

Then $|\Phi_B| = BLx'$ and $\left|\frac{d\Phi_B}{dt}\right| = BLv$. The magnitude of the induced emf is $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = BLv$ and the induced

current is $I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$. Direction of I : Let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive. The flux is positive and increasing in magnitude, so $\frac{d\Phi_B}{dt}$ is positive. Then by Faraday's law $\mathcal{E}$ is negative, and with our choice for direction of $\vec{A}$ a negative $\mathcal{E}$ is counterclockwise. The current induced in the loop is counterclockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26d, for the situation where the loop is entering the field.

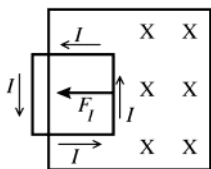


Figure 29.26d

EXECUTE: $\vec{F}_l = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_l$ exerted on the loop by the magnetic field is to the left and has magnitude $F_l = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}$.

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_l$, so is to the right and has this same magnitude.

SET UP: The loop as it leaves the magnetic field region is sketched in Figure 29.26e.

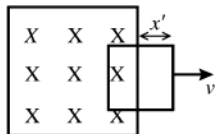


Figure 29.26e

EXECUTE: For $L/2 < x < 3L/2$ the loop is leaving the field region. Let x' be the length of the loop that is outside the field.

Then $|\Phi_B| = BL(L - x')$ and $\left|\frac{d\Phi_B}{dt}\right| = BLv$. The magnitude of the induced emf is $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = BLv$ and the induced

current is $I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$. Direction of I : Again let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive and decreasing in magnitude, so $\frac{d\Phi_B}{dt}$ is negative. Then by Faraday's law $\mathcal{E}$ is positive, and with our choice for direction of $\vec{A}$ a positive $\mathcal{E}$ is clockwise. The current induced in the loop is clockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26f, for the situation where the loop is leaving the field.

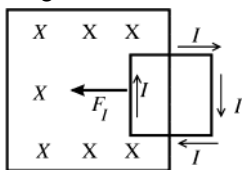


Figure 29.26f

EXECUTE: $\vec{F}_I = I\vec{L} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}$.

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$, so is to the right and has this same magnitude.

(a) The graph of F versus x is given in Figure 29.26g.

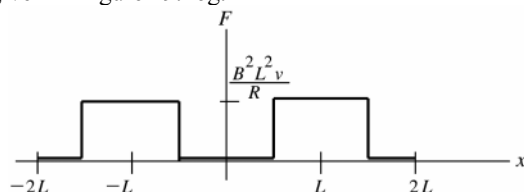


Figure 29.26g

(b) The graph of the induced current I versus x is given in Figure 29.26h.

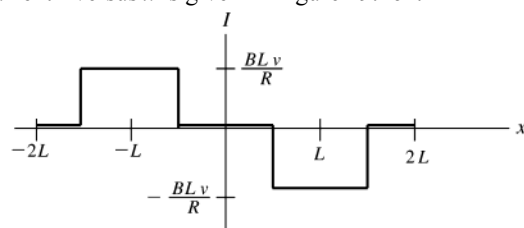


Figure 29.26h

EVALUATE: When the loop is either totally outside or totally inside the magnetic field region the flux isn't changing, there is no induced current, and no external force is needed for the loop to maintain constant speed. When the loop is entering the field the external force required is directed so as to pull the loop in and when the loop is leaving the field the external force required is directed so as to pull the loop out of the field. These directions agree with Lenz's law: the force on the induced current (opposite in direction to the required external force) is directed so as to oppose the loop entering or leaving the field.

29.27. IDENTIFY: A bar moving in a magnetic field has an emf induced across its ends.

SET UP: The induced potential is $\mathcal{E} = vBL \sin \phi$.

EXECUTE: Note that $\phi = 90^\circ$ in all these cases because the bar moved perpendicular to the magnetic field. But the effective length of the bar, $L \sin \theta$, is different in each case.

(a) $\mathcal{E} = vBL \sin \theta = (2.50 \text{ m/s})(1.20 \text{ T})(1.41 \text{ m}) \sin (37.0^\circ) = 2.55 \text{ V}$, with a at the higher potential because positive charges are pushed toward that end.

(b) Same as (a) except $\theta = 53.0^\circ$, giving 3.38 V, with a at the higher potential.

(c) Zero, since the velocity is parallel to the magnetic field.

(d) The bar must move perpendicular to its length, for which the emf is 4.23 V. For $V_b > V_a$, it must move upward and to the left (toward the second quadrant) perpendicular to its length.

EVALUATE: The orientation of the bar affects the potential induced across its ends.

29.28. IDENTIFY: Use Eq.(29.10) to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

(a) **SET UP:** The end view of the solenoid is sketched in Figure 29.28.

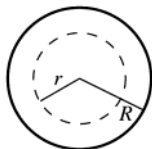


Figure 29.28

Let R be the radius of the solenoid.

Apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r , where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: $\left| \oint \vec{E} \cdot d\vec{l} \right| = E(2\pi r)$

$$\Phi_B = B\pi r^2, \quad \left| -\frac{d\Phi_B}{dt} \right| = \pi r^2 \left| \frac{dB}{dt} \right|$$

$$\left| \oint \vec{E} \cdot d\vec{l} \right| = \left| -\frac{d\Phi_B}{dt} \right| \text{ implies } E(2\pi r) = \pi r^2 \left| \frac{dB}{dt} \right|$$

$$E = \frac{1}{2} r \left| \frac{dB}{dt} \right|$$

$$B = \mu_0 n I, \text{ so } \frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$$

Thus $E = \frac{1}{2} r \mu_0 n \frac{dI}{dt} = \frac{1}{2} (0.00500 \text{ m}) (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (900 \text{ m}^{-1}) (60.0 \text{ A/s}) = 1.70 \times 10^{-4} \text{ V/m}$

(b) $r = 0.0100 \text{ cm}$ is still inside the solenoid so the expression in part (a) applies.

$$E = \frac{1}{2} r \mu_0 n \frac{dI}{dt} = \frac{1}{2} (0.0100 \text{ m}) (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (900 \text{ m}^{-1}) (60.0 \text{ A/s}) = 3.39 \times 10^{-4} \text{ V/m}$$

EVALUATE: Inside the solenoid E is proportional to r , so E doubles when r doubles.

29.29. IDENTIFY: Apply Eqs.(29.9) and (29.10).

SET UP: Evaluate the integral in Eq.(29.10) for a path which is a circle of radius r and concentric with the solenoid. The magnetic field of the solenoid is confined to the region inside the solenoid, so $B(r) = 0$ for $r > R$

EXECUTE: (a) $\frac{d\Phi_B}{dt} = A \frac{dB}{dt} = \pi r_1^2 \frac{dB}{dt}$

(b) $E = \frac{1}{2\pi r_1} \frac{d\Phi_B}{dt} = \frac{\pi r_1^2}{2\pi r_1} \frac{dB}{dt} = \frac{r_1}{2} \frac{dB}{dt}$. The direction of $\vec{E}$ is shown in Figure 29.29a.

(c) All the flux is within $r < R$, so outside the solenoid $E = \frac{1}{2\pi r_2} \frac{d\Phi_B}{dt} = \frac{\pi R^2}{2\pi r_2} \frac{dB}{dt} = \frac{R^2}{2r_2} \frac{dB}{dt}$.

(d) The graph is sketched in Figure 29.29b.

(e) At $r = R/2$, $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi (R/2)^2 \frac{dB}{dt} = \frac{\pi R^2}{4} \frac{dB}{dt}$.

(f) At $r = R$, $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi R^2 \frac{dB}{dt}$.

(g) At $r = 2R$, $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi R^2 \frac{dB}{dt}$.

EVALUATE: The emf is independent of the distance from the center of the cylinder at all points outside it. Even though the magnetic field is zero for $r > R$, the induced electric field is nonzero outside the solenoid and a nonzero emf is induced in a circular turn that has $r > R$.

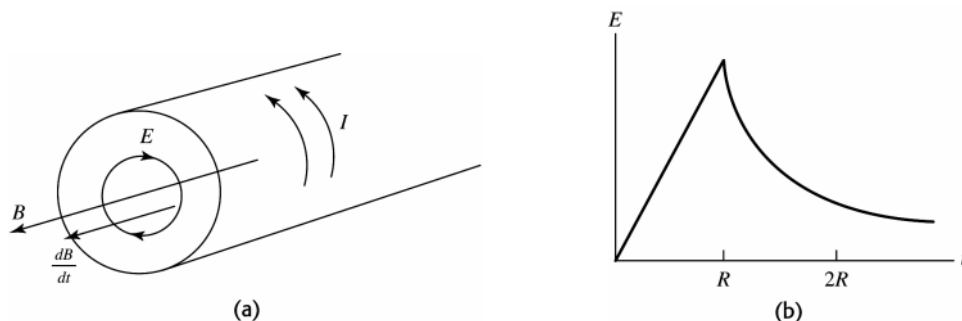


Figure 29.29

29.30. IDENTIFY: Use Eq.(29.10) to calculate the induced electric field E and use this E in Eq.(29.9) to calculate $\mathcal{E}$ between two points.

(a) SET UP: Because of the axial symmetry and the absence of any electric charge, the field lines are concentric circles.

(b) See Figure 29.30.

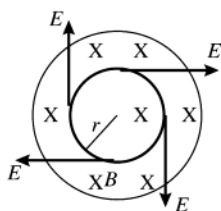


Figure 29.30

$\vec{E}$ is tangent to the ring. The direction of $\vec{E}$ (clockwise or counterclockwise) is the direction in which current will be induced in the ring.

EXECUTE: Use the sign convention for Faraday's law to deduce this direction. Let $\vec{A}$ be into the paper. Then Φ_B is positive. B decreasing then means $\frac{d\Phi_B}{dt}$ is negative, so by $\mathcal{E} = -\frac{d\Phi_B}{dt}$, $\mathcal{E}$ is positive and therefore clockwise. Thus $\vec{E}$ is clockwise around the ring. To calculate E apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to a circular path that coincides with the ring.

$$\oint \vec{E} \cdot d\vec{l} = E(2\pi r)$$

$$\Phi_B = B\pi r^2; \quad \left| \frac{d\Phi_B}{dt} \right| = \pi r^2 \left| \frac{dB}{dt} \right|$$

$$E(2\pi r) = \pi r^2 \left| \frac{dB}{dt} \right| \text{ and } E = \frac{1}{2} r \left| \frac{dB}{dt} \right| = \frac{1}{2} (0.100 \text{ m})(0.0350 \text{ T/s}) = 1.75 \times 10^{-3} \text{ V/m}$$

(c) The induced emf has magnitude $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = E(2\pi r) = (1.75 \times 10^{-3} \text{ V/m})(2\pi)(0.100 \text{ m}) = 1.10 \times 10^{-3} \text{ V}$. Then

$$I = \frac{\mathcal{E}}{R} = \frac{1.10 \times 10^{-3} \text{ V}}{4.00 \Omega} = 2.75 \times 10^{-4} \text{ A}.$$

(d) Points a and b are separated by a distance around the ring of πr so

$$\mathcal{E} = E(\pi r) = (1.75 \times 10^{-3} \text{ V/m})(\pi)(0.100 \text{ m}) = 5.50 \times 10^{-4} \text{ V}$$

(e) The ends are separated by a distance around the ring of $2\pi r$ so $\mathcal{E} = 1.10 \times 10^{-3} \text{ V}$ as calculated in part (c).

EVALUATE: The induced emf, calculated from Faraday's law and used to calculate the induced current, is associated with the induced electric field integrated around the total circumference of the ring.

29.31. IDENTIFY: Apply Eq.(29.1) with $\Phi_B = \mu_0 n i A$.

SET UP: $A = \pi r^2$, where $r = 0.0110 \text{ m}$. In Eq.(29.11), $r = 0.0350 \text{ m}$.

EXECUTE: $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = \left| \frac{d}{dt}(BA) \right| = \left| \frac{d}{dt}(\mu_0 n i A) \right| = \mu_0 n A \left| \frac{di}{dt} \right|$ and $|\mathcal{E}| = E(2\pi r)$. Therefore, $\left| \frac{di}{dt} \right| = \frac{E2\pi r}{\mu_0 n A}$.

$$\left| \frac{di}{dt} \right| = \frac{(8.00 \times 10^{-6} \text{ V/m})2\pi(0.0350 \text{ m})}{\mu_0(400 \text{ m}^{-1})\pi(0.0110 \text{ m})^2} = 9.21 \text{ A/s}.$$

EVALUATE: Outside the solenoid the induced electric field decreases with increasing distance from the axis of the solenoid.

29.32. IDENTIFY: A changing magnetic flux through a coil induces an emf in that coil, which means that an electric field is induced in the material of the coil.

SET UP: According to Faraday's law, the induced electric field obeys the equation $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$.

EXECUTE: (a) For the magnitude of the induced electric field, Faraday's law gives

$$E2\pi r = d(B\pi r^2)/dt = \pi r^2 dB/dt$$

$$E = \frac{r}{2} \frac{dB}{dt} = \frac{0.0225 \text{ m}}{2} (0.250 \text{ T/s}) = 2.81 \times 10^{-3} \text{ V/m}$$

(b) The field points toward the south pole of the magnet and is decreasing, so the induced current is counterclockwise.

EVALUATE: This is a very small electric field compared to most others found in laboratory equipment.

29.33. IDENTIFY: Apply Faraday's law in the form $|\mathcal{E}_{av}| = N \left| \frac{\Delta\Phi_B}{\Delta t} \right|$.

SET UP: The magnetic field of a large straight solenoid is $B = \mu_0 n I$ inside the solenoid and zero outside.

$\Phi_B = BA$, where A is 8.00 cm^2 , the cross-sectional area of the long straight solenoid.

EXECUTE: $|\mathcal{E}_{\text{av}}| = N \left| \frac{\Delta \Phi_B}{\Delta t} \right| = \left| \frac{NA(B_f - B_i)}{\Delta t} \right| = \frac{NA\mu_o nI}{\Delta t}.$

$$\mathcal{E}_{\text{av}} = \frac{\mu_o(12)(8.00 \times 10^{-4} \text{ m}^2)(9000 \text{ m}^{-1})(0.350 \text{ A})}{0.0400 \text{ s}} = 9.50 \times 10^{-4} \text{ V}.$$

EVALUATE: An emf is induced in the second winding even though the magnetic field of the solenoid is zero at the location of the second winding. The changing magnetic field induces an electric field outside the solenoid and that induced electric field produces the emf.

29.34. IDENTIFY: Apply Eq.(29.14).

SET UP: $\epsilon = 3.5 \times 10^{-11} \text{ F/m}$

EXECUTE: $i_D = \epsilon \frac{d\Phi_E}{dt} = (3.5 \times 10^{-11} \text{ F/m})(24.0 \times 10^3 \text{ V} \cdot \text{m/s}^3)t^2.$ $i_D = 21 \times 10^{-6} \text{ A}$ gives $t = 5.0 \text{ s}.$

EVALUATE: i_D depends on the rate at which Φ_E is changing.

29.35. IDENTIFY: Apply Eq.(29.14), where $\epsilon = K\epsilon_o$.

SET UP: $d\Phi_E/dt = 4(8.76 \times 10^3 \text{ V} \cdot \text{m/s}^4)t^3.$ $\epsilon_o = 8.854 \times 10^{-12} \text{ F/m}.$

EXECUTE: $\epsilon = \frac{i_D}{(d\Phi_E/dt)} = \frac{12.9 \times 10^{-12} \text{ A}}{4(8.76 \times 10^3 \text{ V} \cdot \text{m/s}^4)(26.1 \times 10^{-3} \text{ s})^3} = 2.07 \times 10^{-11} \text{ F/m}.$ The dielectric constant is

$$K = \frac{\epsilon}{\epsilon_o} = 2.34.$$

EVALUATE: The larger the dielectric constant, the larger is the displacement current for a given $d\Phi_E/dt$.

29.36. IDENTIFY and SET UP: Eqs.(29.13) and (29.14) show that $i_C = i_D$ and also relate i_D to the rate of change of the electric field flux between the plates. Use this to calculate dE/dt and apply the generalized form of Ampere's law (Eq.29.15) to calculate B .

(a) EXECUTE: $i_C = i_D$, so $j_D = \frac{i_D}{A} = \frac{i_C}{A} = \frac{0.280 \text{ A}}{\pi r^2} = \frac{0.280 \text{ A}}{\pi(0.0400 \text{ m})^2} = 55.7 \text{ A/m}^2$

(b) $j_D = \epsilon_o \frac{dE}{dt}$ so $\frac{dE}{dt} = \frac{j_D}{\epsilon_o} = \frac{55.7 \text{ A/m}^2}{8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 6.29 \times 10^{12} \text{ V/m} \cdot \text{s}$

(c) SET UP: Apply Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_o(i_C + i_D)_{\text{encl}}$ (Eq.(28.20)) to a circular path with radius $r = 0.0200 \text{ m}$. An end view of the solenoid is given in Figure 29.36.

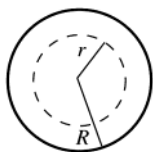


Figure 29.36

By symmetry the magnetic field is tangent to the path and constant around it.

EXECUTE: Thus $\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \int dl = B(2\pi r).$

$i_C = 0$ (no conduction current flows through the air space between the plates)

The displacement current enclosed by the path is $j_D \pi r^2$.

Thus $B(2\pi r) = \mu_o(j_D \pi r^2)$ and $B = \frac{1}{2} \mu_o j_D r = \frac{1}{2} (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(55.7 \text{ A/m}^2)(0.0200 \text{ m}) = 7.00 \times 10^{-7} \text{ T}$

(d) $B = \frac{1}{2} \mu_o j_D r$. Now r is $\frac{1}{2}$ the value in (c), so B is $\frac{1}{2}$ also: $B = \frac{1}{2} (7.00 \times 10^{-7} \text{ T}) = 3.50 \times 10^{-7} \text{ T}$

EVALUATE: The definition of displacement current allows the current to be continuous at the capacitor. The magnetic field between the plates is zero on the axis ($r = 0$) and increases as r increases.

29.37. IDENTIFY: $q = CV$. For a parallel-plate capacitor, $C = \frac{\epsilon A}{d}$, where $\epsilon = K\epsilon_o$. $i_C = dq/dt$. $j_D = \epsilon \frac{E}{dt}$.

SET UP: $E = q/\epsilon A$ so $dE/dt = i_C/\epsilon A$.

EXECUTE: **(a)** $q = CV = \left(\frac{\epsilon A}{d} \right) V = \frac{(4.70)\epsilon_o(3.00 \times 10^{-4} \text{ m}^2)(120 \text{ V})}{2.50 \times 10^{-3} \text{ m}} = 5.99 \times 10^{-10} \text{ C}.$

$$(b) \frac{dq}{dt} = i_C = 6.00 \times 10^{-3} \text{ A.}$$

$$(c) j_D = \epsilon \frac{dE}{dt} = K\epsilon_0 \frac{i_C}{K\epsilon_0 A} = \frac{i_C}{A} = j_C, \text{ so } i_D = i_C = 6.00 \times 10^{-3} \text{ A.}$$

EVALUATE: $i_D = i_C$, so Kirchhoff's junction rule is satisfied where the wire connects to each capacitor plate.

- 29.38. IDENTIFY and SET UP:** Use $i_C = q/t$ to calculate the charge q that the current has carried to the plates in time t . The two equations preceding Eq.(24.2) relate q to the electric field E and the potential difference between the plates. The displacement current density is defined by Eq.(29.16).

EXECUTE: (a) $i_C = 1.80 \times 10^{-3} \text{ A}$

$$q = 0 \text{ at } t = 0$$

The amount of charge brought to the plates by the charging current in time t is

$$q = i_C t = (1.80 \times 10^{-3} \text{ A})(0.500 \times 10^{-6} \text{ s}) = 9.00 \times 10^{-10} \text{ C}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00 \times 10^{-10} \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)} = 2.03 \times 10^5 \text{ V/m}$$

$$V = Ed = (2.03 \times 10^5 \text{ V/m})(2.00 \times 10^{-3} \text{ m}) = 406 \text{ V}$$

$$(b) E = q/\epsilon_0 A$$

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80 \times 10^{-3} \text{ A}}{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)} = 4.07 \times 10^{11} \text{ V/m} \cdot \text{s}$$

Since i_C is constant dE/dt does not vary in time.

$$(c) j_D = \epsilon_0 \frac{dE}{dt} \text{ (Eq.(29.16)), with } \epsilon \text{ replaced by } \epsilon_0 \text{ since there is vacuum between the plates.)}$$

$$j_D = (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4.07 \times 10^{11} \text{ V/m} \cdot \text{s}) = 3.60 \text{ A/m}^2$$

$$i_D = j_D A = (3.60 \text{ A/m}^2)(5.00 \times 10^{-4} \text{ m}^2) = 1.80 \times 10^{-3} \text{ A}; i_D = i_C$$

EVALUATE: $i_C = i_D$. The constant conduction current means the charge q on the plates and the electric field between them both increase linearly with time and i_D is constant.

- 29.39. IDENTIFY:** Ohm's law relates the current in the wire to the electric field in the wire. $j_D = \epsilon \frac{dE}{dt}$. Use Eq.(29.15) to calculate the magnetic fields.

SET UP: Ohm's law says $E = \rho J$. Apply Ohm's law to a circular path of radius r .

$$\text{EXECUTE: (a) } E = \rho J = \frac{\rho I}{A} = \frac{(2.0 \times 10^{-8} \Omega \cdot \text{m})(16 \text{ A})}{2.1 \times 10^{-6} \text{ m}^2} = 0.15 \text{ V/m.}$$

$$(b) \frac{dE}{dt} = \frac{d}{dt} \left(\frac{\rho I}{A} \right) = \frac{\rho}{A} \frac{dI}{dt} = \frac{2.0 \times 10^{-8} \Omega \cdot \text{m}}{2.1 \times 10^{-6} \text{ m}^2} (4000 \text{ A/s}) = 38 \text{ V/m} \cdot \text{s.}$$

$$(c) j_D = \epsilon_0 \frac{dE}{dt} = \epsilon_0 (38 \text{ V/m} \cdot \text{s}) = 3.4 \times 10^{-10} \text{ A/m}^2.$$

$$(d) i_D = j_D A = (3.4 \times 10^{-10} \text{ A/m}^2)(2.1 \times 10^{-6} \text{ m}^2) = 7.14 \times 10^{-16} \text{ A. Eq.(29.15) applied to a circular path of radius } r$$

$$\text{gives } B_D = \frac{\mu_0 i_D}{2\pi r} = \frac{\mu_0 (7.14 \times 10^{-16} \text{ A})}{2\pi (0.060 \text{ m})} = 2.38 \times 10^{-21} \text{ T, and this is a negligible contribution.}$$

$$B_C = \frac{\mu_0 I_C}{2\pi r} = \frac{\mu_0 (16 \text{ A})}{2\pi (0.060 \text{ m})} = 5.33 \times 10^{-5} \text{ T.}$$

EVALUATE: In this situation the displacement current is much less than the conduction current.

- 29.40. IDENTIFY:** Apply Ampere's law to a circular path of radius $r < R$, where R is the radius of the wire.

SET UP: The path is shown in Figure 29.40.

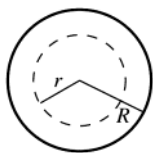


Figure 29.40

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

EXECUTE: There is no displacement current, so $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$

The magnetic field inside the superconducting material is zero, so $\oint \vec{B} \cdot d\vec{l} = 0$. But then Ampere's law says that $I_C = 0$; there can be no conduction current through the path. This same argument applies to any circular path with $r < R$, so all the current must be at the surface of the wire.

EVALUATE: If the current were uniformly spread over the wire's cross section, the magnetic field would be like that calculated in Example 28.9.

29.41. IDENTIFY: A superconducting region has zero resistance.

SET UP: If the superconducting and normal regions each lie along the length of the cylinder, they provide parallel conducting paths.

EXECUTE: Unless some of the regions with resistance completely fill a cross-sectional area of a long type-II superconducting wire, there will still be no total resistance. The regions of no resistance provide the path for the current.

EVALUATE: The situation here is like two resistors in parallel, where one has zero resistance and the other is non-zero. The equivalent resistance is zero.

29.42. IDENTIFY: Apply Eq.(28.29): $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$.

SET UP: For magnetic fields less than the critical field, there is no internal magnetic field. For fields greater than the critical field, $\vec{B}$ is very nearly equal to $\vec{B}_0$.

EXECUTE: (a) The external field is less than the critical field, so inside the superconductor $\vec{B} = 0$ and

$$\vec{M} = -\frac{\vec{B}_0}{\mu_0} = -\frac{(0.130 \text{ T})\hat{i}}{\mu_0} = -(1.03 \times 10^5 \text{ A/m})\hat{i}. \text{ Outside the superconductor, } \vec{B} = \vec{B}_0 = (0.130 \text{ T})\hat{i} \text{ and } \vec{M} = 0.$$

(b) The field is greater than the critical field and $\vec{B} = \vec{B}_0 = (0.260 \text{ T})\hat{i}$, both inside and outside the superconductor.

EVALUATE: Below the critical field the external field is expelled from the superconducting material.

29.43. IDENTIFY: Apply $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$.

SET UP: When the magnetic flux is expelled from the material the magnetic field $\vec{B}$ in the material is zero. When the material is completely normal, the magnetization is close to zero.

EXECUTE: (a) When $\vec{B}_0$ is just under $\vec{B}_{c1}$ (threshold of superconducting phase), the magnetic field in the

$$\text{material must be zero, and } \vec{M} = -\frac{\vec{B}_{c1}}{\mu_0} = -\frac{(55 \times 10^{-3} \text{ T})\hat{i}}{\mu_0} = -(4.38 \times 10^4 \text{ A/m})\hat{i}.$$

(b) When $\vec{B}_0$ is just over $\vec{B}_{c2}$ (threshold of normal phase), there is zero magnetization, and $\vec{B} = \vec{B}_{c2} = (15.0 \text{ T})\hat{i}$.

EVALUATE: Between B_{c1} and B_{c2} there are filaments of normal phase material and there is magnetic field along these filaments.

29.44. IDENTIFY and SET UP: Use Faraday's law to calculate the magnitude of the induced emf and Lenz's law to determine its direction. Apply Ohm's law to calculate I . Use Eq.(25.10) to calculate the resistance of the coil.

(a) **EXECUTE:** The angle ϕ between the normal to the coil and the direction of $\vec{B}$ is 30.0° .

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = (N\pi r^2)(dB/dt) \text{ and } I = |\mathcal{E}|/R.$$

For $t < 0$ and $t > 1.00 \text{ s}$, $dB/dt = 0$, $|\mathcal{E}| = 0$ and $I = 0$.

For $0 \leq t \leq 1.00 \text{ s}$, $dB/dt = (0.120 \text{ T})\pi \sin \pi t$

$$|\mathcal{E}| = (N\pi r^2)\pi(0.120 \text{ T})\sin \pi t = (0.9475 \text{ V})\sin \pi t$$

$$R \text{ for wire: } R_w = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}; \rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}, r = 0.0150 \times 10^{-3} \text{ m}$$

$$L = Nc = N2\pi r = (500)(2\pi)(0.0400 \text{ m}) = 125.7 \text{ m}$$

$$R_w = 3058 \Omega \text{ and the total resistance of the circuit is } R = 3058 \Omega + 600 \Omega = 3658 \Omega$$

$$I = |\mathcal{E}|/R = (0.259 \text{ mA})\sin \pi t. \text{ The graph of } I \text{ versus } t \text{ is sketched in Figure 29.44a.}$$

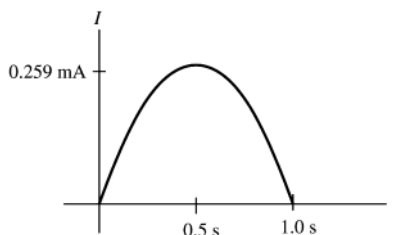
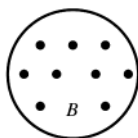


Figure 29.44a

(b) The coil and the magnetic field are shown in Figure 29.44b.



B increasing so Φ_B is $\odot$
and increasing. Φ_B is $\otimes$
so I is clockwise

Figure 29.44b

EVALUATE: The long length of small diameter wire used to make the coil has a rather large resistance, larger than the resistance of the $600\text{-}\Omega$ resistor connected to it in the circuit. The flux has a cosine time dependence so the rate of change of flux and the current have a sine time dependence. There is no induced current for $t < 0$ or $t > 1.00$ s.

29.45. IDENTIFY: Apply Faraday's law and Lenz's law.

SET UP: For a discharging RC circuit, $i(t) = \frac{V_0}{R} e^{-t/RC}$, where V_0 is the initial voltage across the capacitor. The resistance of the small loop is $(25)(0.600\text{ m})(1.0\text{ }\Omega/\text{m}) = 15.0\text{ }\Omega$.

EXECUTE: (a) The large circuit is an RC circuit with a time constant of $\tau = RC = (10\text{ }\Omega)(20 \times 10^{-6}\text{ F}) = 200\text{ }\mu\text{s}$. Thus, the current as a function of time is $i = ((100\text{ V})/(10\text{ }\Omega)) e^{-t/200\text{ }\mu\text{s}}$. At $t = 200\text{ }\mu\text{s}$, we obtain $i = (10\text{ A})(e^{-1}) = 3.7\text{ A}$.

(b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Exercise 29.7 we obtain $\Phi_B = \int_c^{c+a} \frac{\mu_0 i b}{2\pi r} dr = \frac{\mu_0 i b}{2\pi} \ln\left(1 + \frac{a}{c}\right)$. Therefore,

the emf induced in the small loop at $t = 200\text{ }\mu\text{s}$ is $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi} \ln\left(1 + \frac{a}{c}\right) \frac{di}{dt}$.

$\mathcal{E} = -\frac{(4\pi \times 10^{-7}\text{ Wb/A}\cdot\text{m}^2)(0.200\text{ m})}{2\pi} \ln(3.0) \left(-\frac{3.7\text{ A}}{200 \times 10^{-6}\text{ s}}\right) = +0.81\text{ mV}$. Thus, the induced current in the small

loop is $i' = \frac{\mathcal{E}}{R} = \frac{0.81\text{ mV}}{15.0\text{ }\Omega} = 54\text{ }\mu\text{A}$.

(c) The magnetic field from the large loop is directed out of the page within the small loop. The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

EVALUATE: (d) Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop—as can be seen by comparing the distance c to the dimensions of the large loop.

29.46. IDENTIFY: A changing magnetic field causes a changing flux through a coil and therefore induces an emf in the coil.

SET UP: Faraday's law says that the induced emf is $\mathcal{E} = -\frac{d\Phi_B}{dt}$ and the magnetic flux through a coil is defined as $\Phi_B = BA \cos\phi$.

EXECUTE: In this case, $\Phi_B = BA$, where A is constant. So the emf is proportional to the *negative* slope of the magnetic field. The result is shown in Figure 29.46.

EVALUATE: It is the rate at which the magnetic field is *changing*, not the field's magnitude, that determines the induced emf. When the field is constant, even though it may have a large value, the induced emf is zero.

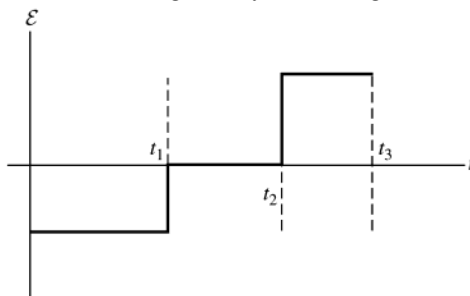


Figure 29.46

29.47. IDENTIFY: Follow the steps specified in the problem.

SET UP: Let the flux through the loop due to the current be positive.

EXECUTE: (a) $\Phi_B = BA = \frac{\mu_0 i}{2a} \pi a^2 = \frac{\mu_0 i \pi a}{2}$.

$$(b) \mathcal{E} = -\frac{d\Phi_B}{dt} = iR \Rightarrow -\frac{d}{dt}\left(\frac{\mu_0 i \pi a}{2}\right) = -\frac{\mu_0 \pi a}{2} \frac{di}{dt} = iR \Rightarrow \frac{di}{dt} = -i \frac{2R}{\mu_0 \pi a}$$

$$(c) \text{ Solving } \frac{di}{i} = -dt \frac{2R}{\mu_0 \pi a} \text{ for } i(t) \text{ yields } i(t) = i_0 e^{-t(2R/\mu_0 \pi a)}.$$

$$(d) \text{ We want } i(t) = i_0(0.010) = i_0 e^{-t(2R/\mu_0 \pi a)}, \text{ so } \ln(0.010) = -t(2R/\mu_0 \pi a) \text{ and}$$

$$t = -\frac{\mu_0 \pi a}{2R} \ln(0.010) = -\frac{\mu_0 \pi (0.50 \text{ m})}{2(0.10 \Omega)} \ln(0.010) = 4.55 \times 10^{-5} \text{ s}.$$

EVALUATE: (e) We can ignore the self-induced currents because it takes only a very short time for them to die out.

29.48. IDENTIFY: A changing magnetic field causes a changing flux through a coil and therefore induces an emf in the coil.

SET UP: Faraday's law says that the induced emf is $\mathcal{E} = -\frac{d\Phi_B}{dt}$ and the magnetic flux through a coil is defined as $\Phi_B = BA \cos \phi$.

EXECUTE: In this case, $\Phi_B = BA$, where A is constant. So the emf is proportional to the *negative* slope of the magnetic field. The result is shown in Figure 29.48.

EVALUATE: It is the rate at which the magnetic field is *changing*, not the field's magnitude, that determines the induced emf. When the field is constant, even though it may have a large value, the induced emf is zero.

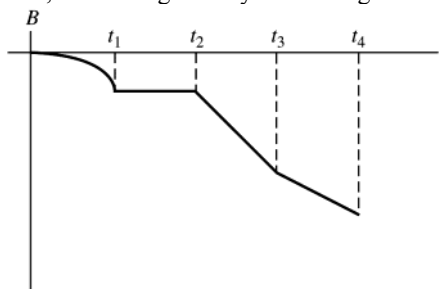


Figure 29.48

29.49. (a) IDENTIFY: (i) $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right|$. The flux is changing because the magnitude of the magnetic field of the wire decreases with distance from the wire. Find the flux through a narrow strip of area and integrate over the loop to find the total flux.

SET UP:

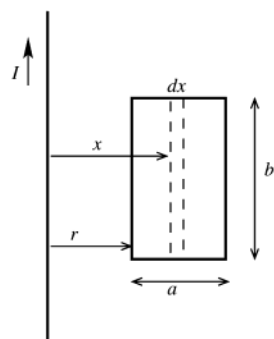


Figure 29.49a

Consider a narrow strip of width dx and a distance x from the long wire, as shown in Figure 29.49a. The magnetic field of the wire at the strip is $B = \mu_0 I / 2\pi x$. The flux through the strip is $d\Phi_B = Bb dx = (\mu_0 Ib / 2\pi)(dx/x)$

EXECUTE: The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 Ib}{2\pi} \right) \int_r^{r+a} \frac{dx}{x}$

$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi} \right) \ln \left(\frac{r+a}{r} \right)$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dr} \frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi} \left(-\frac{a}{r(r+a)} \right) v$$

$$|\mathcal{E}| = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

(ii) **IDENTIFY:** $\mathcal{E} = Bvl$ for a bar of length l moving at speed v perpendicular to a magnetic field B . Calculate the induced emf in each side of the loop, and combine the emfs according to their polarity.

SET UP: The four segments of the loop are shown in Figure 29.49b.

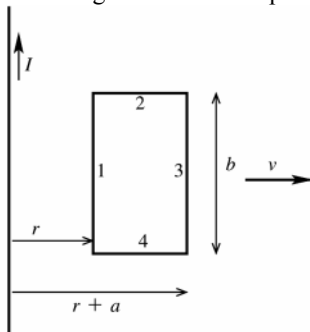


Figure 29.49b

EXECUTE: The emf in each side

of the loop is $\mathcal{E}_1 = \left(\frac{\mu_0 I}{2\pi r} \right) vb$,

$$\mathcal{E}_2 = \left(\frac{\mu_0 I}{2\pi r(r+a)} \right) vb, \quad \mathcal{E}_3 = \mathcal{E}_4 = 0$$

Both emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are directed toward the top of the loop so oppose each other. The net emf is

$$\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2 = \frac{\mu_0 I vb}{2\pi} \left(\frac{1}{r} - \frac{1}{r+a} \right) = \frac{\mu_0 I abv}{2\pi r(r+a)}$$

This expression agrees with what was obtained in (i) using Faraday's law.

(b) (i) IDENTIFY and SET UP: The flux of the induced current opposes the change in flux.

EXECUTE: $\vec{B}$ is $\otimes$. Φ_B is $\otimes$ and decreasing, so the flux Φ_{ind} of the induced current is $\otimes$ and the current is clockwise.

(ii) IDENTIFY and SET UP: Use the right-hand rule to find the force on the positive charges in each side of the loop. The forces on positive charges in segments 1 and 2 of the loop are shown in Figure 29.49c.

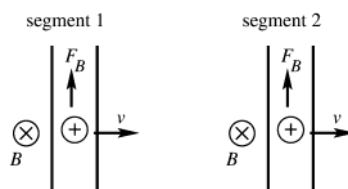


Figure 29.49c

EXECUTE: B is larger at segment 1 since it is closer to the long wire, so F_B is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.

(c) EVALUATE: When $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \rightarrow 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \rightarrow \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

29.50. IDENTIFY: Apply Faraday's law.

SET UP: For rotation about the y -axis the situation is the same as in Examples 29.4 and 29.5 and we can apply the results from those examples.

EXECUTE: (a) Rotating about the y -axis: the flux is given by $\Phi_B = BA \cos \phi$ and

$$\mathcal{E}_{\text{max}} = \left| \frac{d\Phi_B}{dt} \right| = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}.$$

(b) Rotating about the x -axis: $\frac{d\Phi_B}{dt} = 0$ and $\mathcal{E} = 0$.

(c) Rotating about the z -axis: the flux is given by $\Phi_B = BA \cos \phi$ and

$$\mathcal{E}_{\text{max}} = \left| \frac{d\Phi_B}{dt} \right| = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}.$$

EVALUATE: The maximum emf is the same if the loop is rotated about an edge parallel to the z -axis as it is when it is rotated about the z -axis.

29.51. IDENTIFY: Apply the results of Example 29.4, so $\mathcal{E}_{\text{max}} = N\omega BA$ for N loops.

SET UP: For the minimum ω , let the rotating loop have an area equal to the area of the uniform magnetic field, so $A = (0.100 \text{ m})^2$.

EXECUTE: $N = 400$, $B = 1.5 \text{ T}$, $A = (0.100 \text{ m})^2$ and $\mathcal{E}_{\text{max}} = 120 \text{ V}$ gives

$$\omega = \mathcal{E}_{\text{max}} / NBA = (20 \text{ rad/s})(1 \text{ rev}/2\pi \text{ rad})(60 \text{ s}/1 \text{ min}) = 190 \text{ rpm}.$$

EVALUATE: In $\mathcal{E}_{\text{max}} = \omega BA$, ω is in rad/s.

29.52. IDENTIFY: Apply the results of Example 29.4, generalized to N loops: $\mathcal{E}_{\max} = N\omega BA$. $v = r\omega$.

SET UP: In the expression for $\mathcal{E}_{\max}$, ω must be in rad/s. $30 \text{ rpm} = 3.14 \text{ rad/s}$

EXECUTE: (a) Solving for A we obtain $A = \frac{\mathcal{E}_0}{\omega NB} = \frac{9.0 \text{ V}}{(3.14 \text{ rad/s})(2000 \text{ turns})(8.0 \times 10^{-5} \text{ T})} = 18 \text{ m}^2$.

(b) Assuming a point on the coil at maximum distance from the axis of rotation we have

$$v = r\omega = \sqrt{\frac{A}{\pi}}\omega = \sqrt{\frac{18 \text{ m}^2}{\pi}}(3.14 \text{ rad/s}) = 7.5 \text{ m/s}.$$

EVALUATE: The device is not very feasible. The coil would need a rigid frame and the effects of air resistance would be appreciable.

29.53. IDENTIFY: Apply Faraday's law in the form $\mathcal{E}_{\text{av}} = -N \frac{\Delta\Phi_B}{\Delta t}$ to calculate the average emf. Apply Lenz's law to calculate the direction of the induced current.

SET UP: $\Phi_B = BA$. The flux changes because the area of the loop changes.

EXECUTE: (a) $\mathcal{E}_{\text{av}} = \left| \frac{\Delta\Phi_B}{\Delta t} \right| = B \left| \frac{\Delta A}{\Delta t} \right| = B \frac{\pi r^2}{\Delta t} = (0.950 \text{ T}) \frac{\pi(0.0650/2 \text{ m})^2}{0.250 \text{ s}} = 0.0126 \text{ V}$.

(b) Since the magnetic field is directed into the page and the magnitude of the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point a through the resistor to point b .

EVALUATE: Faraday's law can be used to find the direction of the induced current. Let $\vec{A}$ be into the page. Then Φ_B is positive and decreasing in magnitude, so $d\Phi_B/dt < 0$. Therefore $\mathcal{E} > 0$ and the induced current is clockwise around the loop.

29.54. IDENTIFY: By Lenz's law, the induced current flows to oppose the flux change that caused it.

SET UP: When the switch is suddenly closed with an uncharged capacitor, the current in the outer circuit immediately increases from zero to its maximum value. As the capacitor gets charged, the current in the outer circuit gradually decreases to zero.

EXECUTE: (a) (i) The current in the outer circuit is suddenly increasing and is in a counterclockwise direction. The magnetic field through the inner circuit is out of the paper and increasing. The magnetic flux through the inner circuit is increasing, so the induced current in the inner circuit is clockwise (a to b) to oppose the flux increase. (ii) The current in the outer circuit is still counterclockwise but is now decreasing, so the magnetic field through the inner circuit is out of the page but decreasing. The flux through the inner circuit is now decreasing, so the induced current is counterclockwise (b to a) to oppose the flux decrease.

(b) The graph is sketched in Figure 29.54.

EVALUATE: Even though the current in the outer circuit does not change direction, the current in the inner circuit does as the flux through it changes from increasing to decreasing.

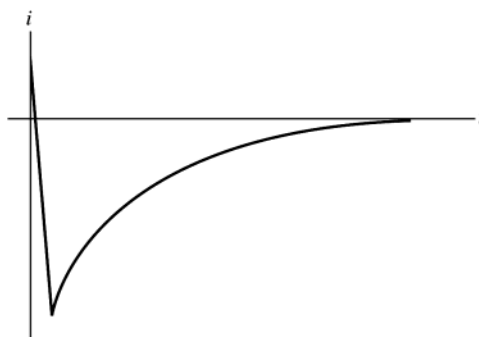


Figure 29.54

29.55. IDENTIFY: Use Faraday's law to calculate the induced emf and Ohm's law to find the induced current. Use Eq.(27.19) to calculate the magnetic force F_l on the induced current. Use the net force $F - F_l$ in Newton's 2nd law to calculate the acceleration of the rod and use that to describe its motion.

(a) SET UP: The forces in the rod are shown in Figure 29.55a.

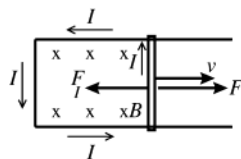


Figure 29.55a

EXECUTE: $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = BLv$

$$I = \frac{BLv}{R}$$

Use $\mathcal{E} = -\frac{d\Phi_B}{dt}$ to find the direction of I : Let $\vec{A}$ be into the page. Then $\Phi_B > 0$. The area of the circuit is

increasing, so $\frac{d\Phi_B}{dt} > 0$. Then $\mathcal{E} < 0$ and with our direction for $\vec{A}$ this means that $\mathcal{E}$ and I are counterclockwise, as shown in the sketch. The force F_I on the rod due to the induced current is given by $\vec{F}_I = I\vec{L} \times \vec{B}$. This gives $\vec{F}_I$ to the left with magnitude $F_I = ILB = (BLv/R)LB = B^2L^2v/R$. Note that $\vec{F}_I$ is directed to oppose the motion of the rod, as required by Lenz's law.

EVALUATE: The net force on the rod is $F - F_I$, so its acceleration is $a = (F - F_I)/m = (F - B^2L^2v/R)/m$. The rod starts with $v = 0$ and $a = F/m$. As the speed v increases the acceleration a decreases. When $a = 0$ the rod has reached its terminal speed v_t . The graph of v versus t is sketched in Figure 29.55b.

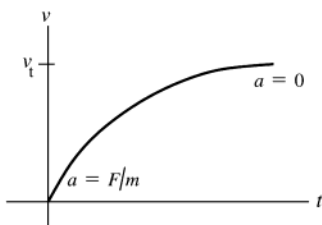


Figure 29.55b

(Recall that a is the slope of the tangent to the v versus t curve.)

(b) EXECUTE: $v = v_t$ when $a = 0$ so $\frac{F - B^2L^2v_t/R}{m} = 0$ and $v_t = \frac{RF}{B^2L^2}$.

EVALUATE: A large F produces a large v_t . If B is larger, or R is smaller, the induced current is larger at a given v so F_I is larger and the terminal speed is less.

29.56. IDENTIFY: Apply Newton's 2nd law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v . At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv . The induced emf is counterclockwise, so it opposes the voltage of the battery, $\mathcal{E}$.

EXECUTE: (a) The net current in the loop is $I = \frac{\mathcal{E} - BLv}{R}$. The acceleration of the bar is

$a = \frac{F}{m} = \frac{ILB \sin(90^\circ)}{m} = \frac{(\mathcal{E} - BLv)LB}{mR}$. To find $v(t)$, set $\frac{dv}{dt} = a = \frac{(\mathcal{E} - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$$\int_0^v \frac{dv}{(\mathcal{E} - BLv)} = \int_0^t \frac{LB}{mR} dt \rightarrow v = \frac{\mathcal{E}}{BL} (1 - e^{-B^2L^2t/mR}) = (10 \text{ m/s})(1 - e^{-t/3.1 \text{ s}})$$

The graph of v versus t is sketched in Figure 29.56. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \mathcal{E}/R = 2.4 \text{ A}$, $F = ILB = 2.88 \text{ N}$ and $a = F/m = 3.2 \text{ m/s}^2$.

(c) When $v = 2.0 \text{ m/s}$, $a = \frac{[12 \text{ V} - (1.5 \text{ T})(0.8 \text{ m})(2.0 \text{ m/s})](0.8 \text{ m})(1.5 \text{ T})}{(0.90 \text{ kg})(5.0 \Omega)} = 2.6 \text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\frac{\mathcal{E}}{BL} = \frac{12 \text{ V}}{(1.5 \text{ T})(0.8 \text{ m})} = 10 \text{ m/s}$, which makes the acceleration zero.

EVALUATE: The current in the circuit is counterclockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

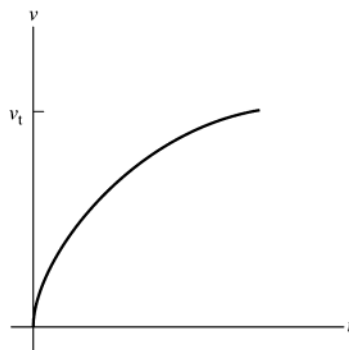


Figure 29.56

29.57. IDENTIFY: Apply $\mathcal{E} = BvL$. Use $\sum \vec{F} = m\vec{a}$ applied to the satellite motion to find the speed v of the satellite.

SET UP: The gravitational force on the satellite is $F_g = G \frac{mm_E}{r^2}$, where m is the mass of the satellite and r is the radius of its orbit.

EXECUTE: $B = 8.0 \times 10^{-5} \text{ T}$, $L = 2.0 \text{ m}$. $G \frac{mm_E}{r^2} = m \frac{v^2}{r}$ and $r = 400 \times 10^3 \text{ m} + R_E$ gives $v = \sqrt{\frac{Gm_E}{r}} = 7.665 \times 10^3 \text{ m/s}$.

Using this v in $\mathcal{E} = vBL$ gives $\mathcal{E} = (8.0 \times 10^{-5} \text{ T})(7.665 \times 10^3 \text{ m/s})(2.0 \text{ m}) = 1.2 \text{ V}$.

EVALUATE: The induced emf is large enough to be measured easily.

29.58. IDENTIFY: The induced emf is $\mathcal{E} = BvL$, where L is measured in a direction that is perpendicular to both the magnetic field and the velocity of the bar.

SET UP: The magnetic force pushed positive charge toward the high potential end of the bullet.

EXECUTE: (a) $\mathcal{E} = BLv = (8 \times 10^{-5} \text{ T})(0.004 \text{ m})(300 \text{ m/s}) = 96 \mu\text{V}$. Since a positive charge moving to the east would be deflected upward, the top of the bullet will be at a higher potential.

(b) For a bullet that travels south, $\vec{v}$ and $\vec{B}$ are along the same line, there is no magnetic force and the induced emf is zero.

(c) If $\vec{v}$ is horizontal, the magnetic force on positive charges in the bullet is either upward or downward, perpendicular to the line between the front and back of the bullet. There is no emf induced between the front and back of the bullet.

EVALUATE: Since the velocity of a bullet is always in the direction from the back to the front of the bullet, and since the magnetic force is perpendicular to the velocity, there is never an induced emf between the front and back of the bullet, no matter what the direction of the magnetic field is.

29.59. IDENTIFY: Find the magnetic field at a distance r from the center of the wire. Divide the rectangle into narrow strips of width dr , find the flux through each strip and integrate to find the total flux.

SET UP: Example 28.8 uses Ampere's law to show that the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 I r / 2\pi R^2$.

EXECUTE: Consider a small strip of length W and width dr that is a distance r from the axis of the wire, as shown in Figure 29.59. The flux through the strip is $d\Phi_B = B(r)W dr = \frac{\mu_0 IW}{2\pi R^2} r dr$. The total flux through the rectangle is

$$\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2} \right) \int_0^R r dr = \frac{\mu_0 IW}{4\pi}.$$

EVALUATE: Note that the result is independent of the radius R of the wire.

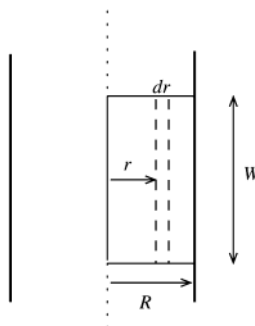


Figure 29.59

29.60. IDENTIFY: Apply Faraday's law to calculate the magnitude and direction of the induced emf.

SET UP: Let $\vec{A}$ be directed out of the page in Figure 29.50 in the textbook. This means that counterclockwise emf is positive.

EXECUTE: (a) $\Phi_B = BA = B_0\pi r_0^2(1 - 3(t/t_0)^2 + 2(t/t_0)^3)$.

$$(b) \mathcal{E} = -\frac{d\Phi_B}{dt} = -B_0\pi r_0^2 \frac{d}{dt}(1 - 3(t/t_0)^2 + 2(t/t_0)^3) = -\frac{B_0\pi r_0^2}{t_0}(-6(t/t_0) + 6(t/t_0)^2). \mathcal{E} = -\frac{6 B_0\pi r_0^2}{t_0} = \left(\left(\frac{t}{t_0}\right)^2 - \left(\frac{t}{t_0}\right)\right). \text{ At}$$

$$t = 5.0 \times 10^{-3} \text{ s}, \mathcal{E} = -\frac{6B_0\pi(0.0420 \text{ m})^2}{0.010 \text{ s}} \left(\left(\frac{5.0 \times 10^{-3} \text{ s}}{0.010 \text{ s}} \right)^2 - \left(\frac{5.0 \times 10^{-3} \text{ s}}{0.010 \text{ s}} \right) \right) = 0.0665 \text{ V}. \mathcal{E} \text{ is positive so it is}$$

counterclockwise.

$$(c) I = \frac{\mathcal{E}}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r + R = \frac{\mathcal{E}}{I} \Rightarrow r = \frac{0.0665 \text{ V}}{3.0 \times 10^{-3} \text{ A}} - 12 \Omega = 10.2 \Omega.$$

(d) Evaluating the emf at $t = 1.21 \times 10^{-2} \text{ s}$ and using the equations of part (b), $\mathcal{E} = -0.0676 \text{ V}$, and the current flows clockwise, from b to a through the resistor.

$$(e) \mathcal{E} = 0 \text{ when } 0 = \left(\left(\frac{t}{t_0} \right)^2 - \left(\frac{t}{t_0} \right) \right). 1 = \frac{t}{t_0} \text{ and } t = t_0 = 0.010 \text{ s}.$$

EVALUATE: At $t = t_0$, $B = 0$. At $t = 5.00 \times 10^{-3} \text{ s}$, $\vec{B}$ is in the $+\hat{k}$ direction and is decreasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is counterclockwise. At $t = 0.0121 \text{ s}$, $\vec{B}$ is in the $+\hat{k}$ direction and is increasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is clockwise. These results for the direction of $\mathcal{E}$ agree with the results we obtained from Faraday's law.

29.61. (a) and (b) IDENTIFY and Set Up:

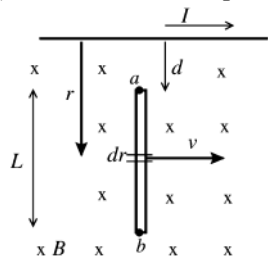


Figure 29.61a

The magnetic field of the wire is given by

$$B = \frac{\mu_0 I}{2\pi r} \text{ and varies along the length of the}$$

bar. At every point along the bar $\vec{B}$ has direction into the page. Divide the bar up into thin slices, as shown in Figure 29.61a.

EXECUTE: The emf $d\mathcal{E}$ induced in each slice is given by $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$. $\vec{v} \times \vec{B}$ is directed toward the wire, so

$$d\mathcal{E} = -vB dr = -v \left(\frac{\mu_0 I}{2\pi r} \right) dr. \text{ The total emf induced in the bar is}$$

$$V_{ba} = \int_a^b d\mathcal{E} = -\int_d^{d+L} \left(\frac{\mu_0 Iv}{2\pi r} \right) dr = -\frac{\mu_0 Iv}{2\pi} \int_d^{d+L} \frac{dr}{r} = -\frac{\mu_0 Iv}{2\pi} [\ln(r)]_d^{d+L}$$

$$V_{ba} = -\frac{\mu_0 Iv}{2\pi} (\ln(d+L) - \ln(d)) = -\frac{\mu_0 Iv}{2\pi} \ln(1 + L/d)$$

EVALUATE: The minus sign means that V_{ba} is negative, point a is at higher potential than point b . (The force $\vec{F} = q\vec{v} \times \vec{B}$ on positive charge carriers in the bar is towards a , so a is at higher potential.) The potential difference increases when I or v increase, or d decreases.

(c) **IDENTIFY:** Use Faraday's law to calculate the induced emf.

SET UP: The wire and loop are sketched in Figure 29.61b.

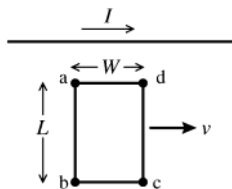


Figure 29.61b

EXECUTE: As the loop moves to the right the magnetic flux through it doesn't change. Thus

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = 0 \text{ and } I = 0.$$

EVALUATE: This result can also be understood as follows. The induced emf in section ab puts point a at higher potential; the induced emf in section dc puts point d at higher potential. If you travel around the loop then these two induced emf's sum to zero. There is no emf in the loop and hence no current.

- 29.62. IDENTIFY:** $\mathcal{E} = vBL$, where v is the component of velocity perpendicular to the field direction and perpendicular to the bar.

SET UP: Wires A and C have a length of 0.500 m and wire D has a length of $\sqrt{2(0.500 \text{ m})^2} = 0.707 \text{ m}$.

EXECUTE: Wire A : $\vec{v}$ is parallel to $\vec{B}$, so the induced emf is zero.

Wire C : $\vec{v}$ is perpendicular to $\vec{B}$. The component of $\vec{v}$ perpendicular to the bar is $v \cos 45^\circ$.

$$\mathcal{E} = (0.350 \text{ m/s})(\cos 45^\circ)(0.120 \text{ T})(0.500 \text{ m}) = 0.0148 \text{ V}.$$

Wire D : $\vec{v}$ is perpendicular to $\vec{B}$. The component of $\vec{v}$ perpendicular to the bar is $v \cos 45^\circ$.

$$\mathcal{E} = (0.350 \text{ m/s})(\cos 45^\circ)(0.120 \text{ T})(0.707 \text{ m}) = 0.0210 \text{ V}.$$

EVALUATE: The induced emf depends on the angle between $\vec{v}$ and $\vec{B}$ and also on the angle between $\vec{v}$ and the bar.

- 29.63. (a) IDENTIFY:** Use the expression for motional emf to calculate the emf induced in the rod.

SET UP: The rotating rod is shown in Figure 29.63a.

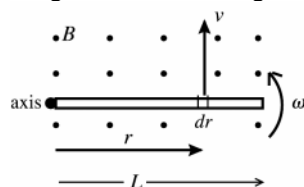


Figure 29.63a

The emf induced in a thin slice is $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{\ell}$.

EXECUTE: Assume that $\vec{B}$ is directed out of the page. Then $\vec{v} \times \vec{B}$ is directed radially outward and

$$d\ell = dr, \text{ so } \vec{v} \times \vec{B} \cdot d\vec{\ell} = vB \, dr$$

$$v = r\omega \text{ so } d\mathcal{E} = \omega B r \, dr.$$

The $d\mathcal{E}$ for all the thin slices that make up the rod are in series so they add:

$$\mathcal{E} = \int d\mathcal{E} = \int_0^L \omega B r \, dr = \frac{1}{2} \omega B L^2 = \frac{1}{2} (8.80 \text{ rad/s})(0.650 \text{ T})(0.240 \text{ m})^2 = 0.165 \text{ V}$$

EVALUATE: $\mathcal{E}$ increases with ω , B or L^2 .

(b) No current flows so there is no IR drop in potential. Thus the potential difference between the ends equals the emf of 0.165 V calculated in part (a).

(c) **SET UP:** The rotating rod is shown in Figure 29.63b.

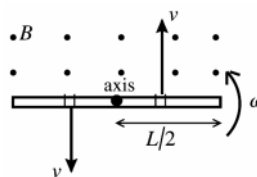


Figure 29.63b

EXECUTE: The emf between the center of the rod and each end is $\mathcal{E} = \frac{1}{2} \omega B (L/2)^2 = \frac{1}{4} (0.165 \text{ V}) = 0.0412 \text{ V}$, with the direction of the emf from the center of the rod toward each end. The emfs in each half of the rod thus oppose each other and there is no net emf between the ends of the rod.

EVALUATE: ω and B are the same as in part (a) but L of each half is $\frac{1}{2}L$ for the whole rod. $\mathcal{E}$ is proportional to L^2 , so is smaller by a factor of $\frac{1}{4}$.

- 29.64. IDENTIFY:** The power applied by the person in moving the bar equals the rate at which the electrical energy is dissipated in the resistance.

SET UP: From Example 29.7, the power required to keep the bar moving at a constant velocity is $P = \frac{(BLv)^2}{R}$.

$$\text{EXECUTE: (a) } R = \frac{(BLv)^2}{P} = \frac{[(0.25 \text{ T})(3.0 \text{ m})(2.0 \text{ m/s})]^2}{25 \text{ W}} = 0.090 \, \Omega.$$

(b) For a 50 W power dissipation we would require that the resistance be decreased to half the previous value.

(c) Using the resistance from part (a) and a bar length of 0.20 m,

$$P = \frac{(BLv)^2}{R} = \frac{[(0.25 \text{ T})(0.20 \text{ m})(2.0 \text{ m/s})]^2}{0.090 \, \Omega} = 0.11 \text{ W}.$$

EVALUATE: When the bar is moving to the right the magnetic force on the bar is to the left and an applied force directed to the right is required to maintain constant speed. When the bar is moving to the left the magnetic force on the bar is to the right and an applied force directed to the left is required to maintain constant speed.

- 29.65. (a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate I , and Eq.(27.19) to calculate the force on the rod due to the induced current.

SET UP: The force on the wire is shown in Figure 29.65.

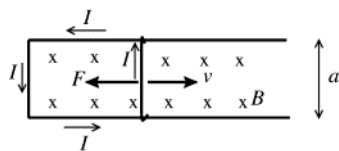


Figure 29.65

EXECUTE: When the wire has speed v the induced emf is $\mathcal{E} = Bva$ and the

induced current is $I = \mathcal{E}/R = \frac{Bva}{R}$

The induced current flows upward in the wire as shown, so the force $\vec{F} = I\vec{L} \times \vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F = IaB = B^2a^2v/R$.

- (b)** Apply $\sum \vec{F} = m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t = 0$, so $x_0 = 0$.

$$\sum F_x = ma_x \text{ says } -F = ma_x$$

$$a_x = -\frac{F}{m} = -\frac{B^2a^2v}{mR}$$

Use this expression to solve for $v(t)$:

$$a_x = \frac{dv}{dt} = -\frac{B^2a^2v}{mR} \text{ and } \frac{dv}{v} = -\frac{B^2a^2}{mR} dt$$

$$\int_{v_0}^v \frac{dv'}{v'} = -\frac{B^2a^2}{mR} \int_0^t dt'$$

$$\ln(v) - \ln(v_0) = -\frac{B^2a^2t}{mR}$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2a^2t}{mR} \text{ and } v = v_0 e^{-B^2a^2t/mR}$$

Note: At $t = 0$, $v = v_0$ and $v \rightarrow 0$ when $t \rightarrow \infty$

Now solve for $x(t)$:

$$v = \frac{dx}{dt} = v_0 e^{-B^2a^2t/mR} \text{ so } dx = v_0 e^{-B^2a^2t/mR} dt$$

$$\int_0^x dx' = \int_0^t v_0 e^{-B^2a^2t'/mR} dt'$$

$$x = v_0 \left(-\frac{mR}{B^2a^2} \right) \left[e^{-B^2a^2t'/mR} \right]_0^t = \frac{mRv_0}{B^2a^2} (1 - e^{-B^2a^2t/mR})$$

Comes to rest implies $v = 0$. This happens when $t \rightarrow \infty$.

$t \rightarrow \infty$ gives $x = \frac{mRv_0}{B^2a^2}$. Thus this is the distance the wire travels before coming to rest.

EVALUATE: The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on v and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2a^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2a^2$. The actual stopping distance is twice this.

- 29.66. IDENTIFY:** Since the bar is straight and the magnetic field is uniform, integrating $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{L}$ along the length of the bar gives $\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{L}$

SET UP: $\vec{v} = (4.20 \text{ m/s})\hat{i}$. $\vec{L} = (0.250 \text{ m})(\cos 36.9^\circ\hat{i} + \sin 36.9^\circ\hat{j})$.

EXECUTE: (a) $\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{L} = (4.20 \text{ m/s})\hat{i} \times ((0.120 \text{ T})\hat{i} - (0.220 \text{ T})\hat{j}) \cdot (0.0900 \text{ T})\hat{k} \cdot \vec{L}$

$$\mathcal{E} = ((0.378 \text{ V/m})\hat{j} - (0.924 \text{ V/m})\hat{k}) \cdot ((0.250 \text{ m})(\cos 36.9^\circ\hat{i} + \sin 36.9^\circ\hat{j}))$$

$$\mathcal{E} = (0.378 \text{ V/m})(0.250 \text{ m})\sin 36.9^\circ = 0.0567 \text{ V}$$

(b) The higher potential end is the end to which positive charges in the rod are pushed by the magnetic force.

$\vec{v} \times \vec{B}$ has a positive y -component, so the end of the rod marked $+$ in Figure 29.66 is at higher potential.

EVALUATE: Since $\vec{v} \times \vec{B}$ has nonzero $\hat{j}$ and $\hat{k}$ components, and $\vec{L}$ has nonzero $\hat{i}$ and $\hat{j}$ components, only the $\hat{k}$ component of $\vec{B}$ contributes to $\mathcal{E}$. In fact, $|\mathcal{E}| = |v_x B_z L_y| = (4.20 \text{ m/s})(0.0900 \text{ T})(0.250 \text{ m})\sin 36.9^\circ = 0.0567 \text{ V}$.

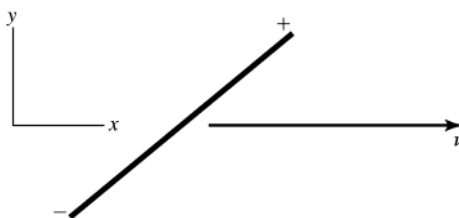


Figure 29.66

29.67. IDENTIFY: Use Eq.(29.10) to calculate the induced electric field at each point and then use $\vec{F} = q\vec{E}$.

SET UP:

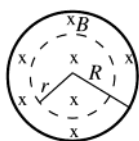


Figure 29.67a

Apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to a concentric circle of radius r , as shown in Figure 29.67a. Take $\vec{A}$ to be into the page, in the direction of $\vec{B}$.

EXECUTE: B increasing then gives $\frac{d\Phi_B}{dt} > 0$, so $\oint \vec{E} \cdot d\vec{l}$ is negative. This means that E is tangent to the circle in the counterclockwise direction, as shown in Figure 29.67b.

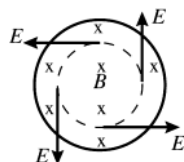


Figure 29.67b

$$\oint \vec{E} \cdot d\vec{l} = -E(2\pi r)$$

$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$$

$$-E(2\pi r) = -\pi r^2 \frac{dB}{dt} \text{ so } E = \frac{1}{2} r \frac{dB}{dt}$$

point a The induced electric field and the force on q are shown in Figure 29.67c.

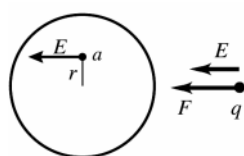


Figure 29.67c

$$F = qE = \frac{1}{2} qr \frac{dB}{dt}$$

$\vec{F}$ is to the left
($\vec{F}$ is in the same direction as $\vec{E}$ since q is positive.)

point b The induced electric field and the force on q are shown in Figure 29.67d.

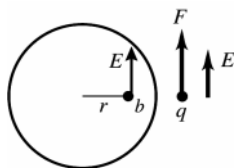


Figure 29.67d

$$F = qE = \frac{1}{2} qr \frac{dB}{dt}$$

$\vec{F}$ is toward the top of the page.

point c $r = 0$ here, so $E = 0$ and $F = 0$.

EVALUATE: If there were a concentric conducting ring of radius r in the magnetic field region, Lenz's law tells us that the increasing magnetic field would induce a counterclockwise current in the ring. This agrees with the direction of the force we calculated for the individual positive point charges.

29.68. IDENTIFY: A bar moving in a magnetic field has an emf induced across its ends. The propeller acts as such a bar.

SET UP: Different parts of the propeller are moving at different speeds, so we must integrate to get the total induced emf. The potential induced across an element of length dx is $d\mathcal{E} = vBdx$, where B is uniform.

EXECUTE: (a) Call x the distance from the center to an element of length dx , and L the length of the propeller.

The speed of dx is $x\omega$, giving $d\mathcal{E} = vBdx = x\omega Bdx$. $\mathcal{E} = \int_0^{L/2} x\omega Bdx = \omega BL^2/8$.

(b) The potential difference is zero since the potential is the same at both ends of the propeller.

$$(c) \mathcal{E} = 2\pi \left(\frac{220 \text{ rev}}{60 \text{ s}} \right) (0.50 \times 10^{-4} \text{ T}) \frac{(2.0 \text{ m})^3}{8} = 5.8 \times 10^{-4} \text{ V} = 0.58 \text{ mV}$$

EVALUATE: A potential difference of about $\frac{1}{2}$ mV is not large enough to be concerned about in a propeller.

29.69. IDENTIFY: Follow the steps specified in the problem.

SET UP: The electric field region is sketched in Figure 29.69.

EXECUTE: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$. If $\vec{B}$ is constant then $\frac{d\Phi_B}{dt} = 0$, so $\oint \vec{E} \cdot d\vec{l} = 0$. $\int_{abcda} \vec{E} \cdot d\vec{l} = E_{ab}L - E_{cd}L = 0$. But

$E_{cd} = 0$, so $E_{ab}L = 0$. But since we assumed $E_{ab} \neq 0$, this contradicts Faraday's law. Thus, we can't have a uniform electric field abruptly drop to zero in a region in which the magnetic field is constant.

EVALUATE: If the magnetic field in the region is constant, then the integral $\oint \vec{E} \cdot d\vec{l}$ must be zero.

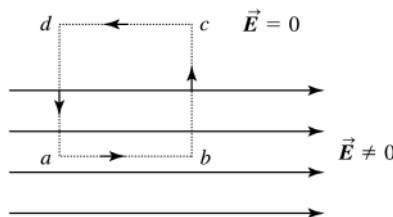


Figure 26.69

29.70. IDENTIFY and SET UP: At the terminal speed v_t , the upward force F_l exerted on the loop due to the induced current equals the downward force of gravity: $F_l = mg$. Use Eq.(29.6) to find the induced emf in the side of the loop that is totally within the magnetic field. There is no induced emf in the other sides of the loop.

EXECUTE: $\mathcal{E} = Bvs$, $I = Bvs/R$ and $F_l = IsB = B^2s^2v/R$

$$\frac{B^2s^2v_t}{R} = mg \text{ and } v_t = \frac{mgR}{B^2s^2}$$

$$m = \rho_m V = \rho_m (4s) \pi (d/2)^2 = \rho_m \pi d^2 s$$

$$R = \frac{\rho L}{A} = \frac{\rho_R 4s}{\frac{1}{4} \pi d^2} = \frac{16 \rho_R s}{\pi d^2}$$

Using these expressions for m and R gives $v_t = 16 \rho_m \rho_R g / B^2$

EVALUATE: We know $\rho_m = 8900 \text{ kg/m}^3$ (Table 14.1) and $\rho_R = 1.72 \times 10^{-8} \Omega \cdot \text{m}$ (Table 25.1). Taking $B = 0.5 \text{ T}$ gives $v_t = 9.6 \text{ cm/s}$.

29.71. IDENTIFY: Follow the steps specified in the problem.

SET UP: (a) The magnetic field region is sketched in Figure 29.71.

EXECUTE: (b) $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $\vec{B} = B_0 \hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

$\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0$ but $B_{cd} = 0$. $B_{ab}L = 0$, but $B_{ab} \neq 0$. This is a contradiction and violates Ampere's Law.

EVALUATE: We often describe a magnetic field as being confined to a region, but this result shows that the edges of such a region can't be sharp.

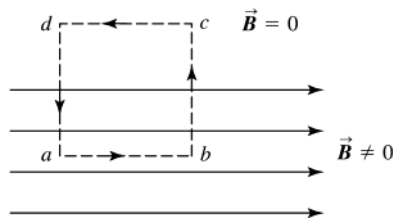


Figure 29.71

- 29.72. IDENTIFY and SET UP:** Apply Ohm's law to the dielectric to relate the current in the dielectric to the charge on the plates. Use Eq.(25.1) for the current and obtain a differential equation for $q(t)$. Integrate this equation to obtain $q(t)$ and $i(t)$. Use $E = q/\epsilon A$ and Eq.(29.16) to calculate j_D .

EXECUTE: (a) Apply Ohm's law to the dielectric: The capacitor is sketched in Figure 29.72.

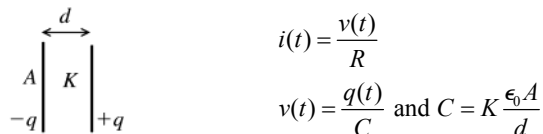


Figure 29.72

$$v(t) = \left(\frac{d}{K\epsilon_0 A} \right) q(t)$$

The resistance R of the dielectric slab is $R = \rho d / A$. Thus $i(t) = \frac{v(t)}{R} = \left(\frac{q(t)d}{K\epsilon_0 A} \right) \left(\frac{A}{\rho d} \right) = \frac{q(t)}{K\epsilon_0 \rho}$. But the current $i(t)$

in the dielectric is related to the rate of change dq/dt of the charge $q(t)$ on the plates by $i(t) = -dq/dt$ (a positive i in the direction from the + to the - plate of the capacitor corresponds to a decrease in the charge). Using this in the above

gives $-\frac{dq}{dt} = \left(\frac{1}{K\epsilon_0 \rho} \right) q(t)$. $\frac{dq}{q} = -\frac{dt}{K\epsilon_0 \rho}$. Integrate both sides of this equation from $t = 0$, where $q = Q_0$, to a later

time t when the charge is $q(t)$. $\int_{Q_0}^q \frac{dq}{q} = -\left(\frac{1}{K\epsilon_0 \rho} \right) \int_0^t dt$. $\ln \left(\frac{q}{Q_0} \right) = -\frac{t}{K\epsilon_0 \rho}$ and $q(t) = Q_0 e^{-t/K\epsilon_0 \rho}$. Then

$i(t) = -\frac{dq}{dt} = \left(\frac{Q_0}{K\epsilon_0 \rho} \right) e^{-t/K\epsilon_0 \rho}$ and $j_C = \frac{i(t)}{A} = \left(\frac{Q_0}{AK\epsilon_0 \rho} \right) e^{-t/K\epsilon_0 \rho}$. The conduction current flows from the positive to

the negative plate of the capacitor.

(b) $E(t) = \frac{q(t)}{\epsilon A} = \frac{q(t)}{K\epsilon_0 A}$

$$j_D(t) = \epsilon \frac{dE}{dt} = K\epsilon_0 \frac{dE}{dt} = K\epsilon_0 \frac{dq(t)/dt}{K\epsilon_0 A} = -\frac{i_C(t)}{A} = -j_C(t)$$

The minus sign means that $j_D(t)$ is directed from the negative to the positive plate. $\vec{E}$ is from + to - but dE/dt is negative (E decreases) so $j_D(t)$ is from - to +.

EVALUATE: There is no conduction current to and from the plates so the concept of displacement current, with $\vec{j}_D = -\vec{j}_C$ in the dielectric, allows the current to be continuous at the capacitor.

- 29.73. IDENTIFY:** The conduction current density is related to the electric field by Ohm's law. The displacement current density is related to the rate of change of the electric field by Eq.(29.16).

SET UP: $dE/dt = \omega E_0 \cos \omega t$

EXECUTE: (a) $j_C(\text{max}) = \frac{E_0}{\rho} = \frac{0.450 \text{ V/m}}{2300 \Omega \cdot \text{m}} = 1.96 \times 10^{-4} \text{ A/m}^2$

(b) $j_D(\text{max}) = \epsilon_0 \left(\frac{dE}{dt} \right)_{\text{max}} = \epsilon_0 \omega E_0 = 2\pi \epsilon_0 f E_0 = 2\pi \epsilon_0 (120 \text{ Hz})(0.450 \text{ V/m}) = 3.00 \times 10^{-9} \text{ A/m}^2$

(c) If $j_C = j_D$ then $\frac{E_0}{\rho} = \omega \epsilon_0 E_0$ and $\omega = \frac{1}{\rho \epsilon_0} = 4.91 \times 10^7 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = \frac{4.91 \times 10^7 \text{ rad/s}}{2\pi} = 7.82 \times 10^6 \text{ Hz.}$$

EVALUATE: (d) The two current densities are out of phase by 90° because one has a sine function and the other has a cosine, so the displacement current leads the conduction current by 90° .

- 29.74. IDENTIFY:** A current is induced in the loop because of its motion and because of this current the magnetic field exerts a torque on the loop.

SET UP: Each side of the loop has mass $m/4$ and the center of mass of each side is at the center of each side. The flux through the loop is $\Phi_B = BA \cos \phi$.

EXECUTE: (a) $\vec{\tau}_g = \sum \vec{r}_{cm} \times m\vec{g}$ summed over each leg.

$$\tau_g = \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g \sin(90^\circ - \phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g \sin(90^\circ - \phi) + (L)\left(\frac{m}{4}\right)g \sin(90^\circ - \phi)$$

$$\tau_g = \frac{mgL}{2} \cos \phi \text{ (clockwise).}$$

$$\tau_B = |\vec{\tau} \times \vec{B}| = IAB \sin \phi \text{ (counterclockwise).}$$

$$I = \frac{\mathcal{E}}{R} = \frac{BA}{R} \frac{d}{dt} \cos \phi = -\frac{BA}{R} \frac{d\phi}{dt} \sin \phi = \frac{BA\omega}{R} \sin \phi. \text{ The current is going counterclockwise looking to the } -\hat{k} \text{ direction.}$$

Therefore, $\tau_B = \frac{B^2 A^2 \omega}{R} \sin^2 \phi = \frac{B^2 L^4 \omega}{R} \sin^2 \phi$. The net torque is $\tau = \frac{mgL}{2} \cos \phi - \frac{B^2 L^4 \omega}{R} \sin^2 \phi$, opposite to the direction of the rotation.

(b) $\tau = I\alpha$ (I being the moment of inertia). About this axis $I = \frac{5}{12} mL^2$. Therefore,

$$\alpha = \frac{12}{5} \frac{1}{mL^2} \left[\frac{mgL}{2} \cos \phi - \frac{B^2 L^4 \omega}{R} \sin^2 \phi \right] = \frac{6g}{5L} \cos \phi - \frac{12B^2 L^2 \omega}{5mR} \sin^2 \phi.$$

EVALUATE: (c) The magnetic torque slows down the fall (since it opposes the gravitational torque).

(d) Some energy is lost through heat from the resistance of the loop.

29.75. IDENTIFY: Apply Eq.(29.10).

SET UP: Use an integration path that is a circle of radius r . By symmetry the induced electric field is tangent to this path and constant in magnitude at all points on the path.

EXECUTE: (a) The induced electric field at these points is shown in Figure 29.75a.

(b) To work out the amount of the electric field that is in the direction of the loop at a general position, we will use the geometry shown in Figure 29.75b. $E_{\text{loop}} = E \cos \theta$ but $E = \frac{\mathcal{E}}{2\pi r} = \frac{\mathcal{E}}{2\pi(a/\cos \theta)} = \frac{\mathcal{E} \cos \theta}{2\pi a}$. Therefore,

$E_{\text{loop}} = \frac{\mathcal{E} \cos^2 \theta}{2\pi a}$. But $\mathcal{E} = \frac{d\Phi_B}{dt} = A \frac{dB}{dt} = \pi r^2 \frac{dB}{dt} = \frac{\pi a^2}{\cos^2 \theta} \frac{dB}{dt}$, so $E_{\text{loop}} = \frac{\pi a^2}{2\pi a} \frac{dB}{dt} = \frac{a}{2} \frac{dB}{dt}$. This is exactly the value for a ring, obtained in Exercise 29.30, and has no dependence on the part of the loop we pick.

$$(c) I = \frac{\mathcal{E}}{R} = \frac{A}{R} \frac{dB}{dt} = \frac{L^2}{R} \frac{dB}{dt} = \frac{(0.20 \text{ m})^2 (0.0350 \text{ T/s})}{1.90 \Omega} = 7.37 \times 10^{-4} \text{ A.}$$

$$(d) \mathcal{E}_{ab} = \frac{1}{8} \mathcal{E} = \frac{1}{8} L^2 \frac{dB}{dt} = \frac{(0.20 \text{ m})^2 (0.0350 \text{ T/s})}{8} = 1.75 \times 10^{-4} \text{ V. But there is potential drop } V = IR = -1.75 \times 10^{-4} \text{ V,}$$

so the potential difference is zero.

EVALUATE: The magnitude of the induced emf between any two points equals the magnitudes of the potential drop due to the current through the resistance of that portion of the loop.

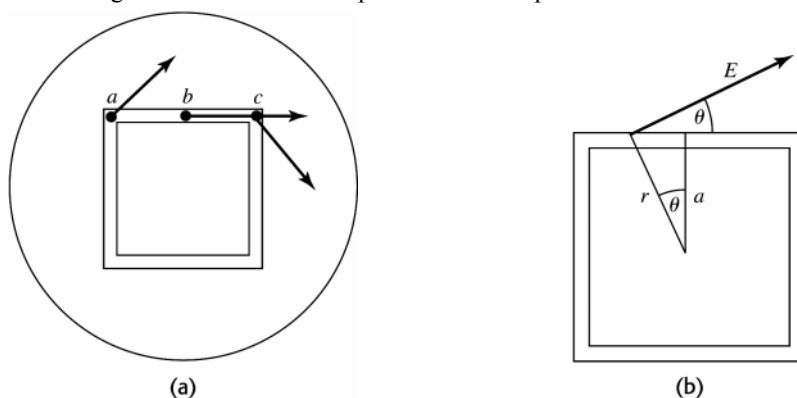


Figure 29.75

29.76. IDENTIFY: Apply Eq.(29.10).

SET UP: Use an integration path that is a circle of radius r . By symmetry the induced electric field is tangent to this path and constant in magnitude at all points on the path.

EXECUTE: (a) The induced emf at these points is shown in Figure 29.76.

(b) The induced emf on the side ac is zero, because the electric field is always perpendicular to the line ac .

$$(c) \text{ To calculate the total emf in the loop, } \mathcal{E} = \frac{d\Phi_B}{dt} = A \frac{dB}{dt} = L^2 \frac{dB}{dt}. \quad \mathcal{E} = (0.20 \text{ m})^2 (0.035 \text{ T/s}) = 1.40 \times 10^{-3} \text{ V.}$$

$$(d) I = \frac{\mathcal{E}}{R} = \frac{1.40 \times 10^{-3} \text{ V}}{1.90 \Omega} = 7.37 \times 10^{-4} \text{ A}$$

(e) Since the loop is uniform, the resistance in length ac is one quarter of the total resistance. Therefore the potential difference between a and c is $V_{ac} = IR_{ac} = (7.37 \times 10^{-4} \text{ A})(1.90 \Omega/4) = 3.50 \times 10^{-4} \text{ V}$ and the point a is at a higher potential since the current is flowing from a to c .

EVALUATE: This loop has the same resistance as the loop in Challenge Problem 29.75 and the induced current is the same.

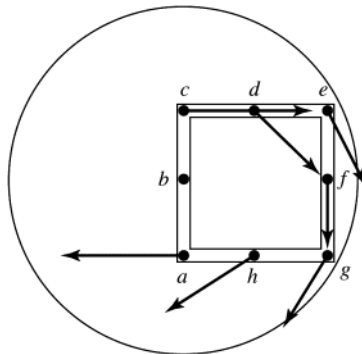


Figure 29.76

29.77. IDENTIFY: The motion of the bar produces an induced current and that results in a magnetic force on the bar.
SET UP: $\vec{F}_B$ is perpendicular to $\vec{B}$, so is horizontal. The vertical component of the normal force equals $mg \cos \phi$, so the horizontal component of the normal force equals $mg \tan \phi$.

EXECUTE: (a) As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from a to b . $F_B = iLB = \frac{LB}{R} \mathcal{E} = \frac{LB}{R} \frac{d\Phi_B}{dt} = \frac{LB}{R} B \frac{dA}{dt} = \frac{LB^2}{R} (vL \cos \phi) = \frac{vL^2 B^2}{R} \cos \phi$. At the terminal speed the horizontal forces balance, so $mg \tan \phi = \frac{vL^2 B^2}{R} \cos \phi$ and $v_t = \frac{Rmg \tan \phi}{L^2 B^2 \cos \phi}$.

$$(c) i = \frac{\mathcal{E}}{R} = \frac{1}{R} \frac{d\Phi_B}{dt} = \frac{1}{R} B \frac{dA}{dt} = \frac{B}{R} (vL \cos \phi) = \frac{vLB \cos \phi}{R} = \frac{mg \tan \phi}{LB}$$

$$(d) P = i^2 R = \frac{Rm^2 g^2 \tan^2 \phi}{L^2 B^2}$$

$$(e) P_g = Fv \cos(90^\circ - \phi) = mg \left(\frac{Rmg \tan \phi}{L^2 B^2 \cos \phi} \right) \sin \phi \text{ and } P_g = \frac{Rm^2 g^2 \tan^2 \phi}{L^2 B^2}$$

EVALUATE: The power in part (e) equals that in part (d), as is required by conservation of energy.

29.78. IDENTIFY: Follow the steps indicated in the problem.

SET UP: The primary assumption throughout the problem is that the square patch is small enough so that the velocity is constant over its whole area, that is, $v = \omega r \approx \omega d$.

EXECUTE: (a) $\omega \rightarrow$ clockwise, $B \rightarrow$ into page. $\mathcal{E} = vBL = \omega dBL$. $I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}A}{\rho L} = \frac{\omega dBA}{\rho}$. Since $\vec{v} \times \vec{B}$ points

outward, A is just the cross-sectional area tL . Therefore, $I = \frac{\omega dBLt}{\rho}$ flowing radially outward since $\vec{v} \times \vec{B}$ points outward.

(b) $\vec{\tau} = \vec{d} \times \vec{F}$ and $\vec{F}_B = I\vec{L} \times \vec{B} = ILB$ pointing counterclockwise. So $\tau = \frac{\omega d^2 B^2 L^2 t}{\rho}$ pointing out of the page (a counterclockwise torque opposing the clockwise rotation).

(c) If $\omega \rightarrow$ counterclockwise and $B \rightarrow$ into page, then $I \rightarrow$ inward radially since $\vec{v} \times \vec{B}$ points inward.

$\tau \rightarrow$ clockwise (again opposing the motion). If $\omega \rightarrow$ counterclockwise and $B \rightarrow$ out of the page, then $I \rightarrow$ radially outward. $\tau \rightarrow$ clockwise (opposing the motion)

The magnitudes of I and τ are the same as in part (a).

EVALUATE: In each case the magnetic torque due to the induced current opposes the rotation of the disk, as is required by conservation of energy.

INDUCTANCE

30.1. IDENTIFY and SET UP: Apply Eq.(30.4).

EXECUTE: (a) $|\mathcal{E}_2| = M \left| \frac{di_1}{dt} \right| = (3.25 \times 10^{-4} \text{ H})(830 \text{ A/s}) = 0.270 \text{ V}$; yes, it is constant.

(b) $|\mathcal{E}_1| = M \left| \frac{di_2}{dt} \right|$; M is a property of the pair of coils so is the same as in part (a). Thus $|\mathcal{E}_1| = 0.270 \text{ V}$.

EVALUATE: The induced emf is the same in either case. A constant di/dt produces a constant emf.

30.2. IDENTIFY: $\mathcal{E}_1 = M \left| \frac{\Delta i_2}{\Delta t} \right|$ and $\mathcal{E}_2 = M \left| \frac{\Delta i_1}{\Delta t} \right|$. $M = \left| \frac{N_2 \Phi_{B2}}{i_1} \right|$, where Φ_{B2} is the flux through one turn of the second coil.

SET UP: M is the same whether we consider an emf induced in coil 1 or in coil 2.

EXECUTE: (a) $M = \frac{\mathcal{E}_2}{|\Delta i_1 / \Delta t|} = \frac{1.65 \times 10^{-3} \text{ V}}{0.242 \text{ A/s}} = 6.82 \times 10^{-3} \text{ H} = 6.82 \text{ mH}$

(b) $\Phi_{B2} = \frac{M i_1}{N_2} = \frac{(6.82 \times 10^{-3} \text{ H})(1.20 \text{ A})}{25} = 3.27 \times 10^{-4} \text{ Wb}$

(c) $\mathcal{E}_1 = M \left| \frac{\Delta i_2}{\Delta t} \right| = (6.82 \times 10^{-3} \text{ H})(0.360 \text{ A/s}) = 2.46 \times 10^{-3} \text{ V} = 2.46 \text{ mV}$

EVALUATE: We can express M either in terms of the total flux through one coil produced by a current in the other coil, or in terms of the emf induced in one coil by a changing current in the other coil.

30.3. IDENTIFY: Replace units of Wb, A and Ω by their equivalents.

SET UP: $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$. $1 \text{ T} = 1 \text{ N}/(\text{A} \cdot \text{m})$. $1 \text{ N} \cdot \text{m} = 1 \text{ J}$. $1 \text{ A} = 1 \text{ C/s}$. $1 \text{ V} = 1 \text{ J/C}$. $1 \text{ V/A} = 1 \Omega$.

EXECUTE: $1 \text{ H} = 1 \text{ Wb/A} = 1 \text{ T} \cdot \text{m}^2 / \text{A} = 1 \text{ N} \cdot \text{m} / \text{A}^2 = 1 \text{ J/A}^2 = 1 (\text{J}/[\text{A} \cdot \text{C}])\text{s} = 1 (\text{V/A})\text{s} = 1 \Omega \cdot \text{s}$.

EVALUATE: We may use whichever equivalent unit is the most convenient in a particular problem.

30.4. IDENTIFY: Changing flux from one object induces an emf in another object.

(a) **SET UP:** The magnetic field due to a solenoid is $B = \mu_0 n I$.

EXECUTE: The above formula gives

$$B_1 = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(300)(0.120 \text{ A})}{0.250 \text{ m}} = 1.81 \times 10^{-4} \text{ T}$$

The average flux through each turn of the inner solenoid is therefore

$$\Phi_B = B_1 A = (1.81 \times 10^{-4} \text{ T})\pi(0.0100 \text{ m})^2 = 5.68 \times 10^{-8} \text{ Wb}$$

(b) **SET UP:** The flux is the same through each turn of both solenoids due to the geometry, so

$$M = \frac{N_2 \Phi_{B,2}}{i_1} = \frac{N_2 \Phi_{B,1}}{i_1}$$

EXECUTE: $M = \frac{(25)(5.68 \times 10^{-8} \text{ Wb})}{0.120 \text{ A}} = 1.18 \times 10^{-5} \text{ H}$

(c) **SET UP:** The induced emf is $\mathcal{E}_2 = -M \frac{di_1}{dt}$.

EXECUTE: $\mathcal{E}_2 = -(1.18 \times 10^{-5} \text{ H})(1750 \text{ A/s}) = -0.0207 \text{ V}$

EVALUATE: A mutual inductance around 10^{-5} H is not unreasonable.

30.5. IDENTIFY and SET UP: Apply Eq.(30.5).

EXECUTE: (a) $M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{400(0.0320 \text{ Wb})}{6.52 \text{ A}} = 1.96 \text{ H}$

(b) $M = \frac{N_1 \Phi_{B1}}{i_2}$ so $\Phi_{B1} = \frac{Mi_2}{N_1} = \frac{(1.96 \text{ H})(2.54 \text{ A})}{700} = 7.11 \times 10^{-3} \text{ Wb}$

EVALUATE: M relates the current in one coil to the flux through the other coil. Eq.(30.5) shows that M is the same for a pair of coils, no matter which one has the current and which one has the flux.

30.6. IDENTIFY: A changing current in an inductor induces an emf in it.

(a) **SET UP:** The self-inductance of a toroidal solenoid is $L = \frac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(500)^2 (6.25 \times 10^{-4} \text{ m}^2)}{2\pi(0.0400 \text{ m})} = 7.81 \times 10^{-4} \text{ H}$

(b) **SET UP:** The magnitude of the induced emf is $\mathcal{E} = L \frac{di}{dt}$.

EXECUTE: $\mathcal{E} = (7.81 \times 10^{-4} \text{ H}) \left(\frac{5.00 \text{ A} - 2.00 \text{ A}}{3.00 \times 10^{-3} \text{ s}} \right) = 0.781 \text{ V}$

(c) The current is decreasing, so the induced emf will be in the same direction as the current, which is from a to b , making b at a higher potential than a .

EVALUATE: This is a reasonable value for self-inductance, in the range of a mH.

30.7. IDENTIFY: $\mathcal{E} = L \left| \frac{\Delta i}{\Delta t} \right|$ and $L = \frac{N\Phi_B}{i}$.

SET UP: $\frac{\Delta i}{\Delta t} = 0.0640 \text{ A/s}$

EXECUTE: (a) $L = \frac{\mathcal{E}}{|\Delta i / \Delta t|} = \frac{0.0160 \text{ V}}{0.0640 \text{ A/s}} = 0.250 \text{ H}$

(b) The average flux through each turn is $\Phi_B = \frac{Li}{N} = \frac{(0.250 \text{ H})(0.720 \text{ A})}{400} = 4.50 \times 10^{-4} \text{ Wb}$.

EVALUATE: The self-induced emf depends on the rate of change of flux and therefore on the rate of change of the current, not on the value of the current.

30.8. IDENTIFY: Combine the two expressions for L : $L = N\Phi_B/i$ and $L = \mathcal{E}/(di/dt)$.

SET UP: Φ_B is the average flux through one turn of the solenoid.

EXECUTE: Solving for N we have $N = \mathcal{E}i/\Phi_B(di/dt) = \frac{(12.6 \times 10^{-3} \text{ V})(1.40 \text{ A})}{(0.00285 \text{ Wb})(0.0260 \text{ A/s})} = 238 \text{ turns}$.

EVALUATE: The induced emf depends on the time rate of change of the total flux through the solenoid.

30.9. IDENTIFY and SET UP: Apply $|\mathcal{E}| = L|di/dt|$. Apply Lenz's law to determine the direction of the induced emf in the coil.

EXECUTE: (a) $|\mathcal{E}| = L(di/dt) = (0.260 \text{ H})(0.0180 \text{ A/s}) = 4.68 \times 10^{-3} \text{ V}$

(b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the $+$ terminal at a .

EVALUATE: The induced emf is directed so as to oppose the decrease in the current.

30.10. IDENTIFY: Apply $\mathcal{E} = -L \frac{di}{dt}$.

SET UP: The induced emf points from low potential to high potential across the inductor.

EXECUTE: (a) The induced emf points from b to a , in the direction of the current. Therefore, the current is decreasing and the induced emf is directed to oppose this decrease.

(b) $|\mathcal{E}| = L|\Delta i / \Delta t|$, so $|\Delta i / \Delta t| = V_{ab}/L = (1.04 \text{ V})/(0.260 \text{ H}) = 4.00 \text{ A/s}$. In 2.00 s the decrease in i is 8.00 A and the current at 2.00 s is $12.0 \text{ A} - 8.0 \text{ A} = 4.0 \text{ A}$.

EVALUATE: When the current is decreasing the end of the inductor where the current enters is at the lower potential. This agrees with our result and with Figure 30.6d in the textbook.

30.11. IDENTIFY and SET UP: Use Eq.(30.6) to relate L to the flux through each turn of the solenoid. Use Eq.(28.23) for the magnetic field through the solenoid.

EXECUTE: $L = \frac{N\Phi_B}{i}$. If the magnetic field is uniform inside the solenoid $\Phi_B = BA$. From Eq.(28.23),

$$B = \mu_0 ni = \mu_0 \left(\frac{N}{l} \right) i \text{ so } \Phi_B = \frac{\mu_0 NiA}{l}. \text{ Then } L = \frac{N}{i} \left(\frac{\mu_0 NiA}{l} \right) = \frac{\mu_0 N^2 A}{l}.$$

EVALUATE: Our result is the same as L for a toroidal solenoid calculated in Example 30.3, except that the average circumference $2\pi r$ of the toroid is replaced by the length l of the straight solenoid.

30.12. IDENTIFY and SET UP: The stored energy is $U = \frac{1}{2}LI^2$. The rate at which thermal energy is developed is $P = I^2R$.

EXECUTE: (a) $U = \frac{1}{2}LI^2 = \frac{1}{2}(12.0 \text{ H})(0.300 \text{ A})^2 = 0.540 \text{ J}$

(b) $P = I^2R = (0.300 \text{ A})^2(180 \Omega) = 16.2 \text{ W} = 16.2 \text{ J/s}$

EVALUATE: (c) No. If I is constant then the stored energy U is constant. The energy being consumed by the resistance of the inductor comes from the emf source that maintains the current; it does not come from the energy stored in the inductor.

30.13. IDENTIFY and SET UP: Use Eq.(30.9) to relate the energy stored to the inductance. Example 30.3 gives the inductance of a toroidal solenoid to be $L = \frac{\mu_0 N^2 A}{2\pi r}$, so once we know L we can solve for N .

EXECUTE: $U = \frac{1}{2}LI^2$ so $L = \frac{2U}{I^2} = \frac{2(0.390 \text{ J})}{(12.0 \text{ A})^2} = 5.417 \times 10^{-3} \text{ H}$

$$N = \sqrt{\frac{2\pi rL}{\mu_0 A}} = \sqrt{\frac{2\pi(0.150 \text{ m})(5.417 \times 10^{-3} \text{ H})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(5.00 \times 10^{-4} \text{ m}^2)}} = 2850.$$

EVALUATE: L and hence U increase according to the square of N .

30.14. IDENTIFY: A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) **SET UP:** The magnetic field inside a toroidal solenoid is $B = \frac{\mu_0 NI}{2\pi r}$.

EXECUTE: $B = \frac{\mu_0(300)(5.00 \text{ A})}{2\pi(0.120 \text{ m})} = 2.50 \times 10^{-3} \text{ T} = 2.50 \text{ mT}$

(b) **SET UP:** The self-inductance of a toroidal solenoid is $L = \frac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(300)^2(4.00 \times 10^{-4} \text{ m}^2)}{2\pi(0.0120 \text{ m})} = 6.00 \times 10^{-5} \text{ H}$

(c) **SET UP:** The energy stored in an inductor is $U_L = \frac{1}{2}LI^2$.

EXECUTE: $U_L = \frac{1}{2}(6.00 \times 10^{-5} \text{ H})(5.00 \text{ A})^2 = 7.50 \times 10^{-4} \text{ J}$

(d) **SET UP:** The energy density in a magnetic field is $u = \frac{B^2}{2\mu_0}$.

EXECUTE: $u = \frac{(2.50 \times 10^{-3} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 2.49 \text{ J/m}^3$

(e) $u = \frac{\text{energy}}{\text{volume}} = \frac{\text{energy}}{2\pi rA} = \frac{7.50 \times 10^{-4} \text{ J}}{2\pi(0.120 \text{ m})(4.00 \times 10^{-4} \text{ m}^2)} = 2.49 \text{ J/m}^3$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.15. IDENTIFY: A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) **SET UP:** The magnetic field inside a solenoid is $B = \mu_0 nI$.

EXECUTE: $B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(400)(80.0 \text{ A})}{0.250 \text{ m}} = 0.161 \text{ T}$

(b) **SET UP:** The energy density in a magnetic field is $u = \frac{B^2}{2\mu_0}$.

EXECUTE: $u = \frac{(0.161 \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 1.03 \times 10^4 \text{ J/m}^3$

(c) **SET UP:** The total stored energy is $U = uV$.

EXECUTE: $U = uV = u(lA) = (1.03 \times 10^4 \text{ J/m}^3)(0.250 \text{ m})(0.500 \times 10^{-4} \text{ m}^2) = 0.129 \text{ J}$

(d) **SET UP:** The energy stored in an inductor is $U = \frac{1}{2}LI^2$.

EXECUTE: Solving for L and putting in the numbers gives

$$L = \frac{2U}{I^2} = \frac{2(0.129 \text{ J})}{(80.0 \text{ A})^2} = 4.02 \times 10^{-5} \text{ H}$$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.16. IDENTIFY: Energy = Pt . $U = \frac{1}{2}LI^2$.

SET UP: $P = 200 \text{ W} = 200 \text{ J/s}$

EXECUTE: (a) Energy = $(200 \text{ W})(24 \text{ h})(3600 \text{ s/h}) = 1.73 \times 10^7 \text{ J}$

$$(b) L = \frac{2U}{I^2} = \frac{2(1.73 \times 10^7 \text{ J})}{(80.0 \text{ A})^2} = 5.41 \times 10^3 \text{ H}$$

EVALUATE: A large value of L and a large current would be required, just for one light bulb. Also, the resistance of the inductor would have to be very small, to avoid a large $P = I^2R$ rate of electrical energy loss.

30.17. IDENTIFY and SET UP: Starting with Eq. (30.9), follow exactly the same steps as in the text except that the magnetic permeability μ is used in place of μ_0 .

EXECUTE: Using $L = \frac{\mu N^2 A}{2\pi r}$ and $B = \frac{\mu NI}{2\pi r}$ gives $u = \frac{B^2}{2\mu}$.

EVALUATE: For a given value of B , the energy density is less when μ is larger than μ_0 .

30.18. IDENTIFY and SET UP: The energy density (energy per unit volume) in a magnetic field (in vacuum) is given by

$$u = \frac{U}{V} = \frac{B^2}{2\mu_0} \text{ (Eq. 30.10).}$$

$$\text{EXECUTE: (a) } V = \frac{2\mu_0 U}{B^2} = \frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(3.60 \times 10^6 \text{ J})}{(0.600 \text{ T})^2} = 25.1 \text{ m}^3.$$

$$(b) u = \frac{U}{V} = \frac{B^2}{2\mu_0}$$

$$B = \sqrt{\frac{2\mu_0 U}{V}} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(3.60 \times 10^6 \text{ J})}{(0.400 \text{ m})^3}} = 11.9 \text{ T}$$

EVALUATE: Large-scale energy storage in a magnetic field is not practical. The volume in part (a) is quite large and the field in part (b) would be very difficult to achieve.

30.19. IDENTIFY: Apply Kirchhoff's loop rule to the circuit. $i(t)$ is given by Eq. (30.14).

SET UP: The circuit is sketched in Figure 30.19.

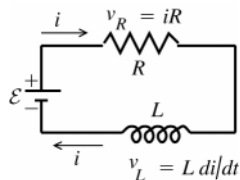


Figure 30.19

$\frac{di}{dt}$ is positive as the current increases from its initial value of zero.

EXECUTE: $\mathcal{E} - v_R - v_L = 0$

$$\mathcal{E} - iR - L \frac{di}{dt} = 0 \text{ so } i = \frac{\mathcal{E}}{R} (1 - e^{-(R/L)t})$$

(a) Initially ($t = 0$), $i = 0$ so $\mathcal{E} - L \frac{di}{dt} = 0$

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} = \frac{6.00 \text{ V}}{2.50 \text{ H}} = 2.40 \text{ A/s}$$

(b) $\mathcal{E} - iR - L \frac{di}{dt} = 0$ (Use this equation rather than Eq. (30.15) since i rather than t is given.)

$$\text{Thus } \frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{6.00 \text{ V} - (0.500 \text{ A})(8.00 \Omega)}{2.50 \text{ H}} = 0.800 \text{ A/s}$$

$$(c) i = \frac{\mathcal{E}}{R} (1 - e^{-(R/L)t}) = \left(\frac{6.00 \text{ V}}{8.00 \Omega} \right) (1 - e^{-(8.00 \Omega / 2.50 \text{ H})(0.250 \text{ s})}) = 0.750 \text{ A} (1 - e^{-0.800}) = 0.413 \text{ A}$$

(d) Final steady state means $t \rightarrow \infty$ and $\frac{di}{dt} \rightarrow 0$, so $\mathcal{E} - iR = 0$.

$$i = \frac{\mathcal{E}}{R} = \frac{6.00 \text{ V}}{8.00 \Omega} = 0.750 \text{ A}$$

EVALUATE: Our results agree with Fig. 30.12 in the textbook. The current is initially zero and increases to its final value of $\mathcal{E}/R$. The slope of the current in the figure, which is di/dt , decreases with t .

30.20. IDENTIFY: The current decays exponentially.

SET UP: After opening the switch, the current is $i = I_0 e^{-tR/L}$, and the time constant is $\tau = L/R$.

EXECUTE: (a) The initial current is $I_0 = (6.30 \text{ V})/(15.0 \Omega) = 0.420 \text{ A}$. Now solve for L and put in the numbers.

$$L = \frac{-tR}{\ln(i/I_0)} = \frac{-(2.00 \text{ ms})(15.0 \Omega)}{\ln\left(\frac{0.210 \text{ A}}{0.420 \text{ A}}\right)} = 43.3 \text{ mH}$$

(b) $\tau = L/R = (43.3 \text{ mH})/(15.0 \Omega) = 2.89 \text{ ms}$

(c) Solve $i = I_0 e^{-t/\tau}$ for t , giving $t = -\tau \ln(i/I_0) = -(2.89 \text{ ms}) \ln(0.0100) = 13.3 \text{ ms}$.

EVALUATE: In less than 5 time constants, the current is only 1% of its initial value.

30.21. IDENTIFY: $i = \mathcal{E}/R(1 - e^{-t/\tau})$, with $\tau = L/R$. The energy stored in the inductor is $U = \frac{1}{2} Li^2$.

SET UP: The maximum current occurs after a long time and is equal to $\mathcal{E}/R$.

EXECUTE: (a) $i_{\max} = \mathcal{E}/R$ so $i = i_{\max}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$ and $e^{-t/\tau} = \frac{1}{2}$. $-t/\tau = \ln(\frac{1}{2})$.

$$t = \frac{L \ln 2}{R} = \frac{(\ln 2)(1.25 \times 10^{-3} \text{ H})}{50.0 \Omega} = 17.3 \mu\text{s}$$

(b) $U = \frac{1}{2} U_{\max}$ when $i = i_{\max}/\sqrt{2}$. $1 - e^{-t/\tau} = 1/\sqrt{2}$, so $e^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$. $t = -L \ln(0.2929)/R = 30.7 \mu\text{s}$.

EVALUATE: $\tau = L/R = 2.50 \times 10^{-5} \text{ s} = 25.0 \mu\text{s}$. The time in part (a) is 0.692τ and the time in part (b) is 1.23τ .

30.22. IDENTIFY: With S_1 closed and S_2 open, $i(t)$ is given by Eq.(30.14). With S_1 open and S_2 closed, $i(t)$ is given by Eq.(30.18).

SET UP: $U = \frac{1}{2} Li^2$. After S_1 has been closed a long time, i has reached its final value of $I = \mathcal{E}/R$.

EXECUTE: (a) $U = \frac{1}{2} LI^2$ and $I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(0.260 \text{ J})}{0.115 \text{ H}}} = 2.13 \text{ A}$. $\mathcal{E} = IR = (2.13 \text{ A})(120 \Omega) = 256 \text{ V}$.

(b) $i = Ie^{-(R/L)t}$ and $U = \frac{1}{2} Li^2 = \frac{1}{2} LI^2 e^{-2(R/L)t} = \frac{1}{2} U_0 = \frac{1}{2} \left(\frac{1}{2} LI^2\right)$. $e^{-2(R/L)t} = \frac{1}{2}$, so

$$t = -\frac{L}{2R} \ln\left(\frac{1}{2}\right) = -\frac{0.115 \text{ H}}{2(120 \Omega)} \ln\left(\frac{1}{2}\right) = 3.32 \times 10^{-4} \text{ s}.$$

EVALUATE: $\tau = L/R = 9.58 \times 10^{-4} \text{ s}$. The time in part (b) is $\tau \ln(2)/2 = 0.347\tau$.

30.23. IDENTIFY: L has units of H and R has units of Ω .

SET UP: $1 \text{ H} = 1 \Omega \cdot \text{s}$

EXECUTE: Units of $L/R = \text{H}/\Omega = (\Omega \cdot \text{s})/\Omega = \text{s} = \text{units of time}$.

EVALUATE: $Rt/L = t/\tau$ is dimensionless.

30.24. IDENTIFY: Apply the loop rule.

SET UP: In applying the loop rule, go around the circuit in the direction of the current. The voltage across the inductor is $-L di/dt$.

EXECUTE: $-L di/dt - iR = 0$. $\frac{di}{dt} = -i \frac{R}{L}$ gives $\int_{I_0}^i \frac{di'}{i'} = -\frac{R}{L} \int_0^t dt'$ and $\ln(i/I_0) = -\frac{R}{L} t$. $i = I_0 e^{-(R/L)t}$.

EVALUATE: di/dt is negative, so there is a potential rise across the inductor; point c is at higher potential than point b . There is a potential drop across the resistor.

30.25. IDENTIFY: Apply the concepts of current decay in an R - L circuit. Apply the loop rule to the circuit. $i(t)$ is given by Eq.(30.18). The voltage across the resistor depends on i and the voltage across the inductor depends on di/dt .

SET UP: The circuit with S_1 closed and S_2 open is sketched in Figure 30.25a.

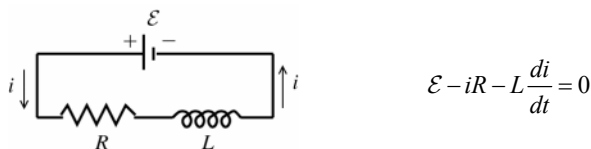
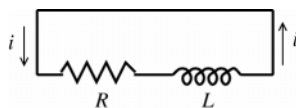


Figure 30.25a

Constant current established means $\frac{di}{dt} = 0$.

EXECUTE: $i = \frac{\mathcal{E}}{R} = \frac{60.0 \text{ V}}{240 \Omega} = 0.250 \text{ A}$

(a) **SET UP:** The circuit with S_2 closed and S_1 open is shown in Figure 30.25b.



$$i = I_0 e^{-(R/L)t}$$

$$\text{At } t = 0, i = I_0 = 0.250 \text{ A}$$

Figure 30.25b

The inductor prevents an instantaneous change in the current; the current in the inductor just after S_2 is closed and S_1 is opened equals the current in the inductor just before this is done.

(b) **EXECUTE:** $i = I_0 e^{-(R/L)t} = (0.250 \text{ A})e^{-(240 \, \Omega / 0.160 \text{ H})(4.00 \times 10^{-4} \text{ s})} = (0.250 \text{ A})e^{-0.600} = 0.137 \text{ A}$

(c) **SET UP:** See Figure 30.25c.

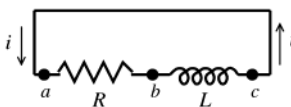


Figure 30.25c

EXECUTE: If we trace around the loop in the direction of the current the potential falls as we travel through the resistor so it must rise as we pass through the inductor: $v_{ab} > 0$ and $v_{bc} < 0$. So point c is at higher potential than point b .

$$v_{ab} + v_{bc} = 0 \text{ and } v_{bc} = -v_{ab}$$

$$\text{Or, } v_{cb} = v_{ab} = iR = (0.137 \text{ A})(240 \, \Omega) = 32.9 \text{ V}$$

(d) $i = I_0 e^{-(R/L)t}$

$$i = \frac{1}{2} I_0 \text{ says } \frac{1}{2} I_0 = I_0 e^{-(R/L)t} \text{ and } \frac{1}{2} = e^{-(R/L)t}$$

Taking natural logs of both sides of this equation gives $\ln(\frac{1}{2}) = -Rt/L$

$$t = \left(\frac{0.160 \text{ H}}{240 \, \Omega} \right) \ln 2 = 4.62 \times 10^{-4} \text{ s}$$

EVALUATE: The current decays, as shown in Fig. 30.13 in the textbook. The time constant is $\tau = L/R = 6.67 \times 10^{-4} \text{ s}$. The values of t in the problem are less than one time constant. At any instant the potential drop across the resistor (in the direction of the current) equals the potential rise across the inductor.

30.26. **IDENTIFY:** Apply Eq.(30.14).

SET UP: $v_{ab} = iR$. $v_{bc} = L \frac{di}{dt}$. The current is increasing, so di/dt is positive.

EXECUTE: (a) At $t = 0$, $i = 0$. $v_{ab} = 0$ and $v_{bc} = 60 \text{ V}$.

(b) As $t \rightarrow \infty$, $i \rightarrow \mathcal{E}/R$ and $di/dt \rightarrow 0$. $v_{ab} \rightarrow 60 \text{ V}$ and $v_{bc} \rightarrow 0$.

(c) When $i = 0.150 \text{ A}$, $v_{ab} = iR = 36.0 \text{ V}$ and $v_{bc} = 60.0 \text{ V} - 36.0 \text{ V} = 24.0 \text{ V}$.

EVALUATE: At all times, $\mathcal{E} = v_{ab} + v_{bc}$, as required by the loop rule.

30.27. **IDENTIFY:** $i(t)$ is given by Eq.(30.14).

SET UP: The power input from the battery is $\mathcal{E}i$. The rate of dissipation of energy in the resistance is $i^2 R$. The voltage across the inductor has magnitude $L di/dt$, so the rate at which energy is being stored in the inductor is $iL di/dt$.

$$\text{EXECUTE: (a) } P = \mathcal{E}i = \mathcal{E}I_0(1 - e^{-(R/L)t}) = \frac{\mathcal{E}^2}{R}(1 - e^{-(R/L)t}) = \frac{(6.00 \text{ V})^2}{8.00 \, \Omega}(1 - e^{-(8.00 \, \Omega / 2.50 \text{ H})t}).$$

$$P = (4.50 \text{ W})(1 - e^{-(3.20 \text{ s}^{-1})t}).$$

$$\text{(b) } P_R = i^2 R = \frac{\mathcal{E}^2}{R}(1 - e^{-(R/L)t})^2 = \frac{(6.00 \text{ V})^2}{8.00 \, \Omega}(1 - e^{-(8.00 \, \Omega / 2.50 \text{ H})t})^2 = (4.50 \text{ W})(1 - e^{-(3.20 \text{ s}^{-1})t})^2$$

$$\text{(c) } P_L = iL \frac{di}{dt} = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t})L \left(\frac{\mathcal{E}}{L} e^{-(R/L)t} \right) = \frac{\mathcal{E}^2}{R}(e^{-(R/L)t} - e^{-2(R/L)t})$$

$$P_L = (4.50 \text{ W})(e^{-(3.20 \text{ s}^{-1})t} - e^{-(6.40 \text{ s}^{-1})t}).$$

EVALUATE: (d) Note that if we expand the square in part (b), then parts (b) and (c) add to give part (a), and the total power delivered is dissipated in the resistor and inductor. Conservation of energy requires that this be so.

30.28. IDENTIFY: An L - C circuit oscillates, with the energy going back and forth between the inductor and capacitor.

(a) SET UP: The frequency is $f = \frac{\omega}{2\pi}$ and $\omega = \frac{1}{\sqrt{LC}}$, giving $f = \frac{1}{2\pi\sqrt{LC}}$.

EXECUTE: $f = \frac{1}{2\pi\sqrt{(0.280 \times 10^{-3} \text{ H})(20.0 \times 10^{-6} \text{ F})}} = 2.13 \times 10^3 \text{ Hz} = 2.13 \text{ kHz}$

(b) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: $U = \frac{1}{2}(20.0 \times 10^{-6} \text{ F})(150.0 \text{ V})^2 = 0.225 \text{ J}$

(c) SET UP: The current in the circuit is $i = -Q\sin\omega t$, and the energy stored in the inductor is $U = \frac{1}{2}Li^2$.

EXECUTE: First find ω and Q . $\omega = 2\pi f = 1.336 \times 10^4 \text{ rad/s}$.

$$Q = CV = (20.0 \times 10^{-6} \text{ F})(150.0 \text{ V}) = 3.00 \times 10^{-3} \text{ C}$$

Now calculate the current:

$$i = -(1.336 \times 10^4 \text{ rad/s})(3.00 \times 10^{-3} \text{ C}) \sin[(1.336 \times 10^4 \text{ rad/s})(1.30 \times 10^{-3} \text{ s})]$$

Notice that the argument of the sine is in *radians*, so convert it to degrees if necessary. The result is $i = -39.92 \text{ A}$

Now find the energy in the inductor: $U = \frac{1}{2}Li^2 = \frac{1}{2}(0.280 \times 10^{-3} \text{ H})(-39.92 \text{ A})^2 = 0.223 \text{ J}$

EVALUATE: At the end of 1.30 ms, nearly all the energy is now in the inductor, leaving very little in the capacitor.

30.29. IDENTIFY: The energy moves back and forth between the inductor and capacitor.

(a) SET UP: The period is $T = \frac{1}{f} = \frac{1}{\omega/2\pi} = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$.

EXECUTE: Solving for L gives

$$L = \frac{T^2}{4\pi^2 C} = \frac{(8.60 \times 10^{-5} \text{ s})^2}{4\pi^2 (7.50 \times 10^{-9} \text{ F})} = 2.50 \times 10^{-2} \text{ H} = 25.0 \text{ mH}$$

(b) SET UP: The charge on a capacitor is $Q = CV$.

EXECUTE: $Q = CV = (7.50 \times 10^{-9} \text{ F})(12.0 \text{ V}) = 9.00 \times 10^{-8} \text{ C}$

(c) SET UP: The stored energy is $U = Q^2/2C$.

EXECUTE: $U = \frac{(9.00 \times 10^{-8} \text{ C})^2}{2(7.50 \times 10^{-9} \text{ F})} = 5.40 \times 10^{-7} \text{ J}$

(d) SET UP: The maximum current occurs when the capacitor is discharged, so the inductor has all the initial energy. $U_L + U_C = U_{\text{Total}}$. $\frac{1}{2}Li^2 + 0 = U_{\text{Total}}$.

EXECUTE: Solve for the current:

$$I = \sqrt{\frac{2U_{\text{Total}}}{L}} = \sqrt{\frac{2(5.40 \times 10^{-7} \text{ J})}{2.50 \times 10^{-2} \text{ H}}} = 6.58 \times 10^{-3} \text{ A} = 6.58 \text{ mA}$$

EVALUATE: The energy oscillates back and forth forever. However if there is any resistance in the circuit, no matter how small, all this energy will eventually be dissipated as heat in the resistor.

30.30. IDENTIFY: The circuit is described in Figure 30.14 of the textbook.

SET UP: The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$ and the energy stored in the capacitor is $U_C = q^2/2C$. Initially,

$U_C = \frac{1}{2}CV^2$, with $V = 12.0 \text{ V}$. The period of oscillation is $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(12.0 \times 10^{-3} \text{ H})(18.0 \times 10^{-6} \text{ F})} = 2.92 \text{ ms}$.

EXECUTE: **(a)** Energy conservation says $U_L(\text{max}) = U_C(\text{max})$, and $\frac{1}{2}Li_{\text{max}}^2 = \frac{1}{2}CV^2$.

$i_{\text{max}} = V\sqrt{C/L} = (22.5 \text{ V})\sqrt{\frac{18 \times 10^{-6} \text{ F}}{12 \times 10^{-3} \text{ H}}} = 0.871 \text{ A}$. The charge on the capacitor is zero because all the energy is in the inductor.

(b) From Figure 30.14 in the textbook, $q = 0$ at $t = T/4 = 0.730 \text{ ms}$ and at $t = 3T/4 = 2.19 \text{ ms}$.

(c) $q_0 = CV = (18 \mu\text{F})(22.5 \text{ V}) = 405 \mu\text{C}$ is the maximum charge on the plates. The graphs are sketched in Figure 30.30. q refers to the charge on one plate and the sign of i indicates the direction of the current.

EVALUATE: If the capacitor is fully charged at $t = 0$ it is fully charged again at $t = T/2$, but with the opposite polarity.

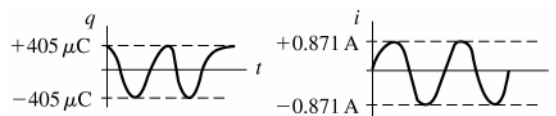


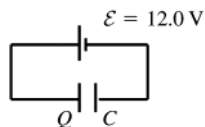
Figure 30.30

- 30.31. IDENTIFY and SET UP:** The angular frequency is given by Eq.(30.22). $q(t)$ and $i(t)$ are given by Eqs.(30.21) and (30.23). The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2 = q^2/2C$. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$.

EXECUTE: (a) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.50 \text{ H})(6.00 \times 10^{-5} \text{ F})}} = 105.4 \text{ rad/s}$, which rounds to 105 rad/s. The period is

given by $T = \frac{2\pi}{\omega} = \frac{2\pi}{105.4 \text{ rad/s}} = 0.0596 \text{ s}$

(b) The circuit containing the battery and capacitor is sketched in Figure 30.31.



$$\mathcal{E} - \frac{Q}{C} = 0$$

$$Q = \mathcal{E}C = (12.0 \text{ V})(6.00 \times 10^{-5} \text{ F}) = 7.20 \times 10^{-4} \text{ C}$$

Figure 30.31

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.00 \times 10^{-5} \text{ F})(12.0 \text{ V})^2 = 4.32 \times 10^{-3} \text{ J}$

(d) $q = Q \cos(\omega t + \phi)$ (Eq.30.21)

$q = Q$ at $t = 0$ so $\phi = 0$

$q = Q \cos \omega t = (7.20 \times 10^{-4} \text{ C}) \cos([105.4 \text{ rad/s}][0.0230 \text{ s}]) = -5.42 \times 10^{-4} \text{ C}$

The minus sign means that the capacitor has discharged fully and then partially charged again by the current maintained by the inductor; the plate that initially had positive charge now has negative charge and the plate that initially had negative charge now has positive charge.

(e) $i = -\omega Q \sin(\omega t + \phi)$ (Eq.30.23)

$i = -(105 \text{ rad/s})(7.20 \times 10^{-4} \text{ C}) \sin([105.4 \text{ rad/s}][0.0230 \text{ s}]) = -0.050 \text{ A}$

The negative sign means the current is counterclockwise in Figure 30.15 in the textbook.

or

$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$ gives $i = \pm \sqrt{\frac{1}{LC} \sqrt{Q^2 - q^2}}$ (Eq.30.26)

$i = \pm(105 \text{ rad/s}) \sqrt{(7.20 \times 10^{-4} \text{ C})^2 - (-5.42 \times 10^{-4} \text{ C})^2} = \pm 0.050 \text{ A}$, which checks.

(f) $U_C = \frac{q^2}{2C} = \frac{(-5.42 \times 10^{-4} \text{ C})^2}{2(6.00 \times 10^{-5} \text{ F})} = 2.45 \times 10^{-3} \text{ J}$

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(1.50 \text{ H})(0.050 \text{ A})^2 = 1.87 \times 10^{-3} \text{ J}$

EVALUATE: Note that $U_C + U_L = 2.45 \times 10^{-3} \text{ J} + 1.87 \times 10^{-3} \text{ J} = 4.32 \times 10^{-3} \text{ J}$.

This agrees with the total energy initially stored in the capacitor, $U = \frac{Q^2}{2C} = \frac{(7.20 \times 10^{-4} \text{ C})^2}{2(6.00 \times 10^{-5} \text{ F})} = 4.32 \times 10^{-3} \text{ J}$.

Energy is conserved. At some times there is energy stored in both the capacitor and the inductor. When $i = 0$ all the energy is stored in the capacitor and when $q = 0$ all the energy is stored in the inductor. But at all times the total energy stored is the same.

30.32. IDENTIFY: $\omega = \frac{1}{\sqrt{LC}} = 2\pi f$

SET UP: ω is the angular frequency in rad/s and f is the corresponding frequency in Hz.

EXECUTE: (a) $L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (1.6 \times 10^6 \text{ Hz})^2 (4.18 \times 10^{-12} \text{ F})} = 2.37 \times 10^{-3} \text{ H}$.

(b) The maximum capacitance corresponds to the minimum frequency.

$C_{\text{max}} = \frac{1}{4\pi^2 f_{\text{min}}^2 L} = \frac{1}{4\pi^2 (5.40 \times 10^5 \text{ Hz})^2 (2.37 \times 10^{-3} \text{ H})} = 3.67 \times 10^{-11} \text{ F} = 36.7 \text{ pF}$

EVALUATE: To vary f by a factor of three (approximately the range in this problem), C must be varied by a factor of nine.

- 30.33. IDENTIFY:** Apply energy conservation and Eqs. (30.22) and (30.23).

SET UP: If I is the maximum current, $\frac{1}{2}LI^2 = \frac{Q^2}{2C}$. For the inductor, $U_L = \frac{1}{2}Li^2$.

EXECUTE: (a) $\frac{1}{2}LI^2 = \frac{Q^2}{2C}$ gives $Q = i\sqrt{LC} = (0.750 \text{ A})\sqrt{(0.0800 \text{ H})(1.25 \times 10^{-9} \text{ F})} = 7.50 \times 10^{-6} \text{ C}$.

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.0800 \text{ H})(1.25 \times 10^{-9} \text{ F})}} = 1.00 \times 10^5 \text{ rad/s}$. $f = \frac{\omega}{2\pi} = 1.59 \times 10^4 \text{ Hz}$.

(c) $q = Q$ at $t = 0$ means $\phi = 0$. $i = -\omega Q \sin(\omega t)$, so

$$i = -(1.00 \times 10^5 \text{ rad/s})(7.50 \times 10^{-6} \text{ C}) \sin[(1.00 \times 10^5 \text{ rad/s}][2.50 \times 10^{-3} \text{ s}]) = -0.7279 \text{ A}.$$

$$U_L = \frac{1}{2} Li^2 = \frac{1}{2} (0.0800 \text{ H})(-0.7279 \text{ A})^2 = 0.0212 \text{ J}.$$

EVALUATE: The total energy of the system is $\frac{1}{2} LI^2 = 0.0225 \text{ J}$. At $t = 2.50 \text{ ms}$, the current is close to its maximum value and most of the system's energy is stored in the inductor.

30.34. IDENTIFY: Apply Eq.(30.25).

SET UP: $q = Q$ when $i = 0$. $i = i_{\max}$ when $q = 0$. $1/\sqrt{LC} = 1917 \text{ s}^{-1}$.

EXECUTE: (a) $\frac{1}{2} Li_{\max}^2 = \frac{Q^2}{2C}$. $Q = i_{\max} \sqrt{LC} = (0.850 \times 10^{-3} \text{ A}) \sqrt{(0.0850 \text{ H})(3.20 \times 10^{-6} \text{ F})} = 4.43 \times 10^{-7} \text{ C}$

(b) $q = \sqrt{Q^2 - LCi^2} = \sqrt{(4.43 \times 10^{-7} \text{ C})^2 - \left(\frac{5.00 \times 10^{-4} \text{ A}}{1917 \text{ s}^{-1}}\right)^2} = 3.58 \times 10^{-7} \text{ C}.$

EVALUATE: The value of q calculated in part (b) is less than the maximum value Q calculated in part (a).

30.35. IDENTIFY: $q = Q \cos(\omega t + \phi)$ and $i = -\omega Q \sin(\omega t + \phi)$

SET UP: $U_C = \frac{q^2}{2C}$. $U_L = \frac{1}{2} Li^2$.

EXECUTE: (a) $U_C = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \frac{Q^2 \cos^2(\omega t + \phi)}{C}$.

$$U_L = \frac{1}{2} Li^2 = \frac{1}{2} L \omega^2 Q^2 \sin^2(\omega t + \phi) = \frac{1}{2} \frac{Q^2 \sin^2(\omega t + \phi)}{C}, \text{ since } \omega^2 = \frac{1}{LC}.$$

(b) $U_{\text{Total}} = U_C + U_L = \frac{1}{2} \frac{Q^2}{C} \cos^2(\omega t + \phi) + \frac{1}{2} L \omega^2 Q^2 \sin^2(\omega t + \phi)$

$$U_{\text{total}} = \frac{1}{2} \frac{Q^2}{C} \cos^2(\omega t + \phi) + \frac{1}{2} L \left(\frac{1}{LC} \right) Q^2 \sin^2(\omega t + \phi) = \frac{1}{2} \frac{Q^2}{C} (\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)) = \frac{1}{2} \frac{Q^2}{C}$$

U_{Total} is a constant.

EVALUATE: Eqs.(30.21) and (30.23) are consistent with conservation of energy in the L - C circuit.

30.36. IDENTIFY: Evaluate $\frac{d^2 q}{dt^2}$ and insert into Eq.(20.20).

SET UP: Equation (30.20) is $\frac{d^2 q}{dt^2} + \frac{1}{LC} q = 0$.

EXECUTE: $q = Q \cos(\omega t + \phi) \Rightarrow \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) \Rightarrow \frac{d^2 q}{dt^2} = -\omega^2 Q \cos(\omega t + \phi).$

$$\frac{d^2 q}{dt^2} + \frac{1}{LC} q = -\omega^2 Q \cos(\omega t + \phi) + \frac{Q}{LC} \cos(\omega t + \phi) = 0 \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

EVALUATE: The value of ϕ depends on the initial conditions, the value of q at $t = 0$.

30.37. IDENTIFY: The unit of L is H and the unit of C is F.

SET UP: $C = q/V_{ab}$ says $1 \text{ F} = 1 \text{ C/V}$. $1 \text{ H} = 1 \text{ V} \cdot \text{s/A} = 1 \text{ V} \cdot \text{s}^2/\text{C}$.

EXECUTE: $1 \text{ H} \cdot \text{F} = (1 \text{ V} \cdot \text{s}^2/\text{C})(1 \text{ C/V}) = 1 \text{ s}^2$. Therefore, LC has units of s^2 and $\sqrt{LC}$ has units of s.

EVALUATE: Our result shows that ωt is dimensionless, since $\omega = 1/\sqrt{LC}$.

30.38. IDENTIFY: The presence of resistance in an L - R - C circuit affects the frequency of oscillation and causes the amplitude of the oscillations to decrease over time.

(a) **SET UP:** The frequency of damped oscillations is $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

EXECUTE: $\omega' = \sqrt{\frac{1}{(22 \times 10^{-3} \text{ H})(15.0 \times 10^{-9} \text{ F})} - \frac{(75.0 \text{ } \Omega)^2}{4(22 \times 10^{-3} \text{ H})^2}} = 5.5 \times 10^4 \text{ rad/s}$

The frequency f is $f = \frac{\omega}{2\pi} = \frac{5.50 \times 10^4 \text{ rad/s}}{2\pi} = 8.76 \times 10^3 \text{ Hz} = 8.76 \text{ kHz}.$

(b) **SET UP:** The amplitude decreases as $A(t) = A_0 e^{-(R/2L)t}$.

EXECUTE: Solving for t and putting in the numbers gives:

$$t = \frac{-2L \ln(A/A_0)}{R} = \frac{-2(22.0 \times 10^{-3} \text{ H}) \ln(0.100)}{75.0 \text{ } \Omega} = 1.35 \times 10^{-3} \text{ s} = 1.35 \text{ ms}$$

(c) **SET UP:** At critical damping, $R = \sqrt{4L/C}$.

EXECUTE: $R = \sqrt{\frac{4(22.0 \times 10^{-3} \text{ H})}{15.0 \times 10^{-9} \text{ F}}} = 2420 \, \Omega$

EVALUATE: The frequency with damping is almost the same as the resonance frequency of this circuit ($1/\sqrt{LC}$), which is plausible because the $75\text{-}\Omega$ resistance is considerably less than the $2420 \, \Omega$ required for critical damping.

30.39. IDENTIFY: Follow the procedure specified in the problem.

SET UP: Make the substitutions $x \rightarrow q$, $m \rightarrow L$, $b \rightarrow R$, $k \rightarrow \frac{1}{C}$.

EXECUTE: (a) Eq. (13.41): $\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{kx}{m} = 0$. This becomes $\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = 0$, which is Eq.(30.27).

(b) Eq. (13.43): $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$. This becomes $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$, which is Eq.(30.29).

(c) Eq. (13.42): $x = Ae^{-(b/2m)t} \cos(\omega't + \phi)$. This becomes $q = Ae^{-(R/2L)t} \cos(\omega't + \phi)$, which is Eq.(30.28).

EVALUATE: Equations for the L - R - C circuit and for a damped harmonic oscillator have the same form.

30.40. IDENTIFY: For part (a), evaluate the derivatives as specified in the problem. For part (b) set $q = Q$ in Eq.(30.28) and set $dq/dt = 0$ in the expression for dq/dt .

SET UP: In terms of ω' , Eq.(30.28) is $q(t) = Ae^{-(R/2L)t} \cos(\omega't + \phi)$.

EXECUTE: (a) $q = Ae^{-(R/2L)t} \cos(\omega't + \phi)$. $\frac{dq}{dt} = -A \frac{R}{2L} e^{-(R/2L)t} \cos(\omega't + \phi) - \omega' A e^{-(R/2L)t} \sin(\omega't + \phi)$.

$$\frac{d^2q}{dt^2} = A \left(\frac{R}{2L} \right)^2 e^{-(R/2L)t} \cos(\omega't + \phi) + 2\omega' A \frac{R}{2L} e^{-(R/2L)t} \sin(\omega't + \phi) - \omega'^2 A e^{-(R/2L)t} \cos(\omega't + \phi)$$

$$\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = q \left(\left(\frac{R}{2L} \right)^2 - \omega'^2 - \frac{R^2}{2L^2} + \frac{1}{LC} \right) = 0, \text{ so } \omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2}.$$

(b) At $t = 0$, $q = Q$, $i = \frac{dq}{dt} = 0$, so $q = A \cos \phi = Q$ and $\frac{dq}{dt} = -\frac{R}{2L} A \cos \phi - \omega' A \sin \phi = 0$. This gives $A = \frac{Q}{\cos \phi}$ and

$$\tan \phi = -\frac{R}{2L\omega'} = -\frac{R}{2L\sqrt{1/LC - R^2/4L^2}}.$$

EVALUATE: If $R = 0$, then $A = Q$ and $\phi = 0$.

30.41. IDENTIFY: Evaluate Eq.(30.29).

SET UP: The angular frequency of the circuit is ω' .

EXECUTE: (a) When $R = 0$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.450 \text{ H})(2.50 \times 10^{-5} \text{ F})}} = 298 \text{ rad/s}$.

(b) We want $\frac{\omega}{\omega_0} = 0.95$, so $\frac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \frac{R^2C}{4L} = (0.95)^2$. This gives

$$R = \sqrt{\frac{4L}{C}(1 - (0.95)^2)} = \sqrt{\frac{4(0.450 \text{ H})(0.0975)}{(2.50 \times 10^{-5} \text{ F})}} = 83.8 \, \Omega.$$

EVALUATE: When R increases, the angular frequency decreases and approaches zero as $R \rightarrow 2\sqrt{L/C}$.

30.42. IDENTIFY: L has units of H and C has units of F.

SET UP: $1 \text{ H} = 1 \, \Omega \cdot \text{s}$. $C = q/V$ says $1 \text{ F} = 1 \text{ C/V}$. $V = IR$ says $1 \text{ V/A} = 1 \, \Omega$.

EXECUTE: The units of L/C are $\frac{\text{H}}{\text{F}} = \frac{\Omega \cdot \text{s}}{\text{C/V}} = \frac{\Omega \cdot \text{V}}{\text{A}} = \Omega^2$. Therefore, the unit of $\sqrt{L/C}$ is Ω .

EVALUATE: For Eq.(30.28) to be valid, $\frac{1}{LC}$ and $\frac{R^2}{4L^2}$ must have the same units, so R and $\sqrt{L/C}$ must have the same units, and we have shown that this is indeed the case.

30.43. IDENTIFY: The emf $\mathcal{E}_2$ in solenoid 2 produced by changing current i_1 in solenoid 1 is given by $\mathcal{E}_2 = M \left| \frac{\Delta i_1}{\Delta t} \right|$. The mutual inductance of two solenoids is derived in Example 30.1. For the two solenoids in this problem

$$M = \frac{\mu_0 AN_1N_2}{l}, \text{ where } A \text{ is the cross-sectional area of the inner solenoid and } l \text{ is the length of the outer solenoid.}$$

SET UP: $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$. Let the outer solenoid be solenoid 1.

EXECUTE: (a) $M = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})\pi(6.00 \times 10^{-4} \text{ m})^2(6750)(15)}{0.500 \text{ m}} = 2.88 \times 10^{-7} \text{ H} = 0.288 \mu\text{H}$

(b) $\mathcal{E}_2 = \left| \frac{\Delta i_1}{\Delta t} \right| = (2.88 \times 10^{-7} \text{ H})(37.5 \text{ A/s}) = 1.08 \times 10^{-5} \text{ V}$

EVALUATE: If current in the inner solenoid changed at 37.5 A/s, the emf induced in the outer solenoid would be $1.08 \times 10^{-5} \text{ V}$.

30.44. IDENTIFY: Apply $\mathcal{E} = -L \frac{di}{dt}$ and $Li = N\Phi_B$.

SET UP: Φ_B is the flux through one turn.

EXECUTE: (a) $\mathcal{E} = -L \frac{di}{dt} = -(3.50 \times 10^{-3} \text{ H}) \frac{d}{dt} ((0.680 \text{ A}) \cos(\pi t / [0.0250 \text{ s}]))$.

$\mathcal{E} = (3.50 \times 10^{-3} \text{ H})(0.680 \text{ A}) \frac{\pi}{0.0250 \text{ s}} \sin(\pi t / [0.0250 \text{ s}])$. Therefore,

$\mathcal{E}_{\max} = (3.50 \times 10^{-3} \text{ H})(0.680 \text{ A}) \frac{\pi}{0.0250 \text{ s}} = 0.299 \text{ V}$.

(b) $\Phi_{B\max} = \frac{Li_{\max}}{N} = \frac{(3.50 \times 10^{-3} \text{ H})(0.680 \text{ A})}{400} = 5.95 \times 10^{-6} \text{ Wb}$.

(c) $\mathcal{E}(t) = -L \frac{di}{dt} = -(3.50 \times 10^{-3} \text{ H})(0.680 \text{ A})(\pi / 0.0250 \text{ s}) \sin(\pi t / 0.0250 \text{ s})$.

$\mathcal{E}(t) = -(0.299 \text{ V}) \sin((125.6 \text{ s}^{-1})t)$. Therefore, at $t = 0.0180 \text{ s}$,

$\mathcal{E}(0.0180 \text{ s}) = -(0.299 \text{ V}) \sin((125.6 \text{ s}^{-1})(0.0180 \text{ s})) = 0.230 \text{ V}$. The magnitude of the induced emf is 0.230 V.

EVALUATE: The maximum emf is when $i = 0$ and at this instant $\Phi_B = 0$.

30.45. IDENTIFY: $\mathcal{E} = -L \frac{di}{dt}$.

SET UP: During an interval in which the graph of i versus t is a straight line, di/dt is constant and equal to the slope of that line.

EXECUTE: (a) The pattern on the oscilloscope is sketched in Figure 30.45.

EVALUATE: (b) Since the voltage is determined by the derivative of the current, the V versus t graph is indeed proportional to the derivative of the current graph.

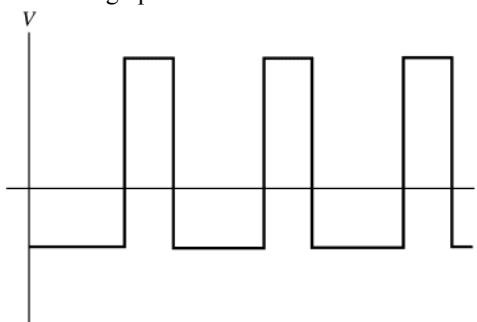


Figure 30.45

30.46. IDENTIFY: Apply $\mathcal{E} = -L \frac{di}{dt}$.

SET UP: $\frac{d}{dt} \cos(\omega t) = -\omega \sin(\omega t)$

EXECUTE: (a) $\mathcal{E} = -L \frac{di}{dt} = -L \frac{d}{dt} ((0.124 \text{ A}) \cos[(240 \pi / \text{s})t])$.

$\mathcal{E} = +(0.250 \text{ H})(0.124 \text{ A})(240 \pi / \text{s}) \sin((240 \pi / \text{s})t) = +(23.4 \text{ V}) \sin((240 \pi / \text{s})t)$.

The graphs are given in Figure 30.46.

(b) $\mathcal{E}_{\max} = 23.4 \text{ V}$; $i = 0$, since the emf and current are 90° out of phase.

(c) $i_{\max} = 0.124 \text{ A}$; $\mathcal{E} = 0$, since the emf and current are 90° out of phase.

EVALUATE: The induced emf depends on the rate at which the current is changing.

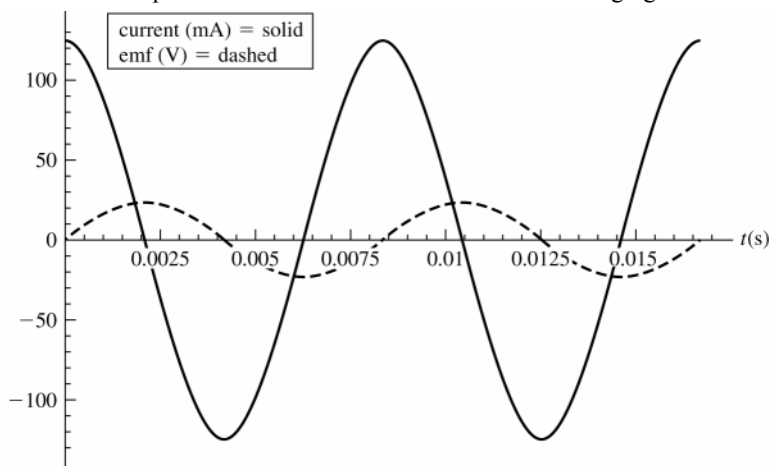


Figure 30.46

30.47. IDENTIFY: Apply $\mathcal{E} = -L \frac{di}{dt}$ to the series and parallel combinations.

SET UP: In series, $i_1 = i_2$ and the voltages add. In parallel the voltages are the same and the currents add.

EXECUTE: (a) Series: $L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} = L_{eq} \frac{di}{dt}$, but $i_1 = i_2 = i$ for series components so $\frac{di_1}{dt} = \frac{di_2}{dt} = \frac{di}{dt}$ and $L_1 + L_2 = L_{eq}$.

(b) Parallel: Now $L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = L_{eq} \frac{di}{dt}$, where $i = i_1 + i_2$. Therefore, $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$. But $\frac{di_1}{dt} = \frac{L_{eq}}{L_1} \frac{di}{dt}$ and

$$\frac{di_2}{dt} = \frac{L_{eq}}{L_2} \frac{di}{dt}. \quad \frac{di}{dt} = \frac{L_{eq}}{L_1} \frac{di}{dt} + \frac{L_{eq}}{L_2} \frac{di}{dt} \quad \text{and} \quad L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} \right)^{-1}.$$

EVALUATE: Inductors in series and parallel combine in the same way as resistors.

30.48. IDENTIFY: Follow the steps outlined in the problem.

SET UP: The energy stored is $U = \frac{1}{2} Li^2$.

EXECUTE: (a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow B 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$.

(b) $d\Phi_B = B dA = \frac{\mu_0 i}{2\pi r} l dr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 i l}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i l}{2\pi} \ln(b/a).$

(d) $L = \frac{N\Phi_B}{i} = l \frac{\mu_0}{2\pi} \ln(b/a).$

(e) $U = \frac{1}{2} Li^2 = \frac{1}{2} l \frac{\mu_0}{2\pi} \ln(b/a) i^2 = \frac{\mu_0 l i^2}{4\pi} \ln(b/a).$

EVALUATE: The magnetic field between the conductors is due only to the current in the inner conductor.

30.49. (a) IDENTIFY and SET UP: An end view is shown in Figure 30.49.

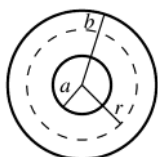


Figure 30.49

Apply Ampere's law to a circular path of radius r .

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$

$I_{encl} = i$, the current in the inner conductor

Thus $B(2\pi r) = \mu_0 i$ and $B = \frac{\mu_0 i}{2\pi r}.$

(b) IDENTIFY and SET UP: Follow the procedure specified in the problem.

EXECUTE: $u = \frac{B^2}{2\mu_0}$

$dU = u dV$, where $dV = 2\pi r l dr$

$$dU = \frac{1}{2\mu_0} \left(\frac{\mu_0 i}{2\pi r} \right)^2 (2\pi r l) dr = \frac{\mu_0 i^2 l}{4\pi} dr$$

(c) $U = \int dU = \frac{\mu_0 i^2 l}{4\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 l}{4\pi} [\ln r]_a^b$

$$U = \frac{\mu_0 i^2 l}{4\pi} (\ln b - \ln a) = \frac{\mu_0 i^2 l}{4\pi} \ln \left(\frac{b}{a} \right)$$

(d) Eq.(30.9): $U = \frac{1}{2} Li^2$

Part (c): $U = \frac{\mu_0 i^2 l}{4\pi} \ln \left(\frac{b}{a} \right)$

$$\frac{1}{2} Li^2 = \frac{\mu_0 i^2 l}{4\pi} \ln \left(\frac{b}{a} \right)$$

$$L = \frac{\mu_0 l}{2\pi} \ln \left(\frac{b}{a} \right).$$

EVALUATE: The value of L we obtain from these energy considerations agrees with L calculated in part (d) of Problem 30.48 by considering flux and Eq.(30.6)

30.50. IDENTIFY: Apply $L = \frac{N\Phi_B}{i}$ to each solenoid, as in Example 30.3. Use $M = \frac{N_2\Phi_{B2}}{i_1}$ to calculate the mutual inductance M .

SET UP: The magnetic field produced by solenoid 1 is confined to the space within its windings and is equal to

$$B_1 = \frac{\mu_0 N_1 i_1}{2\pi r}.$$

EXECUTE: (a) $L_1 = \frac{N_1\Phi_{B1}}{i_1} = \frac{N_1 A}{i_1} \left(\frac{\mu_0 N_1 i_1}{2\pi r} \right) = \frac{\mu_0 N_1^2 A}{2\pi r}$, $L_2 = \frac{N_2\Phi_{B2}}{i_2} = \frac{N_2 A}{i_2} \left(\frac{\mu_0 N_2 i_2}{2\pi r} \right) = \frac{\mu_0 N_2^2 A}{2\pi r}$.

(b) $M = \frac{N_2 AB_1}{i_1} = \frac{\mu_0 N_1 N_2 A}{2\pi r}$. $M^2 = \left(\frac{\mu_0 N_1 N_2 A}{2\pi r} \right)^2 = \frac{\mu_0 N_1^2 A}{2\pi r} \frac{\mu_0 N_2^2 A}{2\pi r} = L_1 L_2$.

EVALUATE: If the two solenoids are identical, so that $N_1 = N_2$, then $M = L$.

30.51. IDENTIFY: $U = \frac{1}{2} LI^2$. The self-inductance of a solenoid is found in Exercise 30.11 to be $L = \frac{\mu_0 AN^2}{l}$.

SET UP: The length l of the solenoid is the number of turns divided by the turns per unit length.

EXECUTE: (a) $L = \frac{2U}{I^2} = \frac{2(10.0 \text{ J})}{(1.50 \text{ A})^2} = 8.89 \text{ H}$

(b) $L = \frac{\mu_0 AN^2}{l}$. If α is the number of turns per unit length, then $N = \alpha l$ and $L = \mu_0 A \alpha^2 l$. For this coil

$$\alpha = 10 \text{ coils/mm} = 10 \times 10^3 \text{ coils/m}. \quad l = \frac{L}{\mu_0 A \alpha^2} = \frac{8.89 \text{ H}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \pi (0.0200 \text{ m})^2 (10 \times 10^3 \text{ coils/m})^2} = 56.3 \text{ m}.$$

This is not a practical length for laboratory use.

EVALUATE: The number of turns is $N = (56.3 \text{ m})(10 \times 10^3 \text{ coils/m}) = 5.63 \times 10^5$ turns. The length of wire in the solenoid is the circumference C of one turn times the number of turns. $C = \pi d = \pi(4.00 \times 10^{-2} \text{ m}) = 0.126 \text{ m}$. The length of wire is $(0.126 \text{ m})(5.63 \times 10^5) = 7.1 \times 10^4 \text{ m} = 71 \text{ km}$. This length of wire will have a large resistance and $I^2 R$ electrical energy losses will be very large.

30.52. IDENTIFY: This is an R - L circuit and $i(t)$ is given by Eq.(30.14).

SET UP: When $t \rightarrow \infty$, $i \rightarrow i_f = V/R$.

EXECUTE: (a) $R = \frac{V}{i_f} = \frac{12.0 \text{ V}}{6.45 \times 10^{-3} \text{ A}} = 1860 \Omega$.

(b) $i = i_f(1 - e^{-(R/L)t})$ so $\frac{Rt}{L} = -\ln(1 - i/i_f)$ and $L = \frac{-Rt}{\ln(1 - i/i_f)} = \frac{-(1860 \Omega)(7.25 \times 10^{-4} \text{ s})}{\ln(1 - (4.86/6.45))} = 0.963 \text{ H}$.

EVALUATE: The current after a long time depends only on R and is independent of L . The value of R/L determines how rapidly the final value of i is reached.

30.53. IDENTIFY and SET UP: Follow the procedure specified in the problem. $L = 2.50 \text{ H}$, $R = 8.00 \Omega$,

$$\mathcal{E} = 6.00 \text{ V}. i = (\mathcal{E}/R)(1 - e^{-t/\tau}), \tau = L/R$$

EXECUTE: (a) Eq.(30.9): $U_L = \frac{1}{2}Li^2$

$$t = \tau \text{ so } i = (\mathcal{E}/R)(1 - e^{-1}) = (6.00 \text{ V}/8.00 \Omega)(1 - e^{-1}) = 0.474 \text{ A}$$

$$\text{Then } U_L = \frac{1}{2}Li^2 = \frac{1}{2}(2.50 \text{ H})(0.474 \text{ A})^2 = 0.281 \text{ J}$$

$$\text{Exercise 30.27 (c): } P_L = \frac{dU_L}{dt} = Li \frac{di}{dt}$$

$$i = \left(\frac{\mathcal{E}}{R}\right)(1 - e^{-t/\tau}); \frac{di}{dt} = \left(\frac{\mathcal{E}}{L}\right)e^{-(R/L)t} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

$$P_L = L \left(\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right) \left(\frac{\mathcal{E}}{L}e^{-t/\tau}\right) = \frac{\mathcal{E}^2}{R}(e^{-t/\tau} - e^{-2t/\tau})$$

$$U_L = \int_0^\tau P_L dt = \frac{\mathcal{E}^2}{R} \int_0^\tau (e^{-t/\tau} - e^{-2t/\tau}) dt = \frac{\mathcal{E}^2}{R} \left[-\tau e^{-t/\tau} + \frac{\tau}{2} e^{-2t/\tau} \right]_0^\tau$$

$$U_L = -\frac{\mathcal{E}^2}{R} \tau \left[e^{-t/\tau} - \frac{1}{2} e^{-2t/\tau} \right]_0^\tau = \frac{\mathcal{E}^2}{R} \tau \left[1 - \frac{1}{2} - e^{-1} + \frac{1}{2} e^{-2} \right]$$

$$U_L = \left(\frac{\mathcal{E}^2}{2R}\right) \left(\frac{L}{R}\right) (1 - 2e^{-1} + e^{-2}) = \frac{1}{2} \left(\frac{\mathcal{E}}{R}\right)^2 L (1 - 2e^{-1} + e^{-2})$$

$$U_L = \frac{1}{2} \left(\frac{6.00 \text{ V}}{8.00 \Omega}\right)^2 (2.50 \text{ H})(0.3996) = 0.281 \text{ J, which checks.}$$

(b) Exercise 30.27(a): The rate at which the battery supplies energy is $P_\mathcal{E} = \mathcal{E}i = \mathcal{E} \left(\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right) = \frac{\mathcal{E}^2}{R}(1 - e^{-t/\tau})$

$$U_\mathcal{E} = \int_0^\tau P_\mathcal{E} dt = \frac{\mathcal{E}^2}{R} \int_0^\tau (1 - e^{-t/\tau}) dt = \frac{\mathcal{E}^2}{R} \left[t + \tau e^{-t/\tau} \right]_0^\tau = \left(\frac{\mathcal{E}^2}{R}\right) (\tau + \tau e^{-1} - \tau)$$

$$U_\mathcal{E} = \left(\frac{\mathcal{E}^2}{R}\right) \tau e^{-1} = \left(\frac{\mathcal{E}^2}{R}\right) \left(\frac{L}{R}\right) e^{-1} = \left(\frac{\mathcal{E}}{R}\right)^2 L e^{-1}$$

$$U_\mathcal{E} = \left(\frac{6.00 \text{ V}}{8.00 \Omega}\right)^2 (2.50 \text{ H})(0.3679) = 0.517 \text{ J}$$

(c) $P_R = i^2 R = \left(\frac{\mathcal{E}}{R}\right)^2 (1 - e^{-t/\tau})^2 = \frac{\mathcal{E}^2}{R} (1 - 2e^{-t/\tau} + e^{-2t/\tau})$

$$U_R = \int_0^\tau P_R dt = \frac{\mathcal{E}^2}{R} \int_0^\tau (1 - 2e^{-t/\tau} + e^{-2t/\tau}) dt = \frac{\mathcal{E}^2}{R} \left[t + 2\tau e^{-t/\tau} - \frac{\tau}{2} e^{-2t/\tau} \right]_0^\tau$$

$$U_R = \frac{\mathcal{E}^2}{R} \left[\tau + 2\tau e^{-1} - \frac{\tau}{2} e^{-2} - 2\tau + \frac{\tau}{2} \right] = \frac{\mathcal{E}^2}{R} \left[-\frac{\tau}{2} + 2\tau e^{-1} - \frac{\tau}{2} e^{-2} \right]$$

$$U_R = \left(\frac{\mathcal{E}^2}{2R}\right) \left(\frac{L}{R}\right) [-1 + 4e^{-1} - e^{-2}]$$

$$U_R = \left(\frac{\mathcal{E}}{R}\right)^2 \left(\frac{1}{2}L\right) [-1 + 4e^{-1} - e^{-2}] = \left(\frac{6.00 \text{ V}}{8.00 \Omega}\right)^2 \left(\frac{1}{2}\right) (2.50 \text{ H})(0.3362) = 0.236 \text{ J}$$

(d) EVALUATE: $U_\mathcal{E} = U_R + U_L$. ($0.517 \text{ J} = 0.236 \text{ J} + 0.281 \text{ J}$)

The energy supplied by the battery equals the sum of the energy stored in the magnetic field of the inductor and the energy dissipated in the resistance of the inductor.

30.54. IDENTIFY: This is a decaying R - L circuit with $I_0 = \mathcal{E}/R$. $i(t) = I_0 e^{-(R/L)t}$.

SET UP: $\mathcal{E} = 60.0 \text{ V}$, $R = 240 \Omega$ and $L = 0.160 \text{ H}$. The rate at which energy stored in the inductor is decreasing is $iL di/dt$.

$$\text{EXECUTE: (a) } U = \frac{1}{2}LI_0^2 = \frac{1}{2}L \left(\frac{\mathcal{E}}{R}\right)^2 = \frac{1}{2}(0.160 \text{ H}) \left(\frac{60 \text{ V}}{240 \Omega}\right)^2 = 5.00 \times 10^{-3} \text{ J.}$$

$$(b) i = \frac{\mathcal{E}}{R} e^{-(R/L)t} \Rightarrow \frac{di}{dt} = -\frac{R}{L} i \Rightarrow \frac{dU_L}{dt} = iL \frac{di}{dt} = -Ri^2 = \frac{\mathcal{E}^2}{R} e^{-2(R/L)t} \cdot \frac{dU_L}{dt} = -\frac{(60 \text{ V})^2}{240 \Omega} e^{-2(240/0.160)(4.00 \times 10^{-4})} = -4.52 \text{ W}.$$

$$(c) \text{ In the resistor, } P_R = \frac{dU_R}{dt} = i^2 R = \frac{\mathcal{E}^2}{R} e^{-2(R/L)t} = \frac{(60 \text{ V})^2}{240 \Omega} e^{-2(240/0.160)(4.00 \times 10^{-4})} = 4.52 \text{ W}.$$

$$(d) P_R(t) = i^2 R = \frac{\mathcal{E}^2}{R} e^{-2(R/L)t}. U_R = \frac{\mathcal{E}^2}{R} \int_0^\infty e^{-2(R/L)t} dt = \frac{\mathcal{E}^2}{R} \frac{L}{2R} = \frac{(60 \text{ V})^2 (0.160 \text{ H})}{2(240 \Omega)^2} = 5.00 \times 10^{-3} \text{ J, which is the same as part (a).}$$

EVALUATE: During the decay of the current all the electrical energy originally stored in the inductor is dissipated in the resistor.

30.55. IDENTIFY and SET UP: Follow the procedure specified in the problem. $\frac{1}{2} Li^2$ is the energy stored in the inductor and $q^2/2C$ is the energy stored in the capacitor. The equation is $-iR - L \frac{di}{dt} - \frac{q}{C} = 0$.

EXECUTE: Multiplying by $-i$ gives $i^2 R + Li \frac{di}{dt} + \frac{qi}{C} = 0$. $\frac{d}{dt} U_L = \frac{d}{dt} \left(\frac{1}{2} Li^2 \right) = \frac{1}{2} L \frac{d}{dt} (i^2) = \frac{1}{2} L \left(2i \frac{di}{dt} \right) = Li \frac{di}{dt}$, the

second term. $\frac{d}{dt} U_C = \frac{d}{dt} \left(\frac{q^2}{2C} \right) = \frac{1}{2C} \frac{d}{dt} (q^2) = \frac{1}{2C} (2q) \frac{dq}{dt} = \frac{qi}{C}$, the third term. $i^2 R = P_R$, the rate at which

electrical energy is dissipated in the resistance. $\frac{d}{dt} U_L = P_L$, the rate at which the amount of energy stored in the

inductor is changing. $\frac{d}{dt} U_C = P_C$, the rate at which the amount of energy stored in the capacitor is changing.

EVALUATE: The equation says that $P_R + P_L + P_C = 0$; the net rate of change of energy in the circuit is zero. Note that at any given time one of P_C or P_L is negative. If the current and U_L are increasing the charge on the capacitor and U_C are decreasing, and vice versa.

30.56. IDENTIFY: The energy stored in a capacitor is $U_C = \frac{1}{2} C v^2$. The energy stored in an inductor is $U_L = \frac{1}{2} Li^2$. Energy conservation requires that the total stored energy be constant.

SET UP: The current is a maximum when the charge on the capacitor is zero and the energy stored in the capacitor is zero.

EXECUTE: (a) Initially $v = 16.0 \text{ V}$ and $i = 0$. $U_L = 0$ and $U_C = \frac{1}{2} C v^2 = \frac{1}{2} (5.00 \times 10^{-6} \text{ F}) (16.0 \text{ V})^2 = 6.40 \times 10^{-4} \text{ J}$. The total energy stored is 0.640 mJ .

(b) The current is maximum when $q = 0$ and $U_C = 0$. $U_C + U_L = 6.40 \times 10^{-4} \text{ J}$ so $U_L = 6.40 \times 10^{-4} \text{ J}$.

$$\frac{1}{2} Li_{\max}^2 = 6.40 \times 10^{-4} \text{ J and } i_{\max} = \sqrt{\frac{2(6.40 \times 10^{-4} \text{ J})}{3.75 \times 10^{-3} \text{ H}}} = 0.584 \text{ A}.$$

EVALUATE: The maximum charge on the capacitor is $Q = CV = 80.0 \mu\text{C}$.

30.57. IDENTIFY and SET UP: Use $U_C = \frac{1}{2} CV_C^2$ (energy stored in a capacitor) to solve for C . Then use Eq.(30.22) and $\omega = 2\pi f$ to solve for the L that gives the desired current oscillation frequency.

EXECUTE: $V_C = 12.0 \text{ V}$; $U_C = \frac{1}{2} CV_C^2$ so $C = 2U_C/V_C^2 = 2(0.0160 \text{ J})/(12.0 \text{ V})^2 = 222 \mu\text{F}$

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ so } L = \frac{1}{(2\pi f)^2 C}$$

$$f = 3500 \text{ Hz gives } L = 9.31 \mu\text{H}$$

EVALUATE: f is in Hz and ω is in rad/s; we must be careful not to confuse the two.

30.58. IDENTIFY: Apply energy conservation to the circuit.

SET UP: For a capacitor $V = q/C$ and $U = q^2/2C$. For an inductor $U = \frac{1}{2} Li^2$

$$\text{EXECUTE: (a) } V_{\max} = \frac{Q}{C} = \frac{6.00 \times 10^{-6} \text{ C}}{2.50 \times 10^{-4} \text{ F}} = 0.0240 \text{ V}.$$

$$(b) \frac{1}{2} Li_{\max}^2 = \frac{Q^2}{2C}, \text{ so } i_{\max} = \frac{Q}{\sqrt{LC}} = \frac{6.00 \times 10^{-6} \text{ C}}{\sqrt{(0.0600 \text{ H})(2.50 \times 10^{-4} \text{ F})}} = 1.55 \times 10^{-3} \text{ A}$$

$$(c) U_{\max} = \frac{1}{2} Li_{\max}^2 = \frac{1}{2} (0.0600 \text{ H})(1.55 \times 10^{-3} \text{ A})^2 = 7.21 \times 10^{-8} \text{ J}.$$

(d) If $i = \frac{1}{2}i_{\max}$ then $U_L = \frac{1}{4}U_{\max} = 1.80 \times 10^{-8} \text{ J}$ and $U_C = \frac{3}{4}U_{\max} = \frac{(\sqrt{3/4}Q)^2}{2C} = \frac{q^2}{2C}$. This gives $q = \sqrt{\frac{3}{4}}Q = 5.20 \times 10^{-6} \text{ C}$.

EVALUATE: $U_{\max} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$ for all times.

30.59. IDENTIFY: Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1 \text{ m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366 \times 10^4 \text{ J/m}^3$. The total energy stored in volume V of the sunspot is $U_B = u_B V$. The mass of the material in volume V of the sunspot is $m = \rho V$.

$K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2 \times 10^4 \text{ m/s}$.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.60. IDENTIFY: $i(t)$ is given by Eq.(30.14).

SET UP: The graph shows $V = 0$ at $t = 0$ and V approaches the constant value of 25 V at large times.

EXECUTE: (a) The voltage behaves the same as the current. Since V_R is proportional to i , the scope must be across the 150Ω resistor.

(b) From the graph, as $t \rightarrow \infty$, $V_R \rightarrow 25 \text{ V}$, so there is no voltage drop across the inductor, so its internal resistance must be zero. $V_R = V_{\max}(1 - e^{-t/\tau})$. When $t = \tau$, $V_R = V_{\max}\left(1 - \frac{1}{e}\right) \approx 0.63V_{\max}$. From the graph, $V = 0.63V_{\max} = 16 \text{ V}$ at $t \approx 0.5 \text{ ms}$. Therefore $\tau = 0.5 \text{ ms}$. $L/R = 0.5 \text{ ms}$ gives $L = (0.5 \text{ ms})(150 \Omega) = 0.075 \text{ H}$.

(c) The graph if the scope is across the inductor is sketched in Figure 30.60.

EVALUATE: At all times $V_R + V_L = 25.0 \text{ V}$. At $t = 0$ all the battery voltage appears across the inductor since $i = 0$. At $t \rightarrow \infty$ all the battery voltage is across the resistance, since $di/dt = 0$.

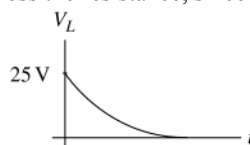


Figure 30.60

30.61. IDENTIFY and SET UP: The current grows in the circuit as given by Eq.(30.14). In an R - L circuit the full emf initially is across the inductance and after a long time is totally across the resistance. A solenoid in a circuit is represented as a resistance in series with an inductance. Apply the loop rule to the circuit; the voltage across a resistance is given by Ohm's law.

EXECUTE: (a) In the R - L circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. In the graph, the voltage drops, so the oscilloscope is across the solenoid.

(b) At $t \rightarrow \infty$ the current in the circuit approaches its final, constant value. The voltage doesn't go to zero because the solenoid has some resistance R_L . The final voltage across the solenoid is IR_L , where I is the final current in the circuit.

(c) The emf of the battery is the initial voltage across the inductor, 50 V. Just after the switch is closed, the current is zero and there is no voltage drop across any of the resistance in the circuit.

(d) As $t \rightarrow \infty$, $\mathcal{E} - IR - IR_L = 0$

$\mathcal{E} = 50 \text{ V}$ and from the graph $IR_L = 15 \text{ V}$ (the final voltage across the inductor), so $IR = 35 \text{ V}$ and $I = (35 \text{ V})/R = 3.5 \text{ A}$

(e) $IR_L = 15 \text{ V}$, so $R_L = (15 \text{ V})/(3.5 \text{ A}) = 4.3 \Omega$

$\mathcal{E} - V_L - iR = 0$, where V_L includes the voltage across the resistance of the solenoid.

$V_L = \mathcal{E} - iR$, $i = \frac{\mathcal{E}}{R_{\text{tot}}}(1 - e^{-t/\tau})$, so $V_L = \mathcal{E}\left[1 - \frac{R}{R_{\text{tot}}}(1 - e^{-t/\tau})\right]$

$\mathcal{E} = 50 \text{ V}$, $R = 10 \Omega$, $R_{\text{tot}} = 14.3 \Omega$, so when $t = \tau$, $V_L = 27.9 \text{ V}$. From the graph, V_L has this value when $t = 3.0 \text{ ms}$ (read approximately from the graph), so $\tau = L/R_{\text{tot}} = 3.0 \text{ ms}$. Then $L = (3.0 \text{ ms})(14.3 \Omega) = 43 \text{ mH}$.

EVALUATE: At $t = 0$ there is no current and the 50 V measured by the oscilloscope is the induced emf due to the inductance of the solenoid. As the current grows, there are voltage drops across the two resistances in the circuit.

We derived an equation for V_L , the voltage across the solenoid. At $t = 0$ it gives $V_L = \mathcal{E}$ and at $t \rightarrow \infty$ it gives $V_L = \mathcal{E}R/R_{\text{tot}} = iR$.

30.62. IDENTIFY: At $t = 0$, $i = 0$ through each inductor. At $t \rightarrow \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \rightarrow \infty$ replace each inductor by a wire.

EXECUTE: (a) Initially the inductor blocks current through it, so the simplified equivalent circuit is shown in Figure 30.62a. $i = \frac{\mathcal{E}}{R} = \frac{50 \text{ V}}{150 \Omega} = 0.333 \text{ A}$. $V_1 = (100 \Omega)(0.333 \text{ A}) = 33.3 \text{ V}$. $V_4 = (50 \Omega)(0.333 \text{ A}) = 16.7 \text{ V}$. $V_3 = 0$

since no current flows through it. $V_2 = V_4 = 16.7 \text{ V}$, since the inductor is in parallel with the 50Ω resistor.

$A_1 = A_3 = 0.333 \text{ A}$, $A_2 = 0$.

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. The simplified circuit is sketched in Figure 30.62b. $i = \mathcal{E}/R = \frac{50 \text{ V}}{130 \Omega} = 0.385 \text{ A}$. $V_1 = (100 \Omega)(0.385 \text{ A}) = 38.5 \text{ V}$; $V_2 = 0$;

$V_3 = V_4 = 50 \text{ V} - 38.5 \text{ V} = 11.5 \text{ V}$. $i_1 = 0.385 \text{ A}$; $i_2 = \frac{11.5 \text{ V}}{75 \Omega} = 0.153 \text{ A}$; $i_3 = \frac{11.5 \text{ V}}{50 \Omega} = 0.230 \text{ A}$.

EVALUATE: Just after the switch is closed the current through the battery is 0.333 A . After a long time the current through the battery is 0.385 A . After a long time there is an additional current path, the equivalent resistance of the circuit is decreased and the current has increased.

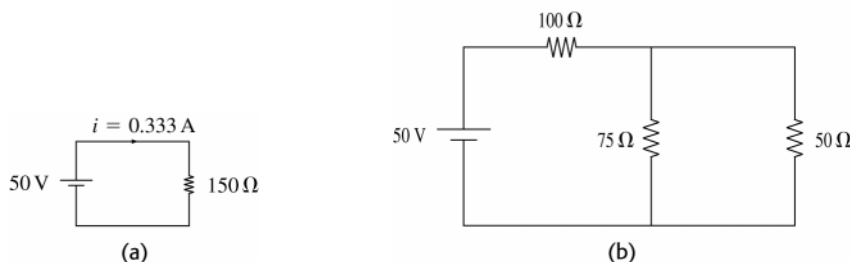


Figure 30.62

30.63. IDENTIFY and SET UP: Just after the switch is closed, the current in each branch containing an inductor is zero and the voltage across any capacitor is zero. The inductors can be treated as breaks in the circuit and the capacitors can be replaced by wires. After a long time there is no voltage across each inductor and no current in any branch containing a capacitor. The inductors can be replaced by wires and the capacitors by breaks in the circuit.

EXECUTE: (a) Just after the switch is closed the voltage V_5 across the capacitor is zero and there is also no current through the inductor, so $V_3 = 0$. $V_2 + V_3 = V_4 = V_5$, and since $V_5 = 0$ and $V_3 = 0$, V_4 and V_2 are also zero.

$V_4 = 0$ means V_3 reads zero. V_1 then must equal 40.0 V , and this means the current read by A_1 is $(40.0 \text{ V})/(50.0 \Omega) = 0.800 \text{ A}$. $A_2 + A_3 + A_4 = A_1$, but $A_2 = A_3 = 0$ so $A_4 = A_1 = 0.800 \text{ A}$. $A_1 = A_4 = 0.800 \text{ A}$; all other ammeters read zero. $V_1 = 40.0 \text{ V}$ and all other voltmeters read zero.

(b) After a long time the capacitor is fully charged so $A_4 = 0$. The current through the inductor isn't changing, so $V_2 = 0$. The currents can be calculated from the equivalent circuit that replaces the inductor by a short circuit, as shown in Figure 30.63a.

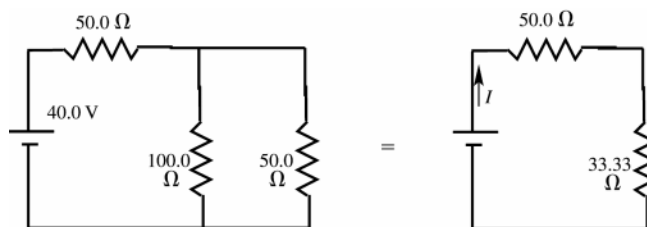


Figure 30.63a

$I = (40.0 \text{ V})/(83.33 \Omega) = 0.480 \text{ A}$; A_1 reads 0.480 A

$V_1 = I(50.0 \Omega) = 24.0 \text{ V}$

The voltage across each parallel branch is $40.0 \text{ V} - 24.0 \text{ V} = 16.0 \text{ V}$

$V_2 = 0$, $V_3 = V_4 = V_5 = 16.0 \text{ V}$

$V_3 = 16.0 \text{ V}$ means A_2 reads 0.160 A . $V_4 = 16.0 \text{ V}$ means A_3 reads 0.320 A . A_4 reads zero. Note that $A_2 + A_3 = A_1$.

(c) $V_5 = 16.0 \text{ V}$ so $Q = CV = (12.0 \mu\text{F})(16.0 \text{ V}) = 192 \mu\text{C}$

(d) At $t = 0$ and $t \rightarrow \infty$, $V_2 = 0$. As the current in this branch increases from zero to 0.160 A the voltage V_2 reflects the rate of change of the current. The graph is sketched in Figure 30.63b.

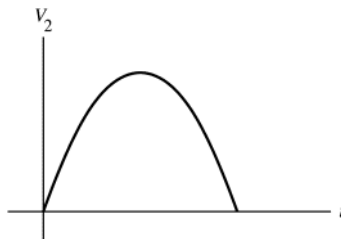


Figure 30.63b

EVALUATE: This reduction of the circuit to resistor networks only apply at $t = 0$ and $t \rightarrow \infty$. At intermediate times the analysis is complicated.

30.64. IDENTIFY: At all times $v_1 + v_2 = 25.0$ V. The voltage across the resistor depends on the current through it and the voltage across the inductor depends on the rate at which the current through it is changing.

SET UP: Immediately after closing the switch the current through the inductor is zero. After a long time the current is no longer changing.

EXECUTE: (a) $i = 0$ so $v_1 = 0$ and $v_2 = 25.0$ V. The ammeter reading is $A = 0$.

(b) After a long time, $v_2 = 0$ and $v_1 = 25.0$ V. $v_1 = iR$ and $i = \frac{v_1}{R} = \frac{25.0 \text{ V}}{15.0 \Omega} = 1.67$ A. The ammeter reading is $A = 1.67$ A.

(c) None of the answers in (a) and (b) depend on L so none of them would change.

EVALUATE: The inductance L of the circuit affects the rate at which current reaches its final value. But after a long time the inductor doesn't affect the circuit and the final current does not depend on L .

30.65. IDENTIFY: At $t = 0$, $i = 0$ through each inductor. At $t \rightarrow \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \rightarrow \infty$ replace each inductor by a wire.

EXECUTE: (a) Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the 40.0Ω and 15.0Ω resistors and is equal to

$$(25.0 \text{ V}) / (40.0 \Omega + 15.0 \Omega) = 0.455 \text{ A}. A_1 = A_4 = 0.455 \text{ A}; A_2 = A_3 = 0.$$

(b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit. The circuit is equivalent to the circuit sketched in Figure 30.65.

$$I = (25.0 \text{ V}) / (42.73 \Omega) = 0.585 \text{ A}. A_1 \text{ reads } 0.585 \text{ A}. \text{ The voltage across each parallel branch is}$$

$$25.0 \text{ V} - (0.585 \text{ A})(40.0 \Omega) = 1.60 \text{ V}. A_2 \text{ reads } (1.60 \text{ V}) / (5.0 \Omega) = 0.320 \text{ A}. A_3 \text{ reads}$$

$$(1.60 \text{ V}) / (10.0 \Omega) = 0.160 \text{ A}. A_4 \text{ reads } (1.60 \text{ V}) / (15.0 \Omega) = 0.107 \text{ A}.$$

EVALUATE: Just after the switch is closed the current through the battery is 0.455 A. After a long time the current through the battery is 0.585 A. After a long time there are additional current paths, the equivalent resistance of the circuit is decreased and the current has increased.

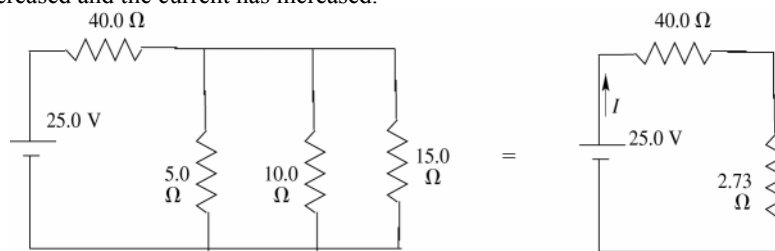


Figure 30.65

30.66. IDENTIFY: Closing S_2 and simultaneously opening S_1 produces an L - C circuit with initial current through the inductor of 3.50 A. When the current is a maximum the charge q on the capacitor is zero and when the charge q is a maximum the current is zero. Conservation of energy says that the maximum energy $\frac{1}{2}Li_{\text{max}}^2$ stored in the inductor equals the maximum energy $\frac{1}{2}\frac{q_{\text{max}}^2}{C}$ stored in the capacitor.

SET UP: $i_{\text{max}} = 3.50$ A, the current in the inductor just after the switch is closed.

EXECUTE: (a) $\frac{1}{2} Li_{\max}^2 = \frac{1}{2} \frac{q_{\max}^2}{C}$.

$$q_{\max} = (\sqrt{LC})i_{\max} = \sqrt{(2.0 \times 10^{-3} \text{ H})(5.0 \times 10^{-6} \text{ F})}(3.50 \text{ A}) = 3.50 \times 10^{-4} \text{ C} = 0.350 \text{ mC}.$$

(b) When q is maximum, $i = 0$.

EVALUATE: In the final circuit the current will oscillate.

- 30.67. IDENTIFY:** Apply the loop rule to each parallel branch. The voltage across a resistor is given by iR and the voltage across an inductor is given by $L|di/dt|$. The rate of change of current through the inductor is limited.

SET UP: With S closed the circuit is sketched in Figure 30.67a.

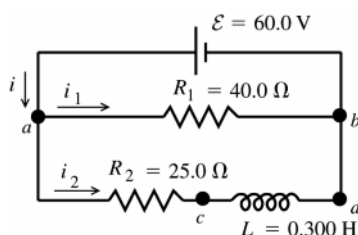


Figure 30.67a

The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is closed the current in the inductor has not had time to increase from zero, so $i_2 = 0$.

EXECUTE: (a) $\mathcal{E} - v_{ab} = 0$, so $v_{ab} = 60.0 \text{ V}$

(b) The voltage drops across R_1 , as we travel through the resistor in the direction of the current, so point a is at higher potential.

(c) $i_2 = 0$ so $v_{R_2} = i_2 R_2 = 0$

$\mathcal{E} - v_{R_2} - v_L = 0$ so $v_L = \mathcal{E} = 60.0 \text{ V}$

(d) The voltage rises when we go from b to a through the emf, so it must drop when we go from a to b through the inductor. Point c must be at higher potential than point d .

(e) After the switch has been closed a long time, $\frac{di_2}{dt} \rightarrow 0$ so $v_L = 0$. Then $\mathcal{E} - v_{R_2} = 0$ and $i_2 R_2 = \mathcal{E}$

$$\text{so } i_2 = \frac{\mathcal{E}}{R_2} = \frac{60.0 \text{ V}}{25.0 \Omega} = 2.40 \text{ A}.$$

SET UP: The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is opened again the current through the inductor hasn't had time to change and is still $i_2 = 2.40 \text{ A}$. The circuit is sketched in Figure 30.67b.

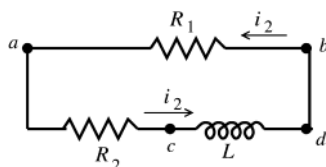


Figure 30.67b

EXECUTE: The current through R_1 is $i_2 = 2.40 \text{ A}$, in the direction b to a . Thus

$$v_{ab} = -i_2 R_1 = -(2.40 \text{ A})(40.0 \Omega)$$

$$v_{ab} = -96.0 \text{ V}$$

(f) Point where current enters resistor is at higher potential; point b is at higher potential.

(g) $v_L - v_{R_1} - v_{R_2} = 0$

$$v_L = v_{R_1} + v_{R_2}$$

$$v_{R_1} = -v_{ab} = 96.0 \text{ V}; v_{R_2} = i_2 R_2 = (2.40 \text{ A})(25.0 \Omega) = 60.0 \text{ V}$$

Then $v_L = v_{R_1} + v_{R_2} = 96.0 \text{ V} + 60.0 \text{ V} = 156 \text{ V}$.

As you travel counterclockwise around the circuit in the direction of the current, the voltage drops across each resistor, so it must rise across the inductor and point d is at higher potential than point c . The current is decreasing, so the induced emf in the inductor is directed in the direction of the current. Thus, $v_{cd} = -156 \text{ V}$.

(h) Point d is at higher potential.

EVALUATE: The voltage across R_1 is constant once the switch is closed. In the branch containing R_2 , just after S is closed the voltage drop is all across L and after a long time it is all across R_2 . Just after S is opened the same current flows in the single loop as had been flowing through the inductor and the sum of the voltage across the resistors equals the voltage across the inductor. This voltage dies away, as the energy stored in the inductor is dissipated in the resistors.

30.68. IDENTIFY: Apply the loop rule to the two loops. The current through the inductor doesn't change abruptly.

SET UP: For the inductor $|\mathcal{E}| = L \left| \frac{di}{dt} \right|$ and $\mathcal{E}$ is directed to oppose the change in current.

EXECUTE: (a) Switch is closed, then at some later time

$$\frac{di}{dt} = 50.0 \text{ A/s} \Rightarrow v_{cd} = L \frac{di}{dt} = (0.300 \text{ H})(50.0 \text{ A/s}) = 15.0 \text{ V}.$$

The top circuit loop: $60.0 \text{ V} = i_1 R_1 \Rightarrow i_1 = \frac{60.0 \text{ V}}{40.0 \Omega} = 1.50 \text{ A}.$

The bottom loop: $60 \text{ V} - i_2 R_2 - 15.0 \text{ V} = 0 \Rightarrow i_2 = \frac{45.0 \text{ V}}{25.0 \Omega} = 1.80 \text{ A}.$

(b) After a long time: $i_2 = \frac{60.0 \text{ V}}{25.0 \Omega} = 2.40 \text{ A}$, and immediately when the switch is opened, the inductor maintains

this current, so $i_1 = i_2 = 2.40 \text{ A}.$

EVALUATE: The current through R_1 changes abruptly when the switch is closed.

30.69. IDENTIFY and SET UP: The circuit is sketched in Figure 30.69a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time i has reached its final value and $di/dt = 0$. The voltage across a resistor depends on i and the voltage across an inductor depends on di/dt .

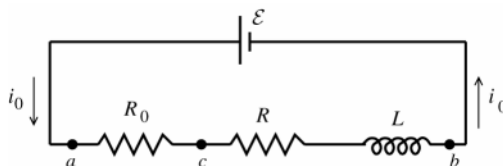


Figure 30.69a

EXECUTE: (a) At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0 R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$, so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0 \text{ V}$. ($i_0 R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\frac{di_0}{dt} \rightarrow 0$ so the potential $-L \frac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives

$$\mathcal{E} - i_0(R_0 + R) = 0.$$

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0 \text{ V}}{50.0 \Omega + 150 \Omega} = 0.180 \text{ A}$$

$$v_{ac} = i_0 R_0 = (0.180 \text{ A})(50.0 \Omega) = 9.0 \text{ V}$$

Thus $v_{cb} = i_0 R + L \frac{di_0}{dt} = (0.180 \text{ A})(150 \Omega) + 0 = 27.0 \text{ V}$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0$

$$\mathcal{E} - iR_0 - iR - L \frac{di}{dt} = 0$$

$$L \frac{di}{dt} = \mathcal{E} - i(R_0 + R) \text{ and } \left(\frac{L}{R + R_0} \right) \frac{di}{dt} = -i + \frac{\mathcal{E}}{R + R_0}$$

$$\frac{di}{-i + \mathcal{E}/(R + R_0)} = \left(\frac{R + R_0}{L} \right) dt$$

$$\text{Integrate from } t = 0, \text{ when } i = 0, \text{ to } t, \text{ when } i = i_0: \int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R + R_0)} = \frac{R + R_0}{L} \int_0^t dt = -\ln \left[-i + \frac{\mathcal{E}}{R + R_0} \right]_0^{i_0} = \left(\frac{R + R_0}{L} \right) t,$$

$$\text{so } \ln \left(-i_0 + \frac{\mathcal{E}}{R + R_0} \right) - \ln \left(\frac{\mathcal{E}}{R + R_0} \right) = - \left(\frac{R + R_0}{L} \right) t$$

$$\ln \left(\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)} \right) = - \left(\frac{R + R_0}{L} \right) t$$

Taking exponentials of both sides gives $\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)} = e^{-(R + R_0)t/L}$ and $i_0 = \frac{\mathcal{E}}{R + R_0} (1 - e^{-(R + R_0)t/L})$

Substituting in the numerical values gives $i_0 = \frac{36.0 \text{ V}}{50 \Omega + 150 \Omega} (1 - e^{-(200 \Omega / 4.00 \text{ H})t}) = (0.180 \text{ A}) (1 - e^{-t/0.020 \text{ s}})$

At $t \rightarrow 0$, $i_0 = (0.180 \text{ A})(1-1) = 0$ (agrees with part (a)). At $t \rightarrow \infty$, $i_0 = (0.180 \text{ A})(1-0) = 0.180 \text{ A}$ (agrees with part (b)).

$$v_{ac} = i_0 R_0 = \frac{\mathcal{E} R_0}{R + R_0} (1 - e^{-(R+R_0)t/L}) = 9.0 \text{ V} (1 - e^{-t/0.020 \text{ s}})$$

$$v_{cb} = \mathcal{E} - v_{ac} = 36.0 \text{ V} - 9.0 \text{ V} (1 - e^{-t/0.020 \text{ s}}) = 9.0 \text{ V} (3.00 + e^{-t/0.020 \text{ s}})$$

At $t \rightarrow 0$, $v_{ac} = 0$, $v_{cb} = 36.0 \text{ V}$ (agrees with part (a)). At $t \rightarrow \infty$, $v_{ac} = 9.0 \text{ V}$, $v_{cb} = 27.0 \text{ V}$ (agrees with part (b)).

The graphs are given in Figure 30.69b.

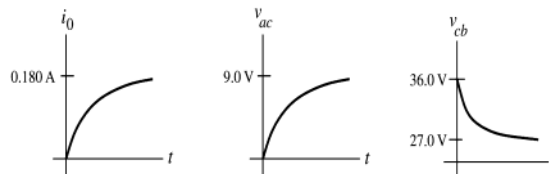


Figure 30.69b

EVALUATE: The expression for $i(t)$ we derived becomes Eq.(30.14) if the two resistors R_0 and R in series are replaced by a single equivalent resistance $R_0 + R$.

30.70. IDENTIFY: Apply the loop rule. The current through the inductor doesn't change abruptly.

SET UP: With S_2 closed, v_{cb} must be zero.

EXECUTE: (a) Immediately after S_2 is closed, the inductor maintains the current $i = 0.180 \text{ A}$ through R . The loop rule around the outside of the circuit yields

$$\mathcal{E} + \mathcal{E}_L - iR - i_0 R_0 = 36.0 \text{ V} + (0.18 \text{ A})(150 \Omega) - (0.18 \text{ A})(150 \Omega) - i_0 (50 \Omega) = 0. \quad i_0 = \frac{36 \text{ V}}{50 \Omega} = 0.720 \text{ A}.$$

$$v_{ac} = (0.72 \text{ A})(50 \text{ V}) = 36.0 \text{ V} \text{ and } v_{cb} = 0.$$

(b) After a long time, $v_{ac} = 36.0 \text{ V}$, and $v_{cb} = 0$. Thus $i_0 = \frac{\mathcal{E}}{R_0} = \frac{36.0 \text{ V}}{50 \Omega} = 0.720 \text{ A}$, $i_R = 0$ and $i_{s2} = 0.720 \text{ A}$.

(c) $i_0 = 0.720 \text{ A}$, $i_R(t) = \frac{\mathcal{E}}{R_{\text{total}}} e^{-(R/L)t}$ and $i_R(t) = (0.180 \text{ A}) e^{-(12.5 \text{ s}^{-1})t}$.

$i_{s2}(t) = (0.720 \text{ A}) - (0.180 \text{ A}) e^{-(12.5 \text{ s}^{-1})t} = (0.180 \text{ A}) (4 - e^{-(12.5 \text{ s}^{-1})t})$. The graphs of the currents are given in Figure 30.70.

EVALUATE: R_0 is in a loop that contains just $\mathcal{E}$ and R_0 , so the current through R_0 is constant. After a long time the current through the inductor isn't changing and the voltage across the inductor is zero. Since v_{cb} is zero, the voltage across R must be zero and i_R becomes zero.

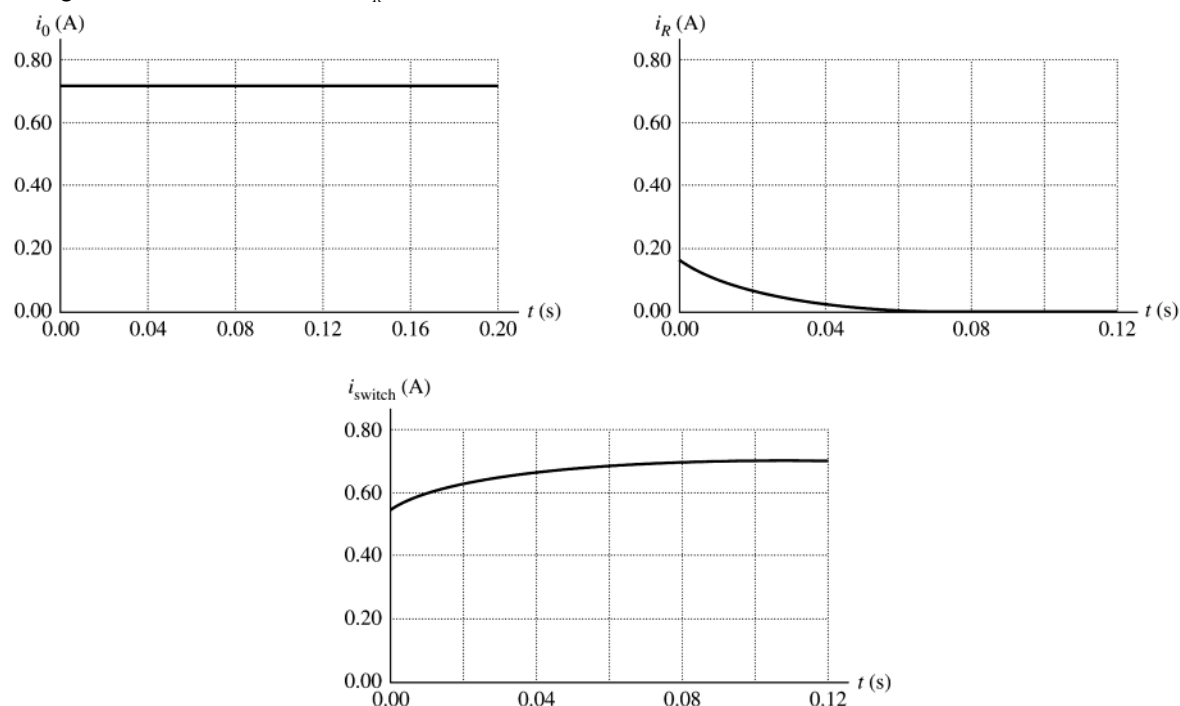


Figure 30.70

30.71. IDENTIFY: The current through an inductor doesn't change abruptly. After a long time the current isn't changing and the voltage across each inductor is zero.

SET UP: Problem 30.47 shows how to find the equivalent inductance of inductors in series and parallel.

EXECUTE: (a) Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

(b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to the circuit shown in Figure 30.71a.

$I = (20.0 \text{ V}) / (75.0 \Omega) = 0.267 \text{ A}$. The voltage between points a and b is zero, so the voltmeter reads zero.

(c) Use the results of Problem 30.49 to combine the inductor network into its equivalent, as shown in Figure 30.71b.

$R = 75.0 \Omega$ is the equivalent resistance. Eq.(30.14) says $i = (\mathcal{E}/R)(1 - e^{-t/\tau})$ with

$\tau = L/R = (10.8 \text{ mH}) / (75.0 \Omega) = 0.144 \text{ ms}$. $\mathcal{E} = 20.0 \text{ V}$, $R = 75.0 \Omega$, $t = 0.115 \text{ ms}$ so $i = 0.147 \text{ A}$.

$V_R = iR = (0.147 \text{ A})(75.0 \Omega) = 11.0 \text{ V}$. $20.0 \text{ V} - V_R - V_L = 0$ and $V_L = 20.0 \text{ V} - V_R = 9.0 \text{ V}$. The ammeter reads 0.147 A and the voltmeter reads 9.0 V.

EVALUATE: The current through the battery increases from zero to a final value of 0.267 A. The voltage across the inductor network drops from 20.0 V to zero.

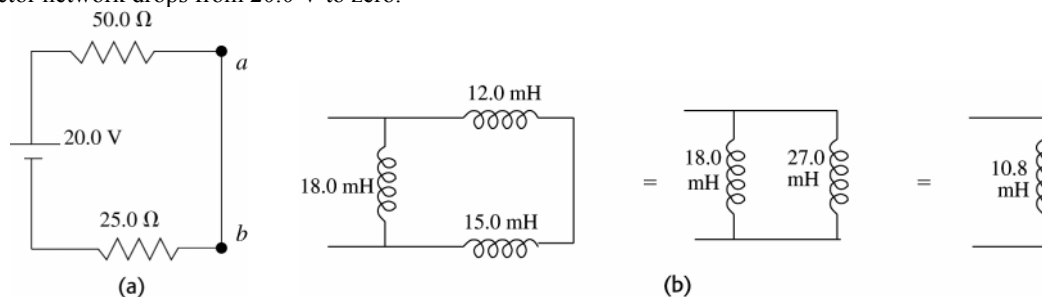


Figure 30.71

30.72. IDENTIFY: At steady state with the switch in position 1, no current flows to the capacitors and the inductors can be replaced by wires. Apply conservation of energy to the circuit with the switch in position 2.

SET UP: Replace the series combinations of inductors and capacitors by their equivalents. For the inductors use the results of Problem 30.47.

EXECUTE: (a) At steady state $i = \frac{\mathcal{E}}{R} = \frac{75.0 \text{ V}}{125 \Omega} = 0.600 \text{ A}$.

(b) The equivalent circuit capacitance of the two capacitors is given by $\frac{1}{C_s} = \frac{1}{25 \mu\text{F}} + \frac{1}{35 \mu\text{F}}$ and $C_s = 14.6 \mu\text{F}$.

$L_s = 15.0 \text{ mH} + 5.0 \text{ mH} = 20.0 \text{ mH}$. The equivalent circuit is sketched in Figure 30.72a.

Energy conservation: $\frac{q^2}{2C} = \frac{1}{2}Li_0^2$. $q = i_0\sqrt{LC} = (0.600 \text{ A})\sqrt{(20 \times 10^{-3} \text{ H})(14.6 \times 10^{-6} \text{ F})} = 3.24 \times 10^{-4} \text{ C}$. As shown in Figure 30.72b, the capacitors have their maximum charge at $t = T/4$.

$t = \frac{1}{4}T = \frac{1}{4}(2\pi\sqrt{LC}) = \frac{\pi}{2}\sqrt{LC} = \frac{\pi}{2}\sqrt{(20 \times 10^{-3} \text{ H})(14.6 \times 10^{-6} \text{ F})} = 8.49 \times 10^{-4} \text{ s}$

EVALUATE: With the switch closed the battery stores energy in the inductors. This then is the energy in the L - C circuit when the switch is in position 2.

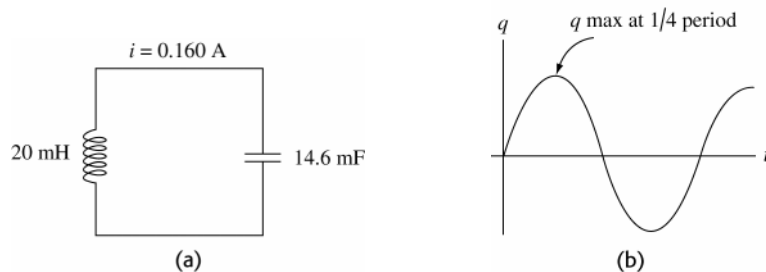


Figure 30.72

30.73. IDENTIFY: Follow the steps specified in the problem.

SET UP: Find the flux through a ring of height h , radius r and thickness dr . Example 28.19 shows that $B = \frac{\mu_0 Ni}{2\pi r}$ inside the toroid.

EXECUTE: (a) $\Phi_B = \int_a^b B(h dr) = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r} \right) (h dr) = \frac{\mu_0 Nih}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 Nih}{2\pi} \ln(b/a).$

(b) $L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi} \ln(b/a).$

(c) $\ln(b/a) = \ln(1 - (b-a)/a) \approx \frac{b-a}{a} + \frac{(b-a)^2}{2a^2} + \dots \Rightarrow L \approx \frac{\mu_0 N^2 h}{2\pi} \left(\frac{b-a}{a} \right).$

EVALUATE: $h(b-a)$ is the cross-sectional area A of the toroid and a is approximately the radius r , so this result is approximately the same as the result derived in Example 30.3.

30.74. IDENTIFY: The direction of the current induced in circuit A is given by Lenz's law.

SET UP: When the switch is closed current flows counterclockwise in the circuit on the left, from the positive plate of the capacitor. The current decreases as a function of time, as the charge and voltage of the capacitor decrease.

EXECUTE: At loop A the magnetic field from the wire of the other circuit adjacent to A is into the page. The magnetic field of this current is decreasing, as the current decreases. Therefore, the magnetic field of the induced current in A is directed into the page inside A and to produce a magnetic field in this direction the induced current is clockwise.

EVALUATE: The magnitude of the emf induced in circuit A decreases with time after the switch is closed, because the rate of change of the current in the other circuit decreases.

30.75. (a) IDENTIFY and SET UP: With switch S closed the circuit is shown in Figure 30.75a.

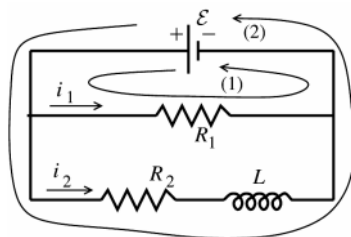


Figure 30.75a

Apply the loop rule to loops 1 and 2.

EXECUTE:

loop 1

$$\mathcal{E} - i_1 R_1 = 0$$

$$i_1 = \frac{\mathcal{E}}{R_1} \text{ (independent of } t\text{)}$$

loop (2)

$$\mathcal{E} - i_2 R_2 - L \frac{di_2}{dt} = 0$$

This is in the form of equation (30.12), so the solution is analogous to Eq.(30.14): $i_2 = \frac{\mathcal{E}}{R_2} (1 - e^{-R_2 t/L})$

(b) **EVALUATE:** The expressions derived in part (a) give that as $t \rightarrow \infty$, $i_1 = \frac{\mathcal{E}}{R_1}$ and $i_2 = \frac{\mathcal{E}}{R_2}$. Since $\frac{di_2}{dt} \rightarrow 0$ at steady-state, the inductance then has no effect on the circuit. The current in R_1 is constant; the current in R_2 starts at zero and rises to $\mathcal{E}/R_2$.

(c) **IDENTIFY and SET UP:** The circuit now is as shown in Figure 30.75b.

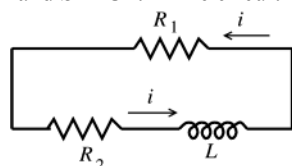


Figure 30.75b

Let $t = 0$ now be when S is opened.

$$\text{At } t = 0, i = \frac{\mathcal{E}}{R_2}.$$

Apply the loop rule to the single current loop.

EXECUTE: $-i(R_1 + R_2) - L \frac{di}{dt} = 0$. (Now $\frac{di}{dt}$ is negative.)

$$L \frac{di}{dt} = -i(R_1 + R_2) \text{ gives } \frac{di}{i} = -\left(\frac{R_1 + R_2}{L} \right) dt$$

Integrate from $t = 0$, when $i = I_0 = \mathcal{E}/R_2$, to t .

$$\int_{I_0}^i \frac{di}{i} = -\left(\frac{R_1 + R_2}{L} \right) \int_0^t dt \text{ and } \ln\left(\frac{i}{I_0} \right) = -\left(\frac{R_1 + R_2}{L} \right) t$$

Taking exponentials of both sides of this equation gives $i = I_0 e^{-(R_1 + R_2)t/L} = \frac{\mathcal{E}}{R_2} e^{-(R_1 + R_2)t/L}$

(d) IDENTIFY and SET UP: Use the equation derived in part (c) and solve for R_2 and $\mathcal{E}$.

EXECUTE: $L = 22.0 \text{ H}$

$$R_{R_1} = \frac{V^2}{P_{R_1}} = 40.0 \text{ W gives } R_1 = \frac{V^2}{P_{R_1}} = \frac{(120 \text{ V})^2}{40.0 \text{ W}} = 360 \Omega.$$

We are asked to find R_2 and $\mathcal{E}$. Use the expression derived in part (c).

$$I_0 = 0.600 \text{ A so } \mathcal{E}/R_2 = 0.600 \text{ A}$$

$$i = 0.150 \text{ A when } t = 0.080 \text{ s, so } i = \frac{\mathcal{E}}{R_2} e^{-(R_1+R_2)t/L} \text{ gives } 0.150 \text{ A} = (0.600 \text{ A}) e^{-(R_1+R_2)t/L}$$

$$\frac{1}{4} = e^{-(R_1+R_2)t/L} \text{ so } \ln 4 = (R_1+R_2)t/L$$

$$R_2 = \frac{L \ln 4}{t} - R_1 = \frac{(22.0 \text{ H}) \ln 4}{0.080 \text{ s}} - 360 \Omega = 381.2 \Omega - 360 \Omega = 21.2 \Omega$$

$$\text{Then } \mathcal{E} = (0.600 \text{ A}) R_2 = (0.600 \text{ A})(21.2 \Omega) = 12.7 \text{ V}.$$

(e) IDENTIFY and SET UP: Use the expressions derived in part (a).

$$\text{EXECUTE: The current through the light bulb before the switch is opened is } i_1 = \frac{\mathcal{E}}{R_1} = \frac{12.7 \text{ V}}{360 \Omega} = 0.0353 \text{ A}$$

EVALUATE: When the switch is opened the current through the light bulb jumps from 0.0353 A to 0.600 A. Since the electrical power dissipated in the bulb (brightness) depend on i^2 , the bulb suddenly becomes much brighter.

30.76. IDENTIFY: Follow the steps specified in the problem.

SET UP: The current in an inductor does not change abruptly.

EXECUTE: (a) Using Kirchhoff's loop rule on the left and right branches:

$$\text{Left: } \mathcal{E} - (i_1 + i_2)R - L \frac{di_1}{dt} = 0 \Rightarrow R(i_1 + i_2) + L \frac{di_1}{dt} = \mathcal{E}.$$

$$\text{Right: } \mathcal{E} - (i_1 + i_2)R - \frac{q_2}{C} = 0 \Rightarrow R(i_1 + i_2) + \frac{q_2}{C} = \mathcal{E}.$$

(b) Initially, with the switch just closed, $i_1 = 0$, $i_2 = \frac{\mathcal{E}}{R}$ and $q_2 = 0$.

(c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise but straightforward exercise. We will show that the initial conditions are satisfied:

$$\text{At } t = 0, q_2 = \frac{\mathcal{E}}{\omega R} e^{-\beta t} \sin(\omega t) = \frac{\mathcal{E}}{\omega R} \sin(0) = 0.$$

$$i_1(t) = \frac{\mathcal{E}}{R} (1 - e^{-\beta t} [(2\omega RC)^{-1} \sin(\omega t) + \cos(\omega t)]) \Rightarrow i_1(0) = \frac{\mathcal{E}}{R} (1 - [\cos(0)]) = 0.$$

$$\text{(d) When does } i_2 \text{ first equal zero? } \omega = \sqrt{\frac{1}{LC} - \frac{1}{(2RC)^2}} = 625 \text{ rad/s}.$$

$$i_2(t) = 0 = \frac{\mathcal{E}}{R} e^{-\beta t} [-(2\omega RC)^{-1} \sin(\omega t) + \cos(\omega t)] \Rightarrow -(2\omega RC)^{-1} \tan(\omega t) + 1 = 0 \text{ and}$$

$$\tan(\omega t) = +2\omega RC = +2(625 \text{ rad/s})(400 \Omega)(2.00 \times 10^{-6} \text{ F}) = +1.00.$$

$$\omega t = \arctan(+1.00) = +0.785 \Rightarrow t = \frac{0.785}{625 \text{ rad/s}} = 1.256 \times 10^{-3} \text{ s}.$$

EVALUATE: As $t \rightarrow \infty$, $i_1 \rightarrow \mathcal{E}/R$, $q_2 \rightarrow 0$ and $i_2 \rightarrow 0$.

30.77. IDENTIFY: Apply $L = \frac{N\Phi_B}{i}$ to calculate L .

SET UP: In the air the magnetic field is $B_{\text{Air}} = \frac{\mu_0 Ni}{W}$. In the liquid, $B_L = \frac{\mu Ni}{W}$

$$\text{EXECUTE: (a) } \Phi_B = BA = B_L A_L + B_{\text{Air}} A_{\text{Air}} = \frac{\mu_0 Ni}{W} ((D-d)W) + \frac{K\mu_0 Ni}{W} (dW) = \mu_0 Ni [(D-d) + Kd].$$

$$L = \frac{N\Phi_B}{i} = \mu_0 N^2 [(D-d) + Kd] = L_0 - L_0 \frac{d}{D} + L_f \frac{d}{D} = L_0 + \left(\frac{L_f - L_0}{D} \right) d.$$

$$d = \left(\frac{L - L_0}{L_f - L_0} \right) D, \text{ where } L_0 = \mu_0 N^2 D, \text{ and } L_f = K\mu_0 N^2 D.$$

(b) Using $K = \chi_m + 1$ we can find the inductance for any height $L = L_0 \left(1 + \chi_m \frac{d}{D} \right)$.

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d = D/4$	0.63024 H	0.63000 H
$d = D/2$	0.63048 H	0.62999 H
$d = 3D/4$	0.63072 H	0.62999 H
$d = D$	0.63096 H	0.62998 H

The values $\chi_m(\text{O}_2) = 1.52 \times 10^{-3}$ and $\chi_m(\text{Hg}) = -2.9 \times 10^{-5}$ have been used.

EVALUATE: (d) The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

30.78. IDENTIFY: The induced emf across the two coils is due to both the self-inductance of each and the mutual inductance of the pair of coils.

SET UP: The equivalent inductance is defined by $\mathcal{E} = L_{\text{eq}} \frac{di}{dt}$, where $\mathcal{E}$ and i are the total emf and current across the combination.

EXECUTE: Series: $L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} \equiv L_{\text{eq}} \frac{di}{dt}$.

But $i = i_1 + i_2 \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$ and $M_{12} = M_{21} \equiv M$, so $(L_1 + L_2 + 2M) \frac{di}{dt} = L_{\text{eq}} \frac{di}{dt}$ and $L_{\text{eq}} = L_1 + L_2 + 2M$.

Parallel: We have $L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} = L_{\text{eq}} \frac{di}{dt}$ and $L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt} = L_{\text{eq}} \frac{di}{dt}$, with $\frac{di_1}{dt} + \frac{di_2}{dt} = \frac{di}{dt}$ and $M_{12} = M_{21} \equiv M$.

To simplify the algebra let $A = \frac{di_1}{dt}$, $B = \frac{di_2}{dt}$, and $C = \frac{di}{dt}$. So $L_1 A + MB = L_{\text{eq}} C$, $L_2 B + MA = L_{\text{eq}} C$, $A + B = C$. Now solve for A and B in terms of C . $(L_1 - M)A + (M - L_2)B = 0$ using $A = C - B$. $(L_1 - M)(C - B) + (M - L_2)B = 0$.

$(L_1 - M)C - (L_1 - M)B + (M - L_2)B = 0$. $(2M - L_1 - L_2)B = (M - L_1)C$ and $B = \frac{(M - L_1)}{(2M - L_1 - L_2)} C$.

But $A = C - B = C - \frac{(M - L_1)C}{(2M - L_1 - L_2)} = \frac{(2M - L_1 - L_2) - M + L_1}{(2M - L_1 - L_2)} C$, or $A = \frac{M - L_2}{2M - L_1 - L_2} C$. Substitute A in B back

into original equation:

$\frac{L_1(M - L_2)C}{2M - L_1 - L_2} + \frac{M(M - L_1)}{(2M - L_1 - L_2)} C = L_{\text{eq}} C$ and $\frac{M^2 - L_1 L_2}{2M - L_1 - L_2} C = L_{\text{eq}} C$. Finally, $L_{\text{eq}} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$.

EVALUATE: If the flux of one coil doesn't pass through the other coil, so $M = 0$, then the results reduce to those of problem 30.47.

30.79. IDENTIFY: Apply Kirchhoff's loop rule to the top and bottom branches of the circuit.

SET UP: Just after the switch is closed the current through the inductor is zero and the charge on the capacitor is zero.

EXECUTE: $\mathcal{E} - i_1 R_1 - L \frac{di_1}{dt} = 0 \Rightarrow i_1 = \frac{\mathcal{E}}{R_1} (1 - e^{-(R_1/L)t})$. $\mathcal{E} - i_2 R_2 - \frac{q_2}{C} = 0 \Rightarrow -\frac{di_2}{dt} R_2 - \frac{i_2}{C} = 0 \Rightarrow i_2 = \frac{\mathcal{E}}{R_2} e^{-(1/R_2 C)t}$.

$q_2 = \int_0^t i_2 dt' = -\frac{\mathcal{E}}{R_2} R_2 C e^{-(1/R_2 C)t} \Big|_0^t = \mathcal{E} C (1 - e^{-(1/R_2 C)t})$.

(b) $i_1(0) = \frac{\mathcal{E}}{R_1} (1 - e^0) = 0$, $i_2 = \frac{\mathcal{E}}{R_2} e^0 = \frac{48.0 \text{ V}}{5000 \Omega} = 9.60 \times 10^{-3} \text{ A}$.

(c) As $t \rightarrow \infty$: $i_1(\infty) = \frac{\mathcal{E}}{R_1} (1 - e^{-\infty}) = \frac{\mathcal{E}}{R_1} = \frac{48.0 \text{ V}}{25.0 \Omega} = 1.92 \text{ A}$, $i_2 = \frac{\mathcal{E}}{R_2} e^{-\infty} = 0$. A good definition of a "long time" is

many time constants later.

(d) $i_1 = i_2 \Rightarrow \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = \frac{\mathcal{E}}{R_2}e^{-(1/R_2C)t} \Rightarrow (1 - e^{-(R_1/L)t}) = \frac{R_1}{R_2}e^{-(1/R_2C)t}$. Expanding the exponentials like

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$, we find: $\frac{R_1}{L}t - \frac{1}{2}\left(\frac{R_1}{L}\right)^2 t^2 + \dots = \frac{R_1}{R_2}\left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2} - \dots\right)$ and

$t\left(\frac{R_1}{L} + \frac{R_1}{R_2^2C}\right) + O(t^2) + \dots = \frac{R_1}{R_2}$, if we have assumed that $t \ll 1$. Therefore:

$$t \approx \frac{1}{R_2}\left(\frac{1}{(1/L) + (1/R_2^2C)}\right) = \left(\frac{LR_2C}{L + R_2^2C}\right) = \left(\frac{(8.0 \text{ H})(5000 \Omega)(2.0 \times 10^{-5} \text{ F})}{8.0 \text{ H} + (5000 \Omega)^2(2.0 \times 10^{-5} \text{ F})}\right) = 1.6 \times 10^{-3} \text{ s}.$$

(e) At $t = 1.57 \times 10^{-3} \text{ s}$: $i_1 = \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = \frac{48 \text{ V}}{25 \Omega}(1 - e^{-(25/8)t}) = 9.4 \times 10^{-3} \text{ A}$.

(f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$$i_{1/2} = 0.960 \text{ A} = i_1 = \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = (1.92 \text{ A})(1 - e^{-(R_1/L)t}). \quad e^{-(R_1/L)t} = 0.500 \Rightarrow t = \frac{L}{R_1} \ln(0.5) = \frac{8.0 \text{ H}}{25 \Omega} \ln(0.5) = 0.22 \text{ s}.$$

EVALUATE: i_1 is initially zero and rises to a final value of 1.92 A. i_2 is initially 9.60 mA and falls to zero, q_2 is initially zero and rises to $q_2 = \mathcal{E}C = 960 \mu\text{C}$.

ALTERNATING CURRENT

31.1. IDENTIFY: $i = I \cos \omega t$ and $I_{\text{rms}} = I/\sqrt{2}$.

SET UP: The specified value is the root-mean-square current; $I_{\text{rms}} = 0.34 \text{ A}$.

EXECUTE: (a) $I_{\text{rms}} = 0.34 \text{ A}$

(b) $I = \sqrt{2}I_{\text{rms}} = \sqrt{2}(0.34 \text{ A}) = 0.48 \text{ A}$.

(c) Since the current is positive half of the time and negative half of the time, its average value is zero.

(d) Since I_{rms} is the square root of the average of i^2 , the average square of the current is $I_{\text{rms}}^2 = (0.34 \text{ A})^2 = 0.12 \text{ A}^2$.

EVALUATE: The current amplitude is larger than its rms value.

31.2. IDENTIFY and SET UP: Apply Eqs.(31.3) and (31.4)

EXECUTE: (a) $I = \sqrt{2}I_{\text{rms}} = \sqrt{2}(2.10 \text{ A}) = 2.97 \text{ A}$.

(b) $I_{\text{rav}} = \frac{2}{\pi}I = \frac{2}{\pi}(2.97 \text{ A}) = 1.89 \text{ A}$.

EVALUATE: (c) The root-mean-square voltage is always greater than the rectified average, because squaring the current before averaging, and then taking the square root to get the root-mean-square value will always give a larger value than just averaging.

31.3. IDENTIFY and SET UP: Apply Eq.(31.5).

EXECUTE: (a) $V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{45.0 \text{ V}}{\sqrt{2}} = 31.8 \text{ V}$.

(b) Since the voltage is sinusoidal, the average is zero.

EVALUATE: The voltage amplitude is larger than V_{rms} .

31.4. IDENTIFY: $V = IX_C$ with $X_C = \frac{1}{\omega C}$.

SET UP: ω is the angular frequency, in rad/s.

EXECUTE: (a) $V = IX_C = \frac{I}{\omega C}$ so $I = V\omega C = (60.0 \text{ V})(100 \text{ rad/s})(2.20 \times 10^{-6} \text{ F}) = 0.0132 \text{ A}$.

(b) $I = V\omega C = (60.0 \text{ V})(1000 \text{ rad/s})(2.20 \times 10^{-6} \text{ F}) = 0.132 \text{ A}$.

(c) $I = V\omega C = (60.0 \text{ V})(10,000 \text{ rad/s})(2.20 \times 10^{-6} \text{ F}) = 1.32 \text{ A}$.

(d) The plot of $\log I$ versus $\log \omega$ is given in Figure 31.4.

EVALUATE: $I = \omega VC$ so $\log I = \log(VC) + \log \omega$. A graph of $\log I$ versus $\log \omega$ should be a straight line with slope +1, and that is what Figure 31.4 shows.

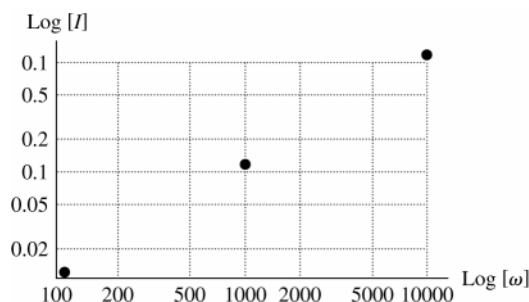


Figure 31.4

31.5. IDENTIFY: $V = IX_L$ with $X_L = \omega L$.

SET UP: ω is the angular frequency, in rad/s.

EXECUTE: (a) $V = IX_L = I\omega L$ and $I = \frac{V}{\omega L} = \frac{60.0 \text{ V}}{(100 \text{ rad/s})(5.00 \text{ H})} = 0.120 \text{ A}$.

(b) $I = \frac{V}{\omega L} = \frac{60.0 \text{ V}}{(1000 \text{ rad/s})(5.00 \text{ H})} = 0.0120 \text{ A}$.

(c) $I = \frac{V}{\omega L} = \frac{60.0 \text{ V}}{(10,000 \text{ rad/s})(5.00 \text{ H})} = 0.00120 \text{ A}$.

(d) The plot of $\log I$ versus $\log \omega$ is given in Figure 31.5.

EVALUATE: $I = \frac{V}{\omega L}$ so $\log I = \log(V/L) - \log \omega$. A graph of $\log I$ versus $\log \omega$ should be a straight line with slope -1 , and that is what Figure 31.5 shows.

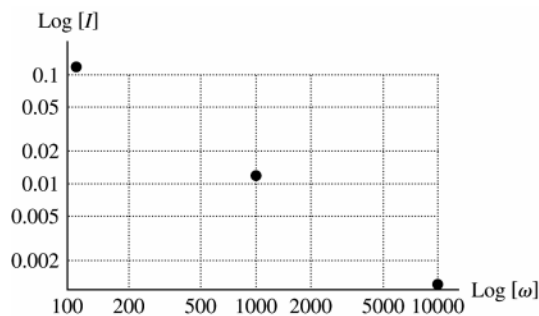


Figure 31.5

31.6. IDENTIFY: The reactance of capacitors and inductors depends on the angular frequency at which they are operated, as well as their capacitance or inductance.

SET UP: The reactances are $X_C = 1/\omega C$ and $X_L = \omega L$.

EXECUTE: (a) Equating the reactances gives $\omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

(b) Using the numerical values we get $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5.00 \text{ mH})(3.50 \mu\text{F})}} = 7560 \text{ rad/s}$

$$X_C = X_L = \omega L = (7560 \text{ rad/s})(5.00 \text{ mH}) = 37.8 \Omega$$

EVALUATE: At other angular frequencies, the two reactances could be very different.

31.7. IDENTIFY and SET UP: For a resistor $v_R = iR$. For an inductor, $v_L = V \cos(\omega t + 90^\circ)$. For a capacitor, $v_C = V \cos(\omega t - 90^\circ)$.

EXECUTE: The graphs are sketched in Figures 31.7a-c. The phasor diagrams are given in Figure 31.7d.

EVALUATE: For a resistor only in the circuit, the current and voltage in phase. For an inductor only, the voltage leads the current by 90° . For a capacitor only, the voltage lags the current by 90° .

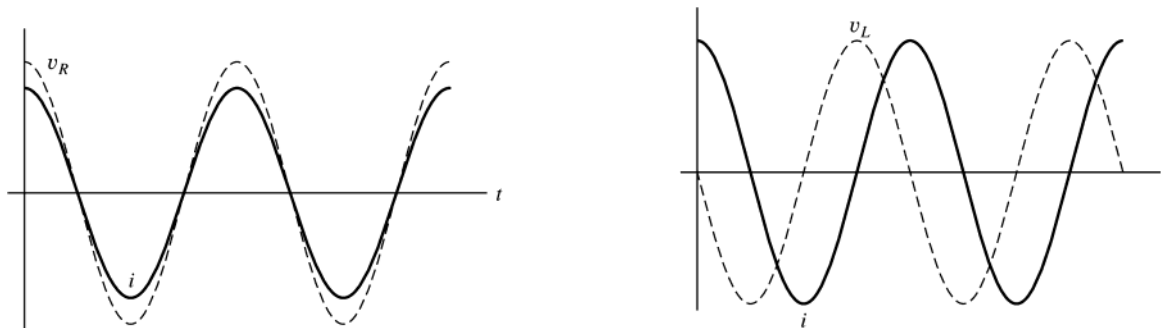


Figure 31.7a and b

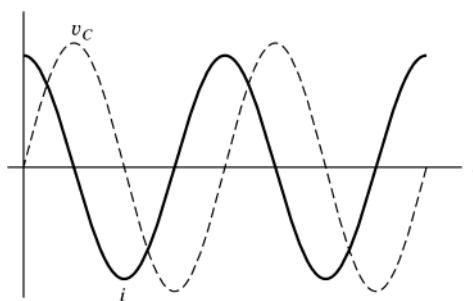


Figure 31.7c

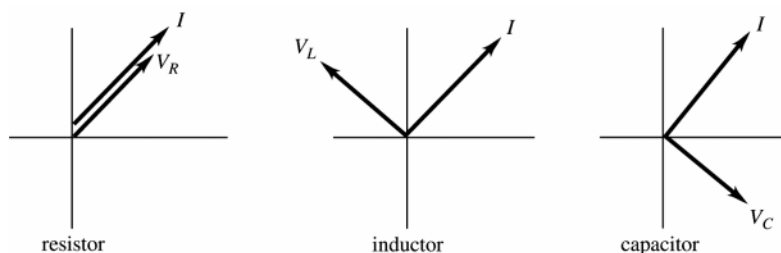


Figure 31.7d

31.8. IDENTIFY: The reactance of an inductor is $X_L = \omega L = 2\pi fL$. The reactance of a capacitor is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$.

SET UP: The frequency f is in Hz.

EXECUTE: (a) At 60.0 Hz, $X_L = 2\pi(60.0 \text{ Hz})(0.450 \text{ H}) = 170 \Omega$. X_L is proportional to f so at 600 Hz, $X_L = 1700 \Omega$.

(b) At 60.0 Hz, $X_C = \frac{1}{2\pi(60.0 \text{ Hz})(2.50 \times 10^{-6} \text{ F})} = 1.06 \times 10^3 \Omega$. X_C is proportional to $1/f$, so at 600 Hz, $X_C = 106 \Omega$.

(c) $X_L = X_C$ says $2\pi fL = \frac{1}{2\pi fC}$ and $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.450 \text{ H})(2.50 \times 10^{-6} \text{ F})}} = 150 \text{ Hz}$.

EVALUATE: X_L increases when f increases. X_C increases when f decreases.

31.9. IDENTIFY and SET UP: Use Eqs.(31.12) and (31.18).

EXECUTE: (a) $X_L = \omega L = 2\pi fL = 2\pi(80.0 \text{ Hz})(3.00 \text{ H}) = 1510 \Omega$

(b) $X_L = 2\pi fL$ gives $L = \frac{X_L}{2\pi f} = \frac{120 \Omega}{2\pi(80.0 \text{ Hz})} = 0.239 \text{ H}$

(c) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi(80.0 \text{ Hz})(4.00 \times 10^{-6} \text{ F})} = 497 \Omega$

(d) $X_C = \frac{1}{2\pi fC}$ gives $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(80.0 \text{ Hz})(120 \Omega)} = 1.66 \times 10^{-5} \text{ F}$

EVALUATE: X_L increases when L increases; X_C decreases when C increases.

31.10. IDENTIFY: $V_L = I\omega L$

SET UP: ω is the angular frequency, in rad/s. $f = \frac{\omega}{2\pi}$ is the frequency in Hz.

EXECUTE: $V_L = I\omega L$ so $f = \frac{V_L}{2\omega IL} = \frac{(12.0 \text{ V})}{2\pi(2.60 \times 10^{-3} \text{ A})(4.50 \times 10^{-4} \text{ H})} = 1.63 \times 10^6 \text{ Hz}$.

EVALUATE: When f is increased, I decreases.

31.11. IDENTIFY and SET UP: Apply Eqs.(31.18) and (31.19).

EXECUTE: $V = IX_C$ so $X_C = \frac{V}{I} = \frac{170 \text{ V}}{0.850 \text{ A}} = 200 \Omega$

$X_C = \frac{1}{\omega C}$ gives $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60.0 \text{ Hz})(200 \Omega)} = 1.33 \times 10^{-5} \text{ F} = 13.3 \mu\text{F}$

EVALUATE: The reactance relates the voltage amplitude to the current amplitude and is similar to Ohm's law.

- 31.12. IDENTIFY:** Compare v_C that is given in the problem to the general form $v_C = \frac{I}{\omega C} \sin \omega t$ and determine ω .

SET UP: $X_C = \frac{1}{\omega C}$, $v_R = iR$ and $i = I \cos \omega t$.

EXECUTE: (a) $X_C = \frac{1}{\omega C} = \frac{1}{(120 \text{ rad/s})(4.80 \times 10^{-6} \text{ F})} = 1736 \Omega$

(b) $I = \frac{V_C}{X_C} = \frac{7.60 \text{ V}}{1736 \Omega} = 4.378 \times 10^{-3} \text{ A}$ and $i = I \cos \omega t = (4.378 \times 10^{-3} \text{ A}) \cos[(120 \text{ rad/s})t]$. Then

$v_R = iR = (4.38 \times 10^{-3} \text{ A})(250 \Omega) \cos((120 \text{ rad/s})t) = (1.10 \text{ V}) \cos((120 \text{ rad/s})t)$.

EVALUATE: The voltage across the resistor has a different phase than the voltage across the capacitor.

- 31.13. IDENTIFY and SET UP:** The voltage and current for a resistor are related by $v_R = iR$. Deduce the frequency of the voltage and use this in Eq.(31.12) to calculate the inductive reactance. Eq.(31.10) gives the voltage across the inductor.

EXECUTE: (a) $v_R = (3.80 \text{ V}) \cos[(720 \text{ rad/s})t]$

$v_R = iR$, so $i = \frac{v_R}{R} = \left(\frac{3.80 \text{ V}}{150 \Omega} \right) \cos[(720 \text{ rad/s})t] = (0.0253 \text{ A}) \cos[(720 \text{ rad/s})t]$

(b) $X_L = \omega L$

$\omega = 720 \text{ rad/s}$, $L = 0.250 \text{ H}$, so $X_L = \omega L = (720 \text{ rad/s})(0.250 \text{ H}) = 180 \Omega$

(c) If $i = I \cos \omega t$ then $v_L = V_L \cos(\omega t + 90^\circ)$ (from Eq.31.10). $V_L = I\omega L = IX_L = (0.02533 \text{ A})(180 \Omega) = 4.56 \text{ V}$
 $v_L = (4.56 \text{ V}) \cos[(720 \text{ rad/s})t + 90^\circ]$

But $\cos(a + 90^\circ) = -\sin a$ (Appendix B), so $v_L = -(4.56 \text{ V}) \sin[(720 \text{ rad/s})t]$.

EVALUATE: The current is the same in the resistor and inductor and the voltages are 90° out of phase, with the voltage across the inductor leading.

- 31.14. IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no capacitor, $Z = \sqrt{R^2 + X_L^2}$ and $\tan \phi = \frac{X_L}{R}$. $X_L = \omega L$. $I = \frac{V}{Z}$. $V_L = IX_L$ and $V_R = IR$. For an inductor, the voltage leads the current.

EXECUTE: (a) $X_L = \omega L = (250 \text{ rad/s})(0.400 \text{ H}) = 100 \Omega$. $Z = \sqrt{(200 \Omega)^2 + (100 \Omega)^2} = 224 \Omega$.

(b) $I = \frac{V}{Z} = \frac{30.0 \text{ V}}{224 \Omega} = 0.134 \text{ A}$

(c) $V_R = IR = (0.134 \text{ A})(200 \Omega) = 26.8 \text{ V}$. $V_L = IX_L = (0.134 \text{ A})(100 \Omega) = 13.4 \text{ V}$.

(d) $\tan \phi = \frac{X_L}{R} = \frac{100 \Omega}{200 \Omega}$ and $\phi = +26.6^\circ$. Since ϕ is positive, the source voltage leads the current.

(e) The phasor diagram is sketched in Figure 31.14.

EVALUATE: Note that $V_R + V_L$ is greater than V . The loop rule is satisfied at each instance of time but the voltages across R and L reach their maxima at different times.

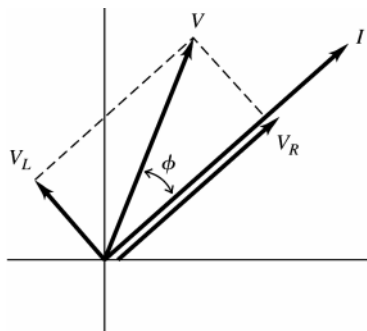


Figure 31.14

- 31.15. IDENTIFY:** $v_R(t)$ is given by Eq.(31.8). $v_L(t)$ is given by Eq.(31.10).

SET UP: From Exercise 31.14, $V = 30.0 \text{ V}$, $V_R = 26.8 \text{ V}$, $V_L = 13.4 \text{ V}$ and $\phi = 26.6^\circ$.

EXECUTE: (a) The graph is given in Figure 31.15.

(b) The different voltages are $v = (30.0 \text{ V}) \cos(250t + 26.6^\circ)$, $v_R = (26.8 \text{ V}) \cos(250t)$,

$v_L = (13.4 \text{ V}) \cos(250t + 90^\circ)$. At $t = 20 \text{ ms}$: $v = 20.5 \text{ V}$, $v_R = 7.60 \text{ V}$, $v_L = 12.85 \text{ V}$. Note that $v_R + v_L = v$.

(c) At $t = 40$ ms: $v = -15.2$ V, $v_R = -22.49$ V, $v_L = 7.29$ V. Note that $v_R + v_L = v$.

EVALUATE: It is important to be careful with radians versus degrees in above expressions!

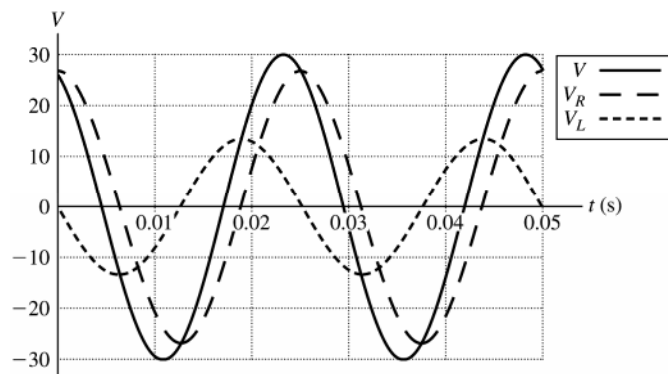


Figure 31.15

- 31.16. IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no resistor, $Z = \sqrt{(X_L - X_C)^2} = |X_L - X_C|$. $\tan \phi = \frac{X_L - X_C}{\text{zero}}$. $X_C = \frac{1}{\omega C}$. $X_L = \omega L$. For an inductor, the voltage leads the current. For a capacitor, the voltage lags the current.

EXECUTE: (a) $X_L = \omega L = (250 \text{ rad/s})(0.400 \text{ H}) = 100 \Omega$. $X_C = \frac{1}{\omega C} = \frac{1}{(250 \text{ rad/s})(6.00 \times 10^{-6} \text{ F})} = 667 \Omega$.

$$Z = |X_L - X_C| = |100 \Omega - 667 \Omega| = 567 \Omega.$$

$$(b) I = \frac{V}{Z} = \frac{30.0 \text{ V}}{567 \Omega} = 0.0529 \text{ A}$$

$$(c) V_C = IX_C = (0.0529 \text{ A})(667 \Omega) = 35.3 \text{ V}. V_L = IX_L = (0.0529 \text{ A})(100 \Omega) = 5.29 \text{ V}.$$

(d) $\tan \phi = \frac{X_L - X_C}{\text{zero}} = \frac{100 \Omega - 667 \Omega}{\text{zero}} = -\infty$ and $\phi = -90^\circ$. The phase angle is negative and the source voltage lags the current.

(e) The phasor diagram is sketched in Figure 31.16.

EVALUATE: When $X_C > X_L$ the phase angle is negative and the source voltage lags the current.

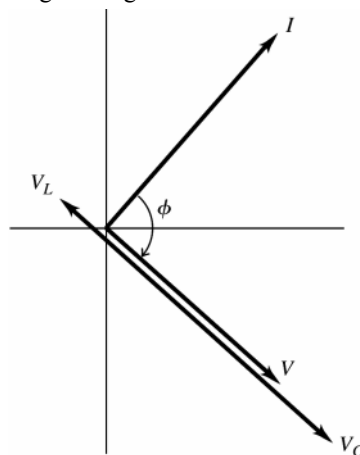


Figure 31.16

- 31.17. IDENTIFY and SET UP:** Calculate the impedance of the circuit and use Eq.(31.22) to find the current amplitude. The voltage amplitudes across each circuit element are given by Eqs.(31.7), (31.13), and (31.19). The phase angle is calculated using Eq.(31.24). The circuit is shown in Figure 31.17a.

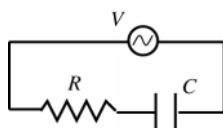


Figure 31.17a

No inductor means $X_L = 0$

$$R = 200 \Omega, C = 6.00 \times 10^{-6} \text{ F},$$

$$V = 30.0 \text{ V}, \omega = 250 \text{ rad/s}$$

EXECUTE: (a) $X_C = \frac{1}{\omega C} = \frac{1}{(250 \text{ rad/s})(6.00 \times 10^{-6} \text{ F})} = 666.7 \, \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200 \, \Omega)^2 + (666.7 \, \Omega)^2} = 696 \, \Omega$$

(b) $I = \frac{V}{Z} = \frac{30.0 \text{ V}}{696 \, \Omega} = 0.0431 \text{ A} = 43.1 \text{ mA}$

(c) Voltage amplitude across the resistor: $V_R = IR = (0.0431 \text{ A})(200 \, \Omega) = 8.62 \text{ V}$

Voltage amplitude across the capacitor: $V_C = IX_C = (0.0431 \text{ A})(666.7 \, \Omega) = 28.7 \text{ V}$

(d) $\tan \phi = \frac{X_L - X_C}{R} = \frac{0 - 666.7 \, \Omega}{200 \, \Omega} = -3.333$ so $\phi = -73.3^\circ$

The phase angle is negative, so the source voltage lags behind the current.

(e) The phasor diagram is sketched qualitatively in Figure 31.17b.

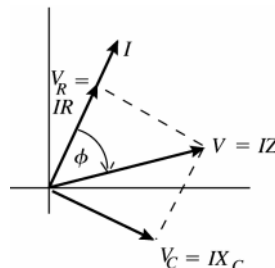


Figure 31.17b

EVALUATE: The voltage across the resistor is in phase with the current and the capacitor voltage lags the current by 90° . The presence of the capacitor causes the source voltage to lag behind the current. Note that $V_R + V_C > V$.

The instantaneous voltages in the circuit obey the loop rule at all times but because of the phase differences the voltage amplitudes do not.

31.18. IDENTIFY: $v_R(t)$ is given by Eq.(31.8). $v_C(t)$ is given by Eq.(31.16).

SET UP: From Exercise 31.17, $V = 30.0 \text{ V}$, $V_R = 8.62 \text{ V}$, $V_C = 28.7 \text{ V}$ and $\phi = -73.3^\circ$.

EXECUTE: (a) The graph is given in Figure 31.18.

(b) The different voltage are:

$$v = (30.0 \text{ V})\cos(250t - 73.3^\circ), \quad v_R = (8.62 \text{ V})\cos(250t), \quad v_C = (28.7 \text{ V})\cos(250t - 90^\circ). \text{ At } t = 20 \text{ ms:}$$

$$v = -25.1 \text{ V}, \quad v_R = 2.45 \text{ V}, \quad v_C = -27.5 \text{ V. Note that } v_R + v_C = v.$$

(c) At $t = 40 \text{ ms}$: $v = -22.9 \text{ V}$, $v_R = -7.23 \text{ V}$, $v_C = -15.6 \text{ V}$. Note that $v_R + v_C = v$.

EVALUATE: It is important to be careful with radians vs. degrees!

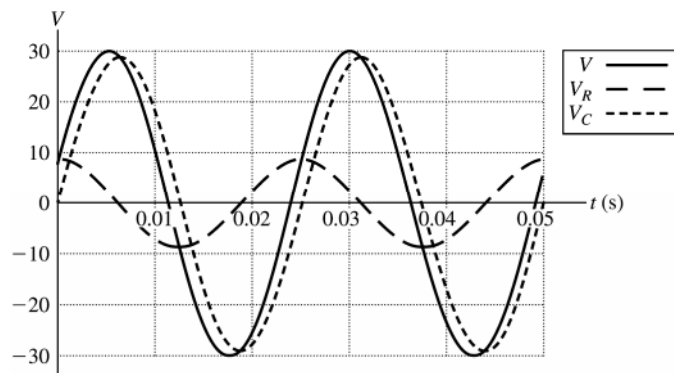


Figure 31.18

31.19. IDENTIFY: Apply the equations in Section 31.3.

SET UP: $\omega = 250 \text{ rad/s}$, $R = 200 \, \Omega$, $L = 0.400 \text{ H}$, $C = 6.00 \, \mu\text{F}$ and $V = 30.0 \text{ V}$.

EXECUTE: (a) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$$Z = \sqrt{(200 \, \Omega)^2 + ((250 \text{ rad/s})(0.400 \text{ H}) - 1/((250 \text{ rad/s})(6.00 \times 10^{-6} \text{ F})))^2} = 601 \, \Omega$$

(b) $I = \frac{V}{Z} = \frac{30 \text{ V}}{601 \, \Omega} = 0.0499 \text{ A}$.

(c) $\phi = \arctan\left(\frac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\frac{100 \, \Omega - 667 \, \Omega}{200 \, \Omega}\right) = -70.6^\circ$, and the voltage lags the current.

(d) $V_R = IR = (0.0499 \text{ A})(200 \Omega) = 9.98 \text{ V}$;

$V_L = I\omega L = (0.0499 \text{ A})(250 \text{ rad/s})(0.400 \text{ H}) = 4.99 \text{ V}$; $V_C = \frac{I}{\omega C} = \frac{(0.0499 \text{ A})}{(250 \text{ rad/s})(6.00 \times 10^{-6} \text{ F})} = 33.3 \text{ V}$.

EVALUATE: (e) At any instant, $v = v_R + v_C + v_L$. But v_C and v_L are 180° out of phase, so v_C can be larger than v at a value of t , if $v_L + v_R$ is negative at that t .

31.20. IDENTIFY: $v_R(t)$ is given by Eq.(31.8). $v_C(t)$ is given by Eq.(31.16). $v_L(t)$ is given by Eq.(31.10).

SET UP: From Exercise 31.19, $V = 30.0 \text{ V}$, $V_L = 4.99 \text{ V}$, $V_R = 9.98 \text{ V}$, $V_C = 33.3 \text{ V}$ and $\phi = -70.6^\circ$.

EXECUTE: (a) The graph is sketched in Figure 31.20. The different voltages plotted in the graph are: $v = (30 \text{ V})\cos(250t - 70.6^\circ)$, $v_R = (9.98 \text{ V})\cos(250t)$, $v_L = (4.99 \text{ V})\cos(250t + 90^\circ)$ and $v_C = (33.3 \text{ V})\cos(250t - 90^\circ)$.

(b) At $t = 20 \text{ ms}$: $v = -24.3 \text{ V}$, $v_R = 2.83 \text{ V}$, $v_L = 4.79 \text{ V}$, $v_C = -31.9 \text{ V}$.

(c) At $t = 40 \text{ ms}$: $v = -23.8 \text{ V}$, $v_R = -8.37 \text{ V}$, $v_L = 2.71 \text{ V}$, $v_C = -18.1 \text{ V}$.

EVALUATE: In both parts (b) and (c), note that the source voltage equals the sum of the other voltages at the given instant. Be careful with degrees versus radians!

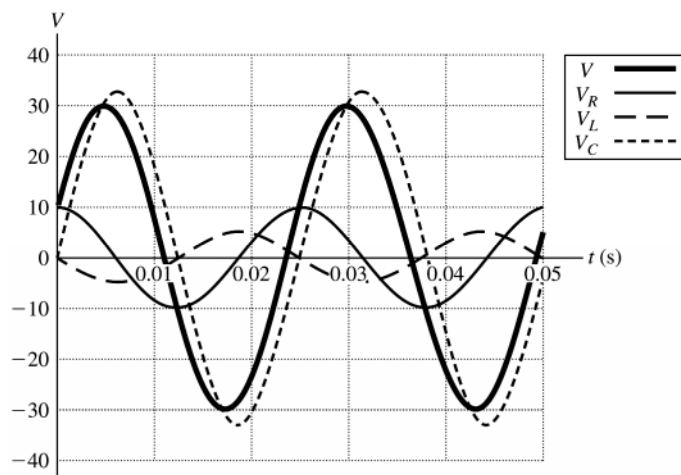


Figure 31.20

31.21. IDENTIFY and SET UP: The current is largest at the resonance frequency. At resonance, $X_L = X_C$ and $Z = R$. For part (b), calculate Z and use $I = V/Z$.

EXECUTE: (a) $f_0 = \frac{1}{2\pi\sqrt{LC}} = 113 \text{ Hz}$. $I = V/R = 15.0 \text{ mA}$.

(b) $X_C = 1/\omega C = 500 \Omega$. $X_L = \omega L = 160 \Omega$. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200 \Omega)^2 + (160 \Omega - 500 \Omega)^2} = 394.5 \Omega$.
 $I = V/Z = 7.61 \text{ mA}$. $X_C > X_L$ so the source voltage lags the current.

EVALUATE: $\omega_0 = 2\pi f_0 = 710 \text{ rad/s}$. $\omega = 400 \text{ rad/s}$ and is less than ω_0 . When $\omega < \omega_0$, $X_C > X_L$. Note that I in part (b) is less than I in part (a).

31.22. IDENTIFY: The impedance and individual reactances depend on the angular frequency at which the circuit is driven.

SET UP: The impedance is $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$, the current amplitude is $I = V/Z$, and the instantaneous values of the potential and current are $v = V \cos(\omega t + \phi)$, where $\tan \phi = (X_L - X_C)/R$, and $i = I \cos \omega t$.

EXECUTE: (a) Z is a minimum when $\omega L = \frac{1}{\omega C}$, which gives $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(8.00 \text{ mH})(12.5 \mu\text{F})}} = 3162 \text{ rad/s} =$

3162 rad/s and $Z = R = 175 \Omega$.

(b) $I = V/Z = (25.0 \text{ V})/(175 \Omega) = 0.143 \text{ A}$

(c) $i = I \cos \omega t = I/2$, so $\cos \omega t = \frac{1}{2}$, which gives $\omega t = 60^\circ = \pi/3 \text{ rad}$. $v = V \cos(\omega t + \phi)$, where $\tan \phi = (X_L - X_C)/R = 0/R = 0$. So, $v = (25.0 \text{ V}) \cos \omega t = (25.0 \text{ V})(1/2) = 12.5 \text{ V}$.

$v_R = Ri = (175 \Omega)(1/2)(0.143 \text{ A}) = 12.5 \text{ V}$.

$v_C = V_C \cos(\omega t - 90^\circ) = IX_C \cos(\omega t - 90^\circ) = \frac{0.143 \text{ A}}{(3162 \text{ rad/s})(12.5 \mu\text{F})} \cos(60^\circ - 90^\circ) = +3.13 \text{ V}$.

$$v_L = V_L \cos(\omega t + 90^\circ) = I X_L \cos(\omega t + 90^\circ) = (0.143 \text{ A})(3162 \text{ rad/s})(8.00 \text{ mH}) \cos(60^\circ + 90^\circ).$$

$$v_L = -3.13 \text{ V}.$$

$$(d) v_R + v_L + v_C = 12.5 \text{ V} + (-3.13 \text{ V}) + 3.13 \text{ V} = 12.5 \text{ V} = v_{\text{source}}$$

EVALUATE: The instantaneous potential differences across all the circuit elements always add up to the value of the source voltage at that instant. In this case (resonance), the potentials across the inductor and capacitor have the same magnitude but are 180° out of phase, so they add to zero, leaving all the potential difference across the resistor.

31.23. IDENTIFY and SET UP: Use the equation that precedes Eq.(31.20): $V^2 = V_R^2 + (V_L - V_C)^2$

$$\text{EXECUTE: } V = \sqrt{(30.0 \text{ V})^2 + (50.0 \text{ V} - 90.0 \text{ V})^2} = 50.0 \text{ V}$$

EVALUATE: The equation follows directly from the phasor diagrams of Fig.31.13 (b or c). Note that the voltage amplitudes do not simply add to give 170.0 V for the source voltage.

31.24. IDENTIFY and SET UP: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.

$$\text{EXECUTE: (a) If } \omega = \omega_0 = \frac{1}{\sqrt{LC}}, \text{ then } X = \omega L - \frac{1}{\omega C} \text{ and } X = \frac{L}{\sqrt{LC}} - \frac{1}{C/\sqrt{LC}} = 0.$$

(b) When $\omega > \omega_0$, $X > 0$

(c) When $\omega < \omega_0$, $X < 0$

(d) The graph of X versus ω is given in Figure 31.24.

$$\text{EVALUATE: } Z = \sqrt{R^2 + X^2} \text{ and } \tan \phi = X/R.$$

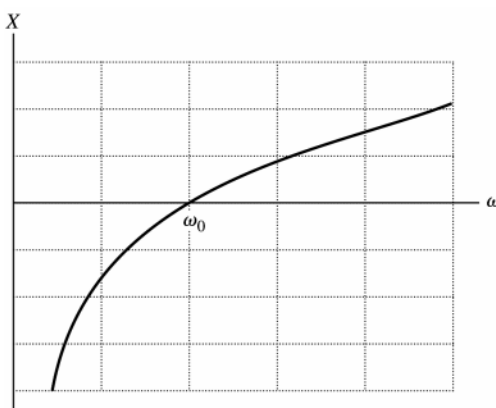


Figure 31.24

31.25. IDENTIFY: For a pure resistance, $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} = I_{\text{rms}}^2 R$.

SET UP: 20.0 W is the average power P_{av} .

EXECUTE: (a) The average power is one-half the maximum power, so the maximum instantaneous power is 40.0 W.

$$(b) I_{\text{rms}} = \frac{P_{\text{av}}}{V_{\text{rms}}} = \frac{20.0 \text{ W}}{120 \text{ V}} = 0.167 \text{ A}$$

$$(c) R = \frac{P_{\text{av}}}{I_{\text{rms}}^2} = \frac{20.0 \text{ W}}{(0.167 \text{ A})^2} = 720 \Omega$$

$$\text{EVALUATE: We can also calculate the average power as } P_{\text{av}} = \frac{V_{R,\text{rms}}^2}{R} = \frac{V_{\text{rms}}^2}{R} = \frac{(120 \text{ V})^2}{720 \Omega} = 20.0 \text{ W}.$$

31.26. IDENTIFY: The average power supplied by the source is $P = V_{\text{rms}} I_{\text{rms}} \cos \phi$. The power consumed in the resistance is $P = I_{\text{rms}}^2 R$.

$$\text{SET UP: } \omega = 2\pi f = 2\pi(1.25 \times 10^3 \text{ Hz}) = 7.854 \times 10^3 \text{ rad/s. } X_L = \omega L = 157 \Omega. X_C = \frac{1}{\omega C} = 909 \Omega.$$

EXECUTE: (a) First, let us find the phase angle between the voltage and the current:

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{157 \Omega - 909 \Omega}{350 \Omega} \text{ and } \phi = -65.04^\circ. \text{ The impedance of the circuit is}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(350 \Omega)^2 + (-752 \Omega)^2} = 830 \Omega. \text{ The average power provided by the generator is then}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\phi) = \frac{V_{\text{rms}}^2}{Z} \cos(\phi) = \frac{(120 \text{ V})^2}{830 \Omega} \cos(-65.04^\circ) = 7.32 \text{ W}$$

(b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\frac{120 \text{ V}}{830 \Omega}\right)^2 (350 \Omega) = 7.32 \text{ W}$.

EVALUATE: Conservation of energy requires that the answers to parts (a) and (b) are equal.

31.27. IDENTIFY: The power factor is $\cos \phi$, where ϕ is the phase angle in Fig. 31.13. The average power is given by Eq. (31.31). Use the result of part (a) to rewrite this expression.

(a) **SET UP:** The phasor diagram is sketched in Figure 31.27.

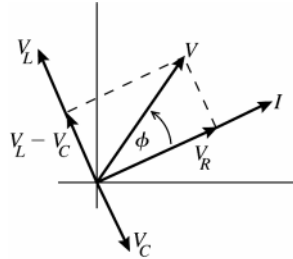


Figure 31.27

EXECUTE:

From the diagram

$$\cos \phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z},$$

as was to be shown.

(b) $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi = V_{\text{rms}} I_{\text{rms}} \left(\frac{R}{Z}\right) = \left(\frac{V_{\text{rms}}}{Z}\right) I_{\text{rms}} R$. But $\frac{V_{\text{rms}}}{Z} = I_{\text{rms}}$, so $P_{\text{av}} = I_{\text{rms}}^2 R$.

EVALUATE: In an L - R - C circuit, electrical energy is stored and released in the inductor and capacitor but none is dissipated in either of these circuit elements. The power delivered by the source equals the power dissipated in the resistor.

31.28. IDENTIFY and SET UP: $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. $\cos \phi = \frac{R}{Z}$.

EXECUTE: $I_{\text{rms}} = \frac{80.0 \text{ V}}{105 \Omega} = 0.762 \text{ A}$. $\cos \phi = \frac{75.0 \Omega}{105 \Omega} = 0.714$. $P_{\text{av}} = (80.0 \text{ V})(0.762 \text{ A})(0.714) = 43.5 \text{ W}$.

EVALUATE: Since the average power consumed by the inductor and by the capacitor is zero, we can also calculate the average power as $P_{\text{av}} = I_{\text{rms}}^2 R = (0.762 \text{ A})^2 (75.0 \Omega) = 43.5 \text{ W}$.

31.29. IDENTIFY and SET UP: Use the equations of Section 31.3 to calculate ϕ , Z and V_{rms} . The average power delivered by the source is given by Eq. (31.31) and the average power dissipated in the resistor is $I_{\text{rms}}^2 R$.

EXECUTE: (a) $X_L = \omega L = 2\pi f L = 2\pi(400 \text{ Hz})(0.120 \text{ H}) = 301.6 \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(400 \text{ Hz})(7.3 \times 10^{-6} \text{ Hz})} = 54.51 \Omega$$

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{301.6 \Omega - 54.51 \Omega}{240 \Omega}, \text{ so } \phi = +45.8^\circ. \text{ The power factor is } \cos \phi = +0.697.$$

$$(b) Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240 \Omega)^2 + (301.6 \Omega - 54.51 \Omega)^2} = 344 \Omega$$

$$(c) V_{\text{rms}} = I_{\text{rms}} Z = (0.450 \text{ A})(344 \Omega) = 155 \text{ V}$$

$$(d) P_{\text{av}} = I_{\text{rms}} V_{\text{rms}} \cos \phi = (0.450 \text{ A})(155 \text{ V})(0.697) = 48.6 \text{ W}$$

$$(e) P_{\text{av}} = I_{\text{rms}}^2 R = (0.450 \text{ A})^2 (240 \Omega) = 48.6 \text{ W}$$

EVALUATE: The average electrical power delivered by the source equals the average electrical power consumed in the resistor.

(f) All the energy stored in the capacitor during one cycle of the current is released back to the circuit in another part of the cycle. There is no net dissipation of energy in the capacitor.

(g) The answer is the same as for the capacitor. Energy is repeatedly being stored and released in the inductor, but no net energy is dissipated there.

31.30. IDENTIFY: The angular frequency and the capacitance can be used to calculate the reactance X_C of the capacitor. The angular frequency and the inductance can be used to calculate the reactance X_L of the inductor. Calculate the phase angle ϕ and then the power factor is $\cos \phi$. Calculate the impedance of the circuit and then the rms current in the circuit. The average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$. On the average no power is consumed in the capacitor or the inductor, it is all consumed in the resistor.

SET UP: The source has rms voltage $V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{45 \text{ V}}{\sqrt{2}} = 31.8 \text{ V}$.

EXECUTE: $X_L = \omega L = (360 \text{ rad/s})(15 \times 10^{-3} \text{ H}) = 5.4 \Omega$. $X_C = \frac{1}{\omega C} = \frac{1}{(360 \text{ rad/s})(3.5 \times 10^{-6} \text{ F})} = 794 \Omega$.

$\tan \phi = \frac{X_L - X_C}{R} = \frac{5.4 \Omega - 794 \Omega}{250 \Omega}$ and $\phi = -72.4^\circ$. The power factor is $\cos \phi = 0.302$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250 \Omega)^2 + (5.4 \Omega - 794 \Omega)^2} = 827 \Omega$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{31.8 \text{ V}}{827 \Omega} = 0.0385 \text{ A}$.

$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi = (31.8 \text{ V})(0.0385 \text{ A})(0.302) = 0.370 \text{ W}$.

(c) The average power delivered to the resistor is $P_{\text{av}} = I_{\text{rms}}^2 R = (0.0385 \text{ A})^2 (250 \Omega) = 0.370 \text{ W}$. The average power delivered to the capacitor and to the inductor is zero.

EVALUATE: On average the power delivered to the circuit equals the power consumed in the resistor. The capacitor and inductor store electrical energy during part of the current oscillation but each return the energy to the circuit during another part of the current cycle.

31.31. IDENTIFY and SET UP: At the resonance frequency, $Z = R$. Use that $V = IZ$, $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$. P_{av} is given by Eq.(31.31).

(a) **EXECUTE:** $V = IZ = IR = (0.500 \text{ A})(300 \Omega) = 150 \text{ V}$

(b) $V_R = IR = 150 \text{ V}$

$X_L = \omega L = L(1/\sqrt{LC}) = \sqrt{L/C} = 2582 \Omega$; $V_L = IX_L = 1290 \text{ V}$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582 \Omega$; $V_C = IX_C = 1290 \text{ V}$

(c) $P_{\text{av}} = \frac{1}{2} VI \cos \phi = \frac{1}{2} I^2 R$, since $V = IR$ and $\cos \phi = 1$ at resonance.

$P_{\text{av}} = \frac{1}{2} (0.500 \text{ A})^2 (300 \Omega) = 37.5 \text{ W}$

EVALUATE: At resonance $V_L = V_C$. Note that $V_L + V_C > V$. However, at any instant $v_L + v_C = 0$.

31.32. IDENTIFY: The current is maximum at the resonance frequency, so choose C such that $\omega = 50.0 \text{ rad/s}$ is the resonance frequency. At the resonance frequency $Z = R$.

SET UP: $V_L = I\omega L$

EXECUTE: (a) The amplitude of the current is given by $I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$. Thus, the current will have a

maximum amplitude when $\omega L = \frac{1}{\omega C}$. Therefore, $C = \frac{1}{\omega^2 L} = \frac{1}{(50.0 \text{ rad/s})^2 (9.00 \text{ H})} = 44.4 \mu\text{F}$.

(b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is

$I = \frac{V}{R} = \frac{120 \text{ V}}{400 \Omega} = 0.300 \text{ A}$. Thus, the amplitude of the voltage across the inductor is

$V_L = I(\omega L) = (0.300 \text{ A})(50.0 \text{ rad/s})(9.00 \text{ H}) = 135 \text{ V}$.

EVALUATE: Note that V_L is greater than the source voltage amplitude.

31.33. IDENTIFY and SET UP: At resonance $X_L = X_C$, $\phi = 0$ and $Z = R$. $R = 150 \Omega$, $L = 0.750 \text{ H}$, $C = 0.0180 \mu\text{F}$, $V = 150 \text{ V}$

EXECUTE: (a) At the resonance frequency $X_L = X_C$ and from $\tan \phi = \frac{X_L - X_C}{R}$ we have that $\phi = 0^\circ$ and the power factor is $\cos \phi = 1.00$.

(b) $P_{\text{av}} = \frac{1}{2} VI \cos \phi$ (Eq.31.31)

At the resonance frequency $Z = R$, so $I = \frac{V}{Z} = \frac{V}{R}$

$P_{\text{av}} = \frac{1}{2} V \left(\frac{V}{R} \right) \cos \phi = \frac{1}{2} \frac{V^2}{R} = \frac{1}{2} \frac{(150 \text{ V})^2}{150 \Omega} = 75.0 \text{ W}$

(c) **EVALUATE:** When C and f are changed but the circuit is kept on resonance, nothing changes in $P_{\text{av}} = V^2/(2R)$, so the average power is unchanged: $P_{\text{av}} = 75.0 \text{ W}$. The resonance frequency changes but since $Z = R$ at resonance the current doesn't change.

31.34. IDENTIFY: $\omega_0 = \frac{1}{\sqrt{LC}}$. $V_C = IX_C$. $V = IZ$.

SET UP: At resonance, $Z = R$.

EXECUTE: (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.350 \text{ H})(0.0120 \times 10^{-6} \text{ F})}} = 1.54 \times 10^4 \text{ rad/s}$

(b) $V = IZ = \left(\frac{V_C}{X_C} \right) Z = \left(\frac{V_C}{X_C} \right) R$. $X_C = \frac{1}{\omega C} = \frac{1}{(1.54 \times 10^4 \text{ rad/s})(0.0120 \times 10^{-6} \text{ F})} = 5.41 \times 10^3 \Omega$.

$V = \left(\frac{550 \text{ V}}{5.41 \times 10^3 \Omega} \right) (400 \Omega) = 40.7 \text{ V}$.

EVALUATE: The voltage amplitude for the capacitor is more than a factor of 10 times greater than the voltage amplitude of the source.

31.35. IDENTIFY and SET UP: The resonance angular frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$. $X_L = \omega L$. $X_C = \frac{1}{\omega C}$ and

$Z = \sqrt{R^2 + (X_L - X_C)^2}$. At the resonance frequency $X_L = X_C$ and $Z = R$.

EXECUTE: (a) $Z = R = 115 \Omega$

(b) $\omega_0 = \frac{1}{\sqrt{(4.50 \times 10^{-3} \text{ H})(1.26 \times 10^{-6} \text{ F})}} = 1.33 \times 10^4 \text{ rad/s}$. $\omega = 2\omega_0 = 2.66 \times 10^4 \text{ rad/s}$.

$X_L = \omega L = (2.66 \times 10^4 \text{ rad/s})(4.50 \times 10^{-3} \text{ H}) = 120 \Omega$. $X_C = \frac{1}{\omega C} = \frac{1}{(2.66 \times 10^4 \text{ rad/s})(1.25 \times 10^{-6} \text{ F})} = 30 \Omega$

$Z = \sqrt{(115 \Omega)^2 + (120 \Omega - 30 \Omega)^2} = 146 \Omega$

(c) $\omega = \omega_0/2 = 6.65 \times 10^3 \text{ rad/s}$. $X_L = 30 \Omega$. $X_C = \frac{1}{\omega C} = 120 \Omega$. $Z = \sqrt{(115 \Omega)^2 + (30 \Omega - 120 \Omega)^2} = 146 \Omega$, the same value as in part (b).

EVALUATE: For $\omega = 2\omega_0$, $X_L > X_C$. For $\omega = \omega_0/2$, $X_L < X_C$. But $(X_L - X_C)^2$ has the same value at these two frequencies, so Z is the same.

31.36. IDENTIFY: At resonance $Z = R$ and $X_L = X_C$.

SET UP: $\omega_0 = \frac{1}{\sqrt{LC}}$. $V = IZ$. $V_R = IR$, $V_L = IX_L$ and $V_C = V_L$.

EXECUTE: (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.280 \text{ H})(4.00 \times 10^{-6} \text{ F})}} = 945 \text{ rad/s}$.

(b) $I = 1.20 \text{ A}$ at resonance, so $R = Z = \frac{V}{I} = \frac{120 \text{ V}}{1.70 \text{ A}} = 70.6 \Omega$

(c) At resonance, $V_R = 120 \text{ V}$, $V_L = V_C = I\omega L = (1.70 \text{ A})(945 \text{ rad/s})(0.280 \text{ H}) = 450 \text{ V}$.

EVALUATE: At resonance, $V_R = V$ and $V_L - V_C = 0$.

31.37. IDENTIFY and SET UP: Eq.(31.35) relates the primary and secondary voltages to the number of turns in each. $I = V/R$ and the power consumed in the resistive load is $I_{\text{rms}}^2 = V_{\text{rms}}^2 / R$.

EXECUTE: (a) $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ so $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{120 \text{ V}}{12.0 \text{ V}} = 10$

(b) $I_2 = \frac{V_2}{R} = \frac{12.0 \text{ V}}{5.00 \Omega} = 2.40 \text{ A}$

(c) $P_{\text{av}} = I_2^2 R = (2.40 \text{ A})^2 (5.00 \Omega) = 28.8 \text{ W}$

(d) The power drawn from the line by the transformer is the 28.8 W that is delivered by the load.

$$P_{\text{av}} = \frac{V^2}{R} \text{ so } R = \frac{V^2}{P_{\text{av}}} = \frac{(120 \text{ V})^2}{28.8 \text{ W}} = 500 \Omega$$

And $\left(\frac{N_1}{N_2} \right)^2 (5.00 \Omega) = (10)^2 (5.00 \Omega) = 500 \Omega$, as was to be shown.

EVALUATE: The resistance is “transformed”. A load of resistance R connected to the secondary draws the same power as a resistance $(N_1/N_2)^2 R$ connected directly to the supply line, without using the transformer.

31.38. IDENTIFY: $P_{\text{av}} = VI$ and $P_{\text{av},1} = P_{\text{av},2}$. $\frac{N_1}{N_2} = \frac{V_1}{V_2}$.

SET UP: $V_1 = 120 \text{ V}$. $V_2 = 13,000 \text{ V}$.

EXECUTE: (a) $\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{13,000 \text{ V}}{120 \text{ V}} = 108$

(b) $P_{\text{av}} = V_2 I_2 = (13,000 \text{ V})(8.50 \times 10^{-3} \text{ A}) = 110 \text{ W}$

(c) $I_1 = \frac{P_{\text{av}}}{V_1} = \frac{110 \text{ W}}{120 \text{ V}} = 0.917 \text{ A}$

EVALUATE: Since the power supplied to the primary must equal the power delivered by the secondary, in a step-up transformer the current in the primary is greater than the current in the secondary.

31.39. IDENTIFY: A transformer transforms voltages according to $\frac{V_2}{V_1} = \frac{N_2}{N_1}$. The effective resistance of a secondary

circuit of resistance R is $R_{\text{eff}} = \frac{R}{(N_2/N_1)^2}$. Resistance R is related to P_{av} and V by $P_{\text{av}} = \frac{V^2}{R}$. Conservation of energy requires $P_{\text{av},1} = P_{\text{av},2}$ so $V_1 I_1 = V_2 I_2$.

SET UP: Let $V_1 = 240 \text{ V}$ and $V_2 = 120 \text{ V}$, so $P_{2,\text{av}} = 1600 \text{ W}$. These voltages are rms.

EXECUTE: (a) $V_1 = 240 \text{ V}$ and we want $V_2 = 120 \text{ V}$, so use a step-down transformer with $N_2/N_1 = \frac{1}{2}$.

(b) $P_{\text{av}} = VI$, so $I = \frac{P_{\text{av}}}{V} = \frac{1600 \text{ W}}{240 \text{ V}} = 6.67 \text{ A}$.

(c) The resistance R of the blower is $R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{1600 \text{ W}} = 9.00 \Omega$. The effective resistance of the blower is

$$R_{\text{eff}} = \frac{9.00 \Omega}{(1/2)^2} = 36.0 \Omega.$$

EVALUATE: $I_2 V_2 = (13.3 \text{ A})(120 \text{ V}) = 1600 \text{ W}$. Energy is provided to the primary at the same rate that it is consumed in the secondary. Step-down transformers step up resistance and the current in the primary is less than the current in the secondary.

31.40. IDENTIFY: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.

SET UP: The woofer has a R and L in series and the tweeter has a R and C in series.

EXECUTE: (a) $Z_{\text{tweeter}} = \sqrt{R^2 + (1/\omega C)^2}$

(b) $Z_{\text{woofer}} = \sqrt{R^2 + (\omega L)^2}$

(c) If $Z_{\text{tweeter}} = Z_{\text{woofer}}$, then the current splits evenly through each branch.

(d) At the crossover point, where currents are equal, $R^2 + (1/\omega C)^2 = R^2 + (\omega L)^2$. $\omega = \frac{1}{\sqrt{LC}}$ and

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

EVALUATE: The crossover frequency corresponds to the resonance frequency of a R - C - L circuit, since the crossover frequency is where $X_L = X_C$.

31.41. IDENTIFY and SET UP: Use Eq.(31.24) to relate L and R to ϕ . The voltage across the coil leads the current in it by 52.3° , so $\phi = +52.3^\circ$.

EXECUTE: $\tan \phi = \frac{X_L - X_C}{R}$. But there is no capacitance in the circuit so $X_C = 0$. Thus $\tan \phi = \frac{X_L}{R}$ and $X_L =$

$$R \tan \phi = (48.0 \Omega) \tan 52.3^\circ = 62.1 \Omega. X_L = \omega L = 2\pi f L \text{ so } L = \frac{X_L}{2\pi f} = \frac{62.1 \Omega}{2\pi(80.0 \text{ Hz})} = 0.124 \text{ H}.$$

EVALUATE: $\phi > 45^\circ$ when $(X_L - X_C) > R$, which is the case here.

31.42. IDENTIFY: $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. $V_{\text{rms}} = I_{\text{rms}} R$. $V_{C,\text{rms}} = I_{\text{rms}} X_C$. $V_{L,\text{rms}} = I_{\text{rms}} X_L$.

SET UP: $V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{30.0 \text{ V}}{\sqrt{2}} = 21.2 \text{ V}$.

EXECUTE: (a) $\omega = 200 \text{ rad/s}$, so $X_L = \omega L = (200 \text{ rad/s})(0.400 \text{ H}) = 80.0 \Omega$ and

$$X_C = \frac{1}{\omega C} = \frac{1}{(200 \text{ rad/s})(6.00 \times 10^{-6} \text{ F})} = 833 \Omega. \quad Z = \sqrt{(200 \Omega)^2 + (80.0 \Omega - 833 \Omega)^2} = 779 \Omega.$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{21.2 \text{ V}}{779 \Omega} = 0.0272 \text{ A}. \quad V_1 \text{ reads } V_{R,\text{rms}} = I_{\text{rms}} R = (0.0272 \text{ A})(200 \Omega) = 5.44 \text{ V}. \quad V_2 \text{ reads}$$

$$V_{L,\text{rms}} = I_{\text{rms}} X_L = (0.0272 \text{ A})(80.0 \Omega) = 2.18 \text{ V}. \quad V_3 \text{ reads } V_{C,\text{rms}} = I_{\text{rms}} X_C = (0.0272 \text{ A})(833 \Omega) = 22.7 \text{ V}. \quad V_4 \text{ reads}$$

$$\left| \frac{V_L - V_C}{\sqrt{2}} \right| = |V_{L,\text{rms}} - V_{C,\text{rms}}| = |2.18 \text{ V} - 22.7 \text{ V}| = 20.5 \text{ V}. \quad V_5 \text{ reads } V_{\text{rms}} = 21.2 \text{ V}.$$

(b) $\omega = 1000 \text{ rad/s}$ so $X_L = \omega L = (5)(80.0 \Omega) = 400 \Omega$ and $X_C = \frac{1}{\omega C} = \frac{833 \Omega}{5} = 167 \Omega$.

$$Z = \sqrt{(200 \Omega)^2 + (400 \Omega - 167 \Omega)^2} = 307 \Omega. \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{21.2 \text{ V}}{307 \Omega} = 0.0691 \text{ A}. \quad V_1 \text{ reads } V_{R,\text{rms}} = 13.8 \text{ V}. \quad V_2 \text{ reads}$$

$$V_{L,\text{rms}} = 27.6 \text{ V}. \quad V_3 \text{ reads } V_{C,\text{rms}} = 11.5 \text{ V}. \quad V_4 \text{ reads } |V_{L,\text{rms}} - V_{C,\text{rms}}| = |27.6 \text{ V} - 11.5 \text{ V}| = 16.1 \text{ V}. \quad V_5 \text{ reads}$$

$$V_{\text{rms}} = 21.2 \text{ V}.$$

EVALUATE: The resonance frequency for this circuit is $\omega_0 = \frac{1}{\sqrt{LC}} = 645 \text{ rad/s}$. 200 rad/s is less than the

resonance frequency and $X_C > X_L$. 1000 rad/s is greater than the resonance frequency and $X_L > X_C$.

31.43. IDENTIFY and SET UP: The rectified current equals the absolute value of the current i . Evaluate the integral as specified in the problem.

EXECUTE: (a) From Fig.31.3b, the rectified current is zero at the same values of t for which the sinusoidal current is zero. At these t , $\cos \omega t = 0$ and $\omega t = \pm \pi/2, \pm 3\pi/2, \dots$. The two smallest positive times are

$$t_1 = \pi/2\omega, \quad t_2 = 3\pi/2\omega.$$

$$(b) \quad A = \left| \int_{t_1}^{t_2} i dt \right| = - \int_{t_1}^{t_2} I \cos \omega t dt = -I \left[\frac{1}{\omega} \sin \omega t \right]_{t_1}^{t_2} = -\frac{I}{\omega} (\sin \omega t_2 - \sin \omega t_1)$$

$$\sin \omega t_1 = \sin[\omega(\pi/2\omega)] = \sin(\pi/2) = 1$$

$$\sin \omega t_2 = \sin[\omega(3\pi/2\omega)] = \sin(3\pi/2) = -1$$

$$A = \left(\frac{I}{\omega} \right) (1 - (-1)) = \frac{2I}{\omega}$$

$$(c) \quad I_{\text{rav}}(t_2 - t_1) = 2I/\omega$$

$$I_{\text{rav}} = \frac{2I}{\omega(t_2 - t_1)} = \frac{2I}{\omega(3\pi/2\omega - \pi/2\omega)} = \frac{2I}{\pi}, \text{ which is Eq.(31.3).}$$

EVALUATE: We have shown that Eq.(31.3) is correct. The average rectified current is less than the current amplitude I , since the rectified current varies between 0 and I . The average of the current is zero, since it has both positive and negative values.

31.44. IDENTIFY: $X_L = \omega L$. $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$

SET UP: $f = 120 \text{ Hz}$; $\omega = 2\pi f$.

EXECUTE: (a) $X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{250 \Omega}{2\pi(120 \text{ Hz})} = 0.332 \Omega$

(b) $Z = \sqrt{R^2 + X_L^2} = \sqrt{(400 \Omega)^2 + (250 \Omega)^2} = 472 \Omega$. $\cos \phi = \frac{R}{Z}$ and $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. $P_{\text{av}} = \frac{V_{\text{rms}}^2}{Z} \frac{R}{Z}$, so

$$V_{\text{rms}} = Z \sqrt{\frac{P_{\text{av}}}{R}} = (472 \Omega) \sqrt{\frac{800 \text{ W}}{400 \Omega}} = 668 \text{ V}.$$

EVALUATE: $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{668 \text{ V}}{472 \Omega} = 1.415 \text{ A}$. We can calculate P_{av} as $I_{\text{rms}}^2 R = (1.415 \text{ A})^2 (400 \Omega) = 800 \text{ W}$, which checks.

31.45. (a) IDENTIFY and SET UP: Source voltage lags current so it must be that $X_C > X_L$ and we must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional L needed to raise X_L to equal X_C .

(b) EXECUTE: power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more X_L is needed: $R = Z \cos\phi = (60.0 \Omega)(0.720) = 43.2 \Omega$

$$\tan\phi = \frac{X_L - X_C}{R} \text{ so } X_L - X_C = R \tan\phi$$

$\cos\phi = 0.720$ gives $\phi = -43.95^\circ$ (ϕ is negative since the voltage lags the current)

Then $X_L - X_C = R \tan\phi = (43.2 \Omega) \tan(-43.95^\circ) = -41.64 \Omega$.

Therefore need to add 41.64Ω of X_L .

$$X_L = \omega L = 2\pi f L \text{ and } L = \frac{X_L}{2\pi f} = \frac{41.64 \Omega}{2\pi(50.0 \text{ Hz})} = 0.133 \text{ H, amount of inductance to add.}$$

EVALUATE: From the information given we can't calculate the original value of L in the circuit, just how much to add. When this L is added the current in the circuit will increase.

31.46. IDENTIFY: Use $V_{\text{rms}} = I_{\text{rms}} Z$ to calculate Z and then find R . $P_{\text{av}} = I_{\text{rms}}^2 R$

SET UP: $X_C = 50.0 \Omega$

$$\text{EXECUTE: } Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{240 \text{ V}}{3.00 \text{ A}} = 80.0 \Omega = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (50.0 \Omega)^2}. \text{ Thus,}$$

$R = \sqrt{(80.0 \Omega)^2 - (50.0 \Omega)^2} = 62.4 \Omega$. The average power supplied to this circuit is equal to the power dissipated by the resistor, which is $P = I_{\text{rms}}^2 R = (3.00 \text{ A})^2 (62.4 \Omega) = 562 \text{ W}$.

$$\text{EVALUATE: } \tan\phi = \frac{X_L - X_C}{R} = \frac{-50.0 \Omega}{62.4 \Omega} \text{ and } \phi = -38.7^\circ.$$

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = (240 \text{ V})(3.00 \text{ A}) \cos(-38.7^\circ) = 562 \text{ W, which checks.}$$

31.47. IDENTIFY: The voltage and current amplitudes are the maximum values of these quantities, not necessarily the instantaneous values.

SET UP: The voltage amplitudes are $V_R = RI$, $V_L = X_L I$, and $V_C = X_C I$, where $I = V/Z$ and

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

EXECUTE: (a) $\omega = 2\pi f = 2\pi(1250 \text{ Hz}) = 7854 \text{ rad/s}$. Carrying extra figures in the calculator gives $X_L = \omega L = (7854 \text{ rad/s})(3.50 \text{ mH}) = 27.5 \Omega$; $X_C = 1/\omega C = 1/[(7854 \text{ rad/s})(10.0 \mu\text{F})] = 12.7 \Omega$;

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0 \Omega)^2 + (27.5 \Omega - 12.7 \Omega)^2} = 52.1 \Omega;$$

$$I = V/Z = (60.0 \text{ V})/(52.1 \Omega) = 1.15 \text{ A}; V_R = RI = (50.0 \Omega)(1.15 \text{ A}) = 57.5 \text{ V};$$

$$V_L = X_L I = (27.5 \Omega)(1.15 \text{ A}) = 31.6 \text{ V}; V_C = X_C I = (12.7 \Omega)(1.15 \text{ A}) = 14.7 \text{ V}.$$

The voltage amplitudes can add to more than 60.0 V because these voltages do not all occur at the same instant of time. At any instant, the instantaneous voltages all add to 60.0 V.

(b) All of them will change because they all depend on ω . $X_L = \omega L$ will double to 55.0Ω , and $X_C = 1/\omega C$ will decrease by half to 6.35Ω . Therefore $Z = \sqrt{(50.0 \Omega)^2 + (55.0 \Omega - 6.35 \Omega)^2} = 69.8 \Omega$; $I = V/Z = (60.0 \text{ V})/(69.8 \Omega) = 0.860 \text{ A}$; $V_R = IR = (0.860 \text{ A})(50.0 \Omega) = 43.0 \text{ V}$;

$$V_L = IX_L = (0.860 \text{ A})(55.0 \Omega) = 47.3 \text{ V}; V_C = IX_C = (0.860 \text{ A})(6.35 \Omega) = 5.47 \text{ V}.$$

EVALUATE: The new amplitudes in part (b) are not simple multiples of the values in part (a) because the impedance and reactances are not all the same simple multiple of the angular frequency.

31.48. IDENTIFY and SET UP: $X_C = \frac{1}{\omega C}$. $X_L = \omega L$.

$$\text{EXECUTE: (a)} \quad \frac{1}{\omega_1 C} = \omega_1 L \text{ and } LC = \frac{1}{\omega_1^2}. \text{ At angular frequency } \omega_2, \quad \frac{X_L}{X_C} = \frac{\omega_2 L}{1/\omega_2 C} = \omega_2^2 LC = (2\omega_1)^2 \frac{1}{\omega_1^2} = 4.$$

$$X_L > X_C.$$

$$\text{(b) At angular frequency } \omega_3, \quad \frac{X_L}{X_C} = \omega_3^2 LC = \left(\frac{\omega_1}{3}\right)^2 \left(\frac{1}{\omega_1^2}\right) = \frac{1}{9}. \quad X_C > X_L.$$

EVALUATE: When ω increases, X_L increases and X_C decreases. When ω decreases, X_L decreases and X_C increases.

(c) The resonance angular frequency ω_0 is the value of ω for which $X_C = X_L$, so $\omega_0 = \omega_1$.

31.49. IDENTIFY and SET UP: Express Z and I in terms of ω , L , C and R . The voltages across the resistor and the inductor are 90° out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

EXECUTE: The circuit is sketched in Figure 31.49.

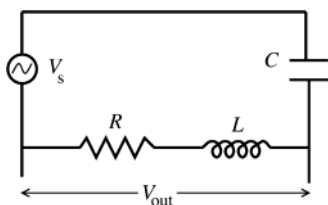


Figure 31.49

$$X_L = \omega L, X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_{\text{out}} = I \sqrt{R^2 + X_L^2} = I \sqrt{R^2 + \omega^2 L^2} = V_s \frac{\sqrt{R^2 + \omega^2 L^2}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

ω small

As ω gets small, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \rightarrow \frac{1}{\omega^2 C^2}$, $R^2 + \omega^2 L^2 \rightarrow R^2$

Therefore $\frac{V_{\text{out}}}{V_s} \rightarrow \sqrt{\frac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as ω becomes small.

ω large

As ω gets large, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \rightarrow R^2 + \omega^2 L^2 \rightarrow \omega^2 L^2$, $R^2 + \omega^2 L^2 \rightarrow \omega^2 L^2$

Therefore, $\frac{V_{\text{out}}}{V_s} \rightarrow \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as ω becomes large.

EVALUATE: $V_{\text{out}}/V_s \rightarrow 0$ as ω becomes small, so there is V_{out} only when the frequency ω of V_s is large. If the source voltage contains a number of frequency components, only the high frequency ones are passed by this filter.

31.50. IDENTIFY: $V = V_C = IX_C$. $I = V/Z$.

SET UP: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.

EXECUTE: $V_{\text{out}} = V_C = \frac{I}{\omega C} \Rightarrow \frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C \sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If ω is large: $\frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C \sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \frac{1}{\omega C \sqrt{(\omega L)^2}} = \frac{1}{(LC)\omega^2}$.

If ω is small: $\frac{V_{\text{out}}}{V_s} \approx \frac{1}{\omega C \sqrt{(1/\omega C)^2}} = \frac{\omega C}{\omega C} = 1$.

EVALUATE: When ω is large, X_C is small and X_L is large so Z is large and the current is small. Both factors in $V_C = IX_C$ are small. When ω is small, X_C is large and the voltage amplitude across the capacitor is much larger than the voltage amplitudes across the resistor and the inductor.

31.51. IDENTIFY: $I = V/Z$ and $P_{\text{av}} = \frac{1}{2} I^2 R$.

SET UP: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

EXECUTE: (a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

$$(b) P_{av} = \frac{1}{2} I^2 R = \frac{1}{2} \left(\frac{V}{Z} \right)^2 R = \frac{V^2 R / 2}{R^2 + (\omega L - 1/\omega C)^2}.$$

(c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \frac{1}{\omega_0 C}$, so $\omega_0 = \frac{1}{\sqrt{LC}}$.

$$(d) P_{av} = \frac{(100 \text{ V})^2 (200 \Omega) / 2}{(200 \Omega)^2 + (\omega(2.00 \text{ H}) - 1/[\omega(0.500 \times 10^{-6} \text{ F})])^2} = \frac{25\omega^2}{40,000\omega^2 + (2\omega^2 - 2,000,000)^2}.$$

The graph of P_{av} versus ω is sketched in Figure 31.51.

EVALUATE: Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light purple curve in Figure 31.19 in the textbook.

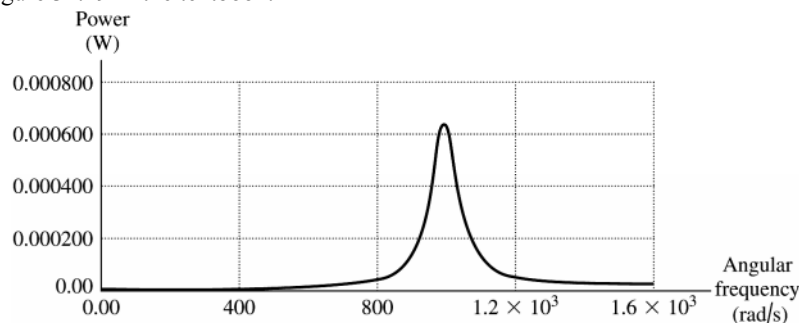


Figure 31.51

31.52. **IDENTIFY:** $V_L = I\omega L$ and $V_C = \frac{I}{\omega C}$.

SET UP: Problem 31.51 shows that $I = \frac{V}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

EXECUTE: (a) $V_L = I\omega L = \frac{V\omega L}{Z} = \frac{V\omega L}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

(b) $V_C = \frac{I}{\omega C} = \frac{I}{\omega C Z} = \frac{1}{\omega C \sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

(c) The graphs are given in Figure 31.52.

EVALUATE: (d) When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \text{ rad/s}$, the two voltages are equal, and are a maximum, 1000 V.

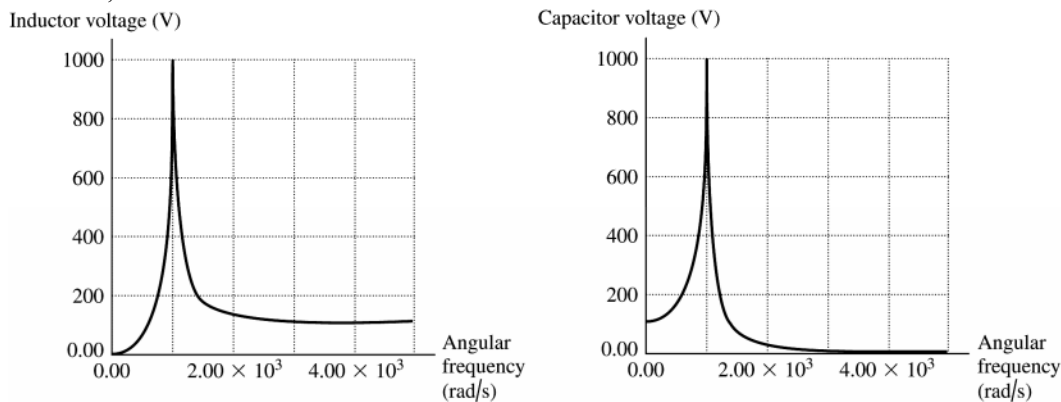


Figure 31.52

31.53. IDENTIFY: $U_B = \frac{1}{2}Li^2$. $U_E = \frac{1}{2}Cv^2$.

SET UP: Let $\langle x \rangle$ denote the average value of the quantity x . $\langle i^2 \rangle = \frac{1}{2}I^2$ and $\langle v_C^2 \rangle = \frac{1}{2}V_C^2$. Problem 31.51 shows that $I = \frac{V}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$. Problem 31.52 shows that $V_C = \frac{V}{\omega C \sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

EXECUTE: (a) $U_B = \frac{1}{2}Li^2 \Rightarrow \langle U_B \rangle = \frac{1}{2}L\langle i^2 \rangle = \frac{1}{2}LI_{\text{rms}}^2 = \frac{1}{2}L\left(\frac{I}{\sqrt{2}}\right)^2 = \frac{1}{4}LI^2$.

$$U_E = \frac{1}{2}Cv_C^2 \Rightarrow \langle U_E \rangle = \frac{1}{2}C\langle v_C^2 \rangle = \frac{1}{2}CV_{C,\text{rms}}^2 = \frac{1}{2}C\left(\frac{V_C}{\sqrt{2}}\right)^2 = \frac{1}{4}CV_C^2$$

(b) Using Problem 31.51a

$$\langle U_B \rangle = \frac{1}{4}LI^2 = \frac{1}{4}L\left(\frac{V^2}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)^2 = \frac{LV^2}{4(R^2 + (\omega L - 1/\omega C)^2)}$$

$$\text{Using Problem (31.47b): } \langle U_E \rangle = \frac{1}{4}CV_C^2 = \frac{1}{4}C\frac{V^2}{\omega^2 C^2(R^2 + (\omega L - 1/\omega C)^2)} = \frac{V^2}{4\omega^2 C(R^2 + (\omega L - 1/\omega C)^2)}$$

(c) The graphs of the magnetic and electric energies are given in Figure 31.53.

EVALUATE: (d) When the angular frequency is zero, the magnetic energy stored in the inductor is zero, while the electric energy in the capacitor is $U_E = CV^2/4$. As the frequency goes to infinity, the energy noted in both

inductor and capacitor go to zero. The energies equal each other at the resonant frequency where $\omega_0 = \frac{1}{\sqrt{LC}}$ and

$$U_B = U_E = \frac{LV^2}{4R^2}$$

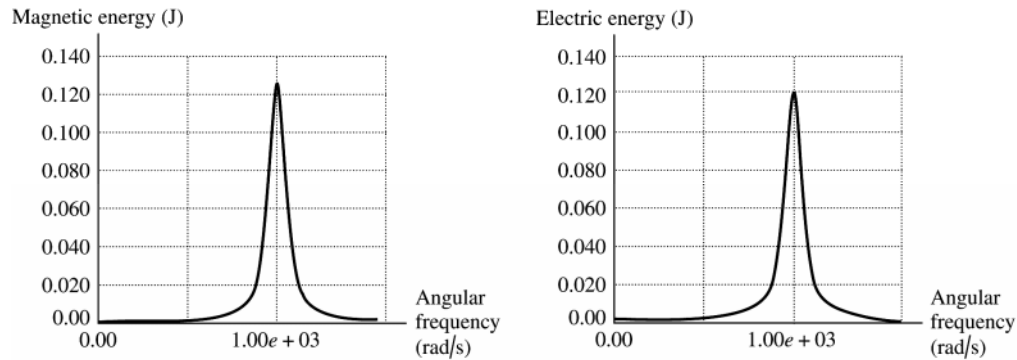


Figure 31.53

31.54. IDENTIFY: At any instant of time the same rules apply to the parallel ac circuit as to parallel dc circuit: the voltages are the same and the currents add.

SET UP: For a resistor the current and voltage in phase. For an inductor the voltage leads the current by 90° and for a capacitor the voltage lags the current by 90° .

EXECUTE: (a) The parallel L - R - C circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

(b) $i_R = \frac{v}{R}$ is always in phase with the voltage. $i_L = \frac{v}{\omega L}$ lags the voltage by 90° , and $i_C = v\omega C$ leads the voltage by 90° . The phase diagram is sketched in Figure 31.54.

(c) From the diagram, $I^2 = I_R^2 + (I_C - I_L)^2 = \left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2$.

(d) From part (c): $I = V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$. But $I = \frac{V}{Z}$, so $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$.

EVALUATE: For large ω , $Z \rightarrow \frac{1}{\omega C}$. The current in the capacitor branch is much larger than the current in the other branches. For small ω , $Z \rightarrow \omega L$. The current in the inductive branch is much larger than the current in the other branches.

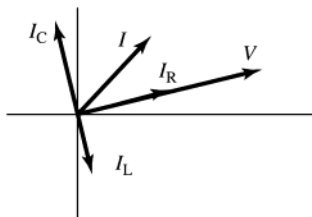


Figure 31.54

31.55. IDENTIFY: Apply the expression for I from problem 31.54 when $\omega_0 = 1/\sqrt{LC}$.

SET UP: From Problem 31.54, $I = V \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$

EXECUTE: (a) At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow I_C = V \omega_0 C = \frac{V}{\omega_0 L} = I_L$ so $I = I_R$ and I is a minimum.

(b) $P_{av} = \frac{V_{rms}^2}{Z} \cos \phi = \frac{V^2}{R}$ at resonance where $R < Z$ so power is a maximum.

(c) At $\omega = \omega_0$, I and V are in phase, so the phase angle is zero, which is the same as a series resonance.

EVALUATE: (d) The parallel circuit is sketched in Figure 31.55. At resonance, $|i_C| = |i_L|$ and at any instant of time these two currents are in opposite directions. Therefore, the net current between a and b is always zero.

(e) If the inductor and capacitor each have some resistance, and these resistances aren't the same, then it is no longer true that $i_C + i_L = 0$ and the statement in part (d) isn't valid.

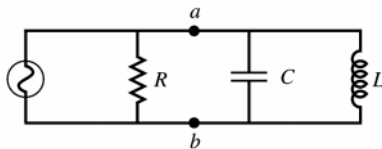


Figure 31.55

31.56. IDENTIFY: Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

SET UP: $V = \sqrt{2}V_{rms} = 311 \text{ V}$

EXECUTE: (a) $I_R = \frac{V}{R} = \frac{311 \text{ V}}{400 \Omega} = 0.778 \text{ A}$.

(b) $I_C = V\omega C = (311 \text{ V})(360 \text{ rad/s})(6.00 \times 10^{-6} \text{ F}) = 0.672 \text{ A}$.

(c) $\phi = \arctan\left(\frac{I_C}{I_R}\right) = \arctan\left(\frac{0.672 \text{ A}}{0.778 \text{ A}}\right) = 40.8^\circ$.

(d) $I = \sqrt{I_R^2 + I_C^2} = \sqrt{(0.778 \text{ A})^2 + (0.672 \text{ A})^2} = 1.03 \text{ A}$.

(e) Leads since $\phi > 0$.

EVALUATE: The phasor diagram shows that the current in the capacitor always leads the source voltage.

31.57. IDENTIFY and SET UP: Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

EXECUTE: (a) $I_R = \frac{V}{R}$; $I_C = V\omega C$; $I_L = \frac{V}{\omega L}$.

(b) The graph of each current versus ω is given in Figure 31.57a.

(c) $\omega \rightarrow 0$: $I_C \rightarrow 0$; $I_L \rightarrow \infty$. $\omega \rightarrow \infty$: $I_C \rightarrow \infty$; $I_L \rightarrow 0$.

At low frequencies, the current is not changing much so the inductor's back-emf doesn't "resist." This allows the current to pass fairly freely. However, the current in the capacitor goes to zero because it tends to "fill up" over the slow period, making it less effective at passing charge. At high frequency, the induced emf in the inductor resists the violent changes and passes little current. The capacitor never gets a chance to fill up so passes charge freely.

(d) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.0 \text{ H})(0.50 \times 10^{-6} \text{ F})}} = 1000 \text{ rad/sec}$ and $f = 159 \text{ Hz}$. The phasor diagram is sketched in

Figure 31.57b.

$$(e) I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2}.$$

$$I = \sqrt{\left(\frac{100 \text{ V}}{200 \Omega}\right)^2 + \left((100 \text{ V})(1000 \text{ s}^{-1})(0.50 \times 10^{-6} \text{ F}) - \frac{100 \text{ V}}{(1000 \text{ s}^{-1})(2.0 \text{ H})}\right)^2} = 0.50 \text{ A}$$

(f) At resonance $I_L = I_C = V\omega C = (100 \text{ V})(1000 \text{ s}^{-1})(0.50 \times 10^{-6} \text{ F}) = 0.0500 \text{ A}$ and $I_R = \frac{V}{R} = \frac{100 \text{ V}}{200 \Omega} = 0.50 \text{ A}$.

EVALUATE: At resonance $i_C = i_L = 0$ at all times and the current through the source equals the current through the resistor.

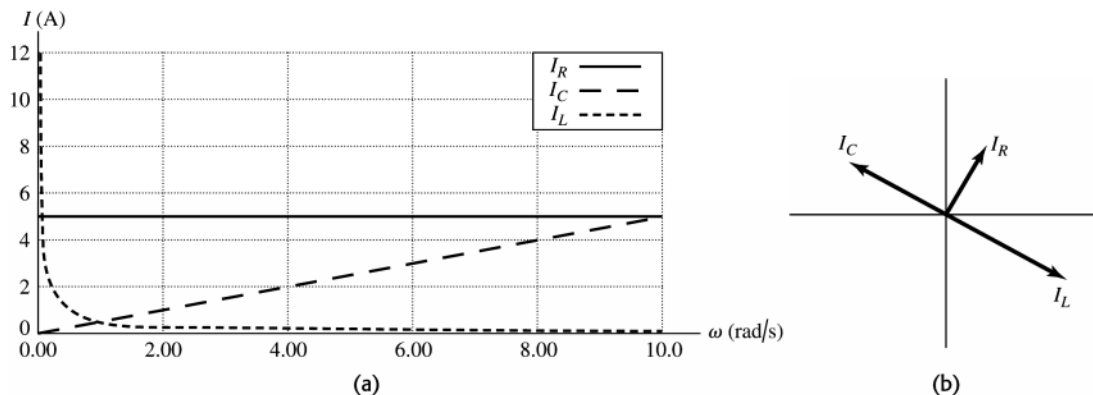


Figure 31.57

31.58. IDENTIFY: The average power depends on the phase angle ϕ .

SET UP: The average power is $P_{av} = V_{rms}I_{rms}\cos\phi$, and the impedance is $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$.

EXECUTE: (a) $P_{av} = V_{rms}I_{rms}\cos\phi = \frac{1}{2}(V_{rms}I_{rms})$, which gives $\cos\phi = \frac{1}{2}$, so $\phi = \pi/3 = 60^\circ$. $\tan\phi = (X_L - X_C)/R$, which gives $\tan 60^\circ = (\omega L - 1/\omega C)/R$. Using $R = 75.0 \Omega$, $L = 5.00 \text{ mH}$, and $C = 2.50 \mu\text{F}$ and solving for ω we get $\omega = 28760 \text{ rad/s} = 28,800 \text{ rad/s}$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L = (28,760 \text{ rad/s})(5.00 \text{ mH}) = 144 \Omega$ and

$X_C = 1/\omega C = 1/[(28,760 \text{ rad/s})(2.50 \mu\text{F})] = 13.9 \Omega$, giving $Z = \sqrt{(75 \Omega)^2 + (144 \Omega - 13.9 \Omega)^2} = 150 \Omega$;

$I = V/Z = (15.0 \text{ V})/(150 \Omega) = 0.100 \text{ A}$ and $P_{av} = \frac{1}{2}VI\cos\phi = \frac{1}{2}(15.0 \text{ V})(0.100 \text{ A})(1/2) = 0.375 \text{ W}$.

EVALUATE: All this power is dissipated in the resistor because the average power delivered to the inductor and capacitor is zero.

31.59. IDENTIFY: We know R , X_C and ϕ so Eq.(31.24) tells us X_L . Use $P_{av} = I_{rms}^2 R$ from Exercise 31.27 to calculate I_{rms} . Then calculate Z and use Eq.(31.26) to calculate V_{rms} for the source.

SET UP: Source voltage lags current so $\phi = -54.0^\circ$. $X_C = 350 \Omega$, $R = 180 \Omega$, $P_{av} = 140 \text{ W}$

EXECUTE: (a) $\tan\phi = \frac{X_L - X_C}{R}$

$X_L = R\tan\phi + X_C = (180 \Omega)\tan(-54.0^\circ) + 350 \Omega = -248 \Omega + 350 \Omega = 102 \Omega$

(b) $P_{av} = V_{rms}I_{rms}\cos\phi = I_{rms}^2 R$ (Exercise 31.27). $I_{rms} = \sqrt{\frac{P_{av}}{R}} = \sqrt{\frac{140 \text{ W}}{180 \Omega}} = 0.882 \text{ A}$

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180 \Omega)^2 + (102 \Omega - 350 \Omega)^2} = 306 \Omega$

$V_{rms} = I_{rms}Z = (0.882 \text{ A})(306 \Omega) = 270 \text{ V}$.

EVALUATE: We could also use Eq.(31.31): $P_{av} = V_{rms}I_{rms}\cos\phi$

$V_{rms} = \frac{P_{av}}{I_{rms}\cos\phi} = \frac{140 \text{ W}}{(0.882 \text{ A})\cos(-54.0^\circ)} = 270 \text{ V}$, which agrees. The source voltage lags the current when

$X_C > X_L$, and this agrees with what we found.

31.60. IDENTIFY and SET UP: Calculate Z and $I = V/Z$.

EXECUTE: (a) For $\omega = 800$ rad/s:

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{(500 \Omega)^2 + ((800 \text{ rad/s})(2.0 \text{ H}) - 1/((800 \text{ rad/s})(5.0 \times 10^{-7} \text{ F})))^2}. \quad Z = 1030 \Omega.$$

$$I = \frac{V}{Z} = \frac{100 \text{ V}}{1030 \Omega} = 0.0971 \text{ A}. \quad V_R = IR = (0.0971 \text{ A})(500 \Omega) = 48.6 \text{ V}, \quad V_C = \frac{1}{\omega C} = \frac{0.0971 \text{ A}}{(800 \text{ rad/s})(5.0 \times 10^{-7} \text{ F})} = 243 \text{ V}$$

and $V_L = I\omega L = (0.0971 \text{ A})(800 \text{ rad/s})(2.00 \text{ H}) = 155 \text{ V}$. $\phi = \arctan\left(\frac{\omega L - 1/(\omega C)}{R}\right) = -60.9^\circ$. The graph of each

voltage versus time is given in Figure 31.60a.

(b) Repeating exactly the same calculations as above for $\omega = 1000$ rad/s:

$Z = R = 500 \Omega$; $\phi = 0$; $I = 0.200 \text{ A}$; $V_R = V = 100 \text{ V}$; $V_C = V_L = 400 \text{ V}$. The graph of each voltage versus time is given in Figure 31.60b.

(c) Repeating exactly the same calculations as part (a) for $\omega = 1250$ rad/s:

$Z = R = 1030 \Omega$; $\phi = +60.9^\circ$; $I = 0.0971 \text{ A}$; $V_R = 48.6 \text{ V}$; $V_C = 155 \text{ V}$; $V_L = 243 \text{ V}$. The graph of each voltage versus time is given in Figure 31.60c.

EVALUATE: The resonance frequency is $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.00 \text{ H})(0.500 \mu\text{F})}} = 1000 \text{ rad/s}$. For $\omega < \omega_0$ the phase angle is negative and for $\omega > \omega_0$ the phase angle is positive.

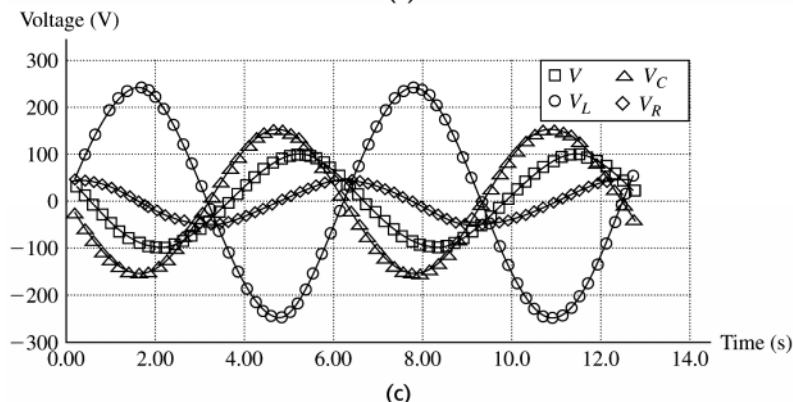
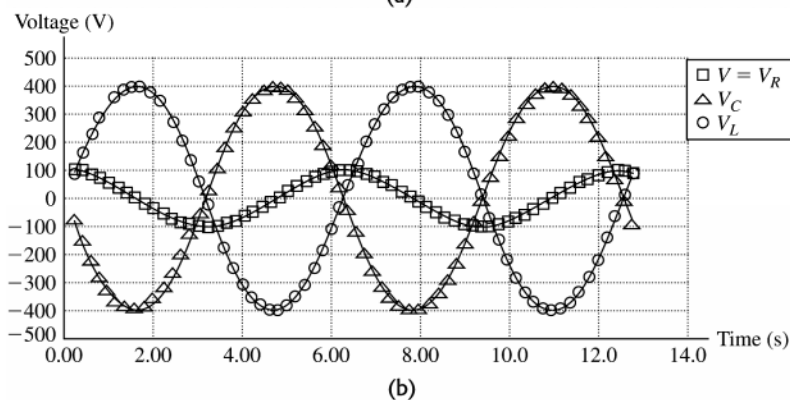
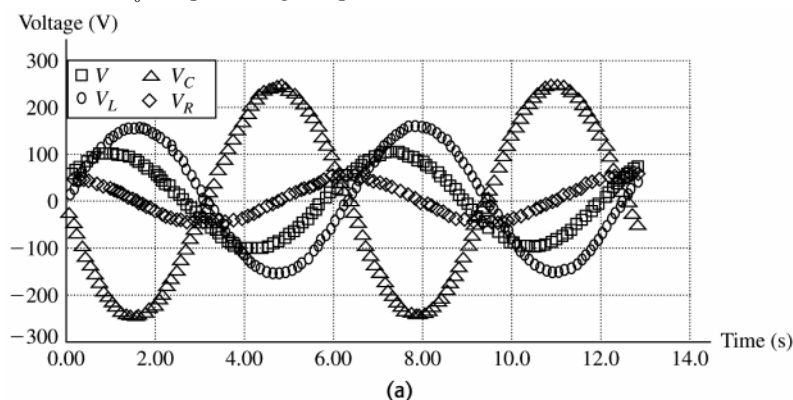


Figure 31.60

31.61. IDENTIFY and SET UP: Eq.(31.19) allows us to calculate I and then Eq.(31.22) gives Z . Solve Eq.(31.21) for L .

EXECUTE: (a) $V_C = IX_C$ so $I = \frac{V_C}{X_C} = \frac{360 \text{ V}}{480 \Omega} = 0.750 \text{ A}$

(b) $V = IZ$ so $Z = \frac{V}{I} = \frac{120 \text{ V}}{0.750 \text{ A}} = 160 \Omega$

(c) $Z^2 = R^2 + (X_L - X_C)^2$

$X_L - X_C = \pm \sqrt{Z^2 - R^2}$, so

$X_L = X_C \pm \sqrt{Z^2 - R^2} = 480 \Omega \pm \sqrt{(160 \Omega)^2 - (80.0 \Omega)^2} = 480 \Omega \pm 139 \Omega$

$X_L = 619 \Omega$ or 341Ω

(d) **EVALUATE:** $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$. At resonance, $X_C = X_L$. As the frequency is lowered below the

resonance frequency X_C increases and X_L decreases. Therefore, for $\omega < \omega_0$, $X_L < X_C$. So for $X_L = 341 \Omega$ the angular frequency is less than the resonance angular frequency. ω is greater than ω_0 when $X_L = 619 \Omega$. But at these two values of X_L , the magnitude of $X_L - X_C$ is the same so Z and I are the same. In one case ($X_L = 619 \Omega$) the source voltage leads the current and in the other ($X_L = 341 \Omega$) the source voltage lags the current.

31.62. IDENTIFY and SET UP: The maximum possible current amplitude occurs at the resonance angular frequency because the impedance is then smallest.

EXECUTE: (a) At the resonance angular frequency $\omega_0 = 1/\sqrt{LC}$, the current is a maximum and $Z = R$, giving $I_{\max} = V/R$. At the required frequency, $I = I_{\max}/3$. $I = V/Z = I_{\max}/3 = (V/R)/3$, which means that $Z = 3R$. Squaring gives $R^2 + (\omega L - 1/\omega C)^2 = 9R^2$. Solving for ω gives $\omega = 3.192 \times 10^5 \text{ rad/s}$ and $\omega = 8.35 \times 10^4 \text{ rad/s}$.

(b) $V = \sqrt{2}V_{\text{rms}} = \sqrt{2}(35.0 \text{ V}) = 49.5 \text{ V}$. $I = \frac{I_{\max}}{3} = \frac{V}{3R} = \frac{49.5 \text{ V}}{3(125 \Omega)} = 0.132 \text{ A}$.

For $\omega = 8.35 \times 10^4 \text{ rad/s}$: $R = 125 \Omega$ and $V_R = IR = 16.5 \text{ V}$; $X_L = \omega L = 125 \Omega$ and $V_L = 16.5 \text{ V}$;

$X_C = \frac{1}{\omega C} = 479 \Omega$ and $V_C = 63.2 \text{ V}$.

For $\omega = 3.192 \times 10^5 \text{ rad/s}$: $R = 125 \Omega$ and $V_R = IR = 16.5 \text{ V}$; $X_L = \omega L = 479 \Omega$ and $V_L = 63.2 \text{ V}$;

$X_C = \frac{1}{\omega C} = 125 \Omega$ and $V_C = 16.5 \text{ V}$.

EVALUATE: For the lower frequency, $X_C > X_L$ and $V_C > V_L$. For the higher frequency, $X_L > X_C$ and $V_L > V_C$.

31.63. IDENTIFY and SET UP: Consider the cycle of the repeating current that lies between $t_1 = \tau/2$ and $t_2 = 3\tau/2$. In

this interval $i = \frac{2I_0}{\tau}(t - \tau)$. $I_{\text{av}} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i dt$ and $I_{\text{rms}}^2 = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i^2 dt$

EXECUTE: $I_{\text{av}} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i dt = \frac{1}{\tau} \int_{\tau/2}^{3\tau/2} \frac{2I_0}{\tau}(t - \tau) dt = \frac{2I_0}{\tau^2} \left[\frac{1}{2}t^2 - \tau t \right]_{\tau/2}^{3\tau/2}$

$I_{\text{av}} = \left(\frac{2I_0}{\tau^2} \right) \left(\frac{9\tau^2}{8} - \frac{3\tau^2}{2} - \frac{\tau^2}{8} + \frac{\tau^2}{2} \right) = (2I_0) \frac{1}{8} (9 - 12 - 1 + 4) = \frac{I_0}{4} (13 - 13) = 0$.

$I_{\text{rms}}^2 = (I^2)_{\text{av}} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i^2 dt = \frac{1}{\tau} \int_{\tau/2}^{3\tau/2} \frac{4I_0^2}{\tau^2} (t - \tau)^2 dt$

$I_{\text{rms}}^2 = \frac{4I_0^2}{\tau^3} \int_{\tau/2}^{3\tau/2} (t - \tau)^2 dt = \frac{4I_0^2}{\tau^3} \left[\frac{1}{3}(t - \tau)^3 \right]_{\tau/2}^{3\tau/2} = \frac{4I_0^2}{3\tau^3} \left[\left(\frac{\tau}{2} \right)^3 - \left(-\frac{\tau}{2} \right)^3 \right]$

$I_{\text{rms}}^2 = \frac{I_0^2}{6} [1 + 1] = \frac{1}{3} I_0^2$

$I_{\text{rms}} = \sqrt{I_{\text{rms}}^2} = \frac{I_0}{\sqrt{3}}$.

EVALUATE: In each cycle the current has as much negative value as positive value and its average is zero. i^2 is always positive and its average is not zero. The relation between I_{rms} and the current amplitude for this current is different from that for a sinusoidal current (Eq.31.4).

31.64. IDENTIFY: Apply $V_{\text{rms}} = I_{\text{rms}}Z$

SET UP: $\omega_0 = \frac{1}{\sqrt{LC}}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

EXECUTE: (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.80 \text{ H})(9.00 \times 10^{-7} \text{ F})}} = 786 \text{ rad/s}$.

(b) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. $Z = \sqrt{(300 \Omega)^2 + ((786 \text{ rad/s})(1.80 \text{ H}) - 1/((786 \text{ rad/s})(9.00 \times 10^{-7} \text{ F})))^2} = 300 \Omega$.

$I_{\text{rms-0}} = \frac{V_{\text{rms}}}{Z} = \frac{60 \text{ V}}{300 \Omega} = 0.200 \text{ A}$.

(c) We want $I = \frac{1}{2}I_{\text{rms-0}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$. $R^2 + (\omega L - 1/\omega C)^2 = \frac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2}$.

$\omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} + R^2 - \frac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2} = 0$ and $(\omega^2)^2 L^2 + \omega^2 \left(R^2 - \frac{2L}{C} - \frac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2} \right) + \frac{1}{C^2} = 0$.

Substituting in the values for this problem, the equation becomes $(\omega^2)^2 (3.24) + \omega^2 (-4.27 \times 10^6) + 1.23 \times 10^{12} = 0$.

Solving this quadratic equation in ω^2 we find $\omega^2 = 8.90 \times 10^5 \text{ rad}^2/\text{s}^2$ or $4.28 \times 10^5 \text{ rad}^2/\text{s}^2$ and

$\omega = 943 \text{ rad/s}$ or 654 rad/s .

(d) (i) $R = 300 \Omega$, $I_{\text{rms-0}} = 0.200 \text{ A}$, $|\omega_1 - \omega_2| = 289 \text{ rad/s}$. (ii) $R = 30 \Omega$, $I_{\text{rms-0}} = 2 \text{ A}$, $|\omega_1 - \omega_2| = 28 \text{ rad/s}$.

(iii) $R = 3 \Omega$, $I_{\text{rms-0}} = 20 \text{ A}$, $|\omega_1 - \omega_2| = 2.88 \text{ rad/s}$.

EVALUATE: The width gets smaller as R gets smaller; $I_{\text{rms-0}}$ gets larger as R gets smaller.

31.65. IDENTIFY: The resonance frequency, the reactances, and the impedance all depend on the values of the circuit elements.

SET UP: The resonance frequency is $\omega_0 = 1/\sqrt{LC}$, the reactances are $X_L = \omega L$ and $X_C = 1/\omega C$, and the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

EXECUTE: (a) $\omega_0 = 1/\sqrt{LC}$ becomes $\frac{1}{\sqrt{2L}\sqrt{2C}} \rightarrow 1/2$, so ω_0 decreases by $\frac{1}{2}$.

(b) Since $X_L = \omega L$, if L is doubled, X_L increases by a factor of 2.

(c) Since $X_C = 1/\omega C$, doubling C decreases X_C by a factor of $\frac{1}{2}$.

(d) $Z = \sqrt{R^2 + (X_L - X_C)^2} \rightarrow Z = \sqrt{(2R)^2 + (2X_L - \frac{1}{2}X_C)^2}$, so Z does not change by a simple factor of 2 or $\frac{1}{2}$.

EVALUATE: The impedance does not change by a simple factor, even though the other quantities do.

31.66. IDENTIFY: A transformer transforms voltages according to $\frac{V_2}{V_1} = \frac{N_2}{N_1}$. The effective resistance of a secondary

circuit of resistance R is $R_{\text{eff}} = \frac{R}{(N_2/N_1)^2}$.

SET UP: $N_2 = 275$ and $V_1 = 25.0 \text{ V}$.

EXECUTE: (a) $V_2 = V_1(N_2/N_1) = (25.0 \text{ V})(834/275) = 75.8 \text{ V}$

(b) $R_{\text{eff}} = \frac{R}{(N_2/N_1)^2} = \frac{125 \Omega}{(834/275)^2} = 13.6 \Omega$

EVALUATE: The voltage across the secondary is greater than the voltage across the primary since $N_2 > N_1$. The effective load resistance of the secondary is less than the resistance R connected across the secondary.

31.67. IDENTIFY: The resonance angular frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$ and the resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

SET UP: ω_0 is independent of R .

EXECUTE: (a) ω_0 (or f_0) depends only on L and C so change these quantities.

(b) To double ω_0 , decrease L and C by multiplying each of them by $\frac{1}{2}$.

EVALUATE: Increasing L and C decreases the resonance frequency; decreasing L and C increases the resonance frequency.

31.68. IDENTIFY: At resonance, $Z = R$. $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_E = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: (a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$. At resonance $\omega L = \frac{1}{\omega C}$ and $I_{\max} = \frac{V}{R}$.

(b) $V_C = IX_C = \frac{V}{R\omega_0 C} = \frac{V}{R} \sqrt{\frac{L}{C}}$.

(c) $V_L = IX_L = \frac{V}{R}\omega_0 L = \frac{V}{R} \sqrt{\frac{L}{C}}$.

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}C \frac{V^2}{R^2} \frac{L}{C} = \frac{1}{2}L \frac{V^2}{R^2}$.

(e) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L \frac{V^2}{R^2}$.

EVALUATE: At resonance $V_C = V_L$ and the maximum energy stored in the inductor equals the maximum energy stored in the capacitor.

31.69. IDENTIFY: $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_E = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: $\omega = \frac{\omega_0}{2}$.

(a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{\omega_0 L}{2} - 2/\omega_0 C\right)^2}} = \frac{V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

(b) $V_C = IX_C = \frac{2}{\omega_0 C} \frac{V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}} = \sqrt{\frac{L}{C}} \frac{2V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

(c) $V_L = IX_L = \frac{\omega_0 L}{2} \frac{V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}} = \sqrt{\frac{L}{C}} \frac{V/2}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{2LV^2}{R^2 + \frac{9}{4} \frac{L}{C}}$.

(e) $U_L = \frac{1}{2}LI^2 = \frac{1}{2} \frac{LV^2}{R^2 + \frac{9}{4} \frac{L}{C}}$.

EVALUATE: For $\omega < \omega_0$, $V_C > V_L$ and the maximum energy stored in the capacitor is greater than the maximum energy stored in the inductor.

31.70. IDENTIFY: $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_E = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: $\omega = 2\omega_0$.

(a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (2\omega_0 L - 1/2\omega_0 C)^2}} = \frac{V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

(b) $V_C = IX_C = \frac{1}{2\omega_0 C} \frac{V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}} = \sqrt{\frac{L}{C}} \frac{V/2}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

(c) $V_L = IX_L = 2\omega_0 L \frac{V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}} = \sqrt{\frac{L}{C}} \frac{2V}{\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{LV^2}{8\sqrt{R^2 + \frac{9}{4} \frac{L}{C}}}$.

$$(e) U_L = \frac{1}{2}LI^2 = \frac{LV^2}{2\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$$

EVALUATE: For $\omega > \omega_0$, $V_L > V_C$ and the maximum energy stored in the inductor is greater than the maximum energy stored in the capacitor.

- 31.71. IDENTIFY and SET UP:** Assume the angular frequency ω of the source and the resistance R of the resistor are known.

EXECUTE: Connect the source, capacitor, resistor, and inductor in series. Measure V_R and V_L . $\frac{V_L}{V_R} = \frac{I\omega L}{IR} = \frac{\omega L}{R}$ and L can be calculated.

EVALUATE: There are a number of other approaches. The frequency could be varied until $V_C = V_L$, and then this frequency is equal to $1/\sqrt{LC}$. If C is known, then L can be calculated.

- 31.72. IDENTIFY:** $P_{av} = V_{rms}I_{rms}\cos\phi$ and $I_{rms} = \frac{V_{rms}}{Z}$. Calculate Z . $R = Z\cos\phi$.

SET UP: $f = 50.0$ Hz and $\omega = 2\pi f$. The power factor is $\cos\phi$.

$$\text{EXECUTE: (a) } P_{av} = \frac{V_{rms}^2}{Z}\cos\phi. \quad Z = \frac{V_{rms}^2\cos\phi}{P_{av}} = \frac{(120\text{ V})^2(0.560)}{(220\text{ W})} = 36.7\ \Omega.$$

$$R = Z\cos\phi = (36.7\ \Omega)(0.560) = 20.6\ \Omega.$$

(b) $Z = \sqrt{R^2 + X_L^2}$. $X_L = \sqrt{Z^2 - R^2} = \sqrt{(36.7\ \Omega)^2 - (20.6\ \Omega)^2} = 30.4\ \Omega$. But $\phi = 0$ is at resonance, so the inductive and capacitive reactances equal each other. Therefore we need to add $X_C = 30.4\ \Omega$. $X_C = \frac{1}{\omega C}$ therefore gives

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50.0\text{ Hz})(30.4\ \Omega)} = 1.05 \times 10^{-4}\text{ F}.$$

$$\text{(c) At resonance, } P_{av} = \frac{V^2}{R} = \frac{(120\text{ V})^2}{20.6\ \Omega} = 699\text{ W}.$$

EVALUATE: $P_{av} = I_{rms}^2 R$ and I_{rms} is maximum at resonance, so the power drawn from the line is maximum at resonance.

- 31.73. IDENTIFY:** $p_R = i^2 R$. $p_L = iL \frac{di}{dt}$. $p_C = \frac{q}{C} i$.

SET UP: $i = I \cos \omega t$

$$\text{EXECUTE: (a) } p_R = i^2 R = I^2 \cos^2(\omega t) R = V_R I \cos^2(\omega t) = \frac{1}{2} V_R I (1 + \cos(2\omega t)).$$

$$P_{av}(R) = \frac{1}{T} \int_0^T p_R dt = \frac{V_R I}{2T} \int_0^T (1 + \cos(2\omega t)) dt = \frac{V_R I}{2T} [t]_0^T = \frac{1}{2} V_R I.$$

$$\text{(b) } p_L = Li \frac{di}{dt} = -\omega L I^2 \cos(\omega t) \sin(\omega t) = -\frac{1}{2} V_L I \sin(2\omega t). \text{ But } \int_0^T \sin(2\omega t) dt = 0 \Rightarrow P_{av}(L) = 0.$$

$$\text{(c) } p_C = \frac{q}{C} i = V_C i \sin(\omega t) \cos(\omega t) = \frac{1}{2} V_C I \sin(2\omega t). \text{ But } \int_0^T \sin(2\omega t) dt = 0 \Rightarrow P_{av}(C) = 0.$$

$$\text{(d) } p = p_R + p_L + p_C = V_R I \cos^2(\omega t) - \frac{1}{2} V_L I \sin(2\omega t) + \frac{1}{2} V_C I \sin(2\omega t) \text{ and}$$

$$p = I \cos(\omega t) (V_R \cos(\omega t) - V_L \sin(\omega t) + V_C \sin(\omega t)). \text{ But } \cos\phi = \frac{V_R}{V} \text{ and } \sin\phi = \frac{V_L - V_C}{V}, \text{ so}$$

$$p = VI \cos(\omega t) (\cos\phi \cos(\omega t) - \sin\phi \sin(\omega t)), \text{ at any instant of time.}$$

EVALUATE: At an instant of time the energy stored in the capacitor and inductor can be changing, but there is no net consumption of electrical energy in these components.

- 31.74. IDENTIFY:** $V_L = IX_L$. $\frac{dV_L}{d\omega} = 0$ at the ω where V_L is a maximum. $V_C = IX_C$. $\frac{dV_C}{d\omega} = 0$ at the ω where V_C is a maximum.

$$\text{SET UP: Problem 31.51 shows that } I = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$$

$$\text{EXECUTE: (a) } V_R = \text{maximum when } V_C = V_L \Rightarrow \omega = \omega_0 = \frac{1}{\sqrt{LC}}.$$

$$\begin{aligned}
 \text{(b) } V_L &= \text{maximum when } \frac{dV_L}{d\omega} = 0. \text{ Therefore: } \frac{dV_L}{d\omega} = 0 = \frac{d}{d\omega} \left(\frac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \right). \\
 0 &= \frac{VL}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}. \quad R^2 + (\omega L - 1/\omega C)^2 = \omega^2(L^2 - 1/\omega^4 C^2). \\
 R^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} &= -\frac{1}{\omega^2 C^2} \cdot \frac{1}{\omega^2} = LC - \frac{R^2 C^2}{2} \text{ and } \omega = \frac{1}{\sqrt{LC - R^2 C^2/2}}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } V_C &= \text{maximum when } \frac{dV_C}{d\omega} = 0. \text{ Therefore: } \frac{dV_C}{d\omega} = 0 = \frac{d}{d\omega} \left(\frac{V}{\omega C \sqrt{R^2 + (\omega L - 1/\omega C)^2}} \right). \\
 0 &= -\frac{V}{\omega^2 C \sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}. \quad R^2 + (\omega L - 1/\omega C)^2 = -\omega^2(L^2 - 1/\omega^4 C^2). \\
 R^2 + \omega^2 L^2 - \frac{2L}{C} &= -\omega^2 L^2 \text{ and } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}. \\
 R^2 + \omega^2 L^2 - \frac{2L}{C} &= -\omega^2 L^2.
 \end{aligned}$$

EVALUATE: V_L is maximum at a frequency greater than the resonance frequency and V_C is a maximum at a frequency less than the resonance frequency. These frequencies depend on R , as well as on L and on C .

31.75. IDENTIFY: Follow the steps specified in the problem.

SET UP: In part (a) use Eq.(31.23) to calculate Z and then $I = V/Z$. ϕ is given by Eq.(31.24). In part (b) let $Z = R + iX$.

EXECUTE: (a) From the current phasors we know that $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$$Z = \sqrt{(400 \Omega)^2 + \left((1000 \text{ rad/s})(0.50 \text{ H}) - \frac{1}{(1000 \text{ rad/s})(1.25 \times 10^{-6} \text{ F})} \right)^2} = 500 \Omega.$$

$$I = \frac{V}{Z} = \frac{200 \text{ V}}{500 \Omega} = 0.400 \text{ A}.$$

$$\text{(b) } \phi = \arctan\left(\frac{\omega L - 1/(\omega C)}{R}\right). \quad \phi = \arctan\left(\frac{(1000 \text{ rad/s})(0.500 \text{ H}) - 1/((1000 \text{ rad/s})(1.25 \times 10^{-6} \text{ F}))}{400 \Omega}\right) = +36.9^\circ$$

$$\text{(c) } Z_{\text{cpx}} = R + i\left(\omega L - \frac{1}{\omega C}\right). \quad Z_{\text{cpx}} = 400 \Omega - i\left((1000 \text{ rad/s})(0.50 \text{ H}) - \frac{1}{(1000 \text{ rad/s})(1.25 \times 10^{-6} \text{ F})}\right) = 400 \Omega - 300 \Omega i.$$

$$Z = \sqrt{(400 \Omega)^2 + (-300 \Omega)^2} = 500 \Omega.$$

$$\text{(d) } I_{\text{cpx}} = \frac{V}{Z_{\text{cpx}}} = \frac{200 \text{ V}}{(400 - 300i) \Omega} = \left(\frac{8 + 6i}{25}\right) \text{ A} = (0.320 \text{ A}) + (0.240 \text{ A})i. \quad I = \sqrt{\left(\frac{8 + 6i}{25}\right) \left(\frac{8 - 6i}{25}\right)} = 0.400 \text{ A}.$$

$$\text{(e) } \tan \phi = \frac{\text{Im}(I_{\text{cpx}})}{\text{Re}(I_{\text{cpx}})} = \frac{6/25}{8/25} = 0.75 \Rightarrow \phi = +36.9^\circ.$$

$$\text{(f) } V_{R\text{cpx}} = I_{\text{cpx}} R = \left(\frac{8 + 6i}{25}\right) (400 \Omega) = (128 + 96i) \text{ V}.$$

$$V_{L\text{cpx}} = iI_{\text{cpx}} \omega L = i\left(\frac{8 + 6i}{25}\right) (1000 \text{ rad/s})(0.500 \text{ H}) = (-120 + 160i) \text{ V}.$$

$$V_{C\text{cpx}} = i\frac{I_{\text{cpx}}}{\omega C} = i\left(\frac{8 + 6i}{25}\right) \frac{1}{(1000 \text{ rad/s})(1.25 \times 10^{-6} \text{ F})} = (+192 - 256i) \text{ V}.$$

$$\text{(g) } V_{\text{cpx}} = V_{R\text{cpx}} + V_{L\text{cpx}} + V_{C\text{cpx}} = (128 + 96i) \text{ V} + (-120 + 160i) \text{ V} + (192 - 256i) \text{ V} = 200 \text{ V}.$$

EVALUATE: Both approaches yield the same value for I and for ϕ .

ELECTROMAGNETIC WAVES

- 32.1. IDENTIFY:** Since the speed is constant, distance $x = ct$.
SET UP: The speed of light is $c = 3.00 \times 10^8$ m/s. $1 \text{ yr} = 3.156 \times 10^7$ s.
EXECUTE: (a) $t = \frac{x}{c} = \frac{3.84 \times 10^8 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \text{ s}$
 (b) $x = ct = (3.00 \times 10^8 \text{ m/s})(8.61 \text{ yr})(3.156 \times 10^7 \text{ s/yr}) = 8.15 \times 10^{16} \text{ m} = 8.15 \times 10^{13} \text{ km}$
EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.
- 32.2. IDENTIFY:** Since the speed is constant the difference in distance is $c\Delta t$.
SET UP: The speed of electromagnetic waves in air is $c = 3.00 \times 10^8$ m/s.
EXECUTE: A total time difference of $0.60 \mu\text{s}$ corresponds to a difference in distance of
 $c\Delta t = (3.00 \times 10^8 \text{ m/s})(0.60 \times 10^{-6} \text{ s}) = 180 \text{ m}$.
EVALUATE: The time delay doesn't depend on the distance from the transmitter to the receiver, it just depends on the difference in the length of the two paths.
- 32.3. IDENTIFY:** Apply $c = f\lambda$.
SET UP: $c = 3.00 \times 10^8$ m/s
EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5000 \text{ m}} = 6.0 \times 10^4 \text{ Hz}$.
 (b) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.0 \text{ m}} = 6.0 \times 10^7 \text{ Hz}$.
 (c) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.0 \times 10^{-6} \text{ m}} = 6.0 \times 10^{13} \text{ Hz}$.
 (d) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.0 \times 10^{-9} \text{ m}} = 6.0 \times 10^{16} \text{ Hz}$.
EVALUATE: f increases when λ decreases.
- 32.4. IDENTIFY:** $c = f\lambda$ and $k = \frac{2\pi}{\lambda}$.
SET UP: $c = 3.00 \times 10^8$ m/s.
EXECUTE: (a) $f = \frac{c}{\lambda}$. UVA: $7.50 \times 10^{14} \text{ Hz}$ to $9.38 \times 10^{14} \text{ Hz}$. UVB: $9.38 \times 10^{14} \text{ Hz}$ to $1.07 \times 10^{15} \text{ Hz}$.
 (b) $k = \frac{2\pi}{\lambda}$. UVA: $1.57 \times 10^7 \text{ rad/m}$ to $1.96 \times 10^7 \text{ rad/m}$. UVB: $1.96 \times 10^7 \text{ rad/m}$ to $2.24 \times 10^7 \text{ rad/m}$.
EVALUATE: Larger λ corresponds to smaller f and k .
- 32.5. IDENTIFY:** $c = f\lambda$. $E_{\text{max}} = cB_{\text{max}}$. $k = 2\pi/\lambda$. $\omega = 2\pi f$.
SET UP: Since the wave is traveling in empty space, its wave speed is $c = 3.00 \times 10^8$ m/s.
EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{432 \times 10^{-9} \text{ m}} = 6.94 \times 10^{14} \text{ Hz}$
 (b) $E_{\text{max}} = cB_{\text{max}} = (3.00 \times 10^8 \text{ m/s})(1.25 \times 10^{-6} \text{ T}) = 375 \text{ V/m}$

$$(c) \quad k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{432 \times 10^{-9} \text{ m}} = 1.45 \times 10^7 \text{ rad/m} . \quad \omega = (2\pi \text{ rad})(6.94 \times 10^{14} \text{ Hz}) = 4.36 \times 10^{15} \text{ rad/s} .$$

$$E = E_{\max} \cos(kx - \omega t) = (375 \text{ V/m}) \cos([1.45 \times 10^7 \text{ rad/m}]x - [4.36 \times 10^{15} \text{ rad/s}]t)$$

$$B = B_{\max} \cos(kx - \omega t) = (1.25 \times 10^{-6} \text{ T}) \cos([1.45 \times 10^7 \text{ rad/m}]x - [4.36 \times 10^{15} \text{ rad/s}]t)$$

EVALUATE: The $\cos(kx - \omega t)$ factor is common to both the electric and magnetic field expressions, since these two fields are in phase.

32.6. IDENTIFY: $c = f\lambda$. $E_{\max} = cB_{\max}$. Apply Eqs.(32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00 \times 10^8 \text{ m/s}$.

$$\text{EXECUTE: (a)} \quad f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{435 \times 10^{-9} \text{ m}} = 6.90 \times 10^{14} \text{ Hz}$$

$$(b) \quad B_{\max} = \frac{E_{\max}}{c} = \frac{2.70 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 9.00 \times 10^{-12} \text{ T}$$

$$(c) \quad k = \frac{2\pi}{\lambda} = 1.44 \times 10^7 \text{ rad/m} . \quad \omega = 2\pi f = 4.34 \times 10^{15} \text{ rad/s} . \text{ If } \vec{E}(z, t) = \hat{i}E_{\max} \cos(kz + \omega t) , \text{ then}$$

$$\vec{B}(z, t) = -\hat{j}B_{\max} \cos(kz + \omega t) , \text{ so that } \vec{E} \times \vec{B} \text{ will be in the } -\hat{k} \text{ direction.}$$

$$\vec{E}(z, t) = \hat{i}(2.70 \times 10^{-3} \text{ V/m}) \cos([1.44 \times 10^7 \text{ rad/s}]z + [4.34 \times 10^{15} \text{ rad/s}]t) \text{ and}$$

$$\vec{B}(z, t) = -\hat{j}(9.00 \times 10^{-12} \text{ T}) \cos([1.44 \times 10^7 \text{ rad/s}]z + [4.34 \times 10^{15} \text{ rad/s}]t) .$$

EVALUATE: The directions of $\vec{E}$ and $\vec{B}$ and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz + \omega t$ since the wave is traveling in the $-z$ -direction. Waves for visible light have very high frequencies.

32.7. IDENTIFY and SET UP: The equations are of the form of Eqs.(32.17), with x replaced by z . $\vec{B}$ is along the y -axis; deduce the direction of $\vec{E}$.

$$\text{EXECUTE: } \omega = 2\pi f = 2\pi(6.10 \times 10^{14} \text{ Hz}) = 3.83 \times 10^{15} \text{ rad/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c} = \frac{3.83 \times 10^{15} \text{ rad/s}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \times 10^7 \text{ rad/m}$$

$$B_{\max} = 5.80 \times 10^{-4} \text{ T}$$

$$E_{\max} = cB_{\max} = (3.00 \times 10^8 \text{ m/s})(5.80 \times 10^{-4} \text{ T}) = 1.74 \times 10^5 \text{ V/m}$$

$\vec{B}$ is along the y -axis. $\vec{E} \times \vec{B}$ is in the direction of propagation (the $+z$ -direction). From this we can deduce the direction of $\vec{E}$, as shown in Figure 32.7.

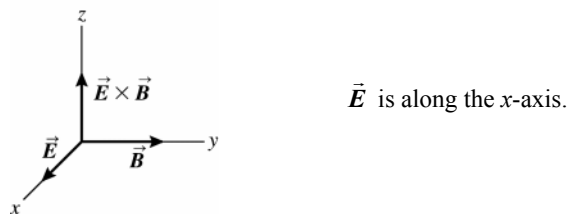


Figure 32.7

$$\vec{E} = E_{\max} \hat{i} \cos(kz - \omega t) = (1.74 \times 10^5 \text{ V/m}) \hat{i} \cos[(1.28 \times 10^7 \text{ rad/m}]z - (3.83 \times 10^{15} \text{ rad/s}]t]$$

$$\vec{B} = B_{\max} \hat{j} \cos(kz - \omega t) = (5.80 \times 10^{-4} \text{ T}) \hat{j} \cos[(1.28 \times 10^7 \text{ rad/m}]z - (3.83 \times 10^{15} \text{ rad/s}]t]$$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.8. IDENTIFY: For an electromagnetic wave propagating in the negative x direction, $E = E_{\max} \cos(kx + \omega t)$. $\omega = 2\pi f$

$$\text{and } k = \frac{2\pi}{\lambda} . \quad T = \frac{1}{f} . \quad E_{\max} = cB_{\max} .$$

SET UP: The wave specified in the problem has a different phase, so $E = -E_{\max} \sin(kx + \omega t)$. $E_{\max} = 375 \text{ V/m}$,

$$k = 1.99 \times 10^7 \text{ rad/m} \text{ and } \omega = 5.97 \times 10^{15} \text{ rad/s} .$$

$$\text{EXECUTE: (a)} \quad B_{\max} = \frac{E_{\max}}{c} = 1.25 \text{ } \mu\text{T} .$$

(b) $f = \frac{\omega}{2\pi} = 9.50 \times 10^{14}$ Hz. $\lambda = \frac{2\pi}{k} = 3.16 \times 10^{-7}$ m = 316 nm. $T = \frac{1}{f} = 1.05 \times 10^{-15}$ s. This wavelength is too short to be visible.

(c) $c = f\lambda = (9.50 \times 10^{14} \text{ Hz})(3.16 \times 10^{-7} \text{ m}) = 3.00 \times 10^8$ m/s. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

EVALUATE: $c = f\lambda = \left(\frac{\omega}{2\pi}\right)\left(\frac{2\pi}{k}\right) = \frac{\omega}{k}$ is an alternative expression for the wave speed.

32.9. IDENTIFY and SET UP: Compare the $\vec{E}(y, t)$ given in the problem to the general form given by Eq.(32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

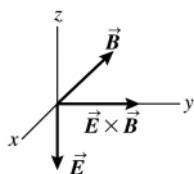
(a) EXECUTE: The equation for the electric field contains the factor $\sin(ky - \omega t)$ so the wave is traveling in the $+y$ -direction. The equation for $\vec{E}(y, t)$ is in terms of $\sin(ky - \omega t)$ rather than $\cos(ky - \omega t)$; the wave is shifted in phase by 90° relative to one with a $\cos(ky - \omega t)$ factor.

(b) $\vec{E}(y, t) = -(3.10 \times 10^5 \text{ V/m})\hat{k} \sin[ky - (2.65 \times 10^{12} \text{ rad/s})t]$

Comparing to Eq.(32.17) gives $\omega = 2.65 \times 10^{12}$ rad/s

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} \text{ so } \lambda = \frac{2\pi c}{\omega} = \frac{2\pi(2.998 \times 10^8 \text{ m/s})}{(2.65 \times 10^{12} \text{ rad/s})} = 7.11 \times 10^{-4} \text{ m}$$

(c)



$\vec{E} \times \vec{B}$ must be in the $+y$ -direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $-z$ -direction then $\vec{B}$ must be in the $-x$ -direction, as shown in Figure 32.9.

Figure 32.9

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2.65 \times 10^{12} \text{ rad/s}}{2.998 \times 10^8 \text{ m/s}} = 8.84 \times 10^3 \text{ rad/m}$$

$$E_{\text{max}} = 3.10 \times 10^5 \text{ V/m}$$

$$\text{Then } B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{3.10 \times 10^5 \text{ V/m}}{2.998 \times 10^8 \text{ m/s}} = 1.03 \times 10^{-3} \text{ T}$$

Using Eq.(32.17) and the fact that $\vec{B}$ is in the $-\hat{i}$ direction when $\vec{E}$ is in the $-\hat{k}$ direction,

$$\vec{B} = -(1.03 \times 10^{-3} \text{ T})\hat{i} \sin[(8.84 \times 10^3 \text{ rad/m})y - (2.65 \times 10^{12} \text{ rad/s})t]$$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.10. IDENTIFY: Apply Eqs.(32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

SET UP: The wave in this problem has a different phase, so $B_y(z, t) = B_{\text{max}} \sin(kx + \omega t)$.

EXECUTE: (a) The phase of the wave is given by $kx + \omega t$, so the wave is traveling in the $-x$ direction.

$$(b) k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}. f = \frac{kc}{2\pi} = \frac{(1.38 \times 10^4 \text{ rad/m})(3.0 \times 10^8 \text{ m/s})}{2\pi} = 6.59 \times 10^{11} \text{ Hz}.$$

(c) Since the magnetic field is in the $+y$ -direction, and the wave is propagating in the $-x$ -direction, then the electric field is in the $+z$ -direction so that $\vec{E} \times \vec{B}$ will be in the $-x$ -direction.

$$\vec{E}(x, t) = +cB(x, t)\hat{k} = cB_{\text{max}} \sin(kx + \omega t)\hat{k}.$$

$$\vec{E}(x, t) = (c(3.25 \times 10^{-9} \text{ T}))\sin((1.38 \times 10^4 \text{ rad/m})x + (4.14 \times 10^{12} \text{ rad/s})t)\hat{k}.$$

$$\vec{E}(x, t) = +(2.48 \text{ V/m})\sin((1.38 \times 10^4 \text{ rad/m})x + (4.14 \times 10^{12} \text{ rad/s})t)\hat{k}.$$

EVALUATE: $\vec{E}$ and $\vec{B}$ have the same phase and are in perpendicular directions.

32.11. IDENTIFY and SET UP: $c = f\lambda$ allows calculation of λ . $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Eq.(32.18) relates the electric and magnetic field amplitudes.

$$\text{EXECUTE: (a) } c = f\lambda \text{ so } \lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{830 \times 10^3 \text{ Hz}} = 361 \text{ m}$$

$$(b) k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{361 \text{ m}} = 0.0174 \text{ rad/m}$$

$$(c) \omega = 2\pi f = (2\pi)(830 \times 10^3 \text{ Hz}) = 5.22 \times 10^6 \text{ rad/s}$$

$$(d) \text{Eq.(32.18): } E_{\max} = cB_{\max} = (2.998 \times 10^8 \text{ m/s})(4.82 \times 10^{-11} \text{ T}) = 0.0144 \text{ V/m}$$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio broadcast band. The electric and magnetic fields in the wave are very weak.

32.12. IDENTIFY: $E_{\max} = cB_{\max}$.

SET UP: The magnetic field of the earth is about 10^{-4} T .

$$\text{EXECUTE: } B = \frac{E}{c} = \frac{3.85 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \times 10^{-11} \text{ T}.$$

EVALUATE: The field is much smaller than the earth's field.

32.13. IDENTIFY and SET UP: $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use Eq.(32.22) to calculate n and K .

$$\text{EXECUTE: (a) } \lambda = \frac{v}{f} = \frac{2.17 \times 10^8 \text{ m/s}}{5.70 \times 10^{14} \text{ Hz}} = 3.81 \times 10^{-7} \text{ m}$$

$$(b) \lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{5.70 \times 10^{14} \text{ Hz}} = 5.26 \times 10^{-7} \text{ m}$$

$$(c) n = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{2.17 \times 10^8 \text{ m/s}} = 1.38$$

$$(d) n = \sqrt{KK_m} \approx \sqrt{K} \text{ so } K = n^2 = (1.38)^2 = 1.90$$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so n is always greater than unity.

32.14. IDENTIFY: Apply Eq.(32.21). $E_{\max} = cB_{\max}$. $v = f\lambda$. Apply Eq.(32.29) with $\mu = K_m\mu_0$ in place of μ_0 .

SET UP: $K = 3.64$. $K_m = 5.18$

$$\text{EXECUTE: (a) } v = \frac{c}{\sqrt{KK_m}} = \frac{(3.00 \times 10^8 \text{ m/s})}{\sqrt{(3.64)(5.18)}} = 6.91 \times 10^7 \text{ m/s}.$$

$$(b) \lambda = \frac{v}{f} = \frac{6.91 \times 10^7 \text{ m/s}}{65.0 \text{ Hz}} = 1.06 \times 10^6 \text{ m}.$$

$$(c) B_{\max} = \frac{E_{\max}}{v} = \frac{7.20 \times 10^{-3} \text{ V/m}}{6.91 \times 10^7 \text{ m/s}} = 1.04 \times 10^{-10} \text{ T}.$$

$$(d) I = \frac{E_{\max} B_{\max}}{2K_m\mu_0} = \frac{(7.20 \times 10^{-3} \text{ V/m})(1.04 \times 10^{-10} \text{ T})}{2(5.18)\mu_0} = 5.75 \times 10^{-8} \text{ W/m}^2.$$

EVALUATE: The wave travels slower in this material than in air.

32.15. IDENTIFY: $I = P/A$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$.

$$\text{EXECUTE: (a) } I = \frac{P}{A} = \frac{(0.05)(75 \text{ W})}{4\pi(3.0 \times 10^{-2} \text{ m})^2} = 330 \text{ W/m}^2.$$

$$(b) E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(330 \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 500 \text{ V/m}. B_{\max} = \frac{E_{\max}}{c} = 1.7 \times 10^{-6} \text{ T} = 1.7 \mu\text{T}.$$

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.16. IDENTIFY and SET UP: The direction of propagation is given by $\vec{E} \times \vec{B}$.

$$\text{EXECUTE: (a) } \hat{S} = \hat{i} \times (-\hat{j}) = -\hat{k}.$$

$$(b) \hat{S} = \hat{j} \times \hat{i} = -\hat{k}.$$

$$(c) \hat{S} = (-\hat{k}) \times (-\hat{i}) = \hat{j}.$$

$$(d) \hat{S} = \hat{i} \times (-\hat{k}) = \hat{j}.$$

EVALUATE: In each case the directions of $\vec{E}$, $\vec{B}$ and the direction of propagation are all mutually perpendicular.

- 32.17. IDENTIFY:** $E_{\max} = cB_{\max}$. $\vec{E} \times \vec{B}$ is in the direction of propagation.
SET UP: $c = 3.00 \times 10^8$ m/s. $E_{\max} = 4.00$ V/m.
EXECUTE: $B_{\max} = E_{\max} / c = 1.33 \times 10^{-8}$ T. For $\vec{E}$ in the +x-direction, $\vec{E} \times \vec{B}$ is in the +z-direction when $\vec{B}$ is in the +y-direction.
EVALUATE: $\vec{E}$, $\vec{B}$ and the direction of propagation are all mutually perpendicular.
- 32.18. IDENTIFY:** The intensity of the electromagnetic wave is given by Eq.(32.29): $I = \frac{1}{2} \epsilon_0 c E_{\max}^2 = \epsilon_0 c E_{\text{rms}}^2$. The total energy passing through a window of area A during a time t is IAt .
SET UP: $\epsilon_0 = 8.85 \times 10^{-12}$ F/m
EXECUTE: Energy $= \epsilon_0 c E_{\text{rms}}^2 At = (8.85 \times 10^{-12} \text{ F/m})(3.00 \times 10^8 \text{ m/s})(0.0200 \text{ V/m})^2 (0.500 \text{ m}^2)(30.0 \text{ s}) = 15.9 \text{ } \mu\text{J}$
EVALUATE: The intensity is proportional to the square of the electric field amplitude.
- 32.19. IDENTIFY and SET UP:** Use Eq.(32.29) to calculate I , Eq.(32.18) to calculate $B_{\max}$, and use $I = P_{\text{av}} / 4\pi r^2$ to calculate P_{av} .
(a) EXECUTE: $I = \frac{1}{2} \epsilon_0 c E_{\max}^2$; $E_{\max} = 0.090$ V/m, so $I = 1.1 \times 10^{-5}$ W/m²
(b) $E_{\max} = cB_{\max}$ so $B_{\max} = E_{\max} / c = 3.0 \times 10^{-10}$ T
(c) $P_{\text{av}} = I(4\pi r^2) = (1.075 \times 10^{-5} \text{ W/m}^2)(4\pi)(2.5 \times 10^3 \text{ m})^2 = 840 \text{ W}$
(d) EVALUATE: The calculation in part (c) assumes that the transmitter emits uniformly in all directions.
- 32.20. IDENTIFY and SET UP:** $I = P_{\text{av}} / A$ and $I = \epsilon_0 c E_{\text{rms}}^2$.
EXECUTE: **(a)** The average power from the beam is $P_{\text{av}} = IA = (0.800 \text{ W/m}^2)(3.0 \times 10^{-4} \text{ m}^2) = 2.4 \times 10^{-4} \text{ W}$.
(b) $E_{\text{rms}} = \sqrt{\frac{I}{\epsilon_0 c}} = \sqrt{\frac{0.800 \text{ W/m}^2}{(8.85 \times 10^{-12} \text{ F/m})(3.00 \times 10^8 \text{ m/s})}} = 17.4 \text{ V/m}$
EVALUATE: The laser emits radiation only in the direction of the beam.
- 32.21. IDENTIFY:** $I = P_{\text{av}} / A$
SET UP: At a distance r from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.
EXECUTE: $P_{\text{av}} = I(4\pi r^2) = (5.0 \times 10^3 \text{ W/m}^2)(4\pi)(2.0 \times 10^{10} \text{ m})^2 = 2.5 \times 10^{25} \text{ J}$
EVALUATE: The intensity decreases with distance from the star as $1/r^2$.
- 32.22. IDENTIFY and SET UP:** $c = f\lambda$, $E_{\max} = cB_{\max}$ and $I = E_{\max} B_{\max} / 2\mu_0$
EXECUTE: **(a)** $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{0.354 \text{ m}} = 8.47 \times 10^8 \text{ Hz}$.
(b) $B_{\max} = \frac{E_{\max}}{c} = \frac{0.0540 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.80 \times 10^{-10} \text{ T}$.
(c) $I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{(0.0540 \text{ V/m})(1.80 \times 10^{-10} \text{ T})}{2\mu_0} = 3.87 \times 10^{-6} \text{ W/m}^2$.
EVALUATE: Alternatively, $I = \frac{1}{2} \epsilon_0 c E_{\max}^2$.
- 32.23. IDENTIFY:** $P_{\text{av}} = IA$ and $I = \frac{1}{2} \epsilon_0 c E_{\max}^2$
SET UP: The surface area of a sphere is $A = 4\pi r^2$.
EXECUTE: $P_{\text{av}} = S_{\text{av}} A = \left(\frac{E_{\max}^2}{2c\mu_0} \right) (4\pi r^2)$. $E_{\max} = \sqrt{\frac{P_{\text{av}} c \mu_0}{2\pi r^2}} = \sqrt{\frac{(60.0 \text{ W})(3.00 \times 10^8 \text{ m/s})\mu_0}{2\pi (5.00 \text{ m})^2}} = 12.0 \text{ V/m}$.
 $B_{\max} = \frac{E_{\max}}{c} = \frac{12.0 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.00 \times 10^{-8} \text{ T}$.
EVALUATE: $E_{\max}$ and $B_{\max}$ are both inversely proportional to the distance from the source.
- 32.24. IDENTIFY:** The Poynting vector is $\vec{S} = \vec{E} \times \vec{B}$.
SET UP: The electric field is in the +y-direction, and the magnetic field is in the +z-direction.
 $\cos^2 \phi = \frac{1}{2}(1 + \cos 2\phi)$
EXECUTE: **(a)** $\hat{S} = \hat{E} \times \hat{B} = (-\hat{j}) \times \hat{k} = -\hat{i}$. The Poynting vector is in the -x-direction, which is the direction of propagation of the wave.

(b) $S(x, t) = \frac{E(x, t)B(x, t)}{\mu_0} = \frac{E_{\max} B_{\max}}{\mu_0} \cos^2(kx + \omega t) = \frac{E_{\max} B_{\max}}{2\mu_0} (1 + \cos(2(\omega t + kx)))$. But over one period, the cosine function averages to zero, so we have $S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0}$. This is Eq.(32.29).

EVALUATE: We can also show that these two results also apply to the wave represented by Eq.(32.17).

32.25. IDENTIFY: Use the radiation pressure to find the intensity, and then $P_{\text{av}} = I(4\pi r^2)$.

SET UP: For a perfectly absorbing surface, $p_{\text{rad}} = \frac{I}{c}$

EXECUTE: $p_{\text{rad}} = I/c$ so $I = cp_{\text{rad}} = 2.70 \times 10^3 \text{ W/m}^2$. Then

$$P_{\text{av}} = I(4\pi r^2) = (2.70 \times 10^3 \text{ W/m}^2)(4\pi)(5.0 \text{ m})^2 = 8.5 \times 10^5 \text{ W}.$$

EVALUATE: Even though the source is very intense the radiation pressure 5.0 m from the surface is very small.

32.26. IDENTIFY: The intensity and the energy density of an electromagnetic wave depends on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I = P_{\text{av}}/A$, and the average power is $P_{\text{av}} = 2I/c$, where $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. The energy density is $u = \epsilon_0 E^2$.

EXECUTE: (a) $I = P_{\text{av}}/A = \frac{316,000 \text{ W}}{2\pi(5000 \text{ m})^2} = 0.00201 \text{ W/m}^2$. $P_{\text{av}} = 2I/c = \frac{2(0.00201 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} = 1.34 \times 10^{-11} \text{ Pa}$

(b) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$ gives

$$E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.00201 \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 1.23 \text{ N/C}$$

$$B_{\max} = E_{\max}/c = (1.23 \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = 4.10 \times 10^{-9} \text{ T}$$

(c) $u = \epsilon_0 E^2$, so $u_{\text{av}} = \epsilon_0 (E_{\text{av}})^2$ and $E_{\text{av}} = \frac{E_{\max}}{\sqrt{2}}$, so

$$u_{\text{av}} = \frac{\epsilon_0 E_{\max}^2}{2} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.23 \text{ N/C})^2}{2} = 6.69 \times 10^{-12} \text{ J/m}^3$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.27. IDENTIFY and SET UP: Use Eqs.(32.30) and (32.31).

EXECUTE: (a) By Eq.(32.30) the average momentum density is $\frac{dp}{dV} = \frac{S_{\text{av}}}{c^2} = \frac{I}{c^2}$

$$\frac{dp}{dV} = \frac{0.78 \times 10^3 \text{ W/m}^2}{(2.998 \times 10^8 \text{ m/s})^2} = 8.7 \times 10^{-15} \text{ kg/m}^2 \cdot \text{s}$$

(b) By Eq.(32.31) the average momentum flow rate per unit area is $\frac{S_{\text{av}}}{c} = \frac{I}{c} = \frac{0.78 \times 10^3 \text{ W/m}^2}{2.998 \times 10^8 \text{ m/s}} = 2.6 \times 10^{-6} \text{ Pa}$

EVALUATE: The radiation pressure that the sunlight would exert on an absorbing or reflecting surface is very small.

32.28. IDENTIFY: Apply Eqs.(32.32) and (32.33). The average momentum density is given by Eq.(32.30), with S replaced by $S_{\text{av}} = I$.

SET UP: $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

EXECUTE: (a) Absorbed light: $p_{\text{rad}} = \frac{I}{c} = \frac{2500 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 8.33 \times 10^{-6} \text{ Pa}$. Then

$$p_{\text{rad}} = \frac{8.33 \times 10^{-6} \text{ Pa}}{1.013 \times 10^5 \text{ Pa/atm}} = 8.23 \times 10^{-11} \text{ atm}.$$

(b) Reflecting light: $p_{\text{rad}} = \frac{2I}{c} = \frac{2(2500 \text{ W/m}^2)}{3.0 \times 10^8 \text{ m/s}} = 1.67 \times 10^{-5} \text{ Pa}$. Then

$$p_{\text{rad}} = \frac{1.67 \times 10^{-5} \text{ Pa}}{1.013 \times 10^5 \text{ Pa/atm}} = 1.65 \times 10^{-10} \text{ atm}.$$

(c) The momentum density is $\frac{dp}{dV} = \frac{S_{\text{av}}}{c^2} = \frac{2500 \text{ W/m}^2}{(3.0 \times 10^8 \text{ m/s})^2} = 2.78 \times 10^{-14} \text{ kg/m}^2 \cdot \text{s}$.

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.29. IDENTIFY: Apply Eq.(32.4) and (32.9).

SET UP: Eq.(32.26) is $S = \epsilon_0 c E^2$.

EXECUTE:
$$S = \frac{\epsilon_0}{\sqrt{\epsilon_0 \mu_0}} E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E c \frac{E}{c} = c \sqrt{\frac{\epsilon_0}{\mu_0}} E B = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \sqrt{\frac{\epsilon_0}{\mu_0}} E B = \frac{E B}{\mu_0} = \frac{E^2}{\mu_0 c} = \epsilon_0 c E^2$$

EVALUATE: We can also write $S = \epsilon_0 c (cB)^2 = \epsilon_0 c^3 B^2$. S can be written solely in terms of E or solely in terms of B .

32.30. IDENTIFY: The electric field at the nodes is zero, so there is no force on a point charge placed at a node.

SET UP: The location of the nodes is given by Eq.(32.36), where x is the distance from one of the planes.
 $\lambda = c/f$.

EXECUTE:
$$\Delta x_{\text{nodes}} = \frac{\lambda}{2} = \frac{c}{2f} = \frac{3.00 \times 10^8 \text{ m/s}}{2(7.50 \times 10^8 \text{ Hz})} = 0.200 \text{ m} = 20.0 \text{ cm}.$$
 There must be nodes at the planes, which

are 80.0 cm apart, and there are two nodes between the planes, each 20.0 cm from a plane. It is at 20 cm, 40 cm, and 60 cm from one plane that a point charge will remain at rest, since the electric fields there are zero.

EVALUATE: The magnetic field amplitude at these points isn't zero, but the magnetic field doesn't exert a force on a stationary charge.

32.31. IDENTIFY and SET UP: Apply Eqs.(32.36) and (32.37).

EXECUTE: (a) By Eq.(32.37) we see that the nodal planes of the $\vec{B}$ field are a distance $\lambda/2$ apart, so $\lambda/2 = 3.55 \text{ mm}$ and $\lambda = 7.10 \text{ mm}$.

(b) By Eq.(32.36) we see that the nodal planes of the $\vec{E}$ field are also a distance $\lambda/2 = 3.55 \text{ mm}$ apart.

(c) $v = f\lambda = (2.20 \times 10^{10} \text{ Hz})(7.10 \times 10^{-3} \text{ m}) = 1.56 \times 10^8 \text{ m/s}$.

EVALUATE: The spacing between the nodes of $\vec{E}$ is the same as the spacing between the nodes of $\vec{B}$. Note that $v < c$, as it must.

32.32. IDENTIFY: The nodal planes of $\vec{E}$ and $\vec{B}$ are located by Eqs.(32.26) and (32.27).

SET UP:
$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{75.0 \times 10^6 \text{ Hz}} = 4.00 \text{ m}$$

EXECUTE: (a)
$$\Delta x = \frac{\lambda}{2} = 2.00 \text{ m}.$$

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is

$$\frac{\lambda}{4} = \frac{\Delta x}{2} = \frac{2.00 \text{ m}}{2} = 1.00 \text{ m}.$$

EVALUATE: The nodal planes of $\vec{B}$ are separated by a distance $\lambda/2$ and are midway between the nodal planes of $\vec{E}$.

32.33. (a) IDENTIFY and SET UP: The distance between adjacent nodal planes of $\vec{B}$ is $\lambda/2$. There is an antinodal plane of $\vec{B}$ midway between any two adjacent nodal planes, so the distance between a nodal plane and an adjacent antinodal plane is $\lambda/4$. Use $v = f\lambda$ to calculate λ .

EXECUTE:
$$\lambda = \frac{v}{f} = \frac{2.10 \times 10^8 \text{ m/s}}{1.20 \times 10^{10} \text{ Hz}} = 0.0175 \text{ m}$$

$$\frac{\lambda}{4} = \frac{0.0175 \text{ m}}{4} = 4.38 \times 10^{-3} \text{ m} = 4.38 \text{ mm}$$

(b) IDENTIFY and SET UP: The nodal planes of $\vec{E}$ are at $x = 0, \lambda/2, \lambda, 3\lambda/2, \dots$, so the antinodal planes of $\vec{E}$ are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$. The nodal planes of $\vec{B}$ are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$, so the antinodal planes of $\vec{B}$ are at $\lambda/2, \lambda, 3\lambda/2, \dots$.

EXECUTE: The distance between adjacent antinodal planes of $\vec{E}$ and antinodal planes of $\vec{B}$ is therefore $\lambda/4 = 4.38 \text{ mm}$.

(c) From Eqs.(32.36) and (32.37) the distance between adjacent nodal planes of $\vec{E}$ and $\vec{B}$ is $\lambda/4 = 4.38 \text{ mm}$.

EVALUATE: The nodes of $\vec{E}$ coincide with the antinodes of $\vec{B}$, and conversely. The nodes of $\vec{B}$ and the nodes of $\vec{E}$ are equally spaced.

32.34. IDENTIFY: Evaluate the derivatives of the expressions for $E_y(x, t)$ and $B_z(x, t)$ that are given in Eqs.(32.34) and (32.35).

SET UP: $\frac{\partial}{\partial x} \sin kx = k \cos kx$, $\frac{\partial}{\partial t} \sin \omega t = \omega \cos \omega t$, $\frac{\partial}{\partial x} \cos kx = -k \sin kx$, $\frac{\partial}{\partial t} \cos \omega t = -\omega \sin \omega t$.

EXECUTE: (a) $\frac{\partial^2 E_y(x, t)}{\partial x^2} = \frac{\partial^2}{\partial x^2} (-2E_{\max} \sin kx \sin \omega t) = \frac{\partial}{\partial x} (-2kE_{\max} \cos kx \sin \omega t)$ and

$$\frac{\partial^2 E_y(x, t)}{\partial x^2} = 2k^2 E_{\max} \sin kx \sin \omega t = \frac{\omega^2}{c^2} 2E_{\max} \sin kx \sin \omega t = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x, t)}{\partial t^2}.$$

Similarly: $\frac{\partial^2 B_z(x, t)}{\partial x^2} = \frac{\partial^2}{\partial x^2} (-2B_{\max} \cos kx \cos \omega t) = \frac{\partial}{\partial x} (+2kB_{\max} \sin kx \cos \omega t)$ and

$$\frac{\partial^2 B_z(x, t)}{\partial x^2} = 2k^2 B_{\max} \cos kx \cos \omega t = \frac{\omega^2}{c^2} 2B_{\max} \cos kx \cos \omega t = \epsilon_0 \mu_0 \frac{\partial^2 B_z(x, t)}{\partial t^2}.$$

(b) $\frac{\partial E_y(x, t)}{\partial x} = \frac{\partial}{\partial x} (-2E_{\max} \sin kx \sin \omega t) = -2kE_{\max} \cos kx \sin \omega t$.

$$\frac{\partial E_y(x, t)}{\partial x} = -\frac{\omega}{c} 2E_{\max} \cos kx \sin \omega t = -\omega 2 \frac{E_{\max}}{c} \cos kx \sin \omega t = -\omega 2B_{\max} \cos kx \sin \omega t.$$

$$\frac{\partial E_y(x, t)}{\partial x} = +\frac{\partial}{\partial t} (2B_{\max} \cos kx \cos \omega t) = -\frac{\partial B_z(x, t)}{\partial t}.$$

Similarly: $-\frac{\partial B_z(x, t)}{\partial x} = \frac{\partial}{\partial x} (+2B_{\max} \cos kx \cos \omega t) = -2kB_{\max} \sin kx \cos \omega t$.

$$-\frac{\partial B_z(x, t)}{\partial x} = -\frac{\omega}{c} 2B_{\max} \sin kx \cos \omega t = -\frac{\omega}{c^2} 2cB_{\max} \sin kx \cos \omega t.$$

$$-\frac{\partial B_z(x, t)}{\partial x} = -\epsilon_0 \mu_0 \omega 2E_{\max} \sin kx \cos \omega t = \epsilon_0 \mu_0 \frac{\partial}{\partial t} (-2E_{\max} \sin kx \sin \omega t) = \epsilon_0 \mu_0 \frac{\partial E_y(x, t)}{\partial t}.$$

EVALUATE: The standing waves are linear superpositions of two traveling waves of the same k and ω .

32.35. IDENTIFY: The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $(2\frac{1}{2})\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: (a) Since $(2\frac{1}{2})\lambda = L$, we have $L = (5/2)(12.2 \text{ cm}) = 30.5 \text{ cm}$.

(b) Solving for the frequency gives $f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(0.122 \text{ m}) = 2.46 \times 10^9 \text{ Hz}$.

(c) $L = 35.5 \text{ cm}$ in this case. $(2\frac{1}{2})\lambda = L$, so $\lambda = 2L/5 = 2(35.5 \text{ cm})/5 = 14.2 \text{ cm}$.

$$f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(0.142 \text{ m}) = 2.11 \times 10^9 \text{ Hz}$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.36. IDENTIFY: Evaluate the partial derivatives of the expressions for $E_y(x, t)$ and $B_z(x, t)$.

SET UP: $\frac{\partial}{\partial x} \sin(kx - \omega t) = k \cos(kx - \omega t)$, $\frac{\partial}{\partial t} \sin(kx - \omega t) = -\omega \cos(kx - \omega t)$, $\frac{\partial}{\partial x} \cos(kx - \omega t) = -k \sin(kx - \omega t)$,

$$\frac{\partial}{\partial t} \cos(kx - \omega t) = \omega \sin(kx - \omega t)$$

EXECUTE: Assume $\vec{E} = E_{\max} \hat{j} \sin(kx - \omega t)$ and $\vec{B} = B_{\max} \hat{k} \sin(kx - \omega t + \phi)$, with $-\pi < \phi < \pi$. Eq. (32.12) is

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}. \text{ This gives } kE_{\max} \cos(kx - \omega t) = +\omega B_{\max} \cos(kx - \omega t + \phi), \text{ so } \phi = 0, \text{ and } kE_{\max} = \omega B_{\max}, \text{ so}$$

$$E_{\max} = \frac{\omega}{k} B_{\max} = \frac{2\pi f}{2\pi/\lambda} B_{\max} = f\lambda B_{\max} = cB_{\max}. \text{ Similarly for Eq.(32.14), } -\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} \text{ gives}$$

$$-kB_{\max} \cos(kx - \omega t + \phi) = -\epsilon_0 \mu_0 \omega E_{\max} \cos(kx - \omega t), \text{ so } \phi = 0 \text{ and } kB_{\max} = \epsilon_0 \mu_0 \omega E_{\max}, \text{ so}$$

$$B_{\max} = \frac{\epsilon_0 \mu_0 \omega}{k} E_{\max} = \frac{2\pi f}{c^2 2\pi/\lambda} E_{\max} = \frac{f\lambda}{c^2} E_{\max} = \frac{1}{c} E_{\max}.$$

EVALUATE: The $\vec{E}$ and $\vec{B}$ fields must oscillate in phase.

32.37. IDENTIFY and SET UP: Take partial derivatives of Eqs.(32.12) and (32.14), as specified in the problem.

EXECUTE: Eq.(32.12): $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

Taking $\frac{\partial}{\partial t}$ of both sides of this equation gives $\frac{\partial^2 E_y}{\partial x \partial t} = -\frac{\partial^2 B_z}{\partial t^2}$. Eq.(32.14) says $-\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$. Taking $\frac{\partial}{\partial x}$ of both sides of this equation gives $-\frac{\partial^2 B_z}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t \partial x}$, so $\frac{\partial^2 E_y}{\partial t \partial x} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 B_z}{\partial x^2}$. But $\frac{\partial^2 E_y}{\partial x \partial t} = \frac{\partial^2 E_y}{\partial t \partial x}$ (The order in which the partial derivatives are taken doesn't change the result.) So $-\frac{\partial^2 B_z}{\partial t^2} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 B_z}{\partial x^2}$ and $\frac{\partial^2 B_z}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 B_z}{\partial t^2}$, as was to be shown.

EVALUATE: Both fields, electric and magnetic, satisfy the wave equation, Eq.(32.10). We have also shown that both fields propagate with the same speed $v = 1/\sqrt{\epsilon_0 \mu_0}$.

32.38. IDENTIFY: The average energy density in the electric field is $u_{E,av} = \frac{1}{2} \epsilon_0 (E^2)_{av}$ and the average energy density in the magnetic field is $u_{B,av} = \frac{1}{2 \mu_0} (B^2)_{av}$.

SET UP: $(\cos^2(kx - \omega t))_{av} = \frac{1}{2}$.

EXECUTE: $E_y(x, t) = E_{max} \cos(kx - \omega t)$. $u_E = \frac{1}{2} \epsilon_0 E_y^2 = \frac{1}{2} \epsilon_0 E_{max}^2 \cos^2(kx - \omega t)$ and $u_{E,av} = \frac{1}{4} \epsilon_0 E_{max}^2$.

$B_z(x, t) = B_{max} \cos(kx - \omega t)$, so $u_B = \frac{1}{2 \mu_0} B_z^2 = \frac{1}{2 \mu_0} B_{max}^2 \cos^2(kx - \omega t)$ and $u_{B,av} = \frac{1}{4 \mu_0} B_{max}^2$. $E_{max} = c B_{max}$, so

$u_{E,av} = \frac{1}{4} \epsilon_0 c^2 B_{max}^2$. $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, so $u_{E,av} = \frac{1}{2 \mu_0} B_{max}^2$, which equals $u_{B,av}$.

EVALUATE: Our result allows us to write $u_{av} = 2u_{E,av} = \frac{1}{2} \epsilon_0 E_{max}^2$ and $u_{av} = 2u_{B,av} = \frac{1}{2 \mu_0} B_{max}^2$.

32.39. IDENTIFY: The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{av}/A = \frac{1}{2} \epsilon_0 c E_{max}^2$, and the force is $F = P_{av} A$ where $P_{av} = I/c$.

EXECUTE: (a) $I = P_{av}/A = (25,000 \text{ W})/[4\pi(575 \text{ m})^2] = 0.00602 \text{ W/m}^2$

(b) $I = \frac{1}{2} \epsilon_0 c E_{max}^2$, so $E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.00602 \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 2.13 \text{ N/C}$.

$B_{max} = E_{max}/c = (2.13 \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = 7.10 \times 10^{-9} \text{ T}$

(c) $F = P_{av} A = (I/c)A = (0.00602 \text{ W/m}^2)(0.150 \text{ m})(0.400 \text{ m})/(3.00 \times 10^8 \text{ m/s}) = 1.20 \times 10^{-12} \text{ N}$

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.40. IDENTIFY: $c = f\lambda$. $E_{max} = c B_{max}$. $I = \frac{1}{2} \epsilon_0 c E_{max}^2$. For a totally absorbing surface the radiation pressure is $\frac{I}{c}$.

SET UP: The wave speed in air is $c = 3.00 \times 10^8 \text{ m/s}$.

EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{3.84 \times 10^{-2} \text{ m}} = 7.81 \times 10^9 \text{ Hz}$

(b) $B_{max} = \frac{E_{max}}{c} = \frac{1.35 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.50 \times 10^{-9} \text{ T}$

(c) $I = \frac{1}{2} \epsilon_0 c E_{max}^2 = \frac{1}{2} (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})(1.35 \text{ V/m})^2 = 2.42 \times 10^{-3} \text{ W/m}^2$

(d) $F = (\text{pressure})A = \frac{IA}{c} = \frac{(2.42 \times 10^{-3} \text{ W/m}^2)(0.240 \text{ m}^2)}{3.00 \times 10^8 \text{ m/s}} = 1.94 \times 10^{-12} \text{ N}$

EVALUATE: The intensity depends only on the amplitudes of the electric and magnetic fields and is independent of the wavelength of the light.

32.41. (a) IDENTIFY and SET UP: Calculate I and then use Eq.(32.29) to calculate $E_{\max}$ and Eq.(32.18) to calculate $B_{\max}$.

EXECUTE: The intensity is power per unit area: $I = \frac{P}{A} = \frac{3.20 \times 10^{-3} \text{ W}}{\pi(1.25 \times 10^{-3} \text{ m})^2} = 652 \text{ W/m}^2$.

$$I = \frac{E_{\max}^2}{2\mu_0 c}, \text{ so } E_{\max} = \sqrt{2\mu_0 c I}$$

$$E_{\max} = \sqrt{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.998 \times 10^8 \text{ m/s})(652 \text{ W/m}^2)} = 701 \text{ V/m}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{701 \text{ V/m}}{2.998 \times 10^8 \text{ m/s}} = 2.34 \times 10^{-6} \text{ T}$$

EVALUATE: The magnetic field amplitude is quite small.

(b) IDENTIFY and SET UP: Eqs.(24.11) and (30.10) give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\max} \cos(kx - \omega t)$ and the average value of $\cos^2(kx - \omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is

$$u_{E,av} = \frac{1}{4}\epsilon_0 E_{\max}^2 = \frac{1}{4}(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(701 \text{ V/m})^2 = 1.09 \times 10^{-6} \text{ J/m}^3. \text{ The energy density in the magnetic field}$$

$$\text{is } u_B = \frac{B^2}{2\mu_0}. \text{ The average value is } u_{B,av} = \frac{B_{\max}^2}{4\mu_0} = \frac{(2.34 \times 10^{-6} \text{ T})^2}{4(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 1.09 \times 10^{-6} \text{ J/m}^3.$$

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and SET UP: The total energy in this length of beam is the total energy density

$$u_{av} = u_{E,av} + u_{B,av} = 2.18 \times 10^{-6} \text{ J/m}^3 \text{ times the volume of this part of the beam.}$$

$$\text{EXECUTE: } U = u_{av} LA = (2.18 \times 10^{-6} \text{ J/m}^3)(1.00 \text{ m})\pi(1.25 \times 10^{-3} \text{ m})^2 = 1.07 \times 10^{-11} \text{ J.}$$

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00 \text{ m}$: $U = P\left(\frac{L}{c}\right) = (3.20 \times 10^{-3} \text{ W})\left(\frac{1.00 \text{ m}}{2.998 \times 10^8 \text{ m/s}}\right) = 1.07 \times 10^{-11} \text{ J}$, which checks.

32.42. IDENTIFY: Use the gaussian surface specified in the hint.

SET UP: The wave is in free space, so in Gauss's law for the electric field, $Q_{\text{encl}} = 0$ and $\oint \vec{E} \cdot d\vec{A} = 0$. Gauss's law for the magnetic field says $\oint \vec{B} \cdot d\vec{A} = 0$

EXECUTE: Use a gaussian surface such that the front surface is ahead of the wave front (no electric or magnetic fields) and the back face is behind the wave front, as shown in Figure 32.42. $\oint \vec{E} \cdot d\vec{A} = E_x A = \frac{Q_{\text{encl}}}{\epsilon_0} = 0$, so $E_x = 0$.

$$\oint \vec{B} \cdot d\vec{A} = B_x A = 0 \text{ and } B_x = 0.$$

EVALUATE: The wave must be transverse, since there are no components of the electric or magnetic field in the direction of propagation.

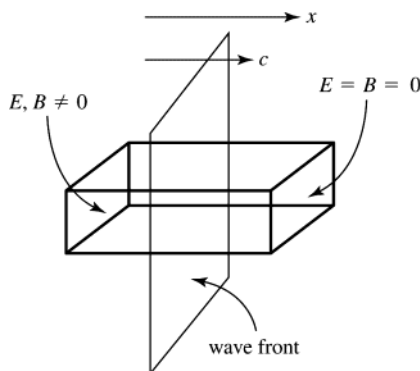


Figure 32.42

32.43. IDENTIFY: $I = P_{av} / A$. For an absorbing surface, the radiation pressure is $p_{\text{rad}} = \frac{I}{c}$

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{av} / (4\pi r^2)$.

EXECUTE: (a) At the sun's surface: $I = \frac{P_{\text{av}}}{4\pi R^2} = \frac{3.9 \times 10^{26} \text{ W}}{4\pi(6.96 \times 10^8 \text{ m})^2} = 6.4 \times 10^7 \text{ W/m}^2$ and

$$p_{\text{rad}} = \frac{I}{c} = \frac{6.4 \times 10^7 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} = 0.21 \text{ Pa}.$$

(b) Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6 \times 10^8 \text{ W/m}^2$ and $p_{\text{rad}} = 0.85 \text{ Pa}$. At the top of the earth's atmosphere, the measured sunlight intensity is $1400 \text{ W/m}^2 = 5 \times 10^{-6} \text{ Pa}$, which is about 100,000 times less than the values above.

EVALUATE: (b) The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.44. IDENTIFY: $I = \frac{P}{A}$. $I = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$.

SET UP: $3.00 \times 10^8 \text{ m/s}$

EXECUTE: $I = \frac{P}{A} = \frac{2.80 \times 10^3 \text{ W}}{36.0 \text{ m}^2} = 77.8 \text{ W/m}^2$.

$$E_{\text{max}} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(77.8 \text{ W/m}^2)}{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 242 \text{ N/C}.$$

EVALUATE: This value of E_{max} is similar to the electric field amplitude in ordinary light sources.

32.45. IDENTIFY: The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = P_{\text{av}} A = (I/c)A$, while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$. Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{\text{net}} = I\alpha$, to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{\text{Abs}} = p_{\text{av}} A = (I/c)A = \frac{\frac{1}{2} \epsilon_0 c E_{\text{max}}^2 A}{c} = \frac{1}{2} \epsilon_0 A E_{\text{max}}^2$

For a totally reflecting surface, the force will be twice as great, which is $\epsilon_0 c E_{\text{max}}^2$. The net torque is therefore

$$\tau_{\text{net}} = F_{\text{refl}}(L/2) - F_{\text{abs}}(L/2) = \epsilon_0 A E_{\text{max}}^2 L/4$$

Newton's 2nd law for rotation gives $\tau_{\text{net}} = I\alpha$. $\epsilon_0 A E_{\text{max}}^2 L/4 = 2m(L/2)^2 \alpha$

$$\text{Solving for } \alpha \text{ gives } \alpha = \epsilon_0 A E_{\text{max}}^2 / (2mL) = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0150 \text{ m})^2 (1.25 \text{ N/C})^2}{(2)(0.00400 \text{ kg})(1.00 \text{ m})} = 3.89 \times 10^{-13} \text{ rad/s}^2$$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.46. IDENTIFY: For light of intensity I_{abs} incident on a totally absorbing surface, the radiation pressure is

$$p_{\text{rad,abs}} = \frac{I_{\text{abs}}}{c}. \text{ For light of intensity } I_{\text{refl}} \text{ incident on a totally reflecting surface, } p_{\text{rad,refl}} = \frac{2I_{\text{refl}}}{c}.$$

SET UP: The total radiation pressure is $p_{\text{rad}} = p_{\text{rad,abs}} + p_{\text{rad,refl}}$. $I_{\text{abs}} = wI$ and $I_{\text{refl}} = (1-w)I$

EXECUTE: (a) $p_{\text{rad}} = p_{\text{rad,abs}} + p_{\text{rad,refl}} = \frac{I_{\text{abs}}}{c} + \frac{2I_{\text{refl}}}{c} = \frac{wI}{c} + \frac{2(1-w)I}{c} = \frac{(2-w)I}{c}$.

(b) (i) For totally absorbing $w = 1$ so $p_{\text{rad}} = \frac{I}{c}$. (ii) For totally reflecting $w = 0$ so $p_{\text{rad}} = \frac{2I}{c}$. These are just equations

32.32 and 32.33.

(c) For $w = 0.9$ and $I = 1.40 \times 10^2 \text{ W/m}^2$, $p_{\text{rad}} = \frac{(2-0.9)(1.40 \times 10^2 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} = 5.13 \times 10^{-6} \text{ Pa}$. For $w = 0.1$ and

$$I = 1.40 \times 10^3 \text{ W/m}^2, p_{\text{rad}} = \frac{(2-0.1)(1.40 \times 10^2 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} = 8.87 \times 10^{-6} \text{ Pa}.$$

EVALUATE: The radiation pressure is greater when a larger fraction is reflected.

32.47. IDENTIFY and SET UP: In the wire the electric field is related to the current density by Eq.(25.7). Use Ampere's law to calculate $\vec{B}$. The Poynting vector is given by Eq.(32.28) and the equation that follows it relates the energy flow through a surface to $\vec{S}$.

EXECUTE: (a) The direction of $\vec{E}$ is parallel to the axis of the cylinder, in the direction of the current. From Eq.(25.7), $E = \rho J = \rho I / \pi a^2$. (E is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.47a; take the current to be coming out of the page.

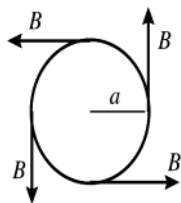


Figure 32.47a

Apply Ampere's law to a circle of radius a .

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi a)$$

$$I_{\text{encl}} = I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \text{ gives } B(2\pi a) = \mu_0 I \text{ and } B = \frac{\mu_0 I}{2\pi a}$$

The direction of $\vec{B}$ is counterclockwise around the circle.

(c) The directions of $\vec{E}$ and $\vec{B}$ are shown in Figure 32.47b.

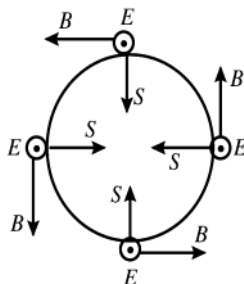


Figure 32.47b

The direction of $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

is radially inward.

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \left(\frac{\rho I}{\pi a^2} \right) \left(\frac{\mu_0 I}{2\pi a} \right)$$

$$S = \frac{\rho I^2}{2\pi^2 a^3}$$

(d) **EVALUATE:** Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi al) = \frac{\rho I^2}{2\pi^2 a^3} (2\pi al) = \frac{\rho I^2 l}{\pi a^2}$. But $R = \frac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since $\vec{S}$ is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.48. IDENTIFY: The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = S_{\text{av}} = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$.

EXECUTE: Since $I = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$, $E_{\text{max}} \propto \sqrt{I}$. A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and $(0.200 \text{ m})/(10.0 \text{ m}) = 1/50$, we have $I_{0.20} = 50^2 I_{10}$. Since $E_{\text{max}} \propto \sqrt{I}$, we have $E_{0.20} = 50 E_{10} = (50)(1.50 \text{ N/C}) = 75.0 \text{ N/C}$.

EVALUATE: While the intensity increases by a factor of $50^2 = 2500$, the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.49. IDENTIFY and SET UP: The magnitude of the induced emf is given by Faraday's law: $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right|$. To calculate $d\Phi_B/dt$ we need dB/dt at the antenna. Use the total power output to calculate I and then combine Eq.(32.29) and (32.18) to calculate B_{max} . The time dependence of B is given by Eq.(32.17).

EXECUTE: $\Phi_B = B\pi R^2$, where $R = 0.0900 \text{ m}$ is the radius of the loop. (This assumes that the magnetic field is uniform across the loop, an excellent approximation.) $|\mathcal{E}| = \pi R^2 \left| \frac{dB}{dt} \right|$

$$B = B_{\text{max}} \cos(kx - \omega t) \text{ so } \left| \frac{dB}{dt} \right| = B_{\text{max}} \omega \sin(kx - \omega t)$$

The maximum value of $\left| \frac{dB}{dt} \right|$ is $B_{\text{max}} \omega$, so $|\mathcal{E}|_{\text{max}} = \pi R^2 B_{\text{max}} \omega$.

$$R = 0.0900 \text{ m}, \omega = 2\pi f = 2\pi(95.0 \times 10^6 \text{ Hz}) = 5.97 \times 10^8 \text{ rad/s}$$

Calculate the intensity I at this distance from the source, and from that the magnetic field amplitude $B_{\max}$:

$$I = \frac{P}{4\pi r^2} = \frac{55.0 \times 10^3 \text{ W}}{4\pi(2.50 \times 10^3 \text{ m})^2} = 7.00 \times 10^{-4} \text{ W/m}^2. \quad I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{(cB_{\max})^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\max}^2$$

$$\text{Thus } B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(7.00 \times 10^{-4} \text{ W/m}^2)}{2.998 \times 10^8 \text{ m/s}}} = 2.42 \times 10^{-9} \text{ T. Then}$$

$$|\mathcal{E}|_{\max} = \pi R^2 B_{\max} \omega = \pi(0.0900 \text{ m})^2 (2.42 \times 10^{-9} \text{ T})(5.97 \times 10^8 \text{ rad/s}) = 0.0368 \text{ V.}$$

EVALUATE: An induced emf of this magnitude is easily detected.

32.50. IDENTIFY: The nodal planes are one-half wavelength apart.

SET UP: The nodal planes of B are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$, which are $\lambda/2$ apart.

EXECUTE: (a) The wavelength is $\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(110.0 \times 10^6 \text{ Hz}) = 2.727 \text{ m}$. So the nodal planes are at $(2.727 \text{ m})/2 = 1.364 \text{ m}$ apart.

(b) For the nodal planes of E , we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.727 \text{ m})/2 = 10.91 \text{ m}$

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.51. IDENTIFY and SET UP: Find the force on you due to the momentum carried off by the light. Express this force in terms of the radiated power of the flashlight. Use this force to calculate your acceleration and use a constant acceleration equation to find the time.

(a) **EXECUTE:** $p_{\text{rad}} = I/c$ and $F = p_{\text{rad}} A$ gives $F = IA/c = P_{\text{av}}/c$

$$a_x = F/m = P_{\text{av}}/(mc) = (200 \text{ W})/[(150 \text{ kg})(3.00 \times 10^8 \text{ m/s})] = 4.44 \times 10^{-9} \text{ m/s}^2$$

$$\text{Then } x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ gives } t = \sqrt{2(x - x_0)/a_x} = \sqrt{2(16.0 \text{ m})/(4.44 \times 10^{-9} \text{ m/s}^2)} = 8.49 \times 10^4 \text{ s} = 23.6 \text{ h}$$

EVALUATE: The radiation force is very small. In the calculation we have ignored any other forces on you.

(b) You could throw the flashlight in the direction away from the ship. By conservation of linear momentum you would move toward the ship with the same magnitude of momentum as you gave the flashlight.

32.52. IDENTIFY: $P_{\text{av}} = IA$ and $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$

SET UP: The power carried by the current i is $P = Vi$.

$$\text{EXECUTE: } I = \frac{P_{\text{av}}}{A} = \frac{1}{2}\epsilon_0 c E_{\max}^2 \text{ and } E_{\max} = \sqrt{\frac{2P_{\text{av}}}{A\epsilon_0 c}} = \sqrt{\frac{2Vi}{A\epsilon_0 c}} = \sqrt{\frac{2(5.00 \times 10^5 \text{ V})(1000 \text{ A})}{(100 \text{ m}^2)\epsilon_0(3.00 \times 10^8 \text{ m/s})}} = 6.14 \times 10^4 \text{ V/m.}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{6.14 \times 10^4 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 2.05 \times 10^{-4} \text{ T.}$$

EVALUATE: $I = Vi/A = \frac{(5.00 \times 10^5 \text{ V})(1000 \text{ A})}{100 \text{ m}^2} = 5.00 \times 10^6 \text{ W/m}^2$. This is a very intense beam spread over a large area.

32.53. IDENTIFY: The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/R = GmM/r^2$, where M is the mass of the Earth and m is the mass of the satellite. The intensity I of the wave is $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\max}^2$, and by definition, $I = P_{\text{av}}/A$.

EXECUTE: (a) The period of the orbit is 12 hr. Applying Newton's 2nd law to the satellite gives $mv^2/R = GmM/r^2$,

which gives $\frac{m(2\pi r/T)^2}{r} = \frac{GmM}{r^2}$. Solving for r , we get

$$r = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} = \left[\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(12 \times 3600 \text{ s})^2}{4\pi^2} \right]^{1/3} = 2.66 \times 10^7 \text{ m}$$

The height above the surface is $h = 2.66 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m} = 2.02 \times 10^7 \text{ m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I = P_{\text{av}}/A = P_{\text{av}}/(2\pi h^2) = (25.0 \text{ W})/[2\pi(2.02 \times 10^7 \text{ m})^2] = 9.75 \times 10^{-15} \text{ W/m}^2$$

$$\text{(b) } I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\max}^2, \text{ so } E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(9.75 \times 10^{-15} \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 2.71 \times 10^{-6} \text{ N/C}$$

$$B_{\max} = E_{\max}/c = (2.71 \times 10^{-6} \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = 9.03 \times 10^{-15} \text{ T}$$

$$t = d/c = (2.02 \times 10^7 \text{ m})/(3.00 \times 10^8 \text{ m/s}) = 0.0673 \text{ s}$$

(c) $P_{\text{av}} = I/c = (9.75 \times 10^{-15} \text{ W/m}^2)/(3.00 \times 10^8 \text{ m/s}) = 3.25 \times 10^{-23} \text{ Pa}$

(d) $\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(1575.42 \times 10^6 \text{ Hz}) = 0.190 \text{ m}$

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.54. IDENTIFY: For a totally reflective surface the radiation pressure is $\frac{2I}{c}$. Find the force due to this pressure and

express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_g = G \frac{mM_{\text{sun}}}{r^2}$.

SET UP: The mass of the sun is $M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg}$. $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

EXECUTE: (a) The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{\text{rad}} = \left(\frac{2I}{c}\right)A$, where A is the area of the sail. $I = \frac{P}{4\pi r^2}$, where r is the distance of the sail from the sun.

$$F_{\text{rad}} = \left(\frac{2A}{c}\right)\left(\frac{P}{4\pi r^2}\right) = \frac{PA}{2\pi r^2 c}. \quad F_{\text{rad}} = F_g \quad \text{so} \quad \frac{PA}{2\pi r^2 c} = G \frac{mM_{\text{sun}}}{r^2}.$$

$$A = \frac{2\pi c G m M_{\text{sun}}}{P} = \frac{2\pi (3.00 \times 10^8 \text{ m/s})(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(10,000 \text{ kg})(1.99 \times 10^{30} \text{ kg})}{3.9 \times 10^{26} \text{ W}}.$$

$$A = 6.42 \times 10^6 \text{ m}^2 = 6.42 \text{ km}^2.$$

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{\text{rad}} = F_g$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.55. IDENTIFY and SET UP: The gravitational force is given by Eq.(12.2). Express the mass of the particle in terms of its density and volume. The radiation pressure is given by Eq.(32.32); relate the power output L of the sun to the intensity at a distance r . The radiation force is the pressure times the cross sectional area of the particle.

EXECUTE: (a) The gravitational force is $F_g = G \frac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho \frac{4}{3} \pi R^3$. Thus

$$F_g = \frac{4\rho G \pi M R^3}{3r^2}.$$

(b) For a totally absorbing surface $p_{\text{rad}} = \frac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation

a distance r from the sun is $I = \frac{L}{4\pi r^2}$. Thus $p_{\text{rad}} = \frac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the

direction of propagation of the radiation, so $F_{\text{rad}} = p_{\text{rad}} A_{\perp}$, where $A_{\perp} = \pi R^2$ is the component of area of the particle

perpendicular to the radiation direction. Thus $F_{\text{rad}} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}$.

(c) $F_g = F_{\text{rad}}$

$$\frac{4\rho G \pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}$$

$$\left(\frac{4\rho G \pi M}{3}\right)R = \frac{L}{4c} \quad \text{and} \quad R = \frac{3L}{16c\rho G \pi M}$$

$$R = \frac{3(3.9 \times 10^{26} \text{ W})}{16(2.998 \times 10^8 \text{ m/s})(3000 \text{ kg/m}^3)(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)\pi(1.99 \times 10^{30} \text{ kg})}$$

$$R = 1.9 \times 10^{-7} \text{ m} = 0.19 \text{ } \mu\text{m}.$$

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R .

(d) $\frac{F_{\text{rad}}}{F_g} = \left(\frac{LR^2}{4cr^2}\right)\left(\frac{3r^2}{4\rho G \pi M R^3}\right) = \frac{3L}{16c\rho G \pi M R}$. F_{rad} is proportional to R^2 and F_g is proportional to R^3 , so this

ratio is proportional to $1/R$. If $R < 0.20 \text{ } \mu\text{m}$ then $F_{\text{rad}} > F_g$ and the radiation force will drive the particles out of the solar system.

32.56. IDENTIFY: The electron has acceleration $a = \frac{v^2}{R}$.

SET UP: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. An electron has $|q| = e = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(13.6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{(9.11 \times 10^{-31} \text{ kg})(5.29 \times 10^{-11} \text{ m})} = 9.03 \times 10^{22} \text{ m/s}^2. \text{ Then using the formula for the rate of energy}$$

emission given in Problem 32.57:

$$\frac{dE}{dt} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = \frac{(1.60 \times 10^{-19} \text{ C})^2 (9.03 \times 10^{22} \text{ m/s}^2)^2}{6\pi\epsilon_0 (3.00 \times 10^8 \text{ m/s})^3} = 4.64 \times 10^{-8} \text{ J/s} = 2.89 \times 10^{11} \text{ eV/s}. \text{ This large value of } \frac{dE}{dt}$$

would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.57 must not apply to electrons in atoms.

32.57. IDENTIFY: The orbiting particle has acceleration $a = \frac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11 \times 10^{-31} \text{ kg}$ and a proton has mass $m_p = 1.67 \times 10^{-27} \text{ kg}$.

EXECUTE: (a) $\left[\frac{q^2 a^2}{6\pi\epsilon_0 c^3} \right] = \frac{C^2 (m/s^2)^2}{(C^2/N \cdot m^2)(m/s)^3} = \frac{N \cdot m}{s} = \frac{J}{s} = W = \left[\frac{dE}{dt} \right]$.

(b) For a proton moving in a circle, the acceleration is

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(6.00 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(1.67 \times 10^{-27} \text{ kg})(0.75 \text{ m})} = 1.53 \times 10^{15} \text{ m/s}^2. \text{ The rate at which it emits energy because of}$$

$$\text{its acceleration is } \frac{dE}{dt} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = \frac{(1.6 \times 10^{-19} \text{ C})^2 (1.53 \times 10^{15} \text{ m/s}^2)^2}{6\pi\epsilon_0 (3.0 \times 10^8 \text{ m/s})^3} = 1.33 \times 10^{-23} \text{ J/s} = 8.32 \times 10^{-5} \text{ eV/s}.$$

$$\text{Therefore, the fraction of its energy that it radiates every second is } \frac{(dE/dt)(1 \text{ s})}{E} = \frac{8.32 \times 10^{-5} \text{ eV}}{6.00 \times 10^6 \text{ eV}} = 1.39 \times 10^{-11}.$$

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses:

$$E_e = E_p \frac{m_e}{m_p} = (6.00 \times 10^6 \text{ eV}) \frac{(9.11 \times 10^{-31} \text{ kg})}{(1.67 \times 10^{-27} \text{ kg})} = 3273 \text{ eV}. \text{ Therefore, the fraction of its energy that it radiates every}$$

$$\text{second is } \frac{(dE/dt)(1 \text{ s})}{E} = \frac{8.32 \times 10^{-5} \text{ eV}}{3273 \text{ eV}} = 2.54 \times 10^{-8}.$$

EVALUATE: The proton has speed $v = \sqrt{\frac{2E}{m_p}} = \sqrt{\frac{2(6.0 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}} = 3.39 \times 10^7 \text{ m/s}$. The electron

has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.56 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.58. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) $E_y(x, t) = E_{\max} e^{-k_C x} \sin(k_C x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max} (-k_C) e^{-k_C x} \sin(k_C x - \omega t) + E_{\max} (+k_C) e^{-k_C x} \cos(k_C x - \omega t)$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max} (+k_C^2) e^{-k_C x} \sin(k_C x - \omega t) + E_{\max} (-k_C^2) e^{-k_C x} \cos(k_C x - \omega t) + E_{\max} (-k_C^2) e^{-k_C x} \cos(k_C x - \omega t) + E_{\max} (-k_C^2) e^{-k_C x} \sin(k_C x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = -2E_{\max} k_C^2 e^{-k_C x} \cos(k_C x - \omega t). \quad \frac{\partial E_y}{\partial t} = E_{\max} e^{-k_C x} \omega \cos(k_C x - \omega t).$$

Setting $\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu \partial E_y}{\rho \partial t}$ gives $2E_{\max} k_C^2 e^{-k_C x} \cos(k_C x - \omega t) = E_{\max} e^{-k_C x} \omega \cos(k_C x - \omega t)$. This will only be true if

$$\frac{2k_C^2}{\omega} = \frac{\mu}{\rho}, \text{ or } k_C = \sqrt{\frac{\omega \mu}{2\rho}}.$$

(b) The energy in the wave is dissipated by the $i^2 R$ heating of the conductor.

(c) $E_y = \frac{E_{y0}}{e} \Rightarrow k_C x = 1, x = \frac{1}{k_C} = \sqrt{\frac{2\rho}{\omega \mu}} = \sqrt{\frac{2(1.72 \times 10^{-8} \Omega \cdot \text{m})}{2\pi(1.0 \times 10^6 \text{ Hz})\mu_0}} = 6.60 \times 10^{-5} \text{ m}.$

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

THE NATURE AND PROPAGATION OF LIGHT

33.1. IDENTIFY: For reflection, $\theta_r = \theta_a$.

SET UP: The desired path of the ray is sketched in Figure 33.1.

EXECUTE: $\tan \phi = \frac{14.0 \text{ cm}}{11.5 \text{ cm}}$, so $\phi = 50.6^\circ$. $\theta_r = 90^\circ - \phi = 39.4^\circ$ and $\theta_r = \theta_a = 39.4^\circ$.

EVALUATE: The angle of incidence is measured from the normal to the surface.

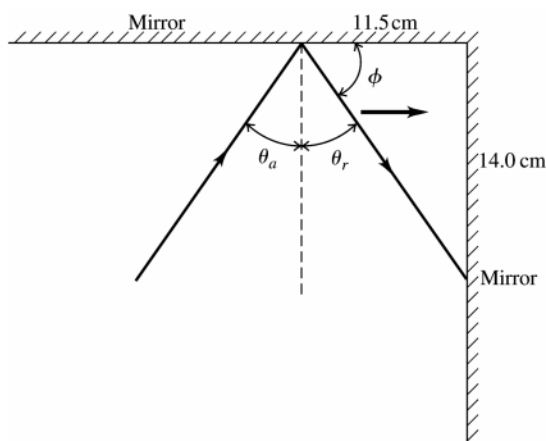


Figure 33.1

33.2 IDENTIFY: For reflection, $\theta_r = \theta_a$.

SET UP: The angles of incidence and reflection at each reflection are shown in Figure 33.2. For the rays to be perpendicular when they cross, $\alpha = 90^\circ$.

EXECUTE: (a) $\theta + \phi = 90^\circ$ and $\beta + \phi = 90^\circ$, so $\beta = \theta$. $\frac{\alpha}{2} + \beta = 90^\circ$ and $\alpha = 180^\circ - 2\theta$.

(b) $\theta = \frac{1}{2}(180^\circ - \alpha) = \frac{1}{2}(180^\circ - 90^\circ) = 45^\circ$.

EVALUATE: As $\theta \rightarrow 0^\circ$, $\alpha \rightarrow 180^\circ$. This corresponds to the incident and reflected rays traveling in nearly the same direction. As $\theta \rightarrow 90^\circ$, $\alpha \rightarrow 0^\circ$. This corresponds to the incident and reflected rays traveling in nearly opposite directions.

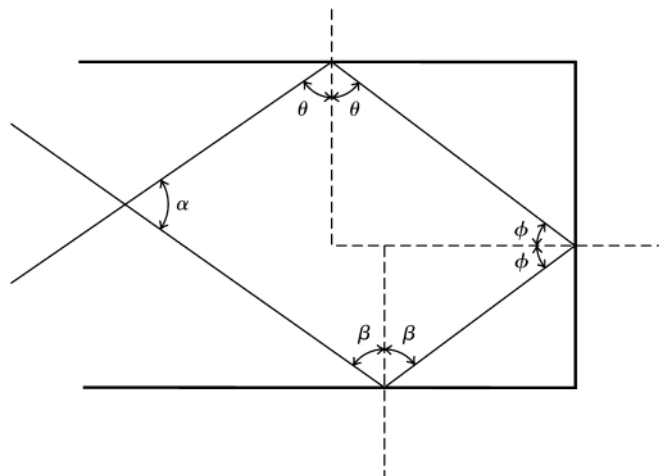


Figure 33.2

33.3. IDENTIFY and SET UP: Use Eqs.(33.1) and (33.5) to calculate v and λ .

EXECUTE: (a) $n = \frac{c}{v}$ so $v = \frac{c}{n} = \frac{2.998 \times 10^8 \text{ m/s}}{1.47} = 2.04 \times 10^8 \text{ m/s}$

(b) $\lambda = \frac{\lambda_0}{n} = \frac{650 \text{ nm}}{1.47} = 442 \text{ nm}$

EVALUATE: Light is slower in the liquid than in vacuum. By $v = f\lambda$, when v is smaller, λ is smaller.

33.4. IDENTIFY: In air, $c = f\lambda_0$. In glass, $\lambda = \frac{\lambda_0}{n}$.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$

EXECUTE: (a) $\lambda_0 = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.80 \times 10^{14} \text{ Hz}} = 517 \text{ nm}$

(b) $\lambda = \frac{\lambda_0}{n} = \frac{517 \text{ nm}}{1.52} = 340 \text{ nm}$

EVALUATE: In glass the light travels slower than in vacuum and the wavelength is smaller.

33.5. IDENTIFY: $n = \frac{c}{v}$. $\lambda = \frac{\lambda_0}{n}$, where λ_0 is the wavelength in vacuum.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$. n for air is only slightly larger than unity.

EXECUTE: (a) $n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{1.94 \times 10^8 \text{ m/s}} = 1.54$.

(b) $\lambda_0 = n\lambda = (1.54)(3.55 \times 10^{-7} \text{ m}) = 5.47 \times 10^{-7} \text{ m}$.

EVALUATE: In quartz the speed is lower and the wavelength is smaller than in air.

33.6. IDENTIFY: $\lambda = \frac{\lambda_0}{n}$.

SET UP: From Table 33.1, $n_{\text{water}} = 1.333$ and $n_{\text{benzene}} = 1.501$.

EXECUTE: (a) $\lambda_{\text{water}} n_{\text{water}} = \lambda_{\text{benzene}} n_{\text{benzene}} = \lambda_0$. $\lambda_{\text{benzene}} = \lambda_{\text{water}} \left(\frac{n_{\text{water}}}{n_{\text{benzene}}} \right) = (438 \text{ nm}) \left(\frac{1.333}{1.501} \right) = 389 \text{ nm}$.

(b) $\lambda_0 = \lambda_{\text{water}} n_{\text{water}} = (438 \text{ nm})(1.333) = 584 \text{ nm}$

EVALUATE: λ is smallest in benzene, since n is largest for benzene.

33.7. IDENTIFY: Apply Eqs.(33.2) and (33.4) to calculate θ_r and θ_b . The angles in these equations are measured with respect to the normal, not the surface.

(a) **SET UP:** The incident, reflected and refracted rays are shown in Figure 33.7.

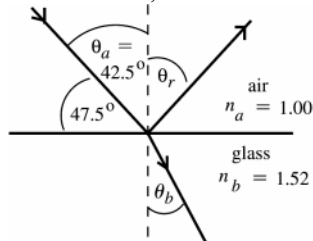


Figure 33.7

EXECUTE: $\theta_r = \theta_a = 42.5^\circ$

The reflected ray makes an angle of $90.0^\circ - \theta_r = 47.5^\circ$ with the surface of the glass.

(b) $n_a \sin \theta_a = n_b \sin \theta_b$, where the angles are measured from the normal to the interface.

$$\sin \theta_b = \frac{n_a \sin \theta_a}{n_b} = \frac{(1.00)(\sin 42.5^\circ)}{1.66} = 0.4070$$

$$\theta_b = 24.0^\circ$$

The refracted ray makes an angle of $90.0^\circ - \theta_b = 66.0^\circ$ with the surface of the glass.

EVALUATE: The light is bent toward the normal when the light enters the material of larger refractive index.

33.8. IDENTIFY: Use the distance and time to find the speed of light in the plastic. $n = \frac{c}{v}$.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$

EXECUTE: $v = \frac{d}{t} = \frac{2.50 \text{ m}}{11.5 \times 10^{-9} \text{ s}} = 2.17 \times 10^8 \text{ m/s}$. $n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{2.17 \times 10^8 \text{ m/s}} = 1.38$.

EVALUATE: In air light travels this same distance in $\frac{2.50 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 8.3 \text{ ns}$.

- 33.9. IDENTIFY and SET UP:** Use Snell's law to find the index of refraction of the plastic and then use Eq.(33.1) to calculate the speed v of light in the plastic.

EXECUTE: $n_a \sin \theta_a = n_b \sin \theta_b$

$$n_b = n_a \left(\frac{\sin \theta_a}{\sin \theta_b} \right) = 1.00 \left(\frac{\sin 62.7^\circ}{\sin 48.1^\circ} \right) = 1.194$$

$$n = \frac{c}{v} \text{ so } v = \frac{c}{n} = (3.00 \times 10^8 \text{ m/s}) / 1.194 = 2.51 \times 10^8 \text{ m/s}$$

EVALUATE: Light is slower in plastic than in air. When the light goes from air into the plastic it is bent toward the normal.

- 33.10. IDENTIFY:** Apply Snell's law at both interfaces.

SET UP: The path of the ray is sketched in Figure 33.10. Table 33.1 gives $n = 1.329$ for the methanol.

EXECUTE: (a) At the air-glass interface $(1.00) \sin 41.3^\circ = n_{\text{glass}} \sin \alpha$. At the glass-methanol interface $n_{\text{glass}} \sin \alpha = (1.329) \sin \theta$. Combining these two equations gives $\sin 41.3^\circ = 1.329 \sin \theta$ and $\theta = 29.8^\circ$.

(b) The same figures applies as for part (a), except $\theta = 20.2^\circ$. $(1.00) \sin 41.3^\circ = n \sin 20.2^\circ$ and $n = 1.91$.

EVALUATE: The angle α is 25.2° . The index of refraction of methanol is less than that of the glass and the ray is bent away from the normal at the glass $\rightarrow$ methanol interface. The unknown liquid has an index of refraction greater than that of the glass, so the ray is bent toward the normal at the glass $\rightarrow$ liquid interface.

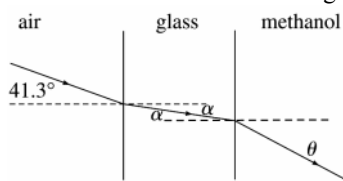


Figure 33.10

- 33.11. IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Let the light initially be in the material with refractive index n_a and let the final slab have refractive index n_b . In part (a) let the middle slab have refractive index n_1 .

EXECUTE: (a) 1st interface: $n_a \sin \theta_a = n_1 \sin \theta_1$. 2nd interface: $n_1 \sin \theta_1 = n_b \sin \theta_b$. Combining the two equations gives $n_a \sin \theta_a = n_b \sin \theta_b$. This is the equation that would apply if the middle slab were absent.

(b) For N slabs, $n_a \sin \theta_a = n_1 \sin \theta_1$, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, ..., $n_{N-2} \sin \theta_{N-2} = n_b \sin \theta_b$. Combining all these equations gives $n_a \sin \theta_a = n_b \sin \theta_b$.

EVALUATE: The final direction of travel depends on the angle of incidence in the first slab and the refractive indices of the first and last slabs.

- 33.12. IDENTIFY:** Apply Snell's law to the refraction at each interface.

SET UP: $n_{\text{air}} = 1.00$. $n_{\text{water}} = 1.333$.

$$\text{EXECUTE: (a) } \theta_{\text{water}} = \arcsin \left(\frac{n_{\text{air}}}{n_{\text{water}}} \sin \theta_{\text{air}} \right) = \arcsin \left(\frac{1.00}{1.333} \sin 35.0^\circ \right) = 25.5^\circ$$

EVALUATE: (b) This calculation has no dependence on the glass because we can omit that step in the chain: $n_{\text{air}} \sin \theta_{\text{air}} = n_{\text{glass}} \sin \theta_{\text{glass}} = n_{\text{water}} \sin \theta_{\text{water}}$.

- 33.13. IDENTIFY:** When a wave passes from one material into another, the number of waves per second that cross the boundary is the same on both sides of the boundary, so the frequency does not change. The wavelength and speed of the wave, however, do change.

SET UP: In a material having index of refraction n , the wavelength is $\lambda = \frac{\lambda_0}{n}$, where λ_0 is the wavelength in vacuum, and the speed is $\frac{c}{n}$.

EXECUTE: (a) The frequency is the same, so it is still f . The wavelength becomes $\lambda = \frac{\lambda_0}{n}$, so $\lambda_0 = n\lambda$. The speed

is $v = \frac{c}{n}$, so $c = nv$.

(b) The frequency is still f . The wavelength becomes $\lambda' = \frac{\lambda_0}{n'} = \frac{n\lambda}{n'} = \left(\frac{n}{n'} \right) \lambda$ and the speed becomes

$$v' = \frac{c}{n'} = \frac{nv}{n'} = \left(\frac{n}{n'} \right) v$$

EVALUATE: These results give the speed and wavelength in a new medium in terms of the original medium without referring them to the values in vacuum (or air).

33.14. IDENTIFY: Apply the law of reflection.

SET UP: The mirror in its original position and after being rotated by an angle θ are shown in Figure 33.14. α is the angle through which the reflected ray rotates when the mirror rotates. The two angles labeled ϕ are equal and the two angles labeled ϕ' are equal because of the law of reflection. The two angles labeled θ are equal because the lines forming one angle are perpendicular to the lines forming the other angle.

EXECUTE: From the diagram, $\alpha = 2\phi' - 2\phi = 2(\phi' - \phi)$ and $\theta = \phi' - \phi$. $\alpha = 2\theta$, as was to be shown.

EVALUATE: This result is independent of the initial angle of incidence.

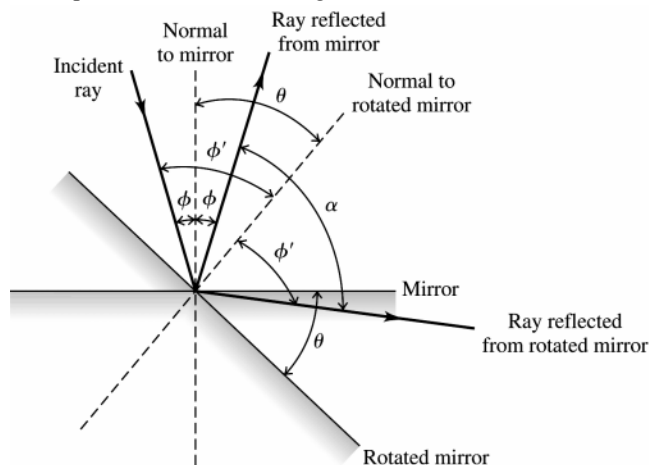


Figure 33.14

33.15. IDENTIFY: Apply $n_a \sin \theta_a = n_b \sin \theta_b$.

SET UP: The light refracts from the liquid into the glass, so $n_a = 1.70$, $\theta_a = 62.0^\circ$. $n_b = 1.58$.

EXECUTE: $\sin \theta_b = \left(\frac{n_a}{n_b} \right) \sin \theta_a = \left(\frac{1.70}{1.58} \right) \sin 62.0^\circ = 0.950$ and $\theta_b = 71.8^\circ$.

EVALUATE: The ray refracts into a material of smaller n , so it is bent away from the normal.

33.16. IDENTIFY: Apply Snell's law.

SET UP: θ_a and θ_b are measured relative to the normal to the surface of the interface. $\theta_a = 60.0^\circ - 15.0^\circ = 45.0^\circ$.

EXECUTE: $\theta_b = \arcsin \left(\frac{n_a}{n_b} \sin \theta_a \right) = \arcsin \left(\frac{1.33}{1.52} \sin 45.0^\circ \right) = 38.2^\circ$. But this is the angle from the normal to the surface,

so the angle from the vertical is an additional 15° because of the tilt of the surface. Therefore, the angle is 53.2° .

EVALUATE: Compared to Example 33.1, θ_a is shifted by 15° but the shift in θ_b is only $53.2^\circ - 49.3^\circ = 3.9^\circ$.

33.17. IDENTIFY: The critical angle for total internal reflection is θ_a that gives $\theta_b = 90^\circ$ in Snell's law.

SET UP: In Figure 33.17 the angle of incidence θ_a is related to angle θ by $\theta_a + \theta = 90^\circ$.

EXECUTE: (a) Calculate θ_a that gives $\theta_b = 90^\circ$. $n_a = 1.60$, $n_b = 1.00$ so $n_a \sin \theta_a = n_b \sin \theta_b$ gives

$$(1.60) \sin \theta_a = (1.00) \sin 90^\circ. \sin \theta_a = \frac{1.00}{1.60} \text{ and } \theta_a = 38.7^\circ. \theta = 90^\circ - \theta_a = 51.3^\circ.$$

(b) $n_a = 1.60$, $n_b = 1.333$. $(1.60) \sin \theta_a = (1.333) \sin 90^\circ$. $\sin \theta_a = \frac{1.333}{1.60}$ and $\theta_a = 56.4^\circ$. $\theta = 90^\circ - \theta_a = 33.6^\circ$.

EVALUATE: The critical angle increases when the ratio $\frac{n_a}{n_b}$ increases.

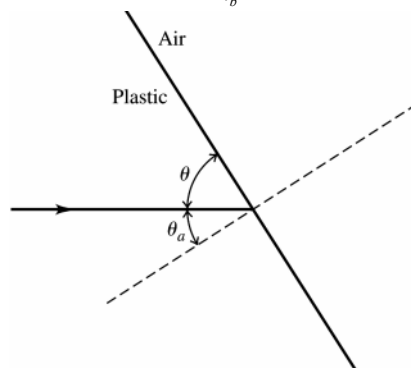


Figure 33.17

33.18. IDENTIFY: Since the refractive index of the glass is greater than that of air or water, total internal reflection will occur at the cube surface if the angle of incidence is greater than or equal to the critical angle.

SET UP: At the critical angle θ_c , Snell's law gives $n_{\text{glass}} \sin \theta_c = n_{\text{air}} \sin 90^\circ$ and likewise for water.

EXECUTE: (a) At the critical angle θ_c , $n_{\text{glass}} \sin \theta_c = n_{\text{air}} \sin 90^\circ$. $1.53 \sin \theta_c = (1.00)(1)$ and $\theta_c = 40.8^\circ$.

(b) Using the same procedure as in part (a), we have $1.53 \sin \theta_c = 1.333 \sin 90^\circ$ and $\theta_c = 60.6^\circ$.

EVALUATE: Since the refractive index of water is closer to the refractive index of glass than the refractive index of air is, the critical angle for glass-to-water is greater than for glass-to-air.

33.19. IDENTIFY: Use the critical angle to find the index of refraction of the liquid.

SET UP: Total internal reflection requires that the light be incident on the material with the larger n , in this case the liquid. Apply $n_a \sin \theta_a = n_b \sin \theta_b$ with $a = \text{liquid}$ and $b = \text{air}$, so $n_a = n_{\text{liq}}$ and $n_b = 1.0$.

EXECUTE: $\theta_a = \theta_{\text{crit}}$ when $\theta_b = 90^\circ$, so $n_{\text{liq}} \sin \theta_{\text{crit}} = (1.0) \sin 90^\circ$

$$n_{\text{liq}} = \frac{1}{\sin \theta_{\text{crit}}} = \frac{1}{\sin 42.5^\circ} = 1.48.$$

(a) $n_a \sin \theta_a = n_b \sin \theta_b$ ($a = \text{liquid}$, $b = \text{air}$)

$$\sin \theta_b = \frac{n_a \sin \theta_a}{n_b} = \frac{(1.48) \sin 35.0^\circ}{1.0} = 0.8489 \text{ and } \theta_b = 58.1^\circ$$

(b) Now $n_a \sin \theta_a = n_b \sin \theta_b$ with $a = \text{air}$, $b = \text{liquid}$

$$\sin \theta_b = \frac{n_a \sin \theta_a}{n_b} = \frac{(1.0) \sin 35.0^\circ}{1.48} = 0.3876 \text{ and } \theta_b = 22.8^\circ$$

EVALUATE: For light traveling liquid $\rightarrow$ air the light is bent away from the normal. For light traveling air $\rightarrow$ liquid the light is bent toward the normal.

33.20. IDENTIFY: The largest angle of incidence for which any light refracts into the air is the critical angle for water $\rightarrow$ air.

SET UP: Figure 33.20 shows a ray incident at the critical angle and therefore at the edge of the ring of light. The radius of this circle is r and $d = 10.0$ m is the distance from the ring to the surface of the water.

EXECUTE: From the figure, $r = d \tan \theta_{\text{crit}}$. θ_{crit} is calculated from $n_a \sin \theta_a = n_b \sin \theta_b$ with $n_a = 1.333$, $\theta_a = \theta_{\text{crit}}$,

$$n_b = 1.00 \text{ and } \theta_b = 90^\circ. \sin \theta_{\text{crit}} = \frac{(1.00) \sin 90^\circ}{1.333} \text{ and } \theta_{\text{crit}} = 48.6^\circ. r = (10.0 \text{ m}) \tan 48.6^\circ = 11.3 \text{ m}.$$

$$A = \pi r^2 = \pi (11.3 \text{ m})^2 = 401 \text{ m}^2.$$

EVALUATE: When the incident angle in the water is larger than the critical angle, no light refracts into the air.

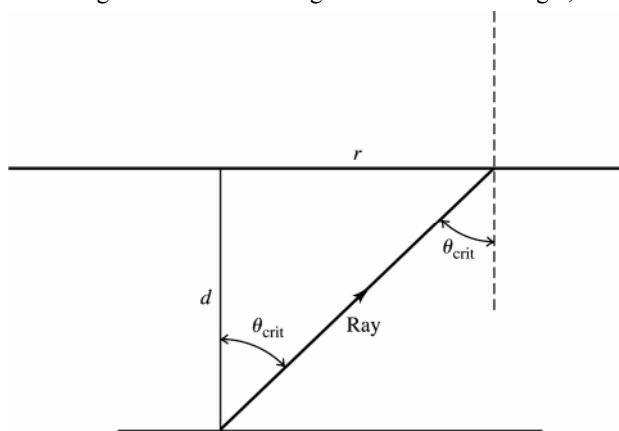


Figure 33.20

33.21. IDENTIFY and SET UP: For glass $\rightarrow$ water, $\theta_{\text{crit}} = 48.7^\circ$. Apply Snell's law with $\theta_a = \theta_{\text{crit}}$ to calculate the index of refraction n_a of the glass.

$$\text{EXECUTE: } n_a \sin \theta_{\text{crit}} = n_b \sin 90^\circ, \text{ so } n_a = \frac{n_b}{\sin \theta_{\text{crit}}} = \frac{1.333}{\sin 48.7^\circ} = 1.77$$

EVALUATE: For total internal reflection to occur the light must be incident in the material of larger refractive index. Our results give $n_{\text{glass}} > n_{\text{water}}$, in agreement with this.

- 33.22. IDENTIFY:** If no light refracts out of the glass at the glass to air interface, then the incident angle at that interface is θ_{crit} .

SET UP: The ray has an angle of incidence of 0° at the first surface of the glass, so enters the glass without being bent, as shown in Figure 33.22. The figure shows that $\alpha + \theta_{\text{crit}} = 90^\circ$.

EXECUTE: (a) For the glass-air interface $\theta_a = \theta_{\text{crit}}$, $n_a = 1.52$, $n_b = 1.00$ and $\theta_b = 90^\circ$. $n_a \sin \theta_a = n_b \sin \theta_b$ gives $\sin \theta_{\text{crit}} = \frac{(1.00)(\sin 90^\circ)}{1.52}$ and $\theta_{\text{crit}} = 41.1^\circ$. $\alpha = 90^\circ - \theta_{\text{crit}} = 48.9^\circ$.

(b) Now the second interface is glass $\rightarrow$ water and $n_b = 1.333$. $n_a \sin \theta_a = n_b \sin \theta_b$ gives $\sin \theta_{\text{crit}} = \frac{(1.333)(\sin 90^\circ)}{1.52}$ and $\theta_{\text{crit}} = 61.3^\circ$. $\alpha = 90^\circ - \theta_{\text{crit}} = 28.7^\circ$.

EVALUATE: The critical angle increases when the air is replaced by water and rays are bent as they refract out of the glass.

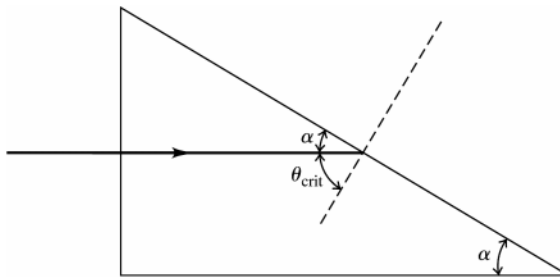


Figure 33.22

- 33.23. IDENTIFY:** Apply $n_a \sin \theta_a = n_b \sin \theta_b$.

SET UP: The light is in diamond and encounters an interface with air, so $n_a = 2.42$ and $n_b = 1.00$. The largest θ_a is when $\theta_b = 90^\circ$.

EXECUTE: $(2.42) \sin \theta_a = (1.00) \sin 90^\circ$. $\sin \theta_a = \frac{1}{2.42}$ and $\theta_a = 24.4^\circ$.

EVALUATE: Diamond has an usually large refractive index, and this results in a small critical angle.

- 33.24. IDENTIFY:** Snell's law is $n_a \sin \theta_a = n_b \sin \theta_b$. $v = \frac{c}{n}$.

SET UP: $a = \text{air}$, $b = \text{glass}$.

EXECUTE: (a) red: $n_b = \frac{n_a \sin \theta_a}{\sin \theta_b} = \frac{(1.00) \sin 57.0^\circ}{\sin 38.1^\circ} = 1.36$. violet: $n_b = \frac{(1.00) \sin 57.0^\circ}{\sin 36.7^\circ} = 1.40$.

(b) red: $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.36} = 2.21 \times 10^8 \text{ m/s}$; violet: $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.40} = 2.14 \times 10^8 \text{ m/s}$.

EVALUATE: n is larger for the violet light and therefore this light is bent more toward the normal, and the violet light has a smaller speed in the glass than the red light.

- 33.25. IDENTIFY:** When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{\text{max}} \cos^2 \phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

SET UP: For the second polarizer $\phi = 60^\circ$. For the third polarizer, $\phi = 90^\circ - 60^\circ = 30^\circ$.

EXECUTE: (a) At point A the intensity is $I_0/2$ and the light is polarized along the vertical direction. At point B the intensity is $(I_0/2)(\cos 60^\circ)^2 = 0.125 I_0$, and the light is polarized along the axis of the second polarizer. At point C the intensity is $(0.125 I_0)(\cos 30^\circ)^2 = 0.0938 I_0$.

(b) Now for the last filter $\phi = 90^\circ$ and $I = 0$.

EVALUATE: Adding the middle filter increases the transmitted intensity.

- 33.26. IDENTIFY:** Apply Snell's law.

SET UP: The incident, reflected and refracted rays are shown in Figure 33.26.

EXECUTE: From the figure, $\theta_b = 37.0^\circ$ and $n_b = n_a \frac{\sin \theta_a}{\sin \theta_b} = 1.33 \frac{\sin 53^\circ}{\sin 37^\circ} = 1.77$.

EVALUATE: The refractive index of b is greater than that of a , and the ray is bent toward the normal when it refracts.

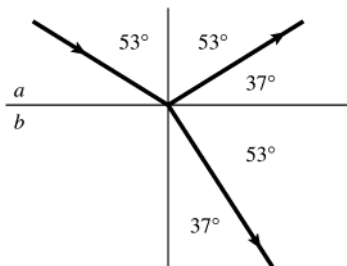


Figure 33.26

- 33.27. IDENTIFY and SET UP:** Reflected beam completely linearly polarized implies that the angle of incidence equals the polarizing angle, so $\theta_p = 54.5^\circ$. Use Eq.(33.8) to calculate the refractive index of the glass. Then use Snell's law to calculate the angle of refraction.

EXECUTE: (a) $\tan \theta_p = \frac{n_b}{n_a}$ gives $n_{\text{glass}} = n_{\text{air}} \tan \theta_p = (1.00) \tan 54.5^\circ = 1.40$.

(b) $n_a \sin \theta_a = n_b \sin \theta_b$

$$\sin \theta_b = \frac{n_a \sin \theta_a}{n_b} = \frac{(1.00) \sin 54.5^\circ}{1.40} = 0.5815 \text{ and } \theta_b = 35.5^\circ$$

EVALUATE:

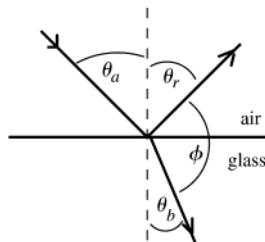


Figure 33.27

Note: $\phi = 180.0^\circ - \theta_r - \theta_b$ and $\theta_r = \theta_a$. Thus $\phi = 180.0^\circ - 54.5^\circ - 35.5^\circ = 90.0^\circ$; the reflected ray and the refracted ray are perpendicular to each other. This agrees with Fig.33.28.

- 33.28. IDENTIFY:** Set $I = I_0/10$, where I is the intensity of light passed by the second polarizer.

SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{\text{max}} \cos^2 \phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

EXECUTE: (a) After the first filter $I = \frac{I_0}{2}$ and the light is polarized along the vertical direction. After the second

filter we want $I = \frac{I_0}{10}$, so $\frac{I_0}{10} = \left(\frac{I_0}{2}\right)(\cos \phi)^2$. $\cos \phi = \sqrt{2/10}$ and $\phi = 63.4^\circ$.

(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter $\frac{I_0}{10} = I_0(\cos \phi)^2$.

$$\cos \phi = \sqrt{1/10} \text{ and } \phi = 71.6^\circ.$$

EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.

- 33.29. IDENTIFY:** From Malus's law, the intensity of the emerging light is proportional to the *square* of the cosine of the angle between the polarizing axes of the two filters.

SET UP: If the angle between the two axes is θ , the intensity of the emerging light is $I = I_{\text{max}} \cos^2 \theta$.

EXECUTE: At angle θ , $I = I_{\text{max}} \cos^2 \theta$, and at the new angle α , $\frac{1}{2} I = I_{\text{max}} \cos^2 \alpha$. Taking the ratio of the intensities

$$\text{gives } \frac{I_{\text{max}} \cos^2 \alpha}{I_{\text{max}} \cos^2 \theta} = \frac{\frac{1}{2} I}{I}, \text{ which gives us } \cos \alpha = \frac{\cos \theta}{\sqrt{2}}. \text{ Solving for } \alpha \text{ yields } \alpha = \arccos\left(\frac{\cos \theta}{\sqrt{2}}\right).$$

EVALUATE: Careful! This result is not $\cos^2 \theta$.

33.30. IDENTIFY: The reflected light is completely polarized when the angle of incidence equals the polarizing angle θ_p , where $\tan \theta_p = \frac{n_b}{n_a}$.

SET UP: $n_b = 1.66$.

EXECUTE: (a) $n_a = 1.00$. $\tan \theta_p = \frac{1.66}{1.00}$ and $\theta_p = 58.9^\circ$.

(b) $n_a = 1.333$. $\tan \theta_p = \frac{1.66}{1.333}$ and $\theta_p = 51.2^\circ$.

EVALUATE: The polarizing angle depends on the refractive indices of both materials at the interface.

33.31. IDENTIFY: When unpolarized light of intensity I_0 is incident on a polarizing filter, the transmitted light has intensity $\frac{1}{2}I_0$ and is polarized along the filter axis. When polarized light of intensity I_0 is incident on a polarizing filter the transmitted light has intensity $I_0 \cos^2 \phi$.

SET UP: For the second filter, $\phi = 62.0^\circ - 25.0^\circ = 37.0^\circ$.

EXECUTE: After the first filter the intensity is $\frac{1}{2}I_0 = 10.0 \text{ W/m}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0 \cos^2 \phi$, where $I_0 = 10.0 \text{ W/m}^2$ and $\phi = 37.0^\circ$. This gives $I = 6.38 \text{ W/m}^2$.

EVALUATE: The transmitted intensity depends on the angle between the axes of the two filters.

33.32. IDENTIFY: After passing through the first filter the light is linearly polarized along the filter axis. After the second filter, $I = I_{\max} (\cos \phi)^2$, where ϕ is the angle between the axes of the two filters.

SET UP: The maximum amount of light is transmitted when $\phi = 0$.

EXECUTE: (a) $I = I_0 (\cos 22.5^\circ)^2 = 0.854 I_0$

(b) $I = I_0 (\cos 45.0^\circ)^2 = 0.500 I_0$

(c) $I = I_0 (\cos 67.5^\circ)^2 = 0.146 I_0$

EVALUATE: As ϕ increases toward 90° the axes of the two filters are closer to being perpendicular to each other and the transmitted intensity decreases.

33.33. IDENTIFY and SET UP: Apply Eq.(33.7) to polarizers #2 and #3. The light incident on the first polarizer is unpolarized, so the transmitted light has half the intensity of the incident light, and the transmitted light is polarized.

(a) **EXECUTE:** The axes of the three filters are shown in Figure 33.33a.

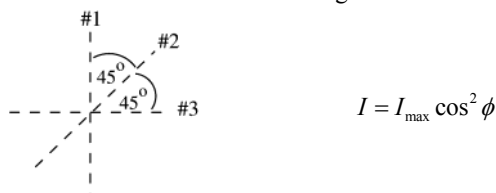


Figure 33.33a

After the first filter the intensity is $I_1 = \frac{1}{2}I_0$ and the light is linearly polarized along the axis of the first polarizer.

After the second filter the intensity is $I_2 = I_1 \cos^2 \phi = (\frac{1}{2}I_0)(\cos 45.0^\circ)^2 = 0.250 I_0$ and the light is linearly polarized along the axis of the second polarizer. After the third filter the intensity is $I_3 = I_2 \cos^2 \phi = 0.250 I_0 (\cos 45.0^\circ)^2 = 0.125 I_0$ and the light is linearly polarized along the axis of the third polarizer.

(b) The axes of the remaining two filters are shown in Figure 33.33b.

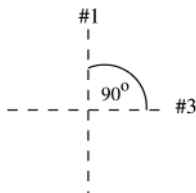


Figure 33.33b

After the first filter the intensity is $I_1 = \frac{1}{2}I_0$ and the light is linearly polarized along the axis of the first polarizer.

After the next filter the intensity is $I_3 = I_1 \cos^2 \phi = (\frac{1}{2}I_0)(\cos 90.0^\circ)^2 = 0$. No light is passed.

EVALUATE: Light is transmitted through all three filters, but no light is transmitted if the middle polarizer is removed.

- 33.34. IDENTIFY:** Use the transmitted intensity when all three polarizers are present to solve for the incident intensity I_0 . Then repeat the calculation with only the first and third polarizers.

SET UP: For unpolarized light incident on a filter, $I = \frac{1}{2}I_0$ and the light is linearly polarized along the filter axis. For polarized light incident on a filter, $I = I_{\max}(\cos\phi)^2$, where $I_{\max}$ is the intensity of the incident light, and the emerging light is linearly polarized along the filter axis.

EXECUTE: With all three polarizers, if the incident intensity is I_0 the transmitted intensity is

$$I = (\tfrac{1}{2}I_0)(\cos 23.0^\circ)^2(\cos[62.0^\circ - 23.0^\circ])^2 = 0.256I_0. \quad I_0 = \frac{I}{0.256} = \frac{75.0 \text{ W/cm}^2}{0.256} = 293 \text{ W/cm}^2.$$

With only the first and third polarizers, $I = (\tfrac{1}{2}I_0)(\cos 62.0^\circ)^2 = 0.110I_0 = (0.110)(293 \text{ W/cm}^2) = 32.2 \text{ W/cm}^2$.

EVALUATE: The transmitted intensity is greater when all three filters are present.

- 33.35. IDENTIFY:** The shorter the wavelength of light, the more it is scattered. The intensity is inversely proportional to the fourth power of the wavelength.

SET UP: The intensity of the scattered light is proportional to $1/\lambda^4$, we can write it as $I = (\text{constant})/\lambda^4$.

EXECUTE: (a) Since I is proportional to $1/\lambda^4$, we have $I = (\text{constant})/\lambda^4$. Taking the ratio of the intensity of the red light to that of the green light gives $\frac{I_R}{I} = \frac{(\text{constant})/\lambda_R^4}{(\text{constant})/\lambda_G^4} = \left(\frac{\lambda_G}{\lambda_R}\right)^4 = \left(\frac{520 \text{ nm}}{665 \text{ nm}}\right)^4 = 0.374$, so $I_R = 0.374I$.

(b) Following the same procedure as in part (a) gives $\frac{I_V}{I} = \left(\frac{\lambda_G}{\lambda_V}\right)^4 = \left(\frac{520 \text{ nm}}{420 \text{ nm}}\right)^4 = 2.35$, so $I_V = 2.35I$.

EVALUATE: In the scattered light, the intensity of the short-wavelength violet light is about 7 times as great as that of the red light, so this scattered light will have a blue-violet color.

- 33.36. IDENTIFY:** As the wave front reaches the sharp object, every point on the front will act as a source of secondary wavelets.

SET UP: Consider a wave front that is just about to go past the corner. Follow it along and draw the successive wave fronts.

EXECUTE: The path of the wavefront is drawn in Figure 33.36.

EVALUATE: The wave fronts clearly bend around the sharp point, just as water waves bend around a rock and light waves bend around the edge of a slit.

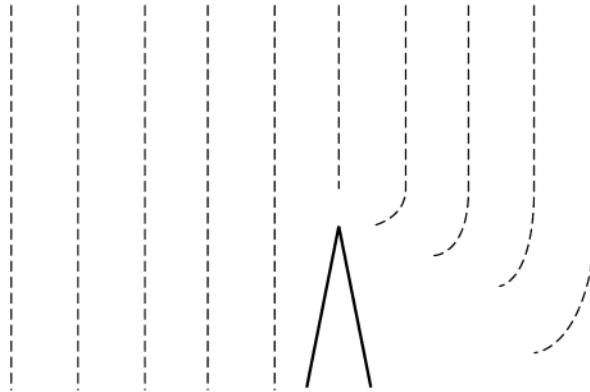


Figure 33.36

- 33.37. IDENTIFY:** Reflection reverses the sign of the component of light velocity perpendicular to the reflecting surface but leaves the other components unchanged.

SET UP: Consider three mirrors, M_1 in the (x,y) -plane, M_2 in the (y,z) -plane, and M_3 in the (x,z) -plane.

EXECUTE: A light ray reflecting from M_1 changes the sign of the z -component of the velocity, reflecting from M_2 changes the x -component, and from M_3 changes the y -component. Thus the velocity, and hence also the path, of the light beam flips by 180° .

EVALUATE: Example 33.3 discusses some uses of corner reflectors.

- 33.38. IDENTIFY:** The light travels slower in the jelly than in the air and hence will take longer to travel the length of the tube when it is filled with jelly than when it contains just air.

SET UP: The definition of the index of refraction is $n = c/v$, where v is the speed of light in the jelly.

EXECUTE: First get the length L of the tube using air. In the air, we have $L = ct = (3.00 \times 10^8 \text{ m/s})(8.72 \text{ ns}) = 2.616 \text{ m}$.

The speed in the jelly is $v = \frac{L}{t} = (2.616 \text{ m})/(8.72 \text{ ns} + 2.04 \text{ ns}) = 2.431 \times 10^8 \text{ m/s}$. $n = \frac{c}{v} =$

$$(3.00 \times 10^8 \text{ m/s})/(2.431 \times 10^8 \text{ m/s}) = 1.23$$

EVALUATE: A high-speed timer would be needed to measure times as short as a few nanoseconds.

33.39. IDENTIFY and SET UP: Apply Snell's law at each interface.

EXECUTE: (a) $n_1 \sin \theta_1 = n_2 \sin \theta_2$ and $n_2 \sin \theta_2 = n_3 \sin \theta_3$, so $n_1 \sin \theta_1 = n_3 \sin \theta_3$ and $\sin \theta_3 = (n_1 \sin \theta_1) / n_3$.

(b) $n_3 \sin \theta_3 = n_2 \sin \theta_2$ and $n_2 \sin \theta_2 = n_1 \sin \theta_1$, so $n_1 \sin \theta_1 = n_3 \sin \theta_3$ and the light makes the same angle with respect to the normal in the material that has refractive index n_1 as it did in part (a).

(c) For reflection, $\theta_r = \theta_a$. These angles are still equal if θ_r becomes the incident angle; reflected rays are also reversible.

EVALUATE: Both the refracted and reflected rays are reversible, in the sense that if the direction of the light is reversed then each of these rays follow the path of the incident ray.

33.40. IDENTIFY: Use the change in transit time to find the speed v of light in the slab, and then apply $n = \frac{c}{v}$ and $\lambda = \frac{\lambda_0}{n}$.

SET UP: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam.

EXECUTE: $\frac{0.840 \text{ m}}{c/n} - \frac{0.840 \text{ m}}{c} = (n-1) \frac{0.840 \text{ m}}{c} = 4.2 \text{ ns}$. We can now solve for the index of refraction:

$$n = \frac{(4.2 \times 10^{-9} \text{ s})(3.00 \times 10^8 \text{ m/s})}{0.840 \text{ m}} + 1 = 2.50. \text{ The wavelength inside of the glass is } \lambda = \frac{490 \text{ nm}}{2.50} = 196 \text{ nm}.$$

EVALUATE: Light travels slower in the slab than in air and the wavelength is shorter.

33.41. IDENTIFY: The angle of incidence at A is to be the critical angle. Apply Snell's law at the air to glass refraction at the top of the block.

SET UP: The ray is sketched in Figure 33.41.

EXECUTE: For glass $\rightarrow$ air at point A , Snell's law gives $(1.38) \sin \theta_{\text{crit}} = (1.00) \sin 90^\circ$ and $\theta_{\text{crit}} = 46.4^\circ$.

$\theta_b = 90^\circ - \theta_{\text{crit}} = 43.6^\circ$. Snell's law applied to the refraction from air to glass at the top of the block gives

$(1.00) \sin \theta_a = (1.38) \sin (43.6^\circ)$ and $\theta_a = 72.1^\circ$.

EVALUATE: If θ_a is larger than 72.1° then the angle of incidence at point A is less than the initial critical angle and total internal reflection doesn't occur.

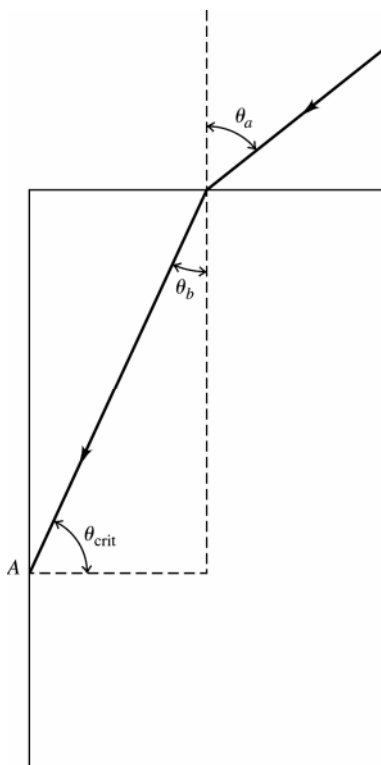


Figure 33.41

33.42. IDENTIFY: As the light crosses the glass-air interface along AB , it is refracted and obeys Snell's law.

SET UP: Snell's law is $n_a \sin \theta_a = n_b \sin \theta_b$ and $n = 1.000$ for air. At point B the angle of the prism is 30.0° .

EXECUTE: Apply Snell's law at AB . The prism angle at A is 60.0° , so for the upper ray, the angle of incidence at AB is $60.0^\circ + 12.0^\circ = 72.0^\circ$. Using this value gives $n_1 \sin 60.0^\circ = \sin 72.0^\circ$ and $n_1 = 1.10$. For the lower ray, the angle of incidence at AB is $60.0^\circ + 12.0^\circ + 8.50^\circ = 80.5^\circ$, giving $n_2 \sin 60.0^\circ = \sin 80.5^\circ$ and $n_2 = 1.14$.

EVALUATE: The lower ray is deflected more than the upper ray because that wavelength has a slightly greater index of refraction than the upper ray.

- 33.43. IDENTIFY:** Circularly polarized light consists of the superposition of light polarized in two perpendicular directions, with a quarter-cycle (90°) phase difference between the two polarization components.
SET UP: A quarter-wave plate shifts the relative phase of the two perpendicular polarization components by 90° .
EXECUTE: In the circularly polarized light the two perpendicular polarization components are 90° out of phase. The quarter-wave plate shifts the relative phase by $\pm 90^\circ$ and then the two components are either in phase or 180° out of phase. Either corresponds to linearly polarized light.
EVALUATE: Either left circularly polarized light or right circularly polarized light is converted to linearly polarized light by the quarter-wave plate.

- 33.44. IDENTIFY:** Apply $\lambda = \frac{\lambda_0}{n}$. The number of wavelengths in a distance d of a material is $\frac{d}{\lambda}$ where λ is the wavelength in the material.
SET UP: The distance in glass is $d_{\text{glass}} = 0.00250 \text{ m}$. The distance in air is $d_{\text{air}} = 0.0180 \text{ m} - 0.00250 \text{ m} = 0.0155 \text{ m}$.
EXECUTE: number of wavelengths = number in air + number in glass.
 number of wavelengths = $\frac{d_{\text{air}}}{\lambda} + \frac{d_{\text{glass}}}{\lambda} n = \frac{0.0155 \text{ m}}{5.40 \times 10^{-7} \text{ m}} + \frac{0.00250 \text{ m}}{5.40 \times 10^{-7} \text{ m}} (1.40) = 3.52 \times 10^4$.
EVALUATE: Without the glass plate the number of wavelengths between the source and screen is $\frac{0.0180 \text{ m}}{5.40 \times 10^{-7} \text{ m}} = 3.33 \times 10^4$. The wavelength is shorter in the glass so there are more wavelengths in a distance in glass than there are in the same distance in air.

- 33.45. IDENTIFY:** Find the critical angle for glass $\rightarrow$ air. Light incident at this critical angle is reflected back to the edge of the halo.

SET UP: The ray incident at the critical angle is sketched in Figure 33.45.

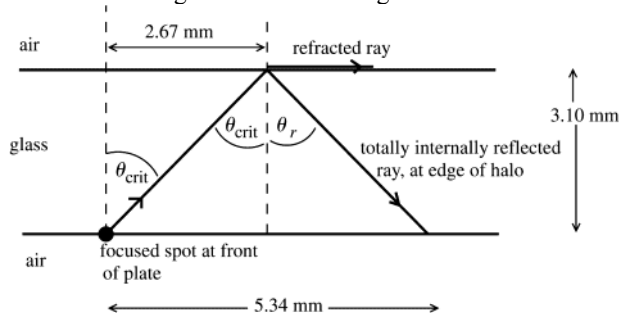


Figure 33.45

EXECUTE: From the distances given in the sketch, $\tan \theta_{\text{crit}} = \frac{2.67 \text{ mm}}{3.10 \text{ mm}} = 0.8613$; $\theta_{\text{crit}} = 40.7^\circ$.

Apply Snell's law to the total internal reflection to find the refractive index of the glass: $n_a \sin \theta_a = n_b \sin \theta_b$

$$n_{\text{glass}} \sin \theta_{\text{crit}} = 1.00 \sin 90^\circ$$

$$n_{\text{glass}} = \frac{1}{\sin \theta_{\text{crit}}} = \frac{1}{\sin 40.7^\circ} = 1.53$$

EVALUATE: Light incident on the back surface is also totally reflected if it is incident at angles greater than θ_{crit} .

If it is incident at less than θ_{crit} it refracts into the air and does not reflect back to the emulsion.

- 33.46. IDENTIFY:** Apply Snell's law to the refraction of the light as it passes from water into air.

SET UP: $\theta_a = \arctan\left(\frac{1.5 \text{ m}}{1.2 \text{ m}}\right) = 51^\circ$. $n_a = 1.00$. $n_b = 1.333$.

EXECUTE: $\theta_b = \arcsin\left(\frac{n_a \sin \theta_a}{n_b}\right) = \arcsin\left(\frac{1.00 \sin 51^\circ}{1.333}\right) = 36^\circ$. Therefore, the distance along the bottom of the

pool from directly below where the light enters to where it hits the bottom is $x = (4.0 \text{ m}) \tan \theta_b = (4.0 \text{ m}) \tan 36^\circ =$

$$2.9 \text{ m}. \quad x_{\text{total}} = 1.5 \text{ m} + x = 1.5 \text{ m} + 2.9 \text{ m} = 4.4 \text{ m}.$$

EVALUATE: The light ray from the flashlight is bent toward the normal when it refracts into the water.

33.47. IDENTIFY: Use Snell's law to determine the effect of the liquid on the direction of travel of the light as it enters the liquid.

SET UP: Use geometry to find the angles of incidence and refraction. Before the liquid is poured in the ray along your line of sight has the path shown in Figure 33.47a.

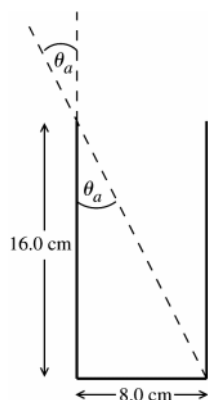


Figure 33.47a

$$\tan \theta_a = \frac{8.0 \text{ cm}}{16.0 \text{ cm}} = 0.500$$

$$\theta_a = 26.57^\circ$$

After the liquid is poured in, θ_a is the same and the refracted ray passes through the center of the bottom of the glass, as shown in Figure 33.47b.

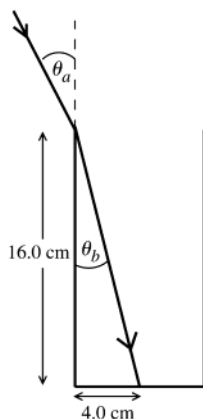


Figure 33.47b

$$\tan \theta_b = \frac{4.0 \text{ cm}}{16.0 \text{ cm}} = 0.250$$

$$\theta_b = 14.04^\circ$$

EXECUTE: Use Snell's law to find n_b , the refractive index of the liquid:

$$n_a \sin \theta_a = n_b \sin \theta_b$$

$$n_b = \frac{n_a \sin \theta_a}{\sin \theta_b} = \frac{(1.00)(\sin 26.57^\circ)}{\sin 14.04^\circ} = 1.84$$

EVALUATE: When the light goes from air to liquid (larger refractive index) it is bent toward the normal.

33.48. IDENTIFY: Apply Snell's law to each refraction and apply the law of reflection at the mirrored bottom.

SET UP: The path of the ray is sketched in Figure 33.48. The problem asks us to calculate θ_b' .

EXECUTE: Apply Snell's law to the air $\rightarrow$ liquid refraction. $(1.00)\sin(42.5^\circ) = (1.63)\sin \theta_b$ and $\theta_b = 24.5^\circ$.

$\theta_b = \phi$ and $\phi = \theta_a'$, so $\theta_a' = \theta_b = 24.5^\circ$. Snell's law applied to the liquid $\rightarrow$ air refraction gives

$$(1.63)\sin(24.5^\circ) = (1.00)\sin \theta_b' \text{ and } \theta_b' = 42.5^\circ.$$

EVALUATE: The light emerges from the liquid at the same angle from the normal as it entered the liquid.

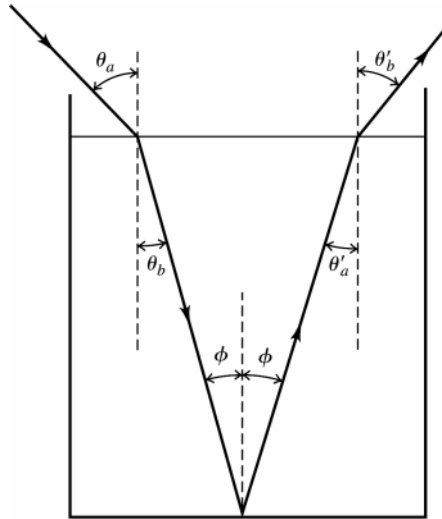


Figure 33.48

33.49. IDENTIFY: Apply Snell's law to the water $\rightarrow$ ice and ice $\rightarrow$ air interfaces.

(a) SET UP: Consider the ray shown in Figure 33.49.

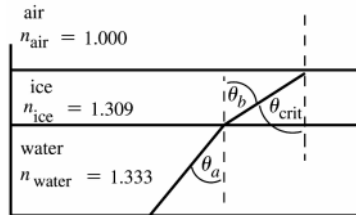


Figure 33.49

We want to find the incident angle θ_a at the water-ice interface that causes the incident angle at the ice-air interface to be the critical angle.

EXECUTE: ice-air interface: $n_{\text{ice}} \sin \theta_{\text{crit}} = 1.0 \sin 90^\circ$

$$n_{\text{ice}} \sin \theta_{\text{crit}} = 1.0 \text{ so } \sin \theta_{\text{crit}} = \frac{1}{n_{\text{ice}}}$$

But from the diagram we see that $\theta_b = \theta_{\text{crit}}$, so $\sin \theta_b = \frac{1}{n_{\text{ice}}}$.

water-ice interface: $n_w \sin \theta_a = n_{\text{ice}} \sin \theta_b$

But $\sin \theta_b = \frac{1}{n_{\text{ice}}}$ so $n_w \sin \theta_a = 1.0$. $\sin \theta_a = \frac{1}{n_w} = \frac{1}{1.333} = 0.7502$ and $\theta_a = 48.6^\circ$.

(b) EVALUATE: The angle calculated in part (a) is the critical angle for a water-air interface; the answer would be the same if the ice layer wasn't there!

33.50. IDENTIFY: The incident angle at the prism $\rightarrow$ water interface is to be the critical angle.

SET UP: The path of the ray is sketched in Figure 33.50. The ray enters the prism at normal incidence so is not bent. For water, $n_{\text{water}} = 1.333$.

EXECUTE: From the figure, $\theta_{\text{crit}} = 45^\circ$. $n_a \sin \theta_a = n_b \sin \theta_b$ gives $n_{\text{glass}} \sin 45^\circ = (1.333) \sin 90^\circ$.

$$n_{\text{glass}} = \frac{1.333}{\sin 45^\circ} = 1.89$$

EVALUATE: For total internal reflection the ray must be incident in the material of greater refractive index. $n_{\text{glass}} > n_{\text{water}}$, so that is the case here.

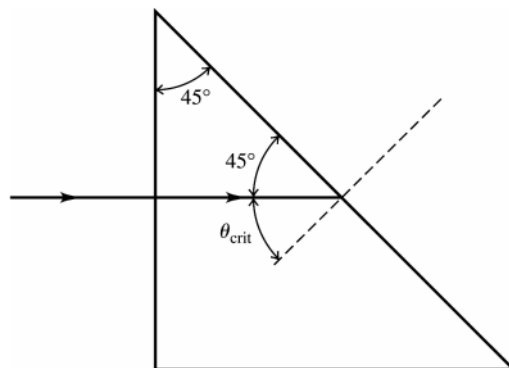


Figure 33.50

33.51. IDENTIFY: Apply Snell's law to the refraction of each ray as it emerges from the glass. The angle of incidence equals the angle $A = 25.0^\circ$.

SET UP: The paths of the two rays are sketched in Figure 33.51.

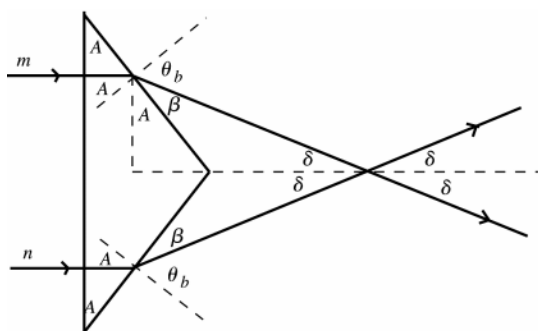


Figure 33.51

EXECUTE: $n_a \sin \theta_a = n_b \sin \theta_b$

$$n_{\text{glass}} \sin 25.0^\circ = 1.00 \sin \theta_b$$

$$\sin \theta_b = n_{\text{glass}} \sin 25.0^\circ$$

$$\sin \theta_b = 1.66 \sin 25.0^\circ = 0.7015$$

$$\theta_b = 44.55^\circ$$

$$\beta = 90.0^\circ - \theta_b = 45.45^\circ$$

Then $\delta = 90.0^\circ - A - \beta = 90.0^\circ - 25.0^\circ - 45.45^\circ = 19.55^\circ$. The angle between the two rays is $2\delta = 39.1^\circ$.

EVALUATE: The light is incident normally on the front face of the prism so the light is not bent as it enters the prism.

33.52. IDENTIFY: The ray shown in the figure that accompanies the problem is to be incident at the critical angle.

SET UP: $\theta_b = 90^\circ$. The incident angle for the ray in the figure is 60° .

$$\text{EXECUTE: } n_a \sin \theta_a = n_b \sin \theta_b \text{ gives } n_b = \left(\frac{n_a \sin \theta_a}{\sin \theta_b} \right) = \left(\frac{1.62 \sin 60^\circ}{\sin 90^\circ} \right) = 1.40.$$

EVALUATE: Total internal reflection occurs only when the light is incident in the material of the greater refractive index.

33.53. IDENTIFY: No light enters the gas because total internal reflection must have occurred at the water-gas interface.

SET UP: At the minimum value of S , the light strikes the water-gas interface at the critical angle. We apply Snell's law, $n_a \sin \theta_a = n_b \sin \theta_b$, at that surface.

EXECUTE: (a) In the water, $\theta = \frac{S}{R} = (1.09 \text{ m}) / (1.10 \text{ m}) = 0.991 \text{ rad} = 56.77^\circ$. This is the critical angle. So, using

the refractive index for water from Table 33.1, we get $n = (1.333) \sin 56.77^\circ = 1.12$

(b) (i) The laser beam stays in the water all the time, so

$$t = 2R/v = 2R / \left(\frac{c}{n_{\text{water}}} \right) = \frac{D n_{\text{water}}}{c} = (2.20 \text{ m})(1.333) / (3.00 \times 10^8 \text{ m/s}) = 9.78 \text{ ns}$$

(ii) The beam is in the water half the time and in the gas the other half of the time.

$$t_{\text{gas}} = \frac{Rn_{\text{gas}}}{c} = (1.10 \text{ m})(1.12)/(3.00 \times 10^8 \text{ m/s}) = 4.09 \text{ ns}$$

The total time is $4.09 \text{ ns} + (9.78 \text{ ns})/2 = 8.98 \text{ ns}$

EVALUATE: The gas must be under considerable pressure to have a refractive index as high as 1.12.

33.54. IDENTIFY: No light enters the water because total internal reflection must have occurred at the glass-water surface.

SET UP: A little geometry tells us that θ is the angle of incidence at the glass-water face in the water. Also, $\theta = 59.2^\circ$ must be the critical angle at that surface, so the angle of refraction is 90.0° . Snell's law, $n_a \sin \theta_a = n_b \sin \theta_b$,

applies at that glass-water surface, and the index of refraction is defined as $n = \frac{c}{v}$.

EXECUTE: Snell's law at the glass-water surface gives $n \sin 59.2^\circ = (1.333)(1.00)$, which gives $n = 1.55$. $v = \frac{c}{n} =$

$(3.00 \times 10^8 \text{ m/s})/1.55 = 1.93 \times 10^8 \text{ m/s}$.

EVALUATE: Notice that θ is *not* the angle of incidence at the reflector, but it is the angle of incidence at the glass-water surface.

33.55. (a) IDENTIFY: Apply Snell's law to the refraction of the light as it enters the atmosphere.

SET UP: The path of a ray from the sun is sketched in Figure 33.55.

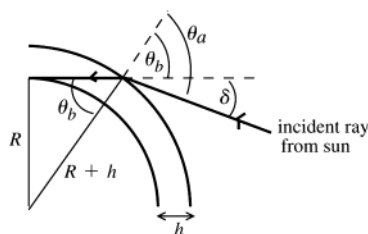


Figure 33.55

$$\delta = \theta_a - \theta_b$$

$$\text{From the diagram } \sin \theta_b = \frac{R}{R+h}$$

$$\theta_b = \arcsin \left(\frac{R}{R+h} \right)$$

EXECUTE: Apply Snell's law to the refraction that occurs at the top of the atmosphere: $n_a \sin \theta_a = n_b \sin \theta_b$

(a = vacuum of space, refractive, index 1.0; b = atmosphere, refractive index n)

$$\sin \theta_a = n \sin \theta_b = n \left(\frac{R}{R+h} \right) \text{ so } \theta_a = \arcsin \left(\frac{nR}{R+h} \right)$$

$$\delta = \theta_a - \theta_b = \arcsin \left(\frac{nR}{R+h} \right) - \arcsin \left(\frac{R}{R+h} \right)$$

$$\text{(b) } \frac{R}{R+h} = \frac{6.38 \times 10^6 \text{ m}}{6.38 \times 10^6 \text{ m} + 20 \times 10^3 \text{ m}} = 0.99688$$

$$\frac{nR}{R+h} = 1.0003(0.99688) = 0.99718$$

$$\theta_b = \arcsin \left(\frac{R}{R+h} \right) = 85.47^\circ$$

$$\theta_a = \arcsin \left(\frac{nR}{R+h} \right) = 85.70^\circ$$

$$\delta = \theta_a - \theta_b = 85.70^\circ - 85.47^\circ = 0.23^\circ$$

EVALUATE: The calculated δ is about the same as the angular radius of the sun.

33.56. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) The distance traveled by the light ray is the sum of the two diagonal segments:

$$d = (x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}. \text{ Then the time taken to travel that distance is } t = \frac{d}{c} = \frac{(x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}}{c}.$$

(b) Taking the derivative with respect to x of the time and setting it to zero yields

$$\frac{dt}{dx} = \frac{1}{c} \frac{d}{dx} \left[(x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2} \right] \frac{dt}{dx} = \frac{1}{c} \left[x(x^2 + y_1^2)^{-1/2} - (l-x)((l-x)^2 + y_2^2)^{-1/2} \right] = 0. \text{ This gives}$$

$$\frac{x}{\sqrt{x^2 + y_1^2}} = \frac{(l-x)}{\sqrt{(l-x)^2 + y_2^2}}, \sin \theta_1 = \sin \theta_2 \text{ and } \theta_1 = \theta_2.$$

EVALUATE: For any other path between points 1 and 2, that includes a point on the reflective surface, the distance traveled and therefore the travel time is greater than for this path.

- 33.57. IDENTIFY and SET UP:** Find the distance that the ray travels in each medium. The travel time in each medium is the distance divided by the speed in that medium.

(a) EXECUTE: The light travels a distance $\sqrt{h_1^2 + x^2}$ in traveling from point A to the interface. Along this path

the speed of the light is v_1 , so the time it takes to travel this distance is $t_1 = \frac{\sqrt{h_1^2 + x^2}}{v_1}$. The light travels a

distance $\sqrt{h_2^2 + (l-x)^2}$ in traveling from the interface to point B. Along this path the speed of the light is v_2 ,

so the time it takes to travel this distance is $t_2 = \frac{\sqrt{h_2^2 + (l-x)^2}}{v_2}$. The total time to go from A to B is

$$t = t_1 + t_2 = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l-x)^2}}{v_2}.$$

$$\text{(b)} \quad \frac{dt}{dx} = \frac{1}{v_1} \left(\frac{1}{2} \right) (h_1^2 + x^2)^{-1/2} (2x) + \frac{1}{v_2} \left(\frac{1}{2} \right) (h_2^2 + (l-x)^2)^{-1/2} 2(l-x)(-1) = 0$$

$$\frac{x}{v_1 \sqrt{h_1^2 + x^2}} = \frac{l-x}{v_2 \sqrt{h_2^2 + (l-x)^2}}$$

Multiplying both sides by c gives $\frac{c}{v_1} \frac{x}{\sqrt{h_1^2 + x^2}} = \frac{c}{v_2} \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$

$$\frac{c}{v_1} = n_1 \text{ and } \frac{c}{v_2} = n_2 \text{ (Eq.33.1)}$$

From Fig.33.55 in the textbook, $\sin \theta_1 = \frac{x}{\sqrt{h_1^2 + x^2}}$ and $\sin \theta_2 = \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$.

So $n_1 \sin \theta_1 = n_2 \sin \theta_2$, which is Snell's law.

EVALUATE: Snell's law is a result of a change in speed when light goes from one material to another.

- 33.58. IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the angles and distances defined in the figure that accompanies the problem.

EXECUTE: **(a)** For light in air incident on a parallel-faced plate, Snell's Law yields:

$$n \sin \theta_a = n' \sin \theta'_a = n' \sin \theta_b = n \sin \theta'_b \Rightarrow \sin \theta_a = \sin \theta'_a \Rightarrow \theta_a = \theta'_a.$$

(b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta'_n = \theta_n$ and the chain of equations can continue.

(c) The lateral displacement of the beam can be calculated using geometry:

$$d = L \sin(\theta_a - \theta'_b) \text{ and } L = \frac{t}{\cos \theta'_b} \Rightarrow d = \frac{t \sin(\theta_a - \theta'_b)}{\cos \theta'_b}.$$

$$\text{(d)} \quad \theta'_b = \arcsin\left(\frac{n \sin \theta_a}{n'}\right) = \arcsin\left(\frac{\sin 66.0^\circ}{1.80}\right) = 30.5^\circ \text{ and } d = \frac{(2.40 \text{ cm}) \sin(66.0^\circ - 30.5^\circ)}{\cos 30.5^\circ} = 1.62 \text{ cm}.$$

EVALUATE: The lateral displacement in part (d) is large, of the same order as the thickness of the plate.

- 33.59. IDENTIFY:** Apply Snell's law to each refraction and apply the law of reflection to each reflection.

SET UP: The paths of rays A and B are sketched in Figure 33.59. Let θ be the angle of incidence for the combined ray.

EXECUTE: For ray A its final direction of travel is at an angle θ with respect to the normal, by the law of reflection. Let the final direction of travel for ray B be at angle ϕ with respect to the normal. At the upper surface, Snell's law gives $n_1 \sin \theta = n_2 \sin \alpha$. The lower surface reflects ray B at angle α . Ray B returns to the upper surface of the film at an angle of incidence α . Snell's law applied to the refraction as ray B leaves the film gives $n_2 \sin \alpha = n_1 \sin \phi$. Combining the two equations gives $n_1 \sin \theta = n_1 \sin \phi$ and $\theta = \phi$; the two rays are parallel after they emerge from the film.

EVALUATE: Ray B is bent toward the normal as it enters the film and away from the normal as it refracts out of the film.

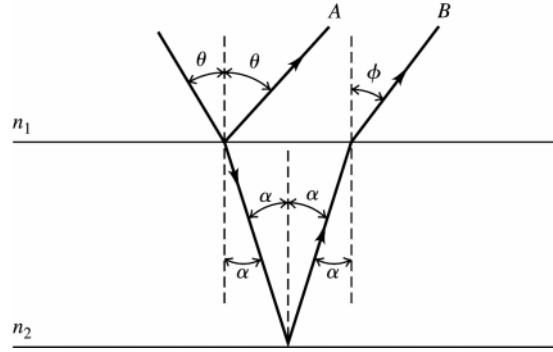


Figure 33.59

33.60. IDENTIFY: Apply Snell's law and the results of Problem 33.58.

SET UP: From Figure 33.58 in the textbook, $n_r = 1.61$ for red light and $n_v = 1.66$ for violet. In the notation of Problem 33.58, t is the thickness of the glass plate and the lateral displacement is d . We want the difference in d for the two colors of light to be 1.0 mm. $\theta_a = 70.0^\circ$. For red light, $n_a \sin \theta_a = n_b \sin \theta'_b$ gives $\sin \theta'_b = \frac{(1.00) \sin 70.0^\circ}{1.61}$

and $\theta'_b = 35.71^\circ$. For violet light, $\sin \theta'_b = \frac{(1.00) \sin 70.0^\circ}{1.66}$ and $\theta'_b = 34.48^\circ$.

EXECUTE: (a) n decreases with increasing λ , so n is smaller for red than for blue. So beam a is the red one.

(b) Problem 33.58 says $d = t \frac{\sin(\theta_a - \theta'_b)}{\cos \theta'_b}$. For red light, $d_r = t \frac{\sin(70^\circ - 35.71^\circ)}{\cos 35.71^\circ} = 0.6938t$ and for violet light,

$$d_v = t \frac{\sin(70^\circ - 35.48^\circ)}{\cos 35.48^\circ} = 0.7048t. \quad d_v - d_r = 1.0 \text{ mm gives } t = \frac{0.10 \text{ cm}}{0.7048 - 0.6958} = 9.1 \text{ cm}.$$

EVALUATE: Our calculation shown that the violet light has greater lateral displacement and this is ray b .

33.61. IDENTIFY: Apply Snell's law to the two refractions of the ray.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: (a) $n_a \sin \theta_a = n_b \sin \theta_b$ gives $\sin \theta_a = n_b \sin \frac{A}{2}$. But $\theta_a = \frac{A}{2} + \alpha$, so $\sin \left(\frac{A}{2} + \alpha \right) = \sin \frac{A + 2\alpha}{2} = n \sin \frac{A}{2}$.

At each face of the prism the deviation is α , so $2\alpha = \delta$ and $\sin \frac{A + \delta}{2} = n \sin \frac{A}{2}$.

(b) From part (a), $\delta = 2 \arcsin \left(n \sin \frac{A}{2} \right) - A$. $\delta = 2 \arcsin \left((1.52) \sin \frac{60.0^\circ}{2} \right) - 60.0^\circ = 38.9^\circ$.

(c) If two colors have different indices of refraction for the glass, then the deflection angles for them will differ:

$$\delta_{\text{red}} = 2 \arcsin \left((1.61) \sin \frac{60.0^\circ}{2} \right) - 60.0^\circ = 47.2^\circ$$

$$\delta_{\text{violet}} = 2 \arcsin \left((1.66) \sin \frac{60.0^\circ}{2} \right) - 60.0^\circ = 52.2^\circ \Rightarrow \Delta \delta = 52.2^\circ - 47.2^\circ = 5.0^\circ$$

EVALUATE: The violet light has a greater refractive index and therefore the angle of deviation is greater for the violet light.

33.62. IDENTIFY: The reflected light is totally polarized when light strikes a surface at Brewster's angle.

SET UP: At the plastic wall, Brewster's angle obeys the equation $\tan \theta_p = n_b/n_a$, and Snell's law, $n_a \sin \theta_a = n_b \sin \theta_b$, applies at the air-water surface.

EXECUTE: To be totally polarized, the reflected sunlight must have struck the wall at Brewster's angle. $\tan \theta_p = n_b/n_a = (1.61)/(1.00)$ and $\theta_p = 58.15^\circ$

This is the angle of incidence at the wall. A little geometry tells us that the angle of incidence at the water surface is $90.00^\circ - 58.15^\circ = 31.85^\circ$. Applying Snell's law at the water surface gives

$$(1.00) \sin 31.85^\circ = 1.333 \sin \theta \text{ and } \theta = 23.3^\circ$$

EVALUATE: We have two different principles involved here: Reflection at Brewster's angle at the wall and Snell's law at the water surface.

- 33.63. IDENTIFY and SET UP:** The polarizer passes $\frac{1}{2}$ of the intensity of the unpolarized component, independent of ϕ . Out of the intensity I_p of the polarized component the polarizer passes intensity $I_p \cos^2(\phi - \theta)$, where $\phi - \theta$ is the angle between the plane of polarization and the axis of the polarizer.
(a) Use the angle where the transmitted intensity is maximum or minimum to find θ . See Figure 33.63.

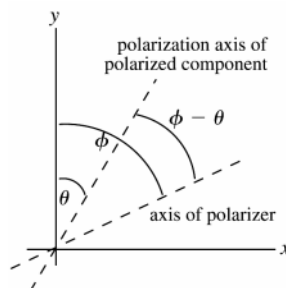


Figure 33.63

EXECUTE: The total transmitted intensity is $I = \frac{1}{2}I_0 + I_p \cos^2(\phi - \theta)$. This is maximum when $\theta = \phi$ and from the table of data this occurs for ϕ between 30° and 40° , say at 35° and $\theta = 35^\circ$. Alternatively, the total transmitted intensity is minimum when $\phi - \theta = 90^\circ$ and from the data this occurs for $\phi = 125^\circ$. Thus, $\theta = \phi - 90^\circ = 125^\circ - 90^\circ = 35^\circ$, in agreement with the above.

(b) IDENTIFY and SET UP: $I = \frac{1}{2}I_0 + I_p \cos^2(\phi - \theta)$

Use data at two values of ϕ to determine the two constants I_0 and I_p . Use data where the I_p term is large ($\phi = 30^\circ$) and where it is small ($\phi = 130^\circ$) to have the greatest sensitivity to both I_0 and I_p :

EXECUTE: $\phi = 30^\circ$ gives $24.8 \text{ W/m}^2 = \frac{1}{2}I_0 + I_p \cos^2(30^\circ - 35^\circ)$

$$24.8 \text{ W/m}^2 = 0.500I_0 + 0.9924I_p$$

$\phi = 130^\circ$ gives $5.2 \text{ W/m}^2 = \frac{1}{2}I_0 + I_p \cos^2(130^\circ - 35^\circ)$

$$5.2 \text{ W/m}^2 = 0.500I_0 + 0.0076I_p$$

Subtracting the second equation from the first gives $19.6 \text{ W/m}^2 = 0.9848I_p$ and $I_p = 19.9 \text{ W/m}^2$. And then

$$I_0 = 2(5.2 \text{ W/m}^2 - 0.0076(19.9 \text{ W/m}^2)) = 10.1 \text{ W/m}^2.$$

EVALUATE: Now that we have I_0 , I_p and θ we can verify that $I = \frac{1}{2}I_0 + I_p \cos^2(\phi - \theta)$ describes that data in the table.

- 33.64. IDENTIFY:** The number of wavelengths in a distance D of material is D/λ , where λ is the wavelength of the light in the material.

SET UP: The condition for a quarter-wave plate is $\frac{D}{\lambda_1} = \frac{D}{\lambda_2} + \frac{1}{4}$, where we have assumed $n_1 > n_2$ so $\lambda_2 > \lambda_1$.

EXECUTE: **(a)** $\frac{n_1 D}{\lambda_0} = \frac{n_2 D}{\lambda_0} + \frac{1}{4}$ and $D = \frac{\lambda_0}{4(n_1 - n_2)}$.

$$\textbf{(b)} D = \frac{\lambda_0}{4(n_1 - n_2)} = \frac{5.89 \times 10^{-7} \text{ m}}{4(1.875 - 1.635)} = 6.14 \times 10^{-7} \text{ m}.$$

EVALUATE: The thickness of the quarter-wave plate in part (b) is 614 nm, which is of the same order as the wavelength in vacuum of the light.

- 33.65. IDENTIFY:** Follow the steps specified in the problem.

SET UP: $\cos(\alpha - \beta) = \sin \alpha \sin \beta + \cos \alpha \cos \beta$. $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$.

EXECUTE: **(a)** Multiplying Eq.(1) by $\sin \beta$ and Eq.(2) by $\sin \alpha$ yields:

$$(1): \frac{x}{a} \sin \beta = \sin \omega t \cos \alpha \sin \beta - \cos \omega t \sin \alpha \sin \beta \quad \text{and} \quad (2): \frac{y}{a} \sin \alpha = \sin \omega t \cos \beta \sin \alpha - \cos \omega t \sin \beta \sin \alpha.$$

Subtracting yields: $\frac{x \sin \beta - y \sin \alpha}{a} = \sin \omega t (\cos \alpha \sin \beta - \cos \beta \sin \alpha)$.

(b) Multiplying Eq. (1) by $\cos \beta$ and Eq. (2) by $\cos \alpha$ yields:

$$(1): \frac{x}{a} \cos \beta = \sin \omega t \cos \alpha \cos \beta - \cos \omega t \sin \alpha \cos \beta \text{ and } (2): \frac{y}{a} \cos \alpha = \sin \omega t \cos \beta \cos \alpha - \cos \omega t \sin \beta \cos \alpha.$$

Subtracting yields: $\frac{x \cos \beta - y \cos \alpha}{a} = -\cos \omega t (\sin \alpha \cos \beta - \sin \beta \cos \alpha).$

(c) Squaring and adding the results of parts (a) and (b) yields:

$$(x \sin \beta - y \sin \alpha)^2 + (x \cos \beta - y \cos \alpha)^2 = a^2 (\sin \alpha \cos \beta - \sin \beta \cos \alpha)^2$$

(d) Expanding the left-hand side, we have:

$$\begin{aligned} x^2 (\sin^2 \beta + \cos^2 \beta) + y^2 (\sin^2 \alpha + \cos^2 \alpha) - 2xy (\sin \alpha \sin \beta + \cos \alpha \cos \beta) \\ = x^2 + y^2 - 2xy (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = x^2 + y^2 - 2xy \cos(\alpha - \beta). \end{aligned}$$

The right-hand side can be rewritten: $a^2 (\sin \alpha \cos \beta - \sin \beta \cos \alpha)^2 = a^2 \sin^2(\alpha - \beta)$. Therefore, $x^2 + y^2 - 2xy \cos(\alpha - \beta) = a^2 \sin^2(\alpha - \beta)$. Or, $x^2 + y^2 - 2xy \cos \delta = a^2 \sin^2 \delta$, where $\delta = \alpha - \beta$.

EVALUATE: (e) $\delta = 0: x^2 + y^2 - 2xy = (x - y)^2 = 0 \Rightarrow x = y$, which is a straight diagonal line

$$\delta = \frac{\pi}{4}: x^2 + y^2 - \sqrt{2}xy = \frac{a^2}{2}, \text{ which is an ellipse}$$

$$\delta = \frac{\pi}{2}: x^2 + y^2 = a^2, \text{ which is a circle. This pattern repeats for the remaining phase differences.}$$

33.66. IDENTIFY: Apply Snell's law to each refraction.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: (a) By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$. Therefore, $\sin \theta_b^C = n \sin \theta_a^C = n \sin \theta_b^A = \sin \theta_a^A = \theta_b^C = \theta_a^A$.

(b) The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi$.

(c) From Snell's Law, $\sin \theta_a^A = n \sin \theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\frac{1}{n} \sin \theta_a^A\right)$.

$$\Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\frac{1}{n} \sin \theta_a^A\right) + \pi.$$

$$(d) \frac{d\Delta}{d\theta_a^A} = 0 = 2 - 4 \frac{d}{d\theta_a^A} \left(\arcsin\left(\frac{1}{n} \sin \theta_a^A\right) \right) \Rightarrow 0 = 2 - \frac{4}{\sqrt{1 - \frac{\sin^2 \theta_1}{n^2}}} \cdot \left(\frac{\cos \theta_1}{n} \right) \cdot 4 \left(1 - \frac{\sin^2 \theta_1}{n^2} \right) = \left(\frac{16 \cos^2 \theta_1}{n^2} \right).$$

$$4 \cos^2 \theta_1 = n^2 - 1 + \cos^2 \theta_1 \cdot 3 \cos^2 \theta_1 = n^2 - 1 \cdot \cos^2 \theta_1 = \frac{1}{3}(n^2 - 1).$$

$$(e) \text{ For violet: } \theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2 - 1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.342^2 - 1)}\right) = 58.89^\circ.$$

$$\Delta_{\text{violet}} = 139.2^\circ \Rightarrow \theta_{\text{violet}} = 40.8^\circ.$$

$$\text{For red: } \theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2 - 1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.330^2 - 1)}\right) = 59.58^\circ. \Delta_{\text{red}} = 137.5^\circ \Rightarrow \theta_{\text{red}} = 42.5^\circ.$$

EVALUATE: The angles we have calculated agree with the values given in Figure 37.20d in the textbook. θ_1 is larger for red than for violet, so red in the rainbow is higher above the horizon.

33.67. IDENTIFY: Follow similar steps to Challenge Problem 33.66.

SET UP: Refer to Figure 33.20e in the textbook.

EXECUTE: The total angular deflection of the ray is

$$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi, \text{ where we have used the fact from the previous problem that all the internal angles are equal and the two external angles are equal. Also using the Snell's Law relationship, we have: } \theta_b^A = \arcsin\left(\frac{1}{n} \sin \theta_a^A\right). \Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\frac{1}{n} \sin \theta_a^A\right) + 2\pi.$$

$$(b) \frac{d\Delta}{d\theta_a^A} = 0 = 2 - 6 \frac{d}{d\theta_a^A} \left(\arcsin \left(\frac{1}{n} \sin \theta_a^A \right) \right) \Rightarrow 0 = 2 - \frac{6}{\sqrt{1 - \frac{\sin^2 \theta_2}{n^2}}} \cdot \left(\frac{\cos \theta_2}{n} \right).$$

$$n^2 \left(1 - \frac{\sin^2 \theta_2}{n^2} \right) = (n^2 - 1 + \cos^2 \theta_2) = 9 \cos^2 \theta_2. \quad \cos^2 \theta_2 = \frac{1}{8} (n^2 - 1).$$

$$(c) \text{ For violet, } \theta_2 = \arccos \left(\sqrt{\frac{1}{8} (n^2 - 1)} \right) = \arccos \left(\sqrt{\frac{1}{8} (1.342^2 - 1)} \right) = 71.55^\circ. \quad \Delta_{\text{violet}} = 233.2^\circ \text{ and } \theta_{\text{violet}} = 53.2^\circ.$$

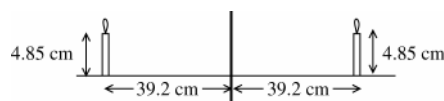
$$\text{For red, } \theta_2 = \arccos \left(\sqrt{\frac{1}{8} (n^2 - 1)} \right) = \arccos \left(\sqrt{\frac{1}{8} (1.330^2 - 1)} \right) = 71.94^\circ. \quad \Delta_{\text{red}} = 230.1^\circ \text{ and } \theta_{\text{red}} = 50.1^\circ.$$

EVALUATE: The angles we calculated agree with those given in Figure 37.20e in the textbook. The color that appears higher above the horizon is violet. The colors appear in reverse order in a secondary rainbow compared to a primary rainbow.

GEOMETRIC OPTICS

- 34.1. IDENTIFY and SET UP:** Plane mirror: $s = -s'$ (Eq.34.1) and $m = y'/y = -s'/s = +1$ (Eq.34.2). We are given s and y and are asked to find s' and y' .

EXECUTE: The object and image are shown in Figure 34.1.



$$\begin{aligned}s' &= -s = -39.2 \text{ cm} \\ |y'| &= |m||y| = (+1)(4.85 \text{ cm}) \\ |y'| &= 4.85 \text{ cm}\end{aligned}$$

Figure 34.1

The image is 39.2 cm to the right of the mirror and is 4.85 cm tall.

EVALUATE: For a plane mirror the image is always the same distance behind the mirror as the object is in front of the mirror. The image always has the same height as the object.

- 34.2. IDENTIFY:** Similar triangles say $\frac{h_{\text{tree}}}{h_{\text{mirror}}} = \frac{d_{\text{tree}}}{d_{\text{mirror}}}$.

SET UP: $d_{\text{mirror}} = 0.350 \text{ m}$, $h_{\text{mirror}} = 0.0400 \text{ m}$ and $d_{\text{tree}} = 28.0 \text{ m} + 0.350 \text{ m}$.

EXECUTE: $h_{\text{tree}} = h_{\text{mirror}} \frac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040 \text{ m} \frac{28.0 \text{ m} + 0.350 \text{ m}}{0.350 \text{ m}} = 3.24 \text{ m}$.

EVALUATE: The image of the tree formed by the mirror is 28.0 m behind the mirror and is 3.24 m tall.

- 34.3. IDENTIFY:** Apply the law of reflection.

SET UP: If up is the $+y$ -direction and right is the $+x$ -direction, then the object is at $(-x_0, -y_0)$ and P'_2 is at $(x_0, -y_0)$.

EXECUTE: Mirror 1 flips the y -values, so the image is at (x_0, y_0) which is P'_3 .

EVALUATE: Mirror 2 uses P'_1 as an object and forms an image at P'_3 .

- 34.4. IDENTIFY:** $f = R/2$

SET UP: For a concave mirror $R > 0$.

EXECUTE: (a) $f = \frac{R}{2} = \frac{34.0 \text{ cm}}{2} = 17.0 \text{ cm}$

EVALUATE: (b) The image formation by the mirror is determined by the law of reflection and that is unaffected by the medium in which the light is traveling. The focal length remains 17.0 cm.

- 34.5. IDENTIFY and SET UP:** Use Eq.(34.6) to calculate s' and use Eq.(34.7) to calculate y' . The image is real if s' is positive and is erect if $m > 0$. Concave means R and f are positive, $R = +22.0 \text{ cm}$; $f = R/2 = +11.0 \text{ cm}$.

EXECUTE: (a)

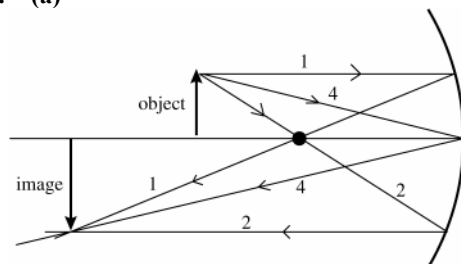


Figure 34.5

Three principal rays, numbered as in Sect. 34.2, are shown in Figure 34.5. The principal ray diagram shows that the image is real, inverted, and enlarged.

$$(b) \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf} \text{ so } s' = \frac{sf}{s-f} = \frac{(16.5 \text{ cm})(11.0 \text{ cm})}{16.5 \text{ cm} - 11.0 \text{ cm}} = +33.0 \text{ cm}$$

$s' > 0$ so real image, 33.0 cm to left of mirror vertex

$$m = -\frac{s'}{s} = -\frac{33.0 \text{ cm}}{16.5 \text{ cm}} = -2.00 \text{ (} m < 0 \text{ means inverted image) } |y'| = |m||y| = 2.00(0.600 \text{ cm}) = 1.20 \text{ cm}$$

EVALUATE: The image is 33.0 cm to the left of the mirror vertex. It is real, inverted, and is 1.20 cm tall (enlarged). The calculation agrees with the image characterization from the principal ray diagram. A concave mirror used alone always forms a real, inverted image if $s > f$ and the image is enlarged if $f < s < 2f$.

34.6. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$.

SET UP: For a convex mirror, $R < 0$. $R = -22.0 \text{ cm}$ and $f = \frac{R}{2} = -11.0 \text{ cm}$.

EXECUTE: (a) The principal-ray diagram is sketched in Figure 34.6.

$$(b) \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ . } s' = \frac{sf}{s-f} = \frac{(16.5 \text{ cm})(-11.0 \text{ cm})}{16.5 \text{ cm} - (-11.0 \text{ cm})} = -6.6 \text{ cm} \text{ . } m = -\frac{s'}{s} = -\frac{-6.6 \text{ cm}}{16.5 \text{ cm}} = +0.400 \text{ .}$$

$|y'| = |m|y = (0.400)(0.600 \text{ cm}) = 0.240 \text{ cm}$. The image is 6.6 cm to the right of the mirror. It is 0.240 cm tall.

$s' < 0$, so the image is virtual. $m > 0$, so the image is erect.

EVALUATE: The calculated image properties agree with the image characterization from the principal-ray diagram.

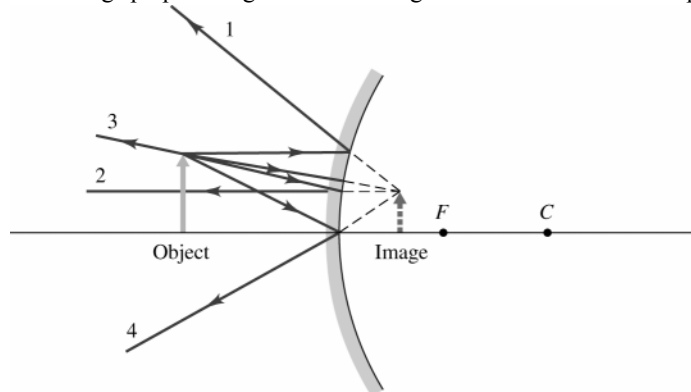


Figure 34.6

34.7. IDENTIFY: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $m = -\frac{s'}{s}$. $|m| = \frac{|y'|}{y}$. Find m and calculate y' .

SET UP: $f = +1.75 \text{ m}$.

EXECUTE: $s \gg f$ so $s' = f = 1.75 \text{ m}$.

$$m = -\frac{s'}{s} = -\frac{1.75 \text{ m}}{5.58 \times 10^{10} \text{ m}} = -3.14 \times 10^{-11} \text{ . } |y'| = |m|y = (3.14 \times 10^{-11})(6.794 \times 10^6 \text{ m}) = 2.13 \times 10^{-4} \text{ m} = 0.213 \text{ mm} \text{ .}$$

EVALUATE: The image is real and is 1.75 m in front of the mirror.

34.8. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$.

SET UP: The mirror surface is convex so $R = -3.00 \text{ cm}$. $s = 24.0 \text{ cm} - 3.00 \text{ cm} = 21.0 \text{ cm}$.

$$\text{EXECUTE: } f = \frac{R}{2} = -1.50 \text{ cm} \text{ . } \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ . } s' = \frac{sf}{s-f} = \frac{(21.0 \text{ cm})(-1.50 \text{ cm})}{21.0 \text{ cm} - (-1.50 \text{ cm})} = -1.40 \text{ cm} \text{ . The image is}$$

1.40 cm behind the surface so it is $3.00 \text{ cm} - 1.40 \text{ cm} = 1.60 \text{ cm}$ from the center of the ornament, on the same side

$$\text{as the object. } m = -\frac{s'}{s} = -\frac{-1.40 \text{ cm}}{21.0 \text{ cm}} = +0.0667 \text{ . } |y'| = |m|y = (0.0667)(3.80 \text{ mm}) = 0.253 \text{ mm} \text{ .}$$

EVALUATE: The image is virtual, upright and smaller than the object.

34.9. IDENTIFY: The shell behaves as a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ , and its magnification is given by } m = -\frac{s'}{s} \text{ .}$$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} = \frac{2}{-18.0 \text{ cm}} - \frac{1}{-6.00 \text{ cm}} \Rightarrow s = 18.0 \text{ cm}$ from the vertex.

$m = -\frac{s'}{s} = -\frac{-6.00 \text{ cm}}{18.0 \text{ cm}} = \frac{1}{3} \Rightarrow y' = \frac{1}{3}(1.5 \text{ cm}) = 0.50 \text{ cm}$. The image is 0.50 cm tall, erect, and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.10. IDENTIFY: The bottom surface of the bowl behaves as a spherical convex mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and its magnification is given by $m = -\frac{s'}{s}$.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{-2}{35 \text{ cm}} - \frac{1}{90 \text{ cm}} \Rightarrow s' = -15 \text{ cm}$ behind bowl.

$m = -\frac{s'}{s} = \frac{15 \text{ cm}}{90 \text{ cm}} = 0.167 \Rightarrow y' = (0.167)(2.0 \text{ cm}) = 0.33 \text{ cm}$. The image is 0.33 cm tall, erect, and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.11. IDENTIFY: We are dealing with a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and its magnification is given by $m = -\frac{s'}{s}$.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs} \Rightarrow s' = \frac{sf}{s-f}$. Also $m = -\frac{s'}{s} = \frac{f}{f-s}$.

(b) The graph is given in Figure 34.11a.

(c) $s' > 0$ for $s > f$, $s < 0$.

(d) $s' < 0$ for $0 < s < f$.

(e) The image is at negative infinity, "behind" the mirror.

(f) At the focal point, $s = f$.

(g) The image is at the mirror, $s' = 0$.

(h) The graph is given in Figure 34.11b.

(i) Erect and larger if $0 < s < f$.

(j) Inverted if $s > f$.

(k) The image is smaller if $s > 2f$ or $s < 0$.

(l) As the object is moved closer and closer to the focal point, the magnification increases to infinite values.

EVALUATE: As the object crosses the focal point, both the image distance and the magnification undergo discontinuities.

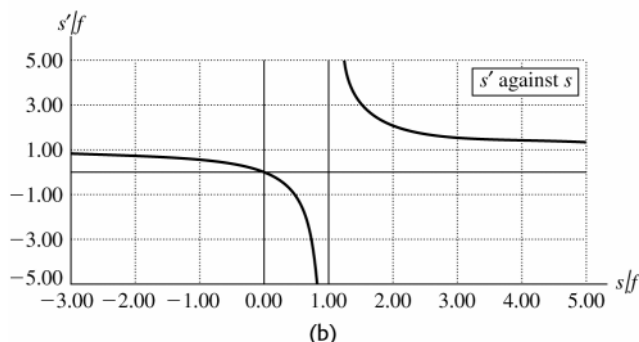
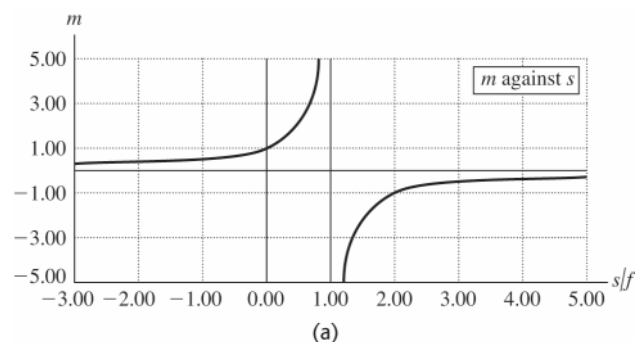


Figure 34.11

34.12. IDENTIFY: $s' = \frac{sf}{s-f}$ and $m = \frac{f}{f-s}$.

SET UP: With $f = -|f|$, $s' = -\frac{s|f|}{s+|f|}$ and $m = \frac{|f|}{s+|f|}$.

EXECUTE: The graphs are given in Figure 34.12.

(a) $s' > 0$ for $-|f| < s < 0$.

(b) $s' < 0$ for $s < -|f|$ and $s < 0$.

(c) If the object is at infinity, the image is at the outward going focal point.

(d) If the object is next to the mirror, then the image is also at the mirror

(e) The image is erect (magnification greater than zero) for $s > -|f|$.

(f) The image is inverted (magnification less than zero) for $s < -|f|$.

(g) The image is larger than the object (magnification greater than one) for $-2|f| < s < 0$.

(h) The image is smaller than the object (magnification less than one) for $s > 0$ and $s < -2|f|$.

EVALUATE: For a real image ($s > 0$), the image formed by a convex mirror is always virtual and smaller than the object.

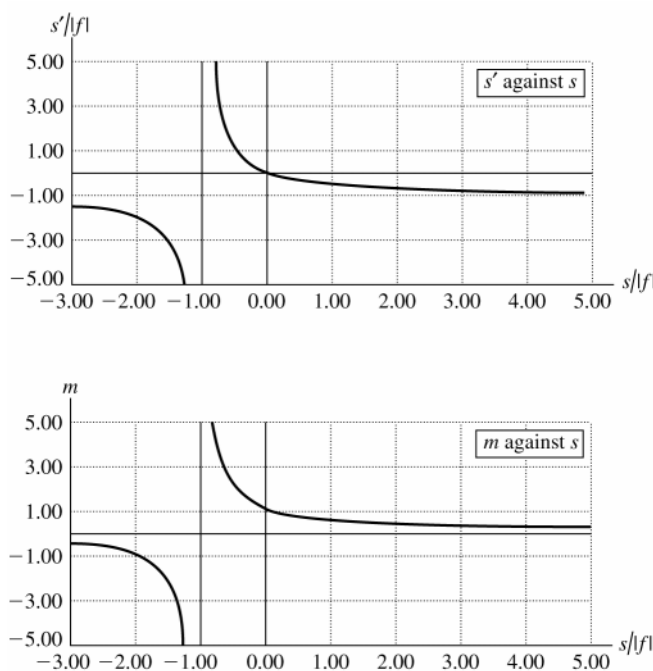


Figure 34.12

34.13. IDENTIFY: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $m = +2.00$ and $s = 1.25$ cm. An erect image must be virtual.

EXECUTE: (a) $s' = \frac{sf}{s-f}$ and $m = -\frac{f}{s-f}$. For a concave mirror, m can be larger than 1.00. For a convex mirror,

$|f| = -f$ so $m = +\frac{|f|}{s+|f|}$ and m is always less than 1.00. The mirror must be concave ($f > 0$).

(b) $\frac{1}{f} = \frac{s'+s}{ss'}$. $f = \frac{ss'}{s+s'}$. $m = -\frac{s'}{s} = +2.00$ and $s' = -2.00s$. $f = \frac{s(-2.00s)}{s-2.00s} = +2.00s = +2.50$ cm.

$R = 2f = +5.00$ cm.

(c) The principal ray diagram is drawn in Figure 34.13.

EVALUATE: The principal-ray diagram agrees with the description from the equations.

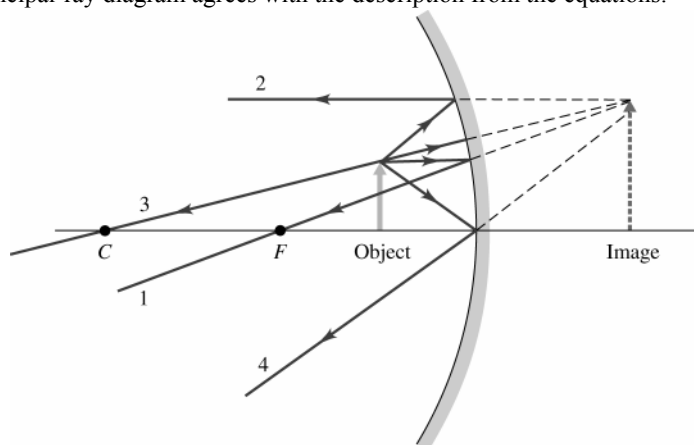


Figure 34.13

34.14. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$.

SET UP: For a concave mirror, $R > 0$. $R = 32.0$ cm and $f = \frac{R}{2} = 16.0$ cm.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $s' = \frac{sf}{s - f} = \frac{(12.0 \text{ cm})(16.0 \text{ cm})}{12.0 \text{ cm} - 16.0 \text{ cm}} = -48.0 \text{ cm}$. $m = -\frac{s'}{s} = -\frac{-48.0 \text{ cm}}{12.0 \text{ cm}} = +4.00$.

(b) $s' = -48.0$ cm, so the image is 48.0 cm to the right of the mirror. $s' < 0$ so the image is virtual.

(c) The principal-ray diagram is sketched in Figure 34.14. The rules for principal rays apply only to paraxial rays. Principal ray 2, that travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.

EVALUATE: A concave mirror forms a virtual image whenever $s < f$.

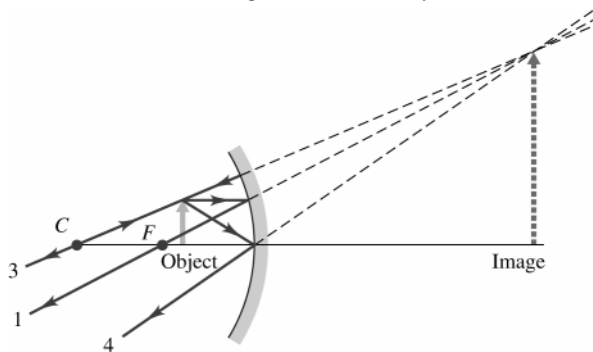


Figure 34.14

34.15. IDENTIFY: Apply Eq.(34.11), with $R \rightarrow \infty$. $|s'|$ is the apparent depth.

SET UP The image and object are shown in Figure 34.15.

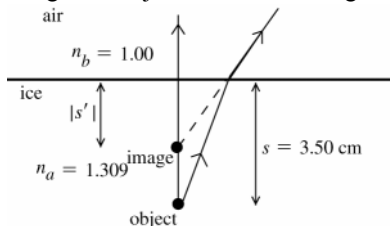


Figure 34.15

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R};$$

$R \rightarrow \infty$ (flat surface), so

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

EXECUTE: $s' = -\frac{n_b s}{n_a} = -\frac{(1.00)(3.50 \text{ cm})}{1.309} = -2.67 \text{ cm}$

The apparent depth is 2.67 cm.

EVALUATE: When the light goes from ice to air (larger to smaller n), it is bent away from the normal and the virtual image is closer to the surface than the object is.

- 34.16. IDENTIFY:** The surface is flat so $R \rightarrow \infty$ and $\frac{n_a}{s} + \frac{n_b}{s'} = 0$.

SET UP: The light travels from the fish to the eye, so $n_a = 1.333$ and $n_b = 1.00$. When the fish is viewed, $s = 7.0$ cm. The fish is 20.0 cm $- 7.0$ cm $= 13.0$ cm above the mirror, so the image of the fish is 13.0 cm below the mirror and 20.0 cm $+ 13.0$ cm $= 33.0$ cm below the surface of the water. When the image is viewed, $s = 33.0$ cm.

EXECUTE: (a) $s' = -\left(\frac{n_b}{n_a}\right)s = -\left(\frac{1.00}{1.333}\right)(7.0 \text{ cm}) = -5.25$ cm. The apparent depth is 5.25 cm.

(b) $s' = -\left(\frac{n_b}{n_a}\right)s = -\left(\frac{1.00}{1.333}\right)(33.0 \text{ cm}) = -24.8$ cm. The apparent depth of the image of the fish in the mirror is 24.8 cm.

EVALUATE: In each case the apparent depth is less than the actual depth of what is being viewed.

- 34.17. IDENTIFY:** $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $m = -\frac{n_a s'}{n_b s}$. Light comes from the fish to the person's eye.

SET UP: $R = -14.0$ cm. $s = +14.0$ cm. $n_a = 1.333$ (water). $n_b = 1.00$ (air). Figure 34.17 shows the object and the refracting surface.

EXECUTE: (a) $\frac{1.333}{14.0 \text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.333}{-14.0 \text{ cm}}$. $s' = -14.0$ cm. $m = -\frac{(1.333)(-14.0 \text{ cm})}{(1.00)(14.0 \text{ cm})} = +1.33$.

The fish's image is 14.0 cm to the left of the bowl surface so is at the center of the bowl and the magnification is 1.33 .

(b) The focal point is at the image location when $s \rightarrow \infty$. $\frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $n_a = 1.00$. $n_b = 1.333$. $R = +14.0$ cm.

$\frac{1.333}{s'} = \frac{1.333 - 1.00}{14.0 \text{ cm}}$. $s' = +56.0$ cm. s' is greater than the diameter of the bowl, so the surface facing the sunlight

does not focus the sunlight to a point inside the bowl. The focal point is outside the bowl and there is no danger to the fish.

EVALUATE: In part (b) the rays refract when they exit the bowl back into the air so the image we calculated is not the final image.

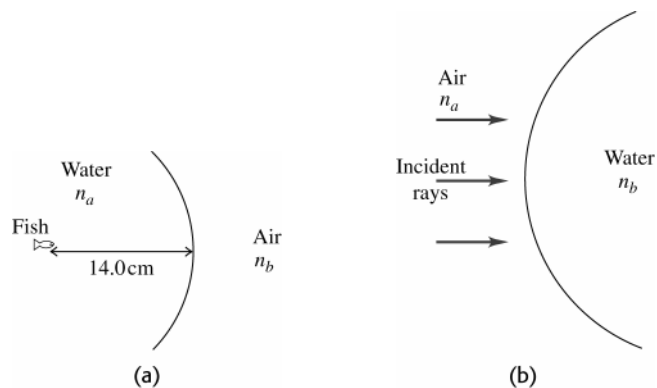


Figure 34.17

- 34.18. IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: For a convex surface, $R > 0$. $R = +3.00$ cm. $n_a = 1.00$, $n_b = 1.60$.

EXECUTE: (a) $s \rightarrow \infty$. $\frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $s' = \left(\frac{n_b}{n_b - n_a}\right)R = \left(\frac{1.60}{1.60 - 1.00}\right)(+3.00 \text{ cm}) = +8.00$ cm. The image is 8.00 cm to the right of the vertex.

(b) $s = 12.0$ cm. $\frac{1.00}{12.0 \text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{3.00 \text{ cm}}$. $s' = +13.7$ cm. The image is 13.7 cm to the right of the vertex.

(c) $s = 2.00$ cm. $\frac{1.00}{2.00 \text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{3.00 \text{ cm}}$. $s' = -5.33$ cm. The image is 5.33 cm to the left of the vertex.

EVALUATE: The image can be either real ($s' > 0$) or virtual ($s' < 0$), depending on the distance of the object from the refracting surface.

- 34.19. IDENTIFY:** The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and oil.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the

equation $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.45}{s} + \frac{1.60}{1.20 \text{ m}} = \frac{0.15}{0.0300 \text{ m}} \Rightarrow s = 0.395 \text{ cm}$

EVALUATE: The presence of the oil changes the location of the image.

34.20. IDENTIFY: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $m = -\frac{n_a s'}{n_b s}$.

SET UP: $R = +4.00 \text{ cm}$. $n_a = 1.00$. $n_b = 1.60$. $s = 24.0 \text{ cm}$.

EXECUTE: $\frac{1}{24.0 \text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{4.00 \text{ cm}}$. $s' = +14.8 \text{ cm}$. $m = -\frac{(1.00)(14.8 \text{ cm})}{(1.60)(24.0 \text{ cm})} = -0.385$.

$|y'| = |m|y = (0.385)(1.50 \text{ mm}) = 0.578 \text{ mm}$. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall.

$m < 0$, so the image is inverted.

EVALUATE: The image is real.

34.21. IDENTIFY: Apply Eqs.(34.11) and (34.12). Calculate s' and y' . The image is erect if $m > 0$.

SET UP: The object and refracting surface are shown in Figure 34.21.

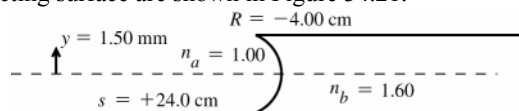


Figure 34.21

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$

$\frac{1.00}{24.0 \text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{-4.00 \text{ cm}}$

Multiplying by 24.0 cm gives $1.00 + \frac{38.4}{s'} = -3.60$

$\frac{38.4 \text{ cm}}{s'} = -4.60$ and $s' = -\frac{38.4 \text{ cm}}{4.60} = -8.35 \text{ cm}$

Eq.(34.12): $m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(-8.35 \text{ cm})}{(1.60)(+24.0 \text{ cm})} = +0.217$

$|y'| = |m|y = (0.217)(1.50 \text{ mm}) = 0.326 \text{ mm}$

EVALUATE: The image is virtual ($s' < 0$) and is 8.35 cm to the left of the vertex. The image is erect ($m > 0$) and is 0.326 mm tall. R is negative since the center of curvature of the surface is on the incoming side.

34.22. IDENTIFY: The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and liquid.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the

equation $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{n_a}{14.0 \text{ cm}} + \frac{1.60}{9.00 \text{ cm}} = \frac{1.60 - n_a}{4.00 \text{ cm}} \Rightarrow n_a = 1.24$.

EVALUATE: The result is a reasonable refractive index for liquids.

34.23. IDENTIFY: Use $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ to calculate f . The apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $R_1 \rightarrow \infty$. $R_2 = -13.0 \text{ cm}$. If the lens is reversed, $R_1 = +13.0 \text{ cm}$ and $R_2 \rightarrow \infty$.

EXECUTE: (a) $\frac{1}{f} = (0.70)\left(\frac{1}{\infty} - \frac{1}{-13.0 \text{ cm}}\right) = \frac{0.70}{13.0 \text{ cm}}$ and $f = 18.6 \text{ cm}$. $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$.

$s' = \frac{sf}{s-f} = \frac{(22.5 \text{ cm})(18.6 \text{ cm})}{22.5 \text{ cm} - 18.6 \text{ cm}} = 107 \text{ cm}$. $m = -\frac{s'}{s} = -\frac{107 \text{ cm}}{22.5 \text{ cm}} = -4.76$.

$y' = my = (-4.76)(3.75 \text{ mm}) = -17.8 \text{ mm}$. The image is 107 cm to the right of the lens and is 17.8 mm tall. The image is real and inverted.

(b) $\frac{1}{f} = (n-1)\left(\frac{1}{13.0 \text{ cm}} - \frac{1}{\infty}\right)$ and $f = 18.6 \text{ cm}$. The image is the same as in part (a).

EVALUATE: Reversing a lens does not change the focal length of the lens.

34.24. IDENTIFY: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The sign of f determines whether the lens is converging or diverging.

SET UP: $s = 16.0 \text{ cm}$. $s' = -12.0 \text{ cm}$.

EXECUTE: (a) $f = \frac{ss'}{s+s'} = \frac{(16.0 \text{ cm})(-12.0 \text{ cm})}{16.0 \text{ cm} + (-12.0 \text{ cm})} = -48.0 \text{ cm}$. $f < 0$ and the lens is diverging.

(b) $m = -\frac{s'}{s} = -\frac{-12.0 \text{ cm}}{16.0 \text{ cm}} = +0.750$. $|y'| = |m|y = (0.750)(8.50 \text{ mm}) = 6.38 \text{ mm}$. $m > 0$ and the image is erect.

(c) The principal-ray diagram is sketched in Figure 34.24.

EVALUATE: A diverging lens always forms an image that is virtual, erect and reduced in size.

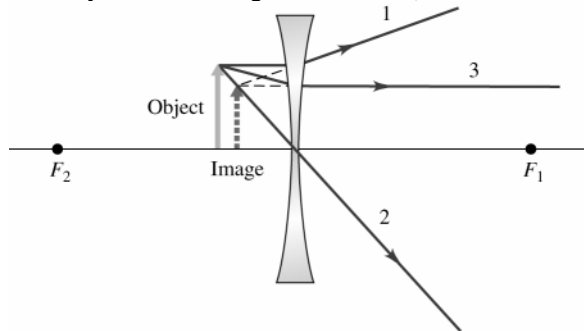


Figure 34.24

34.25. IDENTIFY: The liquid behaves like a lens, so the lensmaker's equation applies.

SET UP: The lensmaker's equation is $\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, and the magnification of the lens is $m = -\frac{s'}{s}$.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{24.0 \text{ cm}} + \frac{1}{s'} = (1.52-1)\left(\frac{1}{-7.00 \text{ cm}} - \frac{1}{-4.00 \text{ cm}}\right)$
 $\Rightarrow s' = 71.2 \text{ cm}$, to the right of the lens.

(b) $m = -\frac{s'}{s} = -\frac{71.2 \text{ cm}}{24.0 \text{ cm}} = -2.97$

EVALUATE: Since the magnification is negative, the image is inverted.

34.26. IDENTIFY: Apply $m = \frac{y'}{y} = -\frac{s'}{s}$ to relate s' and s and then use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: Since the image is inverted, $y' < 0$ and $m < 0$.

EXECUTE: $m = \frac{y'}{y} = \frac{-4.50 \text{ cm}}{3.20 \text{ cm}} = -1.406$. $m = -\frac{s'}{s}$ gives $s' = +1.406s$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives

$\frac{1}{s} + \frac{1}{1.406s} = \frac{1}{90.0 \text{ cm}}$ and $s = 154 \text{ cm}$. $s' = (1.406)(154 \text{ cm}) = 217 \text{ cm}$. The object is 154 cm to the left of the lens. The image is 217 cm to the right of the lens and is real.

EVALUATE: For a single lens an inverted image is always real.

34.27. IDENTIFY: The thin-lens equation applies in this case.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and the magnification is $m = -\frac{s'}{s} = \frac{y'}{y}$.

EXECUTE: $m = \frac{y'}{y} = \frac{34.0 \text{ mm}}{8.00 \text{ mm}} = 4.25 = -\frac{s'}{s} = -\frac{-12.0 \text{ cm}}{s} \Rightarrow s = 2.82 \text{ cm}$. The thin-lens equation gives

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = 3.69 \text{ cm}$.

EVALUATE: Since the focal length is positive, this is a converging lens. The image distance is negative because the object is inside the focal point of the lens.

34.28. IDENTIFY: Apply $m = -\frac{s'}{s}$ to relate s and s' . Then use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: Since the image is to the right of the lens, $s' > 0$. $s' + s = 6.00 \text{ m}$.

EXECUTE: (a) $s' = 80.0s$ and $s + s' = 6.00 \text{ m}$ gives $81.00s = 6.00 \text{ m}$ and $s = 0.0741 \text{ m}$. $s' = 5.93 \text{ m}$.

(b) The image is inverted since both the image and object are real ($s' > 0, s > 0$).

(c) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.0741 \text{ m}} + \frac{1}{5.93 \text{ m}} \Rightarrow f = 0.0732 \text{ m}$, and the lens is converging.

EVALUATE: The object is close to the lens and the image is much farther from the lens. This is typical for slide projectors.

34.29. IDENTIFY: Apply $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

SET UP: For a distant object the image is at the focal point of the lens. Therefore, $f = 1.87 \text{ cm}$. For the double-convex lens, $R_1 = +R$ and $R_2 = -R$, where $R = 2.50 \text{ cm}$.

EXECUTE: $\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(n-1)}{R}$. $n = \frac{R}{2f} + 1 = \frac{2.50 \text{ cm}}{2(1.87 \text{ cm})} + 1 = 1.67$.

EVALUATE: $f > 0$ and the lens is converging. A double-convex lens is always converging.

34.30. IDENTIFY and SET UP: Apply $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

EXECUTE: We have a converging lens if the focal length is positive, which requires

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) > 0 \Rightarrow \left(\frac{1}{R_1} - \frac{1}{R_2}\right) > 0$. This can occur in one of three ways:

(i) R_1 and R_2 both positive and $R_1 < R_2$. (ii) $R_1 \geq 0, R_2 \leq 0$ (double convex and planoconvex).

(iii) R_1 and R_2 both negative and $|R_1| > |R_2|$ (meniscus). The three lenses in Figure 35.32a in the textbook fall into these categories.

We have a diverging lens if the focal length is negative, which requires

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) < 0 \Rightarrow \left(\frac{1}{R_1} - \frac{1}{R_2}\right) < 0$. This can occur in one of three ways:

(i) R_1 and R_2 both positive and $R_1 > R_2$ (meniscus). (ii) R_1 and R_2 both negative and $|R_2| > |R_1|$. (iii) $R_1 \leq 0, R_2 \geq 0$ (planoconcave and double concave). The three lenses in Figure 34.32b in the textbook fall into these categories.

EVALUATE: The converging lenses in Figure 34.32a are all thicker at the center than at the edges. The diverging lenses in Figure 34.32b are all thinner at the center than at the edges.

34.31. IDENTIFY and SET UP: The equations $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$ apply to both thin lenses and spherical mirrors.

EXECUTE: (a) The derivation of the equations in Exercise 34.11 is identical and one gets:

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs} \Rightarrow s' = \frac{sf}{s-f}$, and also $m = -\frac{s'}{s} = \frac{f}{f-s}$.

(b) Again, one gets exactly the same equations for a converging lens rather than a concave mirror because the equations are identical. The difference lies in the interpretation of the results. For a lens, the outgoing side is *not* that on which the object lies, unlike for a mirror. So for an object on the left side of the lens, a positive image distance means that the image is on the right of the lens, and a negative image distance means that the image is on the left side of the lens.

(c) Again, for Exercise 34.12, the change from a convex mirror to a diverging lens changes nothing in the exercises, except for the interpretation of the location of the images, as explained in part (b) above.

EVALUATE: Concave mirrors and converging lenses both have $f > 0$. Convex mirrors and diverging lenses both have $f < 0$.

34.32. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $f = +12.0 \text{ cm}$ and $s' = -17.0 \text{ cm}$.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} = \frac{1}{12.0 \text{ cm}} - \frac{1}{-17.0 \text{ cm}} \Rightarrow s = 7.0 \text{ cm}$.

$m = -\frac{s'}{s} = -\frac{(-17.0)}{7.2} = +2.4 \Rightarrow y = \frac{y'}{m} = \frac{0.800 \text{ cm}}{+2.4} = +0.34 \text{ cm}$, so the object is 0.34 cm tall, erect, same side as the image. The principal-ray diagram is sketched in Figure 34.32.

EVALUATE: When the object is inside the focal point, a converging lens forms a virtual, enlarged image.

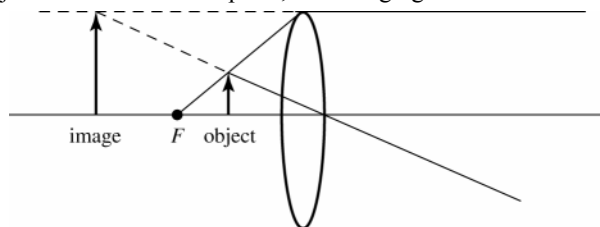


Figure 34.32

34.33. IDENTIFY: Use Eq.(34.16) to calculate the object distance s . m calculated from Eq.(34.17) determines the size and orientation of the image.

SET UP: $f = -48.0$ cm. Virtual image 17.0 cm from lens so $s' = -17.0$ cm.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, so $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s - f}{sf}$

$$s = \frac{s'f}{s' - f} = \frac{(-17.0 \text{ cm})(-48.0 \text{ cm})}{-17.0 \text{ cm} - (-48.0 \text{ cm})} = +26.3 \text{ cm}$$

$$m = -\frac{s'}{s} = -\frac{-17.0 \text{ cm}}{+26.3 \text{ cm}} = +0.646$$

$$m = \frac{y'}{y} \text{ so } |y| = \frac{|y'|}{|m|} = \frac{8.00 \text{ mm}}{0.646} = 12.4 \text{ mm}$$

The principal-ray diagram is sketched in Figure 34.33.

EVALUATE: Virtual image, real object ($s > 0$) so image and object are on same side of lens. $m > 0$ so image is erect with respect to the object. The height of the object is 12.4 mm.

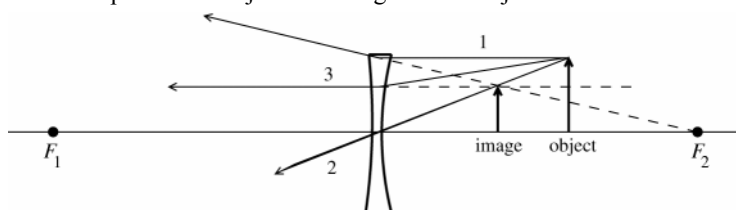


Figure 34.33

34.34. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: The sign of f determines whether the lens is converging or diverging. $s = 16.0$ cm. $s' = +36.0$ cm. Use $m = -\frac{s'}{s}$ to find the size and orientation of the image.

EXECUTE: (a) $f = \frac{ss'}{s + s'} = \frac{(16.0 \text{ cm})(36.0 \text{ cm})}{16.0 \text{ cm} + 36.0 \text{ cm}} = 11.1 \text{ cm}$. $f > 0$ and the lens is converging.

(b) $m = -\frac{s'}{s} = -\frac{36.0 \text{ cm}}{16.0 \text{ cm}} = -2.25$. $|y'| = |m|y = (2.25)(8.00 \text{ mm}) = 18.0 \text{ mm}$. $m < 0$ so the image is inverted.

(c) The principal-ray diagram is sketched in Figure 34.34.

EVALUATE: The image is real so the lens must be converging.

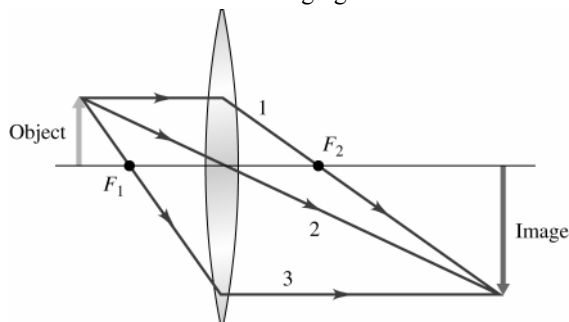


Figure 34.34

34.35. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: The image is to be formed on the film, so $s' = +20.4$ cm.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{20.4 \text{ cm}} = \frac{1}{20.0 \text{ cm}} \Rightarrow s = 1020 \text{ cm} = 10.2 \text{ m}.$

EVALUATE: The object distance is much greater than f , so the image is just outside the focal point of the lens.

34.36. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $s = 3.90$ m. $f = 0.085$ m.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{3.90 \text{ m}} + \frac{1}{s'} = \frac{1}{0.085 \text{ m}} \Rightarrow s' = 0.0869 \text{ m}.$ $y' = -\frac{s'}{s}y = -\frac{0.0869}{3.90}1750 \text{ mm} = -39.0 \text{ mm},$ so

it will not fit on the 24-mm $\times$ 36-mm film.

EVALUATE: The image is just outside the focal point and $s' \approx f$. To have $y' = 36$ mm, so that the image will fit

on the film, $s = -\frac{s'y}{y'} \approx -\frac{(0.085 \text{ m})(1.75 \text{ m})}{-0.036 \text{ m}} = 4.1 \text{ m}.$ The person would need to stand about 4.1 m from the lens.

34.37. IDENTIFY: $|m| = \left| \frac{s'}{s} \right|.$

SET UP: $s \gg f$, so $s' \approx f$.

EXECUTE: (a) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{28 \text{ mm}}{200,000 \text{ mm}} = 1.4 \times 10^{-4}.$

(b) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{105 \text{ mm}}{200,000 \text{ mm}} = 5.3 \times 10^{-4}.$

(c) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{300 \text{ mm}}{200,000 \text{ mm}} = 1.5 \times 10^{-3}.$

EVALUATE: The magnitude of the magnification increases when f increases.

34.38. IDENTIFY: $|m| = \left| \frac{s'}{s} \right| = \left| \frac{y'}{y} \right|$

SET UP: $s \gg f$, so $s' \approx f$.

EXECUTE: $|y'| = \frac{s'}{s}y \approx \frac{f}{s}y = \frac{5.00 \text{ m}}{9.50 \times 10^3 \text{ m}}(70.7 \text{ m}) = 0.0372 \text{ m} = 37.2 \text{ mm}.$

EVALUATE: A very long focal length lens is needed to photograph a distant object.

34.39. IDENTIFY and SET UP: Find the lateral magnification that results in this desired image size. Use Eq.(34.17) to relate m and s' and Eq.(34.16) to relate s and s' to f .

EXECUTE: (a) We need $m = -\frac{24 \times 10^{-3} \text{ m}}{160 \text{ m}} = -1.5 \times 10^{-4}.$ Alternatively, $m = -\frac{36 \times 10^{-3} \text{ m}}{240 \text{ m}} = -1.5 \times 10^{-4}.$

$s \gg f$ so $s' \approx f$

Then $m = -\frac{s'}{s} = -\frac{f}{s} = -1.5 \times 10^{-4}$ and $f = (1.5 \times 10^{-4})(600 \text{ m}) = 0.090 \text{ m} = 90 \text{ mm}.$

A smaller f means a smaller s' and a smaller m , so with $f = 85$ mm the object's image nearly fills the picture area.

(b) We need $m = -\frac{36 \times 10^{-3} \text{ m}}{9.6 \text{ m}} = -3.75 \times 10^{-3}.$ Then, as in part (a), $\frac{f}{s} = 3.75 \times 10^{-3}$ and

$f = (40.0 \text{ m})(3.75 \times 10^{-3}) = 0.15 \text{ m} = 150 \text{ mm}.$ Therefore use the 135 mm lens.

EVALUATE: When $s \gg f$ and $s' \approx f$, $y' = -f(y/s)$. For the mobile home y/s is smaller so a larger f is needed. Note that m is very small; the image is much smaller than the object.

34.40. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to each lens. The image of the first lens serves as the object for the second lens.

SET UP: For a distant object, $s \rightarrow \infty$

EXECUTE: (a) $s_1 = \infty \Rightarrow s'_1 = f_1 = 12 \text{ cm}.$

(b) $s_2 = 4.0 \text{ cm} - 12 \text{ cm} = -8 \text{ cm}.$

$$(c) \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-8 \text{ cm}} + \frac{1}{s'_2} = \frac{1}{-12 \text{ cm}} \Rightarrow s'_2 = 24 \text{ cm, to the right.}$$

$$(d) s_1 = \infty \Rightarrow s'_1 = f_1 = 12 \text{ cm. } s_2 = 8.0 \text{ cm} - 12 \text{ cm} = -4 \text{ cm. } \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-4 \text{ cm}} + \frac{1}{s'_2} = \frac{1}{-12 \text{ cm}} \Rightarrow s'_2 = 6 \text{ cm.}$$

EVALUATE: In each case the image of the first lens serves as a virtual object for the second lens, and $s_2 < 0$.

- 34.41. IDENTIFY:** The f -number of a lens is the ratio of its focal length to its diameter. To maintain the same exposure, the amount of light passing through the lens during the exposure must remain the same.

SET UP: The f -number is f/D .

EXECUTE: (a) $f\text{-number} = \frac{f}{D} \Rightarrow f\text{-number} = \frac{180.0 \text{ mm}}{16.36 \text{ mm}} \Rightarrow f\text{-number} = f/11$. (The f -number is an integer.)

(b) $f/11$ to $f/2.8$ is four steps of 2 in intensity, so one needs $1/16^{\text{th}}$ the exposure. The exposure should be $1/480 \text{ s} = 2.1 \times 10^{-3} \text{ s} = 2.1 \text{ ms}$.

EVALUATE: When opening the lens from $f/11$ to $f/2.8$, the area increases by a factor of 16, so 16 times as much light is allowed in. Therefore the exposure time must be decreased by a factor of $1/16$ to maintain the same exposure on the film or light receptors of a digital camera.

- 34.42. IDENTIFY and SET UP:** The square of the aperture diameter is proportional to the length of the exposure time required.

EXECUTE: $\left(\frac{1}{30} \text{ s}\right) \left(\frac{8 \text{ mm}}{23.1 \text{ mm}}\right)^2 \approx \left(\frac{1}{250} \text{ s}\right)$

EVALUATE: An increase in the aperture diameter decreases the exposure time.

- 34.43. IDENTIFY and SET UP:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s' .

EXECUTE: (a) A real image is formed at the film, so the lens must be convex.

(b) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ so $\frac{1}{s'} = \frac{s-f}{sf}$ and $s' = \frac{sf}{s-f}$, with $f = +50.0 \text{ mm}$. For $s = 45 \text{ cm} = 450 \text{ mm}$, $s' = 56 \text{ mm}$. For $s = \infty$, $s' = f = 50 \text{ mm}$. The range of distances between the lens and film is 50 mm to 56 mm.

EVALUATE: The lens is closer to the film when photographing more distant objects.

- 34.44. IDENTIFY:** The projector lens can be modeled as a thin lens.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and the magnification of the lens is $m = -\frac{s'}{s}$.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{0.150 \text{ m}} + \frac{1}{9.00 \text{ m}} \Rightarrow f = 147.5 \text{ mm}$, so use a $f = 148 \text{ mm}$ lens.

(b) $m = -\frac{s'}{s} \Rightarrow |m| = 60 \Rightarrow \text{Area} = 1.44 \text{ m} \times 2.16 \text{ m}$.

EVALUATE: The lens must produce a real image to be viewed on the screen. Since the magnification comes out negative, the slides to be viewed must be placed upside down in the tray.

- 34.45. (a) IDENTIFY:** The purpose of the corrective lens is to take an object 25 cm from the eye and form a virtual image at the eye's near point. Use Eq.(34.16) to solve for the image distance when the object distance is 25 cm.

SET UP: $\frac{1}{f} = +2.75$ diopters means $f = +\frac{1}{2.75} \text{ m} = +0.3636 \text{ m}$ (converging lens)

$f = 36.36 \text{ cm}; s = 25 \text{ cm}; s' = ?$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ so

$$s' = \frac{sf}{s-f} = \frac{(25 \text{ cm})(36.36 \text{ cm})}{25 \text{ cm} - 36.36 \text{ cm}} = -80.0 \text{ cm}$$

The eye's near point is 80.0 cm from the eye.

(b) **IDENTIFY:** The purpose of the corrective lens is to take an object at infinity and form a virtual image of it at the eye's far point. Use Eq.(34.16) to solve for the image distance when the object is at infinity.

SET UP: $\frac{1}{f} = -1.30$ diopters means $f = -\frac{1}{1.30} \text{ m} = -0.7692 \text{ m}$ (diverging lens)

$f = -76.92 \text{ cm}; s = \infty; s' = ?$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $s = \infty$ says $\frac{1}{s'} = \frac{1}{f}$ and $s' = f = -76.9 \text{ cm}$. The eye's far point is 76.9 cm from the eye.

EVALUATE: In each case a virtual image is formed by the lens. The eye views this virtual image instead of the object. The object is at a distance where the eye can't focus on it, but the virtual image is at a distance where the eye can focus.

34.46. IDENTIFY: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$

SET UP: $n_a = 1.00$, $n_b = 1.40$. $s = 40.0$ cm, $s' = 2.60$ cm.

EXECUTE: $\frac{1}{40.0 \text{ cm}} + \frac{1.40}{2.60 \text{ cm}} = \frac{0.40}{R}$ and $R = 0.710$ cm.

EVALUATE: The cornea presents a convex surface to the object, so $R > 0$.

34.47. IDENTIFY: In each case the lens forms a virtual image at a distance where the eye can focus. Power in diopters equals $1/f$, where f is in meters.

SET UP: In part (a), $s = 25$ cm and in part (b), $s \rightarrow \infty$.

EXECUTE: (a) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.25 \text{ m}} + \frac{1}{-0.600 \text{ m}} \Rightarrow \text{power} = \frac{1}{f} = +2.33$ diopters.

(b) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-0.600 \text{ m}} \Rightarrow \text{power} = \frac{1}{f} = -1.67$ diopters.

EVALUATE: A converging lens corrects the near vision and a diverging lens corrects the far vision.

34.48. IDENTIFY: When the object is at the focal point, $M = \frac{25.0 \text{ cm}}{f}$. In part (b), apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s for $s' = -25.0$ cm.

SET UP: Our calculation assumes the near point is 25.0 cm from the eye.

EXECUTE: (a) Angular magnification $M = \frac{25.0 \text{ cm}}{f} = \frac{25.0 \text{ cm}}{6.00 \text{ cm}} = 4.17$.

(b) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{-25.0 \text{ cm}} = \frac{1}{6.00 \text{ cm}} \Rightarrow s = 4.84$ cm.

EVALUATE: In part (b), $\theta' = \frac{y'}{s}$, $\theta = \frac{y}{25.0 \text{ cm}}$ and $M = \frac{25.0 \text{ cm}}{s} = \frac{25.0 \text{ cm}}{4.84 \text{ cm}} = 5.17$. M is greater when the image is at the near point than when the image is at infinity.

34.49. IDENTIFY: Use Eqs.(34.16) and (34.17) to calculate s and y' .

(a) **SET UP:** $f = 8.00$ cm; $s' = -25.0$ cm; $s = ?$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, so $\frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{s' - f}{s'f}$

EXECUTE: $s = \frac{s'f}{s' - f} = \frac{(-25.0 \text{ cm})(+8.00 \text{ cm})}{-25.0 \text{ cm} - 8.00 \text{ cm}} = +6.06$ cm

(b) $m = -\frac{s'}{s} = -\frac{-25.0 \text{ cm}}{6.06 \text{ cm}} = +4.125$

$|m| = \frac{|y'|}{|y|}$ so $|y'| = |m||y| = (4.125)(1.00 \text{ mm}) = 4.12$ mm

EVALUATE: The lens allows the object to be much closer to the eye than the near point. The lense allows the eye to view an image at the near point rather than the object.

34.50. IDENTIFY: For a thin lens, $-\frac{s'}{s} = \frac{y'}{y}$, so $\left| \frac{y'}{s'} \right| = \left| \frac{y}{s} \right|$, and the angular size of the image equals the angular size of the object.

SET UP: The object has angular size $\theta = \frac{y}{f}$, with θ in radians.

EXECUTE: $\theta = \frac{y}{f} \Rightarrow f = \frac{y}{\theta} = \frac{2.00 \text{ mm}}{0.025 \text{ rad}} = 80.0 \text{ mm} = 8.00$ cm.

EVALUATE: If the insect is at the near point of a normal eye, its angular size is $\frac{2.00 \text{ mm}}{250 \text{ mm}} = 0.0080$ rad.

34.51. IDENTIFY: The thin-lens equation applies to the magnifying lens.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

EXECUTE: The image is behind the lens, so $s' < 0$. The thin-lens equation gives

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} = \frac{1}{5.00 \text{ cm}} - \frac{1}{-25.0 \text{ cm}} \Rightarrow s = 4.17 \text{ cm, on the same side of the lens as the ant.}$$

EVALUATE: Since $s' < 0$, the image will be erect.

34.52. IDENTIFY: Apply Eq.(34.24).

SET UP: $s'_1 = 160 \text{ mm} + 5.0 \text{ mm} = 165 \text{ mm}$

EXECUTE: (a) $M = \frac{(250 \text{ mm})s'_1}{f_1 f_2} = \frac{(250 \text{ mm})(165 \text{ mm})}{(5.00 \text{ mm})(26.0 \text{ mm})} = 317.$

(b) The minimum separation is $\frac{0.10 \text{ mm}}{M} = \frac{0.10 \text{ mm}}{317} = 3.15 \times 10^{-4} \text{ mm}.$

EVALUATE: The angular size of the image viewed by the eye when looking through the microscope is 317 times larger than if the object is viewed at the near-point of the unaided eye.

34.53. (a) IDENTIFY and SET UP:

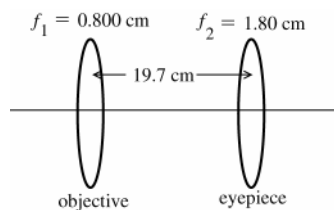


Figure 34.53

Final image is at ∞ so the object for the eyepiece is at its focal point. But the object for the eyepiece is the image of the objective so the image formed by the objective is $19.7 \text{ cm} - 1.80 \text{ cm} = 17.9 \text{ cm}$ to the right of the lens.

Apply Eq.(34.16) to the image formation by the objective, solve for the object distance s .

$f = 0.800 \text{ cm}; s' = 17.9 \text{ cm}; s = ?$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \text{ so } \frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{s' - f}{s'f}$$

EXECUTE: $s = \frac{s'f}{s' - f} = \frac{(17.9 \text{ cm})(+0.800 \text{ cm})}{17.9 \text{ cm} - 0.800 \text{ cm}} = +8.37 \text{ mm}$

(b) **SET UP:** Use Eq.(34.17).

EXECUTE: $m_1 = -\frac{s'}{s} = -\frac{17.9 \text{ cm}}{0.837 \text{ cm}} = -21.4$

The linear magnification of the objective is 21.4.

(c) **SET UP:** Use Eq.(34.23): $M = m_1 M_2$

EXECUTE: $M_2 = \frac{25 \text{ cm}}{f_2} = \frac{25 \text{ cm}}{1.80 \text{ cm}} = 13.9$

$M = m_1 M_2 = (-21.4)(13.9) = -297$

EVALUATE: M is not accurately given by $(25 \text{ cm})s'_1 / f_1 f_2 = 311$, because the object is not quite at the focal point of the objective ($s_1 = 0.837 \text{ cm}$ and $f_1 = 0.800 \text{ cm}$).

34.54. IDENTIFY: Eq.(34.24) can be written $M = |m_1| M_2 = \left| \frac{s'_1}{f_1} \right| M_2.$

SET UP: $s'_1 = f_1 + 120 \text{ mm}$

EXECUTE: $f = 16 \text{ mm} : s' = 120 \text{ mm} + 16 \text{ mm} = 136 \text{ mm}; s = 16 \text{ mm}. |m_1| = \frac{s'}{s} = \frac{136 \text{ mm}}{16 \text{ mm}} = 8.5.$

$f = 4 \text{ mm} : s' = 120 \text{ mm} + 4 \text{ mm} = 124 \text{ mm}; s = 4 \text{ mm} \Rightarrow |m_1| = \frac{s'}{s} = \frac{124 \text{ mm}}{4 \text{ mm}} = 31.$

$f = 1.9 \text{ mm} : s' = 120 \text{ mm} + 1.9 \text{ mm} = 122 \text{ mm}; s = 1.9 \text{ mm} \Rightarrow |m_1| = \frac{s'}{s} = \frac{122 \text{ mm}}{1.9 \text{ mm}} = 64.$

The eyepiece magnifies by either 5 or 10, so:

(a) The maximum magnification occurs for the 1.9-mm objective and 10x eyepiece:

$M = |m_1| M_e = (64)(10) = 640.$

(b) The minimum magnification occurs for the 16-mm objective and 5x eyepiece:

$M = |m_1| M_e = (8.5)(5) = 43.$

EVALUATE: The smaller the focal length of the objective, the greater the overall magnification.

34.55. IDENTIFY: f -number $= f/D$

SET UP: $D = 1.02$ m

EXECUTE: $\frac{f}{D} = 19.0 \Rightarrow f = (19.0)D = (19.0)(1.02 \text{ m}) = 19.4 \text{ m}$.

EVALUATE: Camera lenses can also have an f -number of 19.0. For a camera lens, both the focal length and lens diameter are much smaller, but the f -number is a measure of their ratio.

34.56. IDENTIFY: For a telescope, $M = -\frac{f_1}{f_2}$.

SET UP: $f_2 = 9.0$ cm. The distance between the two lenses equals $f_1 + f_2$.

EXECUTE: $f_1 + f_2 = 1.80 \text{ m} \Rightarrow f_1 = 1.80 \text{ m} - 0.0900 \text{ m} = 1.71 \text{ m}$. $M = -\frac{f_1}{f_2} = -\frac{171}{9.00} = -19.0$.

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.57. (a) IDENTIFY and SET UP: Use Eq.(34.24), with $f_1 = 95.0$ cm (objective) and $f_2 = 15.0$ cm (eyepiece).

EXECUTE: $M = -\frac{f_1}{f_2} = -\frac{95.0 \text{ cm}}{15.0 \text{ cm}} = -6.33$

(b) IDENTIFY and SET UP: Use Eq.(34.17) to calculate y' .

SET UP: $s = 3.00 \times 10^3$ m

$s' = f_1 = 95.0$ cm (since s is very large, $s' \approx f$)

EXECUTE: $m = -\frac{s'}{s} = -\frac{0.950 \text{ m}}{3.00 \times 10^3 \text{ m}} = -3.167 \times 10^{-4}$

$|y'| = |m||y| = (3.167 \times 10^{-4})(60.0 \text{ m}) = 0.0190 \text{ m} = 1.90 \text{ cm}$

(c) IDENTIFY: Use Eq.(34.21) and the angular magnification M obtained in part (a) to calculate θ' . The angular size θ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.

EXECUTE: The angular size of the object for the eyepiece is $\theta = \frac{0.0190 \text{ m}}{0.950 \text{ m}} = 0.0200$ rad.

(Note that this is also the angular size of the object for the objective: $\theta = \frac{60.0 \text{ m}}{3.00 \times 10^3 \text{ m}} = 0.0200$ rad. For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.)

$M = \frac{\theta'}{\theta}$ (Eq.34.21) so the angular size of the image is $\theta' = M\theta = -(6.33)(0.0200 \text{ rad}) = -0.127$ rad (The minus sign shows that the final image is inverted.)

EVALUATE: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00; the angular size of the final image viewed by the eye is 6.33 times larger than the angular size of the original object, as viewed by the unaided eye.

34.58. IDENTIFY: The angle subtended by Saturn with the naked eye is the same as the angle subtended by the image of Saturn formed by the objective lens (see Fig. 34.53 in the textbook).

SET UP: The angle subtended by Saturn is $\theta = \frac{\text{diameter of Saturn}}{\text{distance to Saturn}} = \frac{y'}{f_1}$.

EXECUTE: Putting in the numbers gives $\theta = \frac{y'}{f_1} = \frac{1.7 \text{ mm}}{18 \text{ m}} = \frac{0.0017 \text{ m}}{18 \text{ m}} = 9.4 \times 10^{-5} \text{ rad} = 0.0054^\circ$

EVALUATE: The angle subtended by the final image, formed by the eyepiece, would be much larger than 0.0054° .

34.59. IDENTIFY: $f = R/2$ and $M = -\frac{f_1}{f_2}$.

SET UP: For object and image both at infinity, $f_1 + f_2$ equals the distance d between the two mirrors.

$f_2 = 1.10$ cm. $R_1 = 1.30$ m.

EXECUTE: (a) $f_1 = \frac{R_1}{2} = 0.650 \text{ m} \Rightarrow d = f_1 + f_2 = 0.661 \text{ m}$.

(b) $|M| = \frac{f_1}{f_2} = \frac{0.650 \text{ m}}{0.011 \text{ m}} = 59.1$.

EVALUATE: For a telescope, $f_1 \gg f_2$.

- 34.60. IDENTIFY:** The primary mirror forms an image which then acts as the object for the secondary mirror.
SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$$

EXECUTE: For the first image (formed by the primary mirror):

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{2.5 \text{ m}} - \frac{1}{\infty} \Rightarrow s' = 2.5 \text{ m}.$$

For the second image (formed by the secondary mirror), the distance between the two vertices is x . Assuming that the image formed by the primary mirror is to the right of the secondary mirror, the object distance is $s = x - 2.5 \text{ m}$ and the image distance is $s' = x + 0.15 \text{ m}$. Therefore we have

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{x - 2.5 \text{ m}} + \frac{1}{x + 0.15 \text{ m}} = \frac{1}{-1.5 \text{ m}}$$

The positive root of the quadratic equation gives $x = 1.7 \text{ m}$, which is the distance between the vertices.

EVALUATE: Some light is blocked by the secondary mirror, but usually not enough to make much difference.

- 34.61. IDENTIFY and SET UP:** For a plane mirror $s' = -s$. $v = \frac{ds}{dt}$ and $v' = \frac{ds'}{dt}$, so $v' = -v$.

EXECUTE: The velocities of the object and image relative to the mirror are equal in magnitude and opposite in direction. Thus both you and your image are receding from the mirror surface at 2.40 m/s , in opposite directions. Your image is therefore moving at 4.80 m/s relative to you.

EVALUATE: The result derives from the fact that for a plane mirror the image is the same distance behind the mirror as the object is in front of the mirror.

- 34.62. IDENTIFY:** Apply the law of reflection.

SET UP: The image of one mirror can serve as the object for the other mirror.

EXECUTE: (a) There are three images formed, as shown in Figure 34.62a.

(b) The paths of rays for each image are sketched in Figure 34.62b.

EVALUATE: Our results agree with Figure 34.9 in the textbook.

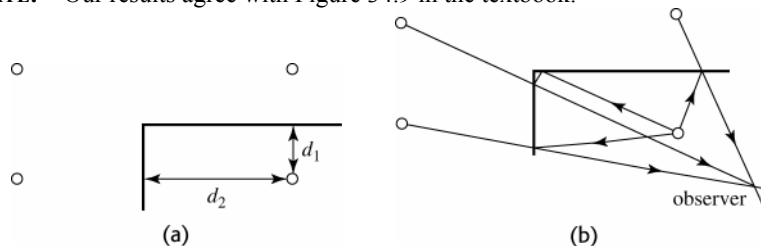


Figure 34.62

- 34.63. IDENTIFY:** Apply the law of reflection for rays from the feet to the eyes and from the top of the head to the eyes.
SET UP: In Figure 34.63, ray 1 travels from the feet of the woman to her eyes and ray 2 travels from the top of her head to her eyes. The total height of the woman is h .
EXECUTE: The two angles labeled θ_1 are equal because of the law of reflection, as are the two angles labeled θ_2 . Since these angles are equal, the two distances labeled y_1 are equal and the two distances labeled y_2 are equal. The height of the woman is $h_w = 2y_1 + 2y_2$. As the drawing shows, the height of the mirror is $h_m = y_1 + y_2$. Comparing, we find that $h_m = h_w / 2$. The minimum height required is half the height of the woman.

EVALUATE: The height of the image is the same as the height of the woman, so the height of the image is twice the height of the mirror.

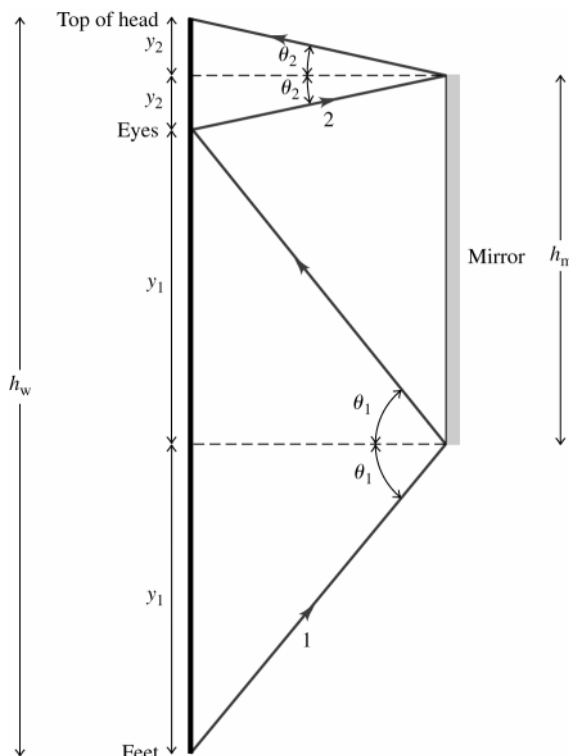


Figure 34.63

34.64. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ and $m = -\frac{s'}{s}$.

SET UP: Since the image is projected onto the wall it is real and $s' > 0$. $m = -\frac{s'}{s}$ so m is negative and $m = -2.25$. The object, mirror and wall are sketched in Figure 34.64. The sketch shows that $s' - s = 400$ cm.

EXECUTE: $m = -2.25 = -\frac{s'}{s}$ and $s' = 2.25s$. $s' - s = 2.25s - s = 400$ cm and $s = 320$ cm.

$s' = 400$ cm + 320 cm = 720 cm. The mirror should be 7.20 m from the wall. $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$. $\frac{1}{320 \text{ cm}} + \frac{1}{720 \text{ cm}} = \frac{2}{R}$. $R = 4.43$ m.

EVALUATE: The focal length of the mirror is $f = R/2 = 222$ cm. $s > f$, as it must if the image is to be real.

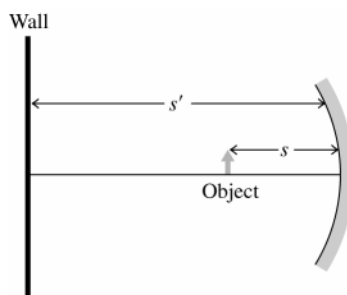


Figure 34.64

34.65. IDENTIFY: We are given the image distance, the image height, and the object height. Use Eq.(34.7) to calculate the object distance s . Then use Eq.(34.4) to calculate R .

(a) SET UP: Image is to be formed on screen so is real image; $s' > 0$. Mirror to screen distance is 8.00 m, so $s' = +800$ cm. $m = -\frac{s'}{s} < 0$ since both s and s' are positive.

EXECUTE: $|m| = \frac{|y'|}{|y|} = \frac{36.0 \text{ m}}{0.600 \text{ cm}} = 60.0$ and $m = -60.0$. Then $m = -\frac{s'}{s}$ gives $s = -\frac{s'}{m} = -\frac{800 \text{ cm}}{-60.0} = +13.3 \text{ cm}$.

(b) $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$, so $\frac{2}{R} = \frac{s+s'}{ss'}$

$$R = 2 \left(\frac{ss'}{s+s'} \right) = 2 \left(\frac{(13.3 \text{ cm})(800 \text{ cm})}{800 \text{ cm} + 13.3 \text{ cm}} \right) = 26.2 \text{ cm}$$

EVALUATE: R is calculated to be positive, which is correct for a concave mirror. Also, in part (a) s is calculated to be positive, as it should be for a real object.

34.66. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s' and then use $m = -\frac{s'}{s} = \frac{y'}{y}$ to find the height of the image.

SET UP: For a convex mirror, $R < 0$, so $R = -18.0 \text{ cm}$ and $f = \frac{R}{2} = -9.00 \text{ cm}$.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $s' = \frac{sf}{s-f} = \frac{(1300 \text{ cm})(-9.00 \text{ cm})}{1300 \text{ cm} - (-9.00 \text{ cm})} = -8.94 \text{ cm}$. $m = -\frac{s'}{s} = -\frac{-8.94 \text{ cm}}{1300 \text{ cm}} = 6.88 \times 10^{-3}$.

$$|y'| = |m|y = (6.88 \times 10^{-3})(1.5 \text{ m}) = 0.0103 \text{ m} = 1.03 \text{ cm}.$$

(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.

EVALUATE: The image formed by a convex mirror is always virtual and smaller than the object.

34.67. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ and $m = -\frac{s'}{s}$.

SET UP: $R = +19.4 \text{ cm}$.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{8.0 \text{ cm}} + \frac{1}{s'} = \frac{2}{19.4 \text{ cm}} \Rightarrow s' = -46 \text{ cm}$, so the image is virtual.

(b) $m = -\frac{s'}{s} = -\frac{-46}{8.0} = 5.8$, so the image is erect, and its height is $y' = (5.8)y = (5.8)(5.0 \text{ mm}) = 29 \text{ mm}$.

EVALUATE: (c) When the filament is 8 cm from the mirror, the image is virtual and cannot be projected onto a wall.

34.68. IDENTIFY: Combine $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ and $m = -\frac{s'}{s}$.

SET UP: $m = +2.50$. $R > 0$.

EXECUTE: $m = -\frac{s'}{s} = +2.50$. $s' = -2.50s$. $\frac{1}{s} + \frac{1}{-2.50s} = \frac{2}{R}$. $\frac{0.600}{s} = \frac{2}{R}$ and $s = 0.300R$.

$s' = -2.50s = (-2.50)(0.300R) = -0.750R$. The object is a distance of $0.300R$ in front of the mirror and the image is a distance of $0.750R$ behind the mirror.

EVALUATE: For a single mirror an erect image is always virtual.

34.69. IDENTIFY and SET UP: Apply Eqs.(34.6) and (34.7). For a virtual object $s < 0$. The image is real if $s' > 0$.

EXECUTE: (a) Convex implies $R < 0$; $R = -24.0 \text{ cm}$; $f = R/2 = -12.0 \text{ cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \text{ so } \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$$

$$s' = \frac{sf}{s-f} = \frac{(-12.0 \text{ cm})s}{s+12.0 \text{ cm}}$$

s is negative, so write as $s = -|s|$; $s' = +\frac{(12.0 \text{ cm})|s|}{12.0 \text{ cm} - |s|}$. Thus $s' > 0$ (real image) for $|s| < 12.0 \text{ cm}$. Since s is negative

this means $-12.0 \text{ cm} < s < 0$. A real image is formed if the virtual object is closer to the mirror than the focus.

(b) $m = -\frac{s'}{s}$; real image implies $s' > 0$; virtual object implies $s < 0$. Thus $m > 0$ and the image is erect.

(c) The principal-ray diagram is given in Figure 34.69.

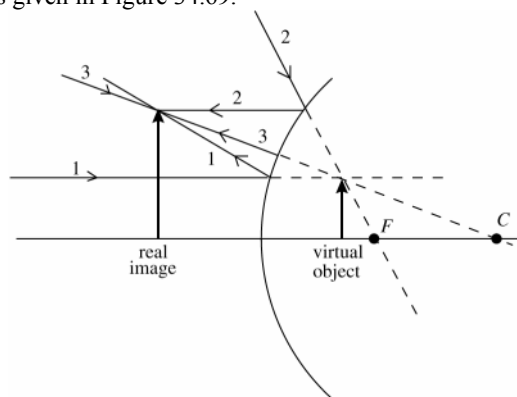


Figure 34.69

EVALUATE: For a real object, only virtual images are formed by a convex mirror. The virtual object considered in this problem must have been produced by some other optical element, by another lens or mirror in addition to the convex one we considered.

34.70. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$, with $R \rightarrow \infty$ since the surfaces are flat.

SET UP: The image formed by the first interface serves as the object for the second interface.

EXECUTE: For the water-benzene interface to get the apparent water depth:

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{1.33}{6.50 \text{ cm}} + \frac{1.50}{s'} = 0 \Rightarrow s' = -7.33 \text{ cm}.$$

For the benzene-air interface, to get the total apparent distance to the bottom:

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{1.50}{(7.33 \text{ cm} + 2.60 \text{ cm})} + \frac{1}{s'} = 0 \Rightarrow s' = -6.62 \text{ cm}.$$

EVALUATE: At the water-benzene interface the light refracts into material of greater refractive index and the overall effect is that the apparent depth is greater than the actual depth.

34.71. IDENTIFY: The focal length is given by $\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$.

SET UP: $R_1 = \pm 4.0 \text{ cm}$ or $\pm 8.0 \text{ cm}$. $R_2 = \pm 8.0 \text{ cm}$ or $\pm 4.0 \text{ cm}$. The signs are determined by the location of the center of curvature for each surface.

EXECUTE: $\frac{1}{f} = (0.60) \left(\frac{1}{\pm 4.00 \text{ cm}} - \frac{1}{\pm 8.00 \text{ cm}} \right)$, so $f = \pm 4.44 \text{ cm}, \pm 13.3 \text{ cm}$. The possible lens shapes are

sketched in Figure 34.71.

$f_1 = +13.3 \text{ cm}; f_2 = +4.44 \text{ cm}; f_3 = 4.44 \text{ cm}; f_4 = -13.3 \text{ cm}; f_5 = -13.3 \text{ cm}; f_6 = +13.3 \text{ cm};$

$f_7 = -4.44 \text{ cm}; f_8 = -4.44 \text{ cm}.$

EVALUATE: f is the same whether the light travels through the lens from right to left or left to right, so for the pairs (1,6), (4,5) and (7,8) the focal lengths are the same.

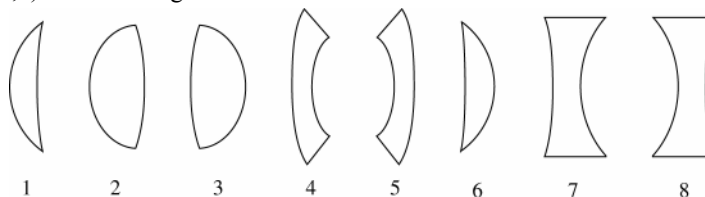


Figure 34.71

34.72. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and the concept of principal rays.

SET UP: $s = 10.0 \text{ cm}$. If extended backwards the ray comes from a point on the optic axis 18.0 cm from the lens and the ray is parallel to the optic axis after it passes through the lens.

EXECUTE: (a) The ray is bent toward the optic axis by the lens so the lens is converging.

(b) The ray is parallel to the optic axis after it passes through the lens so it comes from the focal point; $f = 18.0 \text{ cm}$.

(c) The principal ray diagram is drawn in Figure 34.72. The diagram shows that the image is 22.5 cm to the left of the lens.

(d) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $s' = \frac{sf}{s-f} = \frac{(10.0 \text{ cm})(18.0 \text{ cm})}{10.0 \text{ cm} - 18.0 \text{ cm}} = -22.5 \text{ cm}$. The calculated image position agrees with the principal ray diagram.

EVALUATE: The image is virtual. A converging lens produces a virtual image when the object is inside the focal point.

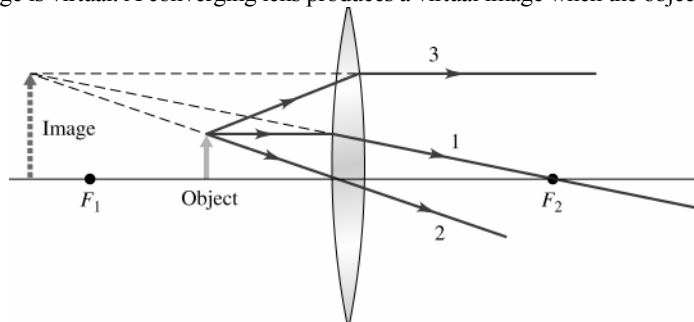


Figure 34.72

34.73. IDENTIFY: Since the truck is moving toward the mirror, its image will also be moving toward the mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \text{ where } f = R/2.$$

EXECUTE: Since the mirror is convex, $f = R/2 = (-1.50 \text{ m})/2 = -0.75 \text{ m}$. Applying the equation for a spherical mirror gives $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{fs}{s-f}$.

Using the chain rule from calculus and the fact that $v = ds/dt$, we have $v' = \frac{ds'}{dt} = \frac{ds'}{ds} \frac{ds}{dt} = v \frac{f^2}{(s-f)^2}$

$$\text{Solving for } v \text{ gives } v = v' \left(\frac{s-f}{f} \right)^2 = (1.5 \text{ m/s}) \left[\frac{2.0 \text{ m} - (-0.75 \text{ m})}{-0.75 \text{ m}} \right]^2 = 20.2 \text{ m/s}.$$

This is the velocity of the truck relative to the mirror, so the truck is approaching the mirror at 20.2 m/s. You are traveling at 25 m/s, so the truck must be traveling at 25 m/s + 20.2 m/s = 45 m/s relative to the highway.

EVALUATE: Even though the truck and car are moving at constant speed, the image of the truck is *not* moving at constant speed because its location depends on the distance from the mirror to the truck.

34.74. IDENTIFY: In this context, the microscope just looks at an image or object. Apply $\frac{n_a}{s} + \frac{n_b}{s'} = 0$ to the image

formed by refraction at the top surface of the second plate. In this calculation the object is the bottom surface of the second plate.

SET UP: The thickness of the second plate is 2.50 mm + 0.78 mm, and this is s . The image is 2.50 mm below the top surface, so $s' = -2.50 \text{ mm}$.

$$\text{EXECUTE: } \frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{n}{s} + \frac{1}{s'} = 0 \Rightarrow n = -\frac{s}{s'} = -\frac{2.50 \text{ mm} + 0.780 \text{ mm}}{-2.50 \text{ mm}} = 1.31.$$

EVALUATE: The object and image distances are measured from the front surface of the second plate, and the image is virtual.

34.75. IDENTIFY and SET UP: In part (a) use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to evaluate ds'/ds . Compare to $m = -\frac{s'}{s}$. In part (b) use

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \text{ to find the location of the image of each face of the cube.}$$

$$\text{EXECUTE: (a) } \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ and taking its derivative with respect to } s \text{ we have } 0 = \frac{d}{ds} \left(\frac{1}{s} + \frac{1}{s'} - \frac{1}{f} \right) = -\frac{1}{s^2} - \frac{1}{s'^2} \frac{ds'}{ds}$$

and $\frac{ds'}{ds} = -\frac{s'^2}{s^2} = -m^2$. But $\frac{ds'}{ds} = m'$, so $m' = -m^2$. Images are always inverted longitudinally.

$$\text{(b) (i) Front face: } \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{200.000 \text{ cm}} + \frac{1}{s'} = \frac{2}{150.000 \text{ cm}} \Rightarrow s' = 120.00 \text{ cm}.$$

Rear face: $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{200.100 \text{ cm}} + \frac{1}{s'} = \frac{2}{150.000 \text{ cm}} \Rightarrow s' = 119.96 \text{ cm}.$

(ii) $m = -\frac{s'}{s} = -\frac{120.000}{200.000} = -0.600.$ $m' = -m^2 = -(-0.600)^2 = -0.360.$

(iii) The two faces perpendicular to the axis (the front and rear faces): squares with side length 0.600 mm. The four faces parallel to the axis (the side faces): rectangles with sides of length 0.360 mm parallel to the axis and 0.600 mm perpendicular to the axis.

EVALUATE: Since the lateral and longitudinal magnifications have different values the image of the cube is not a cube.

34.76. IDENTIFY: $m' = ds'/ds$ and $m = -\frac{n_a s'}{n_b s}.$

SET UP: Use $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to evaluate $ds'/ds.$

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and taking its derivative with respect to s we have

$$0 = \frac{d}{ds} \left(\frac{n_a}{s} + \frac{n_b}{s'} - \frac{n_b - n_a}{R} \right) = -\frac{n_a}{s^2} - \frac{n_b}{s'^2} \frac{ds'}{ds} \text{ and } \frac{ds'}{ds} = -\frac{s'^2}{s^2} \frac{n_a}{n_b} = -\left(\frac{s'^2}{s^2} \frac{n_a^2}{n_b^2} \right) \frac{n_b}{n_a} = -m^2 \frac{n_b}{n_a}.$$

But $\frac{ds'}{ds} = m',$ so $m' = -m^2 \frac{n_b}{n_a}.$

EVALUATE: m' is always negative. This means that images are always inverted longitudinally.

34.77. IDENTIFY and SET UP: Rays that pass through the hole are undeflected. All other rays are blocked. $m = -\frac{s'}{s}.$

EXECUTE: (a) The ray diagram is drawn in Figure 34.77. The ray shown is the only ray from the top of the object that reaches the film, so this ray passes through the top of the image. An inverted image is formed on the far side of the box, no matter how far this side is from the pinhole and no matter how far the object is from the pinhole.

(b) $s = 1.5 \text{ m}.$ $s' = 20.0 \text{ cm}.$ $m = -\frac{s'}{s} = -\frac{20.0 \text{ cm}}{150 \text{ cm}} = -0.133.$ $y' = my = (-0.133)(18 \text{ cm}) = -2.4 \text{ cm}.$ The image is 2.4 cm tall.

EVALUATE: A defect of this camera is that not much light energy passes through the small hole each second, so long exposure times are required.

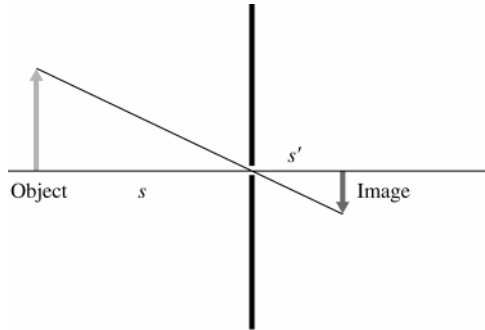


Figure 34.77

34.78. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and $m = -\frac{n_a s'}{n_b s}$ to each refraction. The overall magnification is $m = m_1 m_2.$

SET UP: For the first refraction, $R = +6.0 \text{ cm},$ $n_a = 1.00$ and $n_b = 1.60.$ For the second refraction, $R = -12.0 \text{ cm},$ $n_a = 1.60$ and $n_b = 1.00.$

EXECUTE: (a) The image from the left end acts as the object for the right end of the rod.

(b) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{23.0 \text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{6.0 \text{ cm}} \Rightarrow s' = 28.3 \text{ cm}.$

So the second object distance is $s_2 = 40.0 \text{ cm} - 28.3 \text{ cm} = 11.7 \text{ cm}.$ $m_1 = -\frac{n_a s'}{n_b s} = -\frac{28.3}{(1.60)(23.0)} = -0.769.$

(c) The object is real and inverted.

(d) $\frac{n_a}{s_2} + \frac{n_b}{s'_2} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.60}{11.7 \text{ cm}} + \frac{1}{s'_2} = \frac{-0.60}{-12.0 \text{ cm}} \Rightarrow s'_2 = -11.5 \text{ cm}.$

$m_2 = -\frac{n_a s'_2}{n_b s} = -\frac{(1.60)(-11.5)}{11.7} = 1.57 \Rightarrow m = m_1 m_2 = (-0.769)(1.57) = -1.21.$

(e) The final image is virtual, and inverted.

$$(f) \ y' = (1.50 \text{ mm})(-1.21) = -1.82 \text{ mm}.$$

EVALUATE: The first image is to the left of the second surface, so it serves as a real object for the second surface, with positive object distance.

- 34.79. IDENTIFY:** Apply Eqs.(34.11) and (34.12) to the refraction as the light enters the rod and as it leaves the rod. The image formed by the first surface serves as the object for the second surface. The total magnification is $m_{\text{tot}} = m_1 m_2$, where m_1 and m_2 are the magnifications for each surface.

SET UP: The object and rod are shown in Figure 34.79.

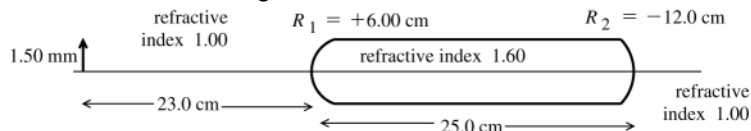


Figure 34.79

(a) image formed by refraction at first surface (left end of rod):

$$s = +23.0 \text{ cm}; n_a = 1.00; n_b = 1.60; R = +6.00 \text{ cm}$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$\text{EXECUTE: } \frac{1}{23.0 \text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{6.00 \text{ cm}}$$

$$\frac{1.60}{s'} = \frac{1}{10.0 \text{ cm}} - \frac{1}{23.0 \text{ cm}} = \frac{23 - 10}{230 \text{ cm}} = \frac{13}{230 \text{ cm}}$$

$$s' = 1.60 \left(\frac{230 \text{ cm}}{13} \right) = +28.3 \text{ cm}; \text{ image is } 28.3 \text{ cm to right of first vertex.}$$

This image serves as the object for the refraction at the second surface (right-hand end of rod). It is $28.3 \text{ cm} - 25.0 \text{ cm} = 3.3 \text{ cm}$ to the right of the second vertex. For the second surface $s = -3.3 \text{ cm}$ (virtual object).

(b) EVALUATE: Object is on side of outgoing light, so is a virtual object.

(c) SET UP: Image formed by refraction at second surface (right end of rod):

$$s = -3.3 \text{ cm}; n_a = 1.60; n_b = 1.00; R = -12.0 \text{ cm}$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$\text{EXECUTE: } \frac{1.60}{-3.3 \text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.60}{-12.0 \text{ cm}}$$

$$s' = +1.9 \text{ cm}; s' > 0 \text{ so image is } 1.9 \text{ cm to right of vertex at right-hand end of rod.}$$

(d) $s' > 0$ so final image is real.

Magnification for first surface:

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(+28.3 \text{ cm})}{(1.00)(+23.0 \text{ cm})} = -0.769$$

Magnification for second surface:

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(+1.9 \text{ cm})}{(1.00)(-3.3 \text{ cm})} = +0.92$$

The overall magnification is $m_{\text{tot}} = m_1 m_2 = (-0.769)(+0.92) = -0.71$ $m_{\text{tot}} < 0$ so final image is inverted with respect to the original object.

$$(e) \ y' = m_{\text{tot}} y = (-0.71)(1.50 \text{ mm}) = -1.06 \text{ mm}$$

The final image has a height of 1.06 mm.

EVALUATE: The two refracting surfaces are not close together and Eq.(34.18) does not apply.

- 34.80. IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$. The type of lens determines the sign of f . The sign of s' determines whether the image is real or virtual.

SET UP: $s = +8.00 \text{ cm}$. $s' = -3.00 \text{ cm}$. s' is negative because the image is on the same side of the lens as the object.

$$\text{EXECUTE: (a) } \frac{1}{f} = \frac{s + s'}{ss'} \text{ and } f = \frac{ss'}{s + s'} = \frac{(8.00 \text{ cm})(-3.00 \text{ cm})}{8.00 \text{ cm} - 3.00 \text{ cm}} = -4.80 \text{ cm} . f \text{ is negative so the lens is diverging.}$$

(b) $m = -\frac{s'}{s} = -\frac{-3.00 \text{ cm}}{8.00 \text{ cm}} = +0.375$. $y' = my = (0.375)(6.50 \text{ mm}) = 2.44 \text{ mm}$. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens can also form a virtual image, if the object distance is less than the focal length. But in that case $|s'| > s$ and the image would be farther from the lens than the object is.

- 34.81. IDENTIFY:** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The type of lens determines the sign of f . $m = \frac{y'}{y} = -\frac{s'}{s}$. The sign of s' depends on whether the image is real or virtual. $s = 16.0 \text{ cm}$.

SET UP: $s' = -22.0 \text{ cm}$; s' is negative because the image is on the same side of the lens as the object.

EXECUTE: (a) $\frac{1}{f} = \frac{s + s'}{ss'}$ and $f = \frac{ss'}{s + s'} = \frac{(16.0 \text{ cm})(-22.0 \text{ cm})}{16.0 \text{ cm} - 22.0 \text{ cm}} = +58.7 \text{ cm}$. f is positive so the lens is converging.

(b) $m = -\frac{s'}{s} = -\frac{-22.0 \text{ cm}}{16.0 \text{ cm}} = 1.38$. $y' = my = (1.38)(3.25 \text{ mm}) = 4.48 \text{ mm}$. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens forms a virtual image when the object is closer to the lens than the focal point.

- 34.82. IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. Use the image distance when viewed from the flat end to determine the refractive index n of the rod.

SET UP: When viewing from the flat end, $n_a = n$, $n_b = 1.00$ and $R \rightarrow \infty$. When viewing from the curved end, $n_a = n$, $n_b = 1.00$ and $R = -10.0 \text{ cm}$.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{n}{15.0 \text{ cm}} + \frac{1}{-9.50 \text{ cm}} = 0 \Rightarrow n = \frac{15.0}{9.50} = 1.58$. When viewed from the curved end of the

rod $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{n}{s} + \frac{1}{s'} = \frac{1 - n}{R} \Rightarrow \frac{1.58}{15.0 \text{ cm}} + \frac{1}{s'} = \frac{-0.58}{-10.0 \text{ cm}}$, and $s' = -21.1 \text{ cm}$. The image is 21.1 cm within the rod from the curved end.

EVALUATE: In each case the image is virtual and on the same side of the surface as the object.

- 34.83. (a) IDENTIFY:** Apply Snell's law to the refraction of a ray at each side of the beam to find where these rays strike the table.

SET UP: The path of a ray is sketched in Figure 34.83.

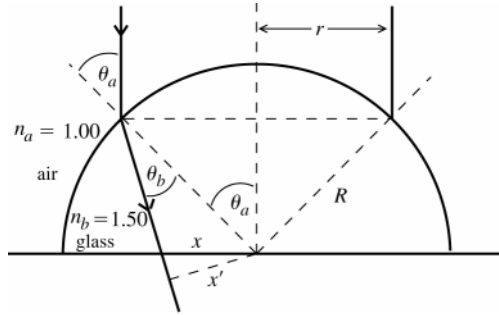


Figure 34.83

The width of the incident beam is exaggerated in the sketch, to make it easier to draw. Since the diameter of the beam is much less than the radius of the hemisphere, angles θ_a and θ_b are small. The diameter of the circle of light formed on the table is $2x$. Note the two right triangles containing the angles θ_a and θ_b .

$r = 0.190 \text{ cm}$ is the radius of the incident beam.

$R = 12.0 \text{ cm}$ is the radius of the glass hemisphere.

EXECUTE: θ_a and θ_b small imply $x \approx x'$; $\sin \theta_a = \frac{r}{R}$, $\sin \theta_b = \frac{x'}{R} \approx \frac{x}{R}$

Snell's law: $n_a \sin \theta_a = n_b \sin \theta_b$

Using the above expressions for $\sin \theta_a$ and $\sin \theta_b$ gives $n_a \frac{r}{R} = n_b \frac{x}{R}$

$$n_a r = n_b x \text{ so } x = \frac{n_a r}{n_b} = \frac{1.00(0.190 \text{ cm})}{1.50} = 0.1267 \text{ cm}$$

The diameter of the circle on the table is $2x = 2(0.1267 \text{ cm}) = 0.253 \text{ cm}$.

(b) **EVALUATE:** R divides out of the expression; the result for the diameter of the spot is independent of the radius R of the hemisphere. It depends only on the diameter of the incident beam and the index of refraction of the glass.

34.84. IDENTIFY and SET UP: Treating each of the goblet surfaces as spherical surfaces, we have to pass, from left to right, through four interfaces. Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to each surface. The image formed by one surface serves as the object for the next surface.

EXECUTE: (a) For the empty goblet:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{\infty} + \frac{1.50}{s'_1} = \frac{0.50}{4.00 \text{ cm}} \Rightarrow s'_1 = 12 \text{ cm}.$$

$$s_2 = 0.60 \text{ cm} - 12 \text{ cm} = -11.4 \text{ cm} \Rightarrow \frac{1.50}{-11.4 \text{ cm}} + \frac{1}{s'_2} = \frac{-0.50}{3.40 \text{ cm}} \Rightarrow s'_2 = -64.6 \text{ cm}.$$

$$s_3 = 64.6 \text{ cm} + 6.80 \text{ cm} = 71.4 \text{ cm} \Rightarrow \frac{1}{71.4 \text{ cm}} + \frac{1.50}{s'_3} = \frac{0.50}{-3.40 \text{ cm}} \Rightarrow s'_3 = -9.31 \text{ cm}.$$

$$s_4 = 9.31 \text{ cm} + 0.60 \text{ cm} = 9.91 \text{ cm} \Rightarrow \frac{1.50}{9.91 \text{ cm}} + \frac{1}{s'_4} = \frac{-0.50}{-4.00 \text{ cm}} \Rightarrow s'_4 = -37.9 \text{ cm}.$$

The final image is $37.9 \text{ cm} - 2(4.0 \text{ cm}) = 29.9 \text{ cm}$ to the left of the goblet.

(b) For the wine-filled goblet:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{\infty} + \frac{1.50}{s'_1} = \frac{0.50}{4.00 \text{ cm}} \Rightarrow s'_1 = 12 \text{ cm}.$$

$$s_2 = 0.60 \text{ cm} - 12 \text{ cm} = -11.4 \text{ cm} \Rightarrow \frac{1.50}{-11.4 \text{ cm}} + \frac{1.37}{s'_2} = \frac{-0.13}{3.40 \text{ cm}} \Rightarrow s'_2 = 14.7 \text{ cm}.$$

$$s_3 = 6.80 \text{ cm} - 14.7 \text{ cm} = -7.9 \text{ cm} \Rightarrow \frac{1.37}{-7.9 \text{ cm}} + \frac{1.50}{s'_3} = \frac{0.13}{-3.40 \text{ cm}} \Rightarrow s'_3 = 11.1 \text{ cm}.$$

$$s_4 = 0.60 \text{ cm} - 11.1 \text{ cm} = -10.5 \text{ cm} \Rightarrow \frac{1.50}{-10.5 \text{ cm}} + \frac{1}{s'_4} = \frac{-0.50}{-4.00 \text{ cm}} \Rightarrow s'_4 = 3.73 \text{ cm}.$$

The final image is 3.73 cm to the right of the goblet.

EVALUATE: If the object for a surface is on the outgoing side of the light, then the object is virtual and the object distance is negative.

34.85. IDENTIFY: The image formed by refraction at the surface of the eye is located by $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.35$. $R > 0$. For a distant object, $s \approx \infty$ and $\frac{1}{s} \approx 0$.

EXECUTE: (a) $s \approx \infty$ and $s' = 2.5 \text{ cm}$: $\frac{1.35}{2.5 \text{ cm}} = \frac{1.35 - 1.00}{R}$ and $R = 0.648 \text{ cm} = 6.48 \text{ mm}$.

(b) $R = 0.648 \text{ cm}$ and $s = 25 \text{ cm}$: $\frac{1.00}{25 \text{ cm}} + \frac{1.35}{s'} = \frac{1.35 - 1.00}{0.648}$. $\frac{1.35}{s'} = 0.500$ and $s' = 2.70 \text{ cm} = 27.0 \text{ mm}$. The image is formed behind the retina.

(c) Calculate s' for $s \approx \infty$ and $R = 0.50 \text{ cm}$: $\frac{1.35}{s'} = \frac{1.35 - 1.00}{0.50 \text{ cm}}$. $s' = 1.93 \text{ cm} = 19.3 \text{ mm}$. The image is formed in front of the retina.

EVALUATE: The cornea alone cannot achieve focus of both close and distant objects.

34.86. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and $m = -\frac{n_a s'}{n_b s}$ to each surface. The overall magnification is $m = m_1 m_2$. The image formed by the first surface is the object for the second surface.

SET UP: For the first surface, $n_a = 1.00$, $n_b = 1.60$ and $R = +15.0 \text{ cm}$. For the second surface, $n_a = 1.60$, $n_b = 1.00$ and $R \Rightarrow \infty$.

EXECUTE: (a) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{12.0 \text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{15.0 \text{ cm}} \Rightarrow s' = -36.9 \text{ cm}$. The object distance for the far end of the rod is $50.0 \text{ cm} - (-36.9 \text{ cm}) = 86.9 \text{ cm}$. The final image is 4.3 cm to the left of the vertex of the

hemispherical surface. $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.60}{86.9 \text{ cm}} + \frac{1}{s'} = 0 \Rightarrow s' = -54.3 \text{ cm}$.

(b) The magnification is the product of the two magnifications:

$$m_1 = -\frac{n_a s'}{n_b s} = -\frac{1.00(-36.9)}{(1.60)(12.0)} = 1.92, m_2 = 1.00 \Rightarrow m = m_1 m_2 = 1.92.$$

EVALUATE: The final image is virtual, erect and larger than the object.

- 34.87. IDENTIFY:** Apply Eq.(34.11) to the image formed by refraction at the front surface of the sphere.
SET UP: Let n_g be the index of refraction of the glass. The image formation is shown in Figure 34.87.

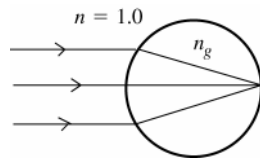


Figure 34.87

$$s = \infty$$

$$s' = +2r, \text{ where } r \text{ is the radius of the sphere}$$

$$n_a = 1.00, n_b = n_g, R = +r$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\frac{1}{\infty} + \frac{n_g}{2r} = \frac{n_g - 1.00}{r}$

$$\frac{n_g}{2r} = \frac{n_g}{r} - \frac{1}{r}; \frac{n_g}{2r} = \frac{1}{r} \text{ and } n_g = 2.00$$

EVALUATE: The required refractive index of the glass does not depend on the radius of the sphere.

- 34.88. IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to each surface. The image of the first surface is the object for the second surface. The relation between s'_1 and s_2 involves the length d of the rod.

SET UP: For the first surface, $n_a = 1.00$, $n_b = 1.55$ and $R = +6.00$ cm. For the second surface, $n_a = 1.55$, $n_b = 1.00$ and $R = -6.00$ cm.

EXECUTE: We have images formed from both ends. From the first surface:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{25.0 \text{ cm}} + \frac{1.55}{s'} = \frac{0.55}{6.00 \text{ cm}} \Rightarrow s' = 30.0 \text{ cm}.$$

This image becomes the object for the second end: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.55}{d - 30.0 \text{ cm}} + \frac{1}{65.0 \text{ cm}} = \frac{-0.55}{-6.00 \text{ cm}}.$

$$d - 30.0 \text{ cm} = 20.3 \text{ cm} \Rightarrow d = 50.3 \text{ cm}.$$

EVALUATE: The final image is real. The first image is 20.3 cm to the right of the second surface and serves as a real object.

- 34.89. IDENTIFY:** The first lens forms an image which then acts as the object for the second lens.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and the magnification is $m = -\frac{s'}{s}$.

EXECUTE: (a) For the first lens: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{5.00 \text{ cm}} + \frac{1}{s'} = \frac{1}{-15.0 \text{ cm}} \Rightarrow s' = -3.75 \text{ cm}$, to the left of the lens (virtual image).

(b) For the second lens, $s = 12.0 \text{ cm} + 3.75 \text{ cm} = 15.75 \text{ cm}$.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{15.75 \text{ cm}} + \frac{1}{s'} = \frac{1}{15.0 \text{ cm}} \Rightarrow s' = 315 \text{ cm}, \text{ or } 332 \text{ cm from the object}.$$

(c) The final image is real.

(d) $m = -\frac{s'}{s}$, $m_1 = 0.750$, $m_2 = -20.0$, $m_{\text{total}} = -15.0 \Rightarrow y' = -6.00 \text{ cm}$, inverted.

EVALUATE: Note that the total magnification is the product of the individual magnifications.

- 34.90. IDENTIFY and SET UP:** Use $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ to calculate the focal length of the lenses. The image formed

by the first lens serves as the object for the second lens. $m_{\text{tot}} = m_1 m_2$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $s' = \frac{sf}{s-f}$.

EXECUTE: (a) $\frac{1}{f} = (0.60)\left(\frac{1}{12.0 \text{ cm}} - \frac{1}{28.0 \text{ cm}}\right)$ and $f = +35.0 \text{ cm}$.

Lens 1: $f_1 = +35.0 \text{ cm}$. $s_1 = +45.0 \text{ cm}$. $s'_1 = \frac{s_1 f_1}{s_1 - f_1} = \frac{(45.0 \text{ cm})(35.0 \text{ cm})}{45.0 \text{ cm} - 35.0 \text{ cm}} = +158 \text{ cm}$.

$m_1 = -\frac{s'_1}{s_1} = -\frac{158 \text{ cm}}{45.0 \text{ cm}} = -3.51$. $|y'_1| = |m_1| y_1 = (3.51)(5.00 \text{ mm}) = 17.6 \text{ mm}$. The image of the first lens is 158 cm to the right of lens 1 and is 17.6 mm tall.

(b) The image of lens 1 is $315 \text{ cm} - 158 \text{ cm} = 157 \text{ cm}$ to the left of lens 2. $f_2 = +35.0 \text{ cm}$. $s_2 = +157 \text{ cm}$.

$$s_2' = \frac{s_2 f_2}{s_2 - f_2} = \frac{(157 \text{ cm})(35.0 \text{ cm})}{157 \text{ cm} - 35.0 \text{ cm}} = +45.0 \text{ cm}. \quad m_2 = -\frac{s_2'}{s_2} = -\frac{45.0 \text{ cm}}{157 \text{ cm}} = -0.287.$$

$m_{\text{tot}} = m_1 m_2 = (-3.51)(-0.287) = +1.00$. The final image is 45.0 cm to the right of lens 2. The final image is 5.00 mm tall. $m_{\text{tot}} > 0$. So the final image is erect.

EVALUATE: The final image is real. It is erect because each lens produces an inversion of the image, and two inversions return the image to the orientation of the object.

- 34.91. IDENTIFY and SET UP:** Apply Eq.(34.16) for each lens position. The lens to screen distance in each case is the image distance. There are two unknowns, the original object distance x and the focal length f of the lens. But each lens position gives an equation, so there are two equations for these two unknowns. The object, lens and screen before and after the lens is moved are shown in Figure 34.91.

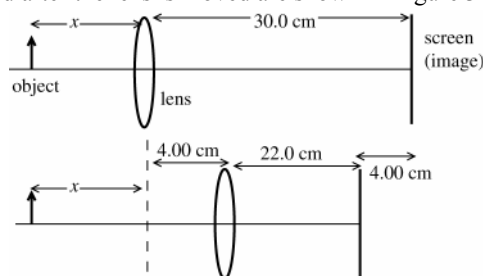


Figure 34.91

$$s = x; \quad s' = 30.0 \text{ cm}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{x} + \frac{1}{30.0 \text{ cm}} = \frac{1}{f}$$

$$s = x + 4.00 \text{ cm}; \quad s' = 22.0 \text{ cm}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ gives } \frac{1}{x + 4.00 \text{ cm}} + \frac{1}{22.0 \text{ cm}} = \frac{1}{f}$$

EXECUTE: Equate these two expressions for $1/f$:

$$\frac{1}{x} + \frac{1}{30.0 \text{ cm}} = \frac{1}{x + 4.00 \text{ cm}} + \frac{1}{22.0 \text{ cm}}$$

$$\frac{1}{x} - \frac{1}{x + 4.00 \text{ cm}} = \frac{1}{22.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}}$$

$$\frac{x + 4.00 \text{ cm} - x}{x(x + 4.00 \text{ cm})} = \frac{30.0 - 22.0}{660 \text{ cm}} \text{ and } \frac{4.00 \text{ cm}}{x(x + 4.00 \text{ cm})} = \frac{8}{660 \text{ cm}}$$

$$x^2 + (4.00 \text{ cm})x - 330 \text{ cm}^2 = 0 \text{ and } x = \frac{1}{2}(-4.00 \pm \sqrt{16.0 + 4(330)}) \text{ cm}$$

$$x \text{ must be positive so } x = \frac{1}{2}(-4.00 + 36.55) \text{ cm} = 16.28 \text{ cm}$$

$$\text{Then } \frac{1}{x} + \frac{1}{30.0 \text{ cm}} = \frac{1}{f} \text{ and } \frac{1}{f} = \frac{1}{16.28 \text{ cm}} + \frac{1}{30.0 \text{ cm}}$$

$f = +10.55 \text{ cm}$, which rounds to 10.6 cm . $f > 0$; the lens is converging.

EVALUATE: We can check that $s = 16.28 \text{ cm}$ and $f = 10.55 \text{ cm}$ gives $s' = 30.0 \text{ cm}$ and that

$s = (16.28 + 4.00) \text{ cm} = 20.28 \text{ cm}$ and $f = 10.55 \text{ cm}$ gives $s' = 22.0 \text{ cm}$.

- 34.92. IDENTIFY and SET UP:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

$$\text{EXECUTE: (a) } \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{n_a}{f} + \frac{n_b}{\infty} = \frac{n_b - n_a}{R} \text{ and } \frac{n_a}{\infty} + \frac{n_b}{f'} = \frac{n_b - n_a}{R}.$$

$$\frac{n_a}{f} = \frac{n_b - n_a}{R} \text{ and } \frac{n_b}{f'} = \frac{n_b - n_a}{R}. \text{ Therefore, } \frac{n_a}{f} = \frac{n_b}{f'} \text{ and } n_a/n_b = \frac{f}{f'}.$$

$$\text{(b) } \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{n_b f}{s f'} + \frac{n_b}{s'} = \frac{n_b(1 - f/f')}{R}. \text{ Therefore, } \frac{f}{s} + \frac{f'}{s'} = \frac{f'(1 - f/f')}{R} = \frac{f' - f}{R} = 1.$$

EVALUATE: For a thin lens the first and second focal lengths are equal.

- 34.93. (a) IDENTIFY:** Use Eq.(34.6) to locate the image formed by each mirror. The image formed by the first mirror serves as the object for the 2nd mirror.

SET UP: The positions of the object and the two mirrors are shown in Figure 34.93a.

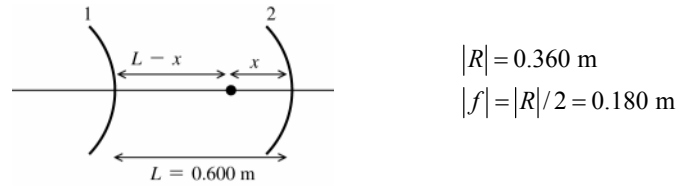


Figure 34.93a

EXECUTE: Image formed by convex mirror (mirror #1):

convex means $f_1 = -0.180$ m; $s_1 = L - x$

$$s'_1 = \frac{s_1 f_1}{s_1 - f_1} = \frac{(L - x)(-0.180 \text{ m})}{L - x + 0.180 \text{ m}} = -(0.180 \text{ m}) \left(\frac{0.600 \text{ m} - x}{0.780 \text{ m} - x} \right) < 0$$

The image is $(0.180 \text{ m}) \left(\frac{0.600 \text{ m} - x}{0.780 \text{ m} - x} \right)$ to the left of mirror #1 so is

$$0.600 \text{ m} + (0.180 \text{ m}) \left(\frac{0.600 \text{ m} - x}{0.780 \text{ m} - x} \right) = \frac{0.576 \text{ m}^2 - (0.780 \text{ m})x}{0.780 \text{ m} - x} \text{ to the left of mirror #2.}$$

Image formed by concave mirror (mirror #2):

concave implies $f_2 = +0.180$ m

$$s_2 = \frac{0.576 \text{ m}^2 - (0.780 \text{ m})x}{0.780 \text{ m} - x}$$

Rays return to the source implies $s'_2 = x$. Using these expressions in $s_2 = \frac{s'_2 f_2}{s'_2 - f_2}$ gives

$$\frac{0.576 \text{ m}^2 - (0.780 \text{ m})x}{0.780 \text{ m} - x} = \frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}}$$

$$0.600x^2 - (0.576 \text{ m})x + 0.10368 \text{ m}^2 = 0$$

$$x = \frac{1}{1.20} (0.576 \pm \sqrt{(0.576)^2 - 4(0.600)(0.10368)}) \text{ m} = \frac{1}{1.20} (0.576 \pm 0.288) \text{ m}$$

$x = 0.72$ m (impossible; can't have $x > L = 0.600$ m) or $x = 0.24$ m.

(b) SET UP: Which mirror is #1 and which is #2 is now reversed from part (a). This is shown in Figure 34.93b.

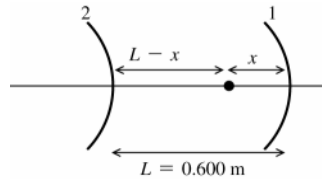


Figure 34.93b

EXECUTE: Image formed by concave mirror (mirror #1):

concave means $f_1 = +0.180$ m; $s_1 = x$

$$s'_1 = \frac{s_1 f_1}{s_1 - f_1} = \frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}}$$

The image is $\frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}}$ to the left of mirror #1, so $s_2 = 0.600 \text{ m} - \frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}} = \frac{(0.420 \text{ m})x - 0.180 \text{ m}^2}{x - 0.180 \text{ m}}$

Image formed by convex mirror (mirror #2):

convex means $f_2 = -0.180$ m

rays return to the source means $s'_2 = L - x = 0.600 \text{ m} - x$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ gives}$$

$$\frac{x - 0.180 \text{ m}}{(0.420 \text{ m})x - 0.180 \text{ m}^2} + \frac{1}{0.600 \text{ m} - x} = -\frac{1}{0.180 \text{ m}}$$

$$\frac{x - 0.180 \text{ m}}{(0.420 \text{ m})x - 0.180 \text{ m}^2} = -\left(\frac{0.780 \text{ m} - x}{0.180 \text{ m}^2 - (0.180 \text{ m})x} \right)$$

$$0.600x^2 - (0.576 \text{ m})x + 0.1036 \text{ m}^2 = 0$$

This is the same quadratic equation as obtained in part (a), so again $x = 0.24$ m.

EVALUATE: For $x = 0.24$ m the image is at the location of the source, both for rays that initially travel from the source toward the left and for rays that travel from the source toward the right.

34.94. IDENTIFY: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $s' = \frac{sf}{s-f}$, for both the mirror and the lens.

SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_m = +10.0$ cm. For the lens, $f = 32.0$ cm. The center of curvature of the mirror is $R = 2f_m = 20.0$ cm to the right of the mirror vertex.

EXECUTE: (a) The principal-ray diagrams from the two images are sketched in Figures 34.94a-b. In Figure 34.94b, only the image formed by the mirror is shown. This image is at the location of the candle so the principal ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.94a and is not drawn.

(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens.

$$s' = \frac{sf}{s-f} = \frac{(85.0 \text{ cm})(32.0 \text{ cm})}{85.0 \text{ cm} - 32.0 \text{ cm}} = +51.3 \text{ cm}. \quad m = -\frac{s'}{s} = -\frac{51.3 \text{ cm}}{85.0 \text{ cm}} = -0.604. \text{ This image is 51.3 cm to the right of the lens.}$$

$s' > 0$ so the image is real. $m < 0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm to the right of the mirror, so

$$s = +20.0 \text{ cm}. \quad s' = \frac{sf}{s-f} = \frac{(20.0 \text{ cm})(10.0 \text{ cm})}{20.0 \text{ cm} - 10.0 \text{ cm}} = 20.0 \text{ cm}. \quad m_1 = -\frac{s'_1}{s_1} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = -1.00. \text{ The image formed by}$$

the mirror is at the location of the candle, so $s_2 = +85.0$ cm and $s'_2 = 51.3$ cm. $m_2 = -0.604$. $m_{\text{tot}} = m_1 m_2 =$

$(-1.00)(-0.604) = 0.604$. The second image is 51.3 cm to the right of the lens. $s'_2 > 0$, so the final image is real.

$m_{\text{tot}} > 0$, so the final image is erect.

EVALUATE: The two images are at the same place. They are the same size. One is erect and one is inverted.

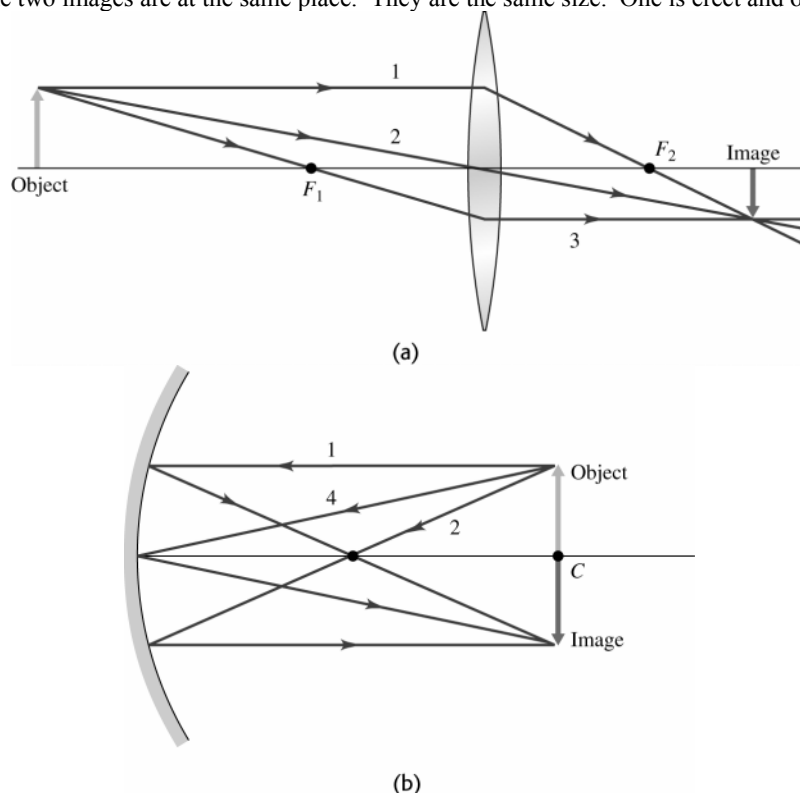


Figure 34.94

34.95. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to each case.

SET UP: $s = 20.0$ cm. $R > 0$. Use $s' = +9.12$ cm to find R . For this calculation, $n_a = 1.00$ and $n_b = 1.55$. Then repeat the calculation with $n_a = 1.33$.

$$\text{EXECUTE: } \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \text{ gives } \frac{1.00}{20.0 \text{ cm}} + \frac{1.55}{9.12 \text{ cm}} = \frac{1.55 - 1.00}{R}. \quad R = 2.50 \text{ cm}.$$

Then $\frac{1.33}{20.0 \text{ cm}} + \frac{1.55}{s'} = \frac{1.55 - 1.33}{2.50 \text{ cm}}$ gives $s' = -72.1 \text{ cm}$. The image is 72.1 cm to the left of the surface vertex.

EVALUATE: With the rod in air the image is real and with the rod in water the image is virtual.

34.96. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to each lens. The image formed by the first lens serves as the object for the second

lens. The focal length of the lens combination is defined by $\frac{1}{s_1} + \frac{1}{s'_2} = \frac{1}{f}$. In part (b) use $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ to

calculate f for the meniscus lens and for the CCl_4 , treated as a thin lens.

SET UP: With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens.

EXECUTE: (a) $\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f_1} \Rightarrow \frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1}$ and $\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{-s'_1} + \frac{1}{s'_2} = \left(\frac{1}{s_1} - \frac{1}{f_1}\right) + \frac{1}{s'_2} = \frac{1}{f_2}$. But overall for the lens

system, $\frac{1}{s_1} + \frac{1}{s'_2} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1}$.

(b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a).

For the meniscus lens $\frac{1}{f_m} = (n_b - n_a)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (0.55)\left(\frac{1}{4.50 \text{ cm}} - \frac{1}{9.00 \text{ cm}}\right) = 0.061 \text{ cm}^{-1}$ and $f_m = 16.4 \text{ cm}$.

For the CCl_4 : $\frac{1}{f_w} = (n_b - n_a)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (0.46)\left(\frac{1}{9.00 \text{ cm}} - \frac{1}{\infty}\right) = 0.051 \text{ cm}^{-1}$ and $f_w = 19.6 \text{ cm}$.

$\frac{1}{f} = \frac{1}{f_w} + \frac{1}{f_m} = 0.112 \text{ cm}^{-1}$ and $f = 8.93 \text{ cm}$.

EVALUATE: $f = \frac{f_1 f_2}{f_1 + f_2}$, so f for the combination is less than either f_1 or f_2 .

34.97. IDENTIFY: Apply Eq.(34.11) with $R \rightarrow \infty$ to the refraction at each surface. For refraction at the first surface the point P serves as a virtual object. The image formed by the first refraction serves as the object for the second refraction.

SET UP: The glass plate and the two points are shown in Figure 37.97.

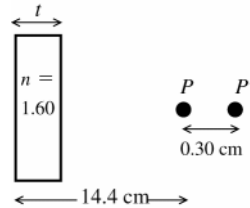


Figure 34.97

plane faces means $R \rightarrow \infty$ and

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

$$s' = -\frac{n_b}{n_a}s$$

EXECUTE: refraction at the first (left-hand) surface of the piece of glass:

The rays converging toward point P constitute a virtual object for this surface, so $s = -14.4 \text{ cm}$.

$n_a = 1.00$, $n_b = 1.60$.

$$s' = -\frac{1.60}{1.00}(-14.4 \text{ cm}) = +23.0 \text{ cm}$$

This image is 23.0 cm to the right of the first surface so is a distance $23.0 \text{ cm} - t$ to the right of the second surface.

This image serves as a virtual object for the second surface.

refraction at the second (right-hand) surface of the piece of glass:

The image is at P' so $s' = 14.4 \text{ cm} + 0.30 \text{ cm} - t = 14.7 \text{ cm} - t$. $s = -(23.0 \text{ cm} - t)$; $n_a = 1.60$; $n_b = 1.00$ $s' = -\frac{n_b}{n_a}s$

gives $14.7 \text{ cm} - t = -\left(\frac{1.00}{1.60}\right)(-23.0 \text{ cm} - t)$. $14.7 \text{ cm} - t = +14.4 \text{ cm} - 0.625t$.

$0.375t = 0.30 \text{ cm}$ and $t = 0.80 \text{ cm}$

EVALUATE: The overall effect of the piece of glass is to diverge the rays and move their convergence point to the right. For a real object, refraction at a plane surface always produces a virtual image, but with a virtual object the image can be real.

34.98. IDENTIFY: Apply the two equations $\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{n_b - n_a}{R_1}$ and $\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{n_c - n_b}{R_2}$.

SET UP: $n_a = n_{\text{liq}} = n_c$, $n_b = n$, and $s'_1 = -s_2$.

EXECUTE: (a) $\frac{n_{\text{liq}}}{s_1} + \frac{n}{s'_1} = \frac{n - n_{\text{liq}}}{R_1}$ and $\frac{n}{-s'_1} + \frac{n_{\text{liq}}}{s'_2} = \frac{n_{\text{liq}} - n}{R_2}$. $\frac{1}{s_1} + \frac{1}{s'_2} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{f'} = (n/n_{\text{liq}} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$.

(b) Comparing the equations for focal length in and out of air we have:

$$f(n-1) = f'(n/n_{\text{liq}} - 1) = f' \left(\frac{n - n_{\text{liq}}}{n_{\text{liq}}} \right) \Rightarrow f' = \left[\frac{n_{\text{liq}}(n-1)}{n - n_{\text{liq}}} \right] f.$$

EVALUATE: When $n_{\text{liq}} = 1$, $f' = f$, as it should.

34.99. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected 15 cm + 19.2 cm = 34.2 cm from the diverging lens.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-15.0 \text{ cm}} + \frac{1}{34.2 \text{ cm}} = \frac{1}{f} \Rightarrow f = -26.7 \text{ cm}$.

EVALUATE: Our calculation yields a negative value of f , which should be the case for a diverging lens.

34.100. IDENTIFY: The spherical mirror forms an image of the object. It forms another image when the image of the plane mirror serves as an object.

SET UP: For the convex mirror $f = -24.0 \text{ cm}$. The image formed by the plane mirror is 10.0 cm to the right of the plane mirror, so is 20.0 cm + 10.0 cm = 30.0 cm from the vertex of the spherical mirror.

EXECUTE: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{10.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{-24.0 \text{ cm}} \Rightarrow s' = -7.06 \text{ cm}, \text{ and the image height}$$

$$\text{is } y' = -\frac{s'}{s} y = -\frac{-7.06}{10.0} (0.250 \text{ cm}) = 0.177 \text{ cm}.$$

The second image is of the plane mirror image is located 30.0 cm from the vertex of the spherical mirror.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{30.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{-24.0 \text{ cm}} \Rightarrow s' = -13.3 \text{ cm} \text{ and the image height is}$$

$$y' = -\frac{s'}{s} y = -\frac{-13.3}{30.0} (0.250 \text{ cm}) = 0.111 \text{ cm}.$$

EVALUATE: Other images are formed by additional reflections from the two mirrors.

34.101. IDENTIFY: In the sketch in Figure 34.101 the light travels upward from the object. Apply Eq.(34.11) with $R \rightarrow \infty$ to the refraction at each surface. The image formed by the first surface serves as the object for the second surface.

SET UP: The locations of the object and the glass plate are shown in Figure 34.101.

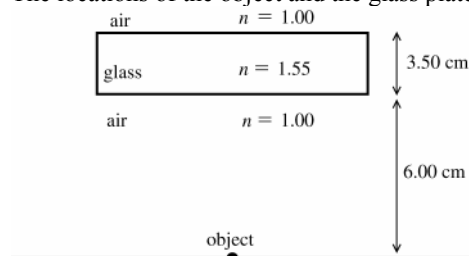


Figure 34.101

For a plane (flat) surface

$$R \rightarrow \infty \text{ so } \frac{n_a}{s} + \frac{n_b}{s'} = 0$$

$$s' = -\frac{n_b}{n_a} s$$

EXECUTE: First refraction (air $\rightarrow$ glass):

$$n_a = 1.00; n_b = 1.55; s = 6.00 \text{ cm}$$

$$s' = -\frac{n_b}{n_a} s = -\frac{1.55}{1.00} (6.00 \text{ cm}) = -9.30 \text{ cm}$$

The image is 9.30 cm below the lower surface of the glass, so is 9.30 cm + 3.50 cm = 12.8 cm below the upper surface.

Second refraction (glass $\rightarrow$ air):

$$n_a = 1.55; n_b = 1.00; s = +12.8 \text{ cm}$$

$$s' = -\frac{n_b}{n_a}s = -\frac{1.00}{1.55}(12.8 \text{ cm}) = -8.26 \text{ cm}$$

The image of the page is 8.26 cm below the top surface of the glass plate and therefore $9.50 \text{ cm} - 8.26 \text{ cm} = 1.24 \text{ cm}$ above the page.

EVALUATE: The image is virtual. If you view the object by looking down from above the plate, the image of the page that you see is closer to your eye than the page is.

- 34.102. IDENTIFY:** Light refracts at the front surface of the lens, refracts at the glass-water interface, reflects from the plane mirror and passes through the two interfaces again, now traveling in the opposite direction.

SET UP: Use the focal length in air to find the radius of curvature R of the lens surfaces.

EXECUTE: (a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{40 \text{ cm}} = 0.52\left(\frac{2}{R}\right) \Rightarrow R = 41.6 \text{ cm}.$

At the air-lens interface: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{70.0 \text{ cm}} + \frac{1.52}{s'_1} = \frac{0.52}{41.6 \text{ cm}}$ and $s'_1 = -851 \text{ cm}$ and $s_2 = 851 \text{ cm}.$

At the lens-water interface: $\Rightarrow \frac{1.52}{851 \text{ cm}} + \frac{1.33}{s'_2} = \frac{-0.187}{-41.6 \text{ cm}}$ and $s'_2 = 491 \text{ cm}.$

The mirror reflects the image back (since there is just 90 cm between the lens and mirror.) So, the position of the image is 401 cm to the left of the mirror, or 311 cm to the left of the lens.

At the water-lens interface: $\Rightarrow \frac{1.33}{-311 \text{ cm}} + \frac{1.52}{s'_3} = \frac{0.187}{41.6 \text{ cm}}$ and $s'_3 = +173 \text{ cm}.$

At the lens-air interface: $\Rightarrow \frac{1.52}{-173 \text{ cm}} + \frac{1}{s'_4} = \frac{-0.52}{-41.6 \text{ cm}}$ and $s'_4 = +47.0 \text{ cm}$, to the left of lens.

$$m = m_1 m_2 m_3 m_4 = \left(\frac{n_{a1}s'_1}{n_{b1}s_1}\right)\left(\frac{n_{a2}s'_2}{n_{b2}s_2}\right)\left(\frac{n_{a3}s'_3}{n_{b3}s_3}\right)\left(\frac{n_{a4}s'_4}{n_{b4}s_4}\right) = \left(\frac{-851}{70}\right)\left(\frac{491}{-851}\right)\left(\frac{+173}{-311}\right)\left(\frac{+47.0}{-173}\right) = -1.06.$$

(Note all the indices of refraction cancel out.)

(b) The image is real.

(c) The image is inverted.

(d) The final height is $y' = my = (1.06)(4.00 \text{ mm}) = 4.24 \text{ mm}.$

EVALUATE: The final image is real even though it is on the same side of the lens as the object!

- 34.103. IDENTIFY:** The camera lens can be modeled as a thin lens that forms an image on the film.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and the magnification of the lens is $m = -\frac{s'}{s}.$

EXECUTE: (a) $m = -\frac{s'}{s} = \frac{y'}{y} = \frac{1(0.0360 \text{ m})}{4(12.0 \text{ m})} \Rightarrow s' = (7.50 \times 10^{-4})s,$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{(7.50 \times 10^{-4})s} = \frac{1}{s}\left(1 + \frac{1}{7.50 \times 10^{-4}}\right) = \frac{1}{f} = \frac{1}{0.0350 \text{ m}} \Rightarrow s = 46.7 \text{ m}.$$

(b) To just fill the frame, the magnification must be 3.00×10^{-3} so:

$$\frac{1}{s}\left(1 + \frac{1}{3.00 \times 10^{-3}}\right) = \frac{1}{f} = \frac{1}{0.0350 \text{ m}} \Rightarrow s = 11.7 \text{ m}.$$

Since the boat is originally 46.7 m away, the distance you must move closer to the boat is $46.7 \text{ m} - 11.7 \text{ m} = 35.0 \text{ m}.$

EVALUATE: This result seems to imply that if you are 4 times as far, the image is $\frac{1}{4}$ as large on the film. However this result is only an approximation, and would not be true for very close distances. It is a better approximation for large distances.

- 34.104. IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}.$

SET UP: $s + s' = 18.0 \text{ cm}$

EXECUTE: (a) $\frac{1}{18.0 \text{ cm} - s'} + \frac{1}{s'} = \frac{1}{3.00 \text{ cm}}. (s')^2 - (18.0 \text{ cm})s' + 54.0 \text{ cm}^2 = 0$ so $s' = 14.2 \text{ cm}$ or $3.80 \text{ cm}.$

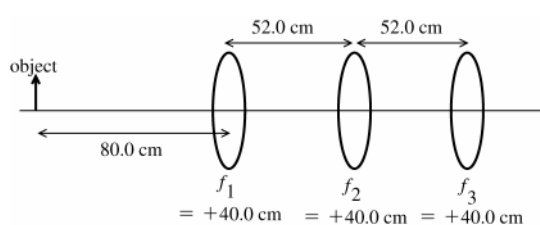
$s = 3.80 \text{ cm}$ or 14.2 cm , so the screen must either be 3.80 cm or 14.2 cm from the object.

$$(b) \ s = 3.80 \text{ cm} : m = -\frac{s'}{s} = -\frac{3.80}{14.2} = -0.268. \quad s = 14.2 \text{ cm} : m = -\frac{s'}{s} = -\frac{14.2}{3.80} = -3.74.$$

EVALUATE: Since the image is projected onto the screen, the image is real and s' is positive. We assumed this when we wrote the condition $s + s' = 18.0 \text{ cm}$.

34.105. IDENTIFY: Apply Eq.(34.16) to calculate the image distance for each lens. The image formed by the 1st lens serves as the object for the 2nd lens, and the image formed by the 2nd lens serves as the object for the 3rd lens.

SET UP: The positions of the object and lenses are shown in Figure 34.105.



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$$

$$s' = \frac{sf}{s-f}$$

Figure 34.105

EXECUTE: lens #1

$$s = +80.0 \text{ cm}; f = +40.0 \text{ cm}$$

$$s' = \frac{sf}{s-f} = \frac{(+80.0 \text{ cm})(+40.0 \text{ cm})}{+80.0 \text{ cm} - 40.0 \text{ cm}} = +80.0 \text{ cm}$$

The image formed by the first lens is 80.0 cm to the right of the first lens, so it is $80.0 \text{ cm} - 52.0 \text{ cm} = 28.0 \text{ cm}$ to the right of the second lens.

lens #2

$$s = -28.0 \text{ cm}; f = +40.0 \text{ cm}$$

$$s' = \frac{sf}{s-f} = \frac{(-28.0 \text{ cm})(+40.0 \text{ cm})}{-28.0 \text{ cm} - 40.0 \text{ cm}} = +16.47 \text{ cm}$$

The image formed by the second lens is 16.47 cm to the right of the second lens, so it is $52.0 \text{ cm} - 16.47 \text{ cm} = 35.53 \text{ cm}$ to the left of the third lens.

lens #3

$$s = +35.53 \text{ cm}; f = +40.0 \text{ cm}$$

$$s' = \frac{sf}{s-f} = \frac{(+35.53 \text{ cm})(+40.0 \text{ cm})}{+35.53 \text{ cm} - 40.0 \text{ cm}} = -318 \text{ cm}$$

The final image is 318 cm to the left of the third lens, so it is $318 \text{ cm} - 52 \text{ cm} - 52 \text{ cm} - 80 \text{ cm} = 134 \text{ cm}$ to the left of the object.

EVALUATE: We used the separation between the lenses and the sign conventions for s and s' to determine the object distances for the 2nd and 3rd lenses. The final image is virtual since the final s' is negative.

34.106. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and calculate s' for each s .

SET UP: $f = 90 \text{ mm}$

$$\text{EXECUTE: } \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{1300 \text{ mm}} + \frac{1}{s'} = \frac{1}{90 \text{ mm}} \Rightarrow s' = 96.7 \text{ mm}.$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{6500 \text{ mm}} + \frac{1}{s'} = \frac{1}{90 \text{ mm}} \Rightarrow s' = 91.3 \text{ mm}.$$

$$\Rightarrow \Delta s' = 96.7 \text{ mm} - 91.3 \text{ mm} = 5.4 \text{ mm} \text{ toward the film}$$

EVALUATE: $s' = \frac{sf}{s-f}$. For $f > 0$ and $s > f$, s' decreases as s increases.

34.107. IDENTIFY and SET UP: The generalization of Eq.(34.22) is $M = \frac{\text{near point}}{f}$, so $f = \frac{\text{near point}}{M}$.

EXECUTE: (a) age 10, near point = 7 cm

$$f = \frac{7 \text{ cm}}{2.0} = 3.5 \text{ cm}$$

(b) age 30, near point = 14 cm

$$f = \frac{14 \text{ cm}}{2.0} = 7.0 \text{ cm}$$

(c) age 60, near point = 200 cm

$$f = \frac{200 \text{ cm}}{2.0} = 100 \text{ cm}$$

(d) $f = 3.5 \text{ cm}$ (from part (a)) and near point = 200 cm (for 60-year-old)

$$M = \frac{200 \text{ cm}}{3.5 \text{ cm}} = 57$$

(e) **EVALUATE:** No. The reason $f = 3.5 \text{ cm}$ gives a larger M for a 60-year-old than for a 10-year-old is that the eye of the older person can't focus on as close an object as the younger person can. The unaided eye of the 60-year-old must view a much smaller angular size, and that is why the same f gives a much larger M . The angular size of the image depends only on f and is the same for the two ages.

34.108. IDENTIFY: Use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s that gives $s' = -25 \text{ cm}$. $M = \frac{\theta'}{\theta}$.

SET UP: Let the height of the object be y , so $\theta' = \frac{y}{s}$ and $\theta = \frac{y}{25 \text{ cm}}$.

EXECUTE: (a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{-25 \text{ cm}} = \frac{1}{f} \Rightarrow s = \frac{f(25 \text{ cm})}{f + 25 \text{ cm}}$.

(b) $\theta' = \arctan\left(\frac{y}{s}\right) = \arctan\left(\frac{y(f + 25 \text{ cm})}{f(25 \text{ cm})}\right) \approx \frac{y(f + 25 \text{ cm})}{f(25 \text{ cm})}$.

(c) $M = \frac{\theta'}{\theta} = \frac{y(f + 25 \text{ cm})}{f(25 \text{ cm})} \cdot \frac{1}{y/25 \text{ cm}} = \frac{f + 25 \text{ cm}}{f}$.

(d) If $f = 10 \text{ cm} \Rightarrow M = \frac{10 \text{ cm} + 25 \text{ cm}}{10 \text{ cm}} = 3.5$. This is 1.4 times greater than the magnification obtained if the image

if formed at infinity ($M_\infty = \frac{25 \text{ cm}}{f} = 2.5$).

EVALUATE: (e) Having the first image form just within the focal length puts one in the situation described above, where it acts as a source that yields an enlarged virtual image. If the first image fell just outside the second focal point, then the image would be real and diminished.

34.109. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The near point is at infinity, so that is where the image must be formed for any objects that are close.

SET UP: The power in diopters equals $\frac{1}{f}$, with f in meters.

EXECUTE: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{24 \text{ cm}} + \frac{1}{-\infty} = \frac{1}{0.24 \text{ m}} = 4.17 \text{ diopters}$.

EVALUATE: To focus on closer objects, the power must be increased.

34.110. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.40$.

EXECUTE: $\frac{1}{36.0 \text{ cm}} + \frac{1.40}{s'} = \frac{0.40}{0.75 \text{ cm}} \Rightarrow s' = 2.77 \text{ cm}$.

EVALUATE: This distance is greater than the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.111. IDENTIFY: Use similar triangles in Figure 34.63 in the textbook and Eq.(34.16) to derive the expressions called for in the problem.

(a) **SET UP:** The effect of the converging lens on the ray bundle is sketched in Figure 34.111.

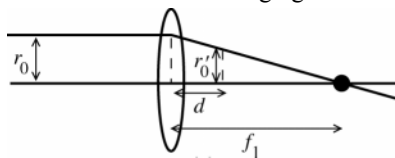


Figure 34.111a

EXECUTE: From similar triangles in Figure 34.111a,

$$\frac{r_0}{f_1} = \frac{r'_0}{f_1 - d}$$

Thus $r'_0 = \left(\frac{f_1 - d}{f_1} \right) r_0$, as was to be shown.

(b) SET UP: The image at the focal point of the first lens, a distance f_1 to the right of the first lens, serves as the object for the second lens. The image is a distance $f_1 - d$ to the right of the second lens, so $s_2 = -(f_1 - d) = d - f_1$.

EXECUTE: $s'_2 = \frac{s_2 f_2}{s_2 - f_2} = \frac{(d - f_1) f_2}{d - f_1 - f_2}$

$f_2 < 0$ so $|f_2| = -f_2$ and $s'_2 = \frac{(f_1 - d)|f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(c) SET UP: The effect of the diverging lens on the ray bundle is sketched in Figure 34.111b.

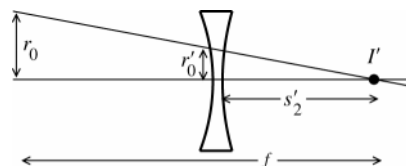


Figure 34.111b

EXECUTE: From similar triangles in the sketch, $\frac{r_0}{f} = \frac{r'_0}{s'_2}$

Thus $\frac{r_0}{r'_0} = \frac{f}{s'_2}$

From the results of part (a), $\frac{r_0}{r'_0} = \frac{f_1}{f_1 - d}$. Combining the two results gives $\frac{f_1}{f_1 - d} = \frac{f}{s'_2}$

$f = s'_2 \left(\frac{f_1}{f_1 - d} \right) = \frac{(f_1 - d)|f_2|f_1}{(|f_2| - f_1 + d)(f_1 - d)} = \frac{f_1|f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(d) SET UP: Put the numerical values into the expression derived in part (c).

EXECUTE: $f = \frac{f_1|f_2|}{|f_2| - f_1 + d}$

$f_1 = 12.0$ cm, $|f_2| = 18.0$ cm, so $f = \frac{216 \text{ cm}^2}{6.0 \text{ cm} + d}$

$d = 0$ gives $f = 36.0$ cm; maximum f

$d = 4.0$ cm gives $f = 21.6$ cm; minimum f

$f = 30.0$ cm says $30.0 \text{ cm} = \frac{216 \text{ cm}^2}{6.0 \text{ cm} + d}$

$6.0 \text{ cm} + d = 7.2 \text{ cm}$ and $d = 1.2 \text{ cm}$

EVALUATE: Changing d produces a range of effective focal lengths. The effective focal length can be both smaller and larger than $f_1 + |f_2|$.

34.112. IDENTIFY: $|M| = \frac{\theta'}{\theta}$. $\theta = \frac{y'_1}{f_1}$, and $\theta' = \frac{y'_2}{s'_2}$. This gives $|M| = \left| \frac{y'_2}{s'_2} \cdot \frac{f_1}{y'_1} \right|$.

SET UP: Since the image formed by the objective is used as the object for the eyepiece, $y'_1 = y_2$.

EXECUTE: $|M| = \left| \frac{y'_2}{s'_2} \cdot \frac{f_1}{y_2} \right| = \left| \frac{y'_2}{y_2} \cdot \frac{f_1}{s'_2} \right| = \left| \frac{s'_2}{s_2} \cdot \frac{f_1}{s'_2} \right| = \left| \frac{f_1}{s_2} \right|$. Therefore, $s_2 = \frac{f_1}{|M|} = \frac{48.0 \text{ cm}}{36} = 1.33 \text{ cm}$, and this is just outside the eyepiece focal point.

Now the distance from the mirror vertex to the lens is $f_1 + s_2 = 49.3 \text{ cm}$, and so

$\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f_2} \Rightarrow s'_2 = \left(\frac{1}{1.20 \text{ cm}} - \frac{1}{1.33 \text{ cm}} \right)^{-1} = 12.3 \text{ cm}$. Thus we have a final image which is real and 12.3 cm from the eyepiece. (Take care to carry plenty of figures in the calculation because two close numbers are subtracted.)

EVALUATE: Eq.(34.25) gives $|M| = 40$, somewhat larger than $|M|$ for this telescope.

34.113. IDENTIFY and SET UP: The image formed by the objective is the object for the eyepiece. The total lateral magnification is $m_{\text{tot}} = m_1 m_2$. $f_1 = 8.00 \text{ mm}$ (objective); $f_2 = 7.50 \text{ cm}$ (eyepiece)

(a) The locations of the object, lenses and screen are shown in Figure 34.113.

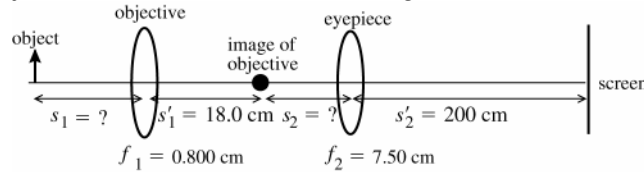


Figure 34.113

EXECUTE: Find the object distance s_1 for the objective:

$$s'_1 = +18.0 \text{ cm}, f_1 = 0.800 \text{ cm}, s_1 = ?$$

$$\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f_1}, \text{ so } \frac{1}{s_1} = \frac{1}{f_1} - \frac{1}{s'_1} = \frac{s'_1 - f_1}{s'_1 f_1}$$

$$s_1 = \frac{s'_1 f_1}{s'_1 - f_1} = \frac{(18.0 \text{ cm})(0.800 \text{ cm})}{18.0 \text{ cm} - 0.800 \text{ cm}} = 0.8372 \text{ cm}$$

Find the object distance s_2 for the eyepiece:

$$s'_2 = +200 \text{ cm}, f_2 = 7.50 \text{ cm}, s_2 = ?$$

$$\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f_2}$$

$$s_2 = \frac{s'_2 f_2}{s'_2 - f_2} = \frac{(200 \text{ cm})(7.50 \text{ cm})}{200 \text{ cm} - 7.50 \text{ cm}} = 7.792 \text{ cm}$$

Now we calculate the magnification for each lens:

$$m_1 = -\frac{s'_1}{s_1} = -\frac{18.0 \text{ cm}}{0.8372 \text{ cm}} = -21.50$$

$$m_2 = -\frac{s'_2}{s_2} = -\frac{200 \text{ cm}}{7.792 \text{ cm}} = -25.67$$

$$m_{\text{tot}} = m_1 m_2 = (-21.50)(-25.67) = 552.$$

(b) From the sketch we can see that the distance between the two lenses is $s'_1 + s_2 = 18.0 \text{ cm} + 7.792 \text{ cm} = 25.8 \text{ cm}$.

EVALUATE: The microscope is not being used in the conventional way; it merely serves as a two-lens system. In particular, the final image formed by the eyepiece in the problem is real, not virtual as is the case normally for a microscope. Eq.(34.23) does not apply here, and in any event gives the angular not the lateral magnification.

34.114. IDENTIFY: For u and u' as defined in Figure 34.64 in the textbook, $M = \frac{u'}{u}$.

SET UP: f_2 is negative. From Figure 34.64, the length of the telescope is $f_1 + f_2$.

EXECUTE: (a) From the figure, $u = \frac{y}{f_1}$ and $u' = \frac{y}{|f_2|} = -\frac{y}{f_2}$. The angular magnification is $M = \frac{u'}{u} = -\frac{f_1}{f_2}$.

$$(b) M = -\frac{f_1}{f_2} \Rightarrow f_2 = -\frac{f_1}{M} = -\frac{95.0 \text{ cm}}{6.33} = -15.0 \text{ cm}.$$

(c) The length of the telescope is $95.0 \text{ cm} - 15.0 \text{ cm} = 80.0 \text{ cm}$, compared to the length of 110 cm for the telescope in Exercise 34.57.

EVALUATE: An advantage of this construction is that the telescope is somewhat shorter.

34.115. IDENTIFY: Use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s' (the distance of each point from the lens), for points A , B and C .

SET UP: The object and lens are shown in Figure 34.115a.

EXECUTE: (a) For point C : $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{45.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{20.0 \text{ cm}} \Rightarrow s' = 36.0 \text{ cm}$.

$y' = -\frac{s'}{s}y = -\frac{36.0}{45.0}(15.0 \text{ cm}) = -12.0 \text{ cm}$, so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point A : $s = 45.0 \text{ cm} + 8.00 \text{ cm}(\cos 45^\circ) = 50.7 \text{ cm}$.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{50.7 \text{ cm}} + \frac{1}{s'} = \frac{1}{20.0 \text{ cm}} \Rightarrow s' = 33.0 \text{ cm}. y' = -\frac{s'}{s}y = -\frac{33.0}{45.0}(15.0 \text{ cm} - 8.00 \text{ cm}(\sin 45^\circ)) = -6.10 \text{ cm},$$

so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis.

For point B : $s = 45.0 \text{ cm} - 8.00 \text{ cm}(\cos 45^\circ) = 39.3 \text{ cm}$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{39.3 \text{ cm}} + \frac{1}{s'} = \frac{1}{20.0 \text{ cm}} \Rightarrow s' = 40.7 \text{ cm}$.

$y' = -\frac{s'}{s}y = -\frac{40.7}{39.3}(15.0 \text{ cm} + 8.00 \text{ cm}(\sin 45^\circ)) = -21.4 \text{ cm}$, so the image of point B is 40.7 cm to the right of the

lens, and 21.4 cm below the axis. The image is shown in Figure 34.115b.

(b) The length of the pencil is the distance from point A to B :

$$L = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(33.0 \text{ cm} - 40.7 \text{ cm})^2 + (6.10 \text{ cm} - 21.4 \text{ cm})^2} = 17.1 \text{ cm}$$

EVALUATE: The image is below the optic axis and is larger than the object.

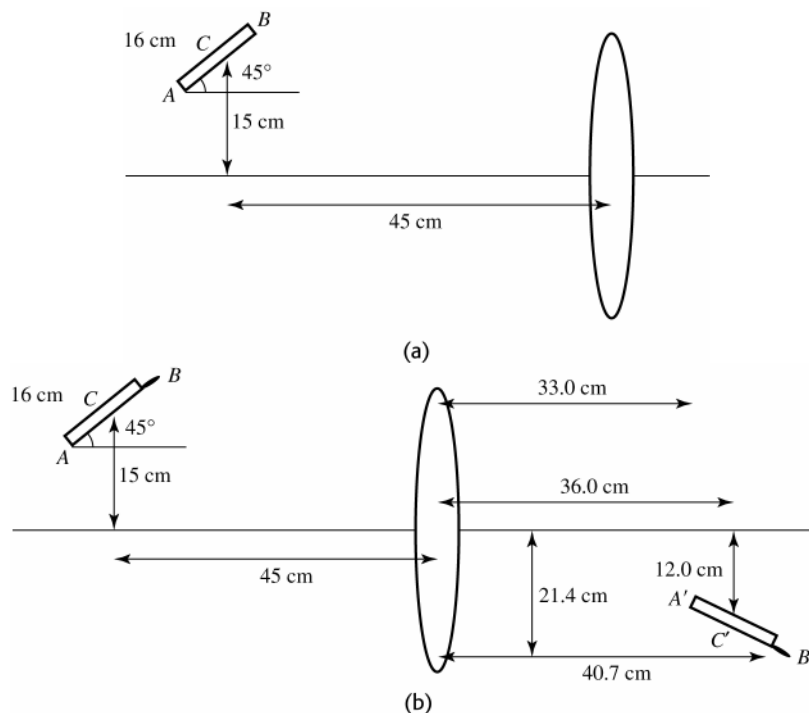


Figure 34.115

34.116. IDENTIFY and SET UP: Consider the ray diagram drawn in Figure 34.116.

EXECUTE: (a) Using the diagram and law of sines, $\frac{\sin \theta}{(R-f)} = \frac{\sin \alpha}{g}$ but $\sin \theta = \frac{h}{R} = \sin \alpha$ (law of

reflection), $g = (R-f)$. Bisecting the triangle: $\cos \theta = \frac{R/2}{(R-f)} \Rightarrow R \cos \theta - f \cos \theta = \frac{R}{2}$.

$f = \frac{R}{2} \left[2 - \frac{1}{\cos \theta} \right] = f_0 \left[2 - \frac{1}{\cos \theta} \right]$. $f_0 = \frac{R}{2}$ is the value of f for θ near zero (incident ray near the axis). When θ increases, $(2 - 1/\cos \theta)$ decreases and f decreases.

(b) $\frac{f-f_0}{f_0} = -0.02 \Rightarrow \frac{f}{f_0} = 0.98$ so $2 - \frac{1}{\cos \theta} = 0.98$. $\cos \theta = \frac{1}{2-0.98} = 0.98$ and $\theta = 11.4^\circ$.

EVALUATE: For $\theta = 45^\circ$, $f = 0.586 f_0$, and f approaches zero as θ approaches 60° .

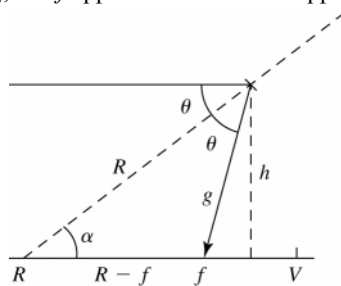


Figure 34.116

34.117. IDENTIFY: The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

SET UP: For a real image $s' > 0$ and the distance between the object and the image is $D = s + s'$. For a real image must have $s > f$.

EXECUTE: $D = s + s'$ but $s' = \frac{sf}{s-f} \Rightarrow D = s + \frac{sf}{s-f} = \frac{s^2}{s-f}$.

$$\frac{dD}{ds} = \frac{d}{ds} \left(\frac{s^2}{s-f} \right) = \frac{2s}{s-f} - \frac{s^2}{(s-f)^2} = \frac{s^2 - 2sf}{(s-f)^2} = 0. \quad s^2 - 2sf = 0. \quad s(s-2f) = 0. \quad s = 2f \text{ is the solution for which } s > f. \text{ For } s = 2f, \quad s' = 2f. \text{ Therefore, the minimum separation is } 2f + 2f = 4f.$$

(b) A graph of D/f versus s/f is sketched in Figure 34.117. Note that the minimum does occur for $D = 4f$.

EVALUATE: If, for example, $s = 3f/2$, then $s' = 3f$ and $D = s + s' = 4.5f$, greater than the minimum value.

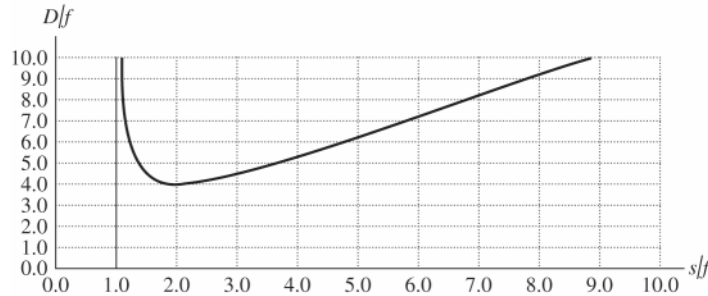


Figure 34.117

34.118. IDENTIFY and SET UP: For a plane mirror, $s' = -s$.

EXECUTE: (a) By the symmetry of image production, any image must be the same distance D as the object from the mirror intersection point. But if the images and the object are equal distances from the mirror intersection, they lie on a circle with radius equal to D .

(b) The center of the circle lies at the mirror intersection as discussed above.

(c) The diagram is sketched in Figure 34.118.

EVALUATE: To see the image, light from the object must be able to reflect from each mirror and reach the person's eyes.

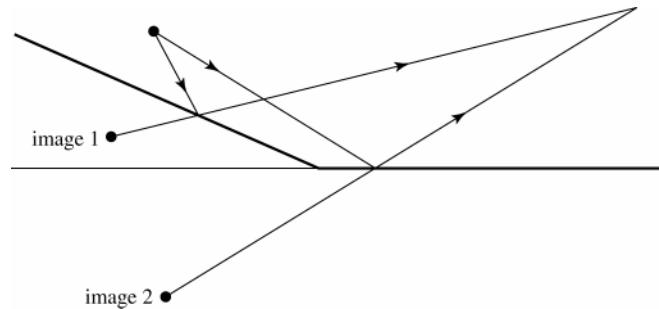


Figure 34.118

34.119. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to refraction at the cornea to find where the object for the cornea must be in

order for the image to be at the retina. Then use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate f so that the lens produces an image of a distant object at this point.

SET UP: For refraction at the cornea, $n_a = 1.33$ and $n_b = 1.40$. The distance from the cornea to the retina in this model of the eye is 2.60 cm. From Problem 34.46, $R = 0.71$ cm.

EXECUTE: (a) People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

(b) When introducing glasses, let's first consider what happens at the eye:

$$\frac{n_a}{s_2} + \frac{n_b}{s'_2} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.33}{s_2} + \frac{1.40}{2.6 \text{ cm}} = \frac{0.07}{0.71 \text{ cm}} \Rightarrow s_2 = -3.02 \text{ cm. That is, the object for the cornea must be } 3.02 \text{ cm}$$

behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, so $s'_1 = 2.00 \text{ cm} + s_2 = 5.02 \text{ cm}$.

$\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f'_1}$ gives $\frac{1}{\infty} + \frac{1}{5.02 \text{ cm}} = \frac{1}{f'_1}$ and $f'_1 = 5.02 \text{ cm}$. This is the focal length in water, but to get it in air, we use

the formula from Problem 34.98: $f_1 = f'_1 \left[\frac{n - n_{\text{liq}}}{n_{\text{liq}}(n - 1)} \right] = (5.02 \text{ cm}) \left[\frac{1.52 - 1.333}{1.333(1.52 - 1)} \right] = 1.35 \text{ cm}$.

EVALUATE: A converging lens is needed.

INTERFERENCE

35.1. IDENTIFY: Compare the path difference to the wavelength.

SET UP: The separation between sources is 5.00 m, so for points between the sources the largest possible path difference is 5.00 m.

EXECUTE: (a) For constructive interference the path difference is $m\lambda$, $m = 0, \pm 1, \pm 2, \dots$. Thus only the path difference of zero is possible. This occurs midway between the two sources, 2.50 m from A.

(b) For destructive interference the path difference is $(m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2, \dots$

A path difference of $\pm\lambda/2 = 3.00$ m is possible but a path difference as large as $3\lambda/2 = 9.00$ m is not possible. For a point a distance x from A and $5.00 - x$ from B the path difference is

$x - (5.00 \text{ m} - x)$. $x - (5.00 \text{ m} - x) = +3.00$ m gives $x = 4.00$ m. $x - (5.00 \text{ m} - x) = -3.00$ m gives $x = 1.00$ m.

EVALUATE: The point of constructive interference is midway between the points of destructive interference.

35.2. IDENTIFY: For destructive interference the path difference is $(m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2, \dots$. The longest wavelength is for $m = 0$. For constructive interference the path difference is $m\lambda$, $m = 0, \pm 1, \pm 2, \dots$. The longest wavelength is for $m = 1$.

SET UP: The path difference is 120 m.

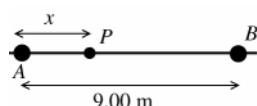
EXECUTE: (a) For destructive interference $\frac{\lambda}{2} = 120 \text{ m} \Rightarrow \lambda = 240 \text{ m}$.

(b) The longest wavelength for constructive interference is $\lambda = 120 \text{ m}$.

EVALUATE: The path difference doesn't depend on the distance of point Q from B.

35.3. IDENTIFY: Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from A to P and from B to P. For constructive interference this path difference is an integer multiple of the wavelength.

SET UP: Consider Figure 35.3



The distance of point P from each coherent source is $r_A = x$ and $r_B = 9.00 \text{ m} - x$.

Figure 35.3

EXECUTE: The path difference is $r_B - r_A = 9.00 \text{ m} - 2x$.

$$r_B - r_A = m\lambda, \quad m = 0, \pm 1, \pm 2, \dots$$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{120 \times 10^6 \text{ Hz}} = 2.50 \text{ m}$$

Thus $9.00 \text{ m} - 2x = m(2.50 \text{ m})$ and $x = \frac{9.00 \text{ m} - m(2.50 \text{ m})}{2} = 4.50 \text{ m} - (1.25 \text{ m})m$. x must lie in the range 0 to

9.00 m since P is said to be between the two antennas.

$m = 0$ gives $x = 4.50 \text{ m}$

$m = +1$ gives $x = 4.50 \text{ m} - 1.25 \text{ m} = 3.25 \text{ m}$

$m = +2$ gives $x = 4.50 \text{ m} - 2.50 \text{ m} = 2.00 \text{ m}$

$m = +3$ gives $x = 4.50 \text{ m} - 3.75 \text{ m} = 0.75 \text{ m}$

$m = -1$ gives $x = 4.50 \text{ m} + 1.25 \text{ m} = 5.75 \text{ m}$

$m = -2$ gives $x = 4.50 \text{ m} + 2.50 \text{ m} = 7.00 \text{ m}$

$m = -3$ gives $x = 4.50 \text{ m} + 3.75 \text{ m} = 8.25 \text{ m}$

All other values of m give values of x out of the allowed range. Constructive interference will occur for $x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, and 8.25 m.

EVALUATE: Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

- 35.4. IDENTIFY:** For constructive interference the path difference d is related to λ by $d = m\lambda$, $m = 0, 1, 2, \dots$. For destructive interference $d = (m + \frac{1}{2})\lambda$, $m = 0, 1, 2, \dots$

SET UP: $d = 2040$ nm

EXECUTE: (a) The brightest wavelengths are when constructive interference occurs:

$$d = m\lambda_m \Rightarrow \lambda_m = \frac{d}{m} \Rightarrow \lambda_3 = \frac{2040 \text{ nm}}{3} = 680 \text{ nm}, \lambda_4 = \frac{2040 \text{ nm}}{4} = 510 \text{ nm and}$$

$$\lambda_5 = \frac{2040 \text{ nm}}{5} = 408 \text{ nm.}$$

(b) The path-length difference is the same, so the wavelengths are the same as part (a).

(c) $d = (m + \frac{1}{2})\lambda_m$ so $\lambda_m = \frac{d}{m + \frac{1}{2}} = \frac{2040 \text{ nm}}{m + \frac{1}{2}}$. The visible wavelengths are $\lambda_3 = 583$ nm and $\lambda_4 = 453$ nm.

EVALUATE: The wavelengths for constructive interference are between those for destructive interference.

- 35.5. IDENTIFY:** If the path difference between the two waves is equal to a whole number of wavelengths, constructive interference occurs, but if it is an odd number of half-wavelengths, destructive interference occurs.

SET UP: We calculate the distance traveled by both waves and subtract them to find the path difference.

EXECUTE: Call P_1 the distance from the right speaker to the observer and P_2 the distance from the left speaker to the observer.

(a) $P_1 = 8.0$ m and $P_2 = \sqrt{(6.0 \text{ m})^2 + (8.0 \text{ m})^2} = 10.0$ m. The path distance is

$$\Delta P = P_2 - P_1 = 10.0 \text{ m} - 8.0 \text{ m} = 2.0 \text{ m}$$

(b) The path distance is one wavelength, so constructive interference occurs.

(c) $P_1 = 17.0$ m and $P_2 = \sqrt{(6.0 \text{ m})^2 + (17.0 \text{ m})^2} = 18.0$ m. The path difference is $18.0 \text{ m} - 17.0 \text{ m} = 1.0$ m, which is one-half wavelength, so destructive interference occurs.

EVALUATE: Constructive interference also occurs if the path difference 2λ , 3λ , 4λ , etc., and destructive interference occurs if it is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.

- 35.6. IDENTIFY:** At an antinode the interference is constructive and the path difference is an integer number of wavelengths; path difference $= m\lambda$, $m = 0, \pm 1, \pm 2, \dots$ at an antinode.

SET UP: The maximum magnitude of the path difference is the separation d between the two sources.

EXECUTE: (a) At $S_1, r_2 - r_1 = 4\lambda$, and this path difference stays the same all along the y -axis, so

$m = +4$. At $S_2, r_2 - r_1 = -4\lambda$, and the path difference below this point, along the negative y -axis, stays the same, so $m = -4$.

(b) The wave pattern is sketched in Figure 35.6.

(c) The maximum and minimum m -values are determined by the largest integer less than or equal to $\frac{d}{\lambda}$.

(d) If $d = 7\frac{1}{2}\lambda \Rightarrow -7 \leq m \leq +7$, so there will be a total of 15 antinodes between the sources.

EVALUATE: We are considering points close to the two sources and the antinodal curves are not straight lines.

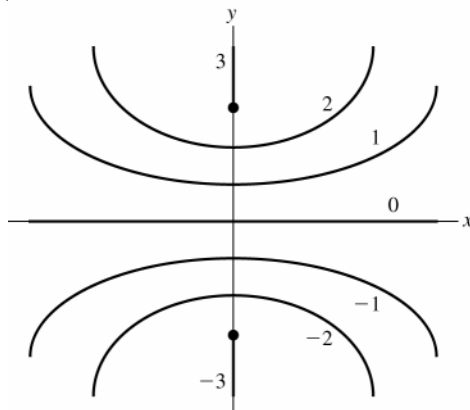


Figure 35.6

35.7. IDENTIFY: At an antinodal point the path difference is equal to an integer number of wavelengths.

SET UP: For $m = 3$, the path difference is 3λ .

EXECUTE: Measuring with a ruler from both S_1 and S_2 to the different points in the antinodal line labeled $m = 3$, we find that the difference in path length is three times the wavelength of the wave, as measured from one crest to the next on the diagram.

EVALUATE: There is a whole curve of points where the path difference is 3λ .

35.8. IDENTIFY: The value of y_{20} is much smaller than R and the approximate expression $y_m = R \frac{m\lambda}{d}$ is accurate.

SET UP: $y_{20} = 10.6 \times 10^{-3} \text{ m}$.

EXECUTE: $d = \frac{20R\lambda}{y_{20}} = \frac{(20)(1.20 \text{ m})(502 \times 10^{-9} \text{ m})}{10.6 \times 10^{-3} \text{ m}} = 1.14 \times 10^{-3} \text{ m} = 1.14 \text{ mm}$

EVALUATE: $\tan \theta_{20} = \frac{y_{20}}{R}$ so $\theta_{20} = 0.51^\circ$ and the approximation $\sin \theta_{20} \approx \tan \theta_{20}$ is very accurate.

35.9. IDENTIFY and SET UP: The dark lines correspond to destructive interference and hence are located by Eq.(35.5):

$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda \text{ so } \sin \theta = \frac{\left(m + \frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm 1, \pm 2, \dots$$

Solve for θ that locates the second and third dark lines. Use $y = R \tan \theta$ to find the distance of each of the dark lines from the center of the screen.

EXECUTE: 1st dark line is for $m = 0$

2nd dark line is for $m = 1$ and $\sin \theta_1 = \frac{3\lambda}{2d} = \frac{3(500 \times 10^{-9} \text{ m})}{2(0.450 \times 10^{-3} \text{ m})} = 1.667 \times 10^{-3}$ and $\theta_1 = 1.667 \times 10^{-3} \text{ rad}$

3rd dark line is for $m = 2$ and $\sin \theta_2 = \frac{5\lambda}{2d} = \frac{5(500 \times 10^{-9} \text{ m})}{2(0.450 \times 10^{-3} \text{ m})} = 2.778 \times 10^{-3}$ and $\theta_2 = 2.778 \times 10^{-3} \text{ rad}$

(Note that θ_1 and θ_2 are small so that the approximation $\theta \approx \sin \theta \approx \tan \theta$ is valid.) The distance of each dark line from the center of the central bright band is given by $y_m = R \tan \theta$, where $R = 0.850 \text{ m}$ is the distance to the screen.

$\tan \theta \approx \theta$ so $y_m = R\theta_m$

$y_1 = R\theta_1 = (0.750 \text{ m})(1.667 \times 10^{-3} \text{ rad}) = 1.25 \times 10^{-3} \text{ m}$

$y_2 = R\theta_2 = (0.750 \text{ m})(2.778 \times 10^{-3} \text{ rad}) = 2.08 \times 10^{-3} \text{ m}$

$\Delta y = y_2 - y_1 = 2.08 \times 10^{-3} \text{ m} - 1.25 \times 10^{-3} \text{ m} = 0.83 \text{ mm}$

EVALUATE: Since θ_1 and θ_2 are very small we could have used Eq.(35.6), generalized to destructive

interference: $y_m = R \left(m + \frac{1}{2}\right) \lambda / d$.

35.10. IDENTIFY: Since the dark fringes are equally spaced, $R \gg y_m$, the angles are small and the dark bands are located

by $y_{m+\frac{1}{2}} = R \frac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: The separation between adjacent dark bands is $\Delta y = \frac{R\lambda}{d}$.

EXECUTE: $\Delta y = \frac{R\lambda}{d} \Rightarrow d = \frac{R\lambda}{\Delta y} = \frac{(1.80 \text{ m})(4.50 \times 10^{-7} \text{ m})}{4.20 \times 10^{-3} \text{ m}} = 1.93 \times 10^{-4} \text{ m} = 0.193 \text{ mm}$.

EVALUATE: When the separation between the slits decreases, the separation between dark fringes increases.

35.11. IDENTIFY and SET UP: The positions of the bright fringes are given by Eq.(35.6): $y_m = R(m\lambda/d)$. For each fringe the adjacent fringe is located at $y_{m+1} = R(m+1)\lambda/d$. Solve for λ .

EXECUTE: The separation between adjacent fringes is $\Delta y = y_{m+1} - y_m = R\lambda/d$.

$\lambda = \frac{d\Delta y}{R} = \frac{(0.460 \times 10^{-3} \text{ m})(2.82 \times 10^{-3} \text{ m})}{2.20 \text{ m}} = 5.90 \times 10^{-7} \text{ m} = 590 \text{ nm}$

EVALUATE: Eq.(35.6) requires that the angular position on the screen be small. The angular position of bright fringes is given by $\sin \theta = m\lambda/d$. The slit separation is much larger than the wavelength ($\lambda/d = 1.3 \times 10^{-3}$), so θ is small so long as m is not extremely large.

- 35.12. IDENTIFY:** The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference $y_m = R \frac{(m + \frac{1}{2})\lambda}{d}$.

SET UP: $d = 0.200 \times 10^{-3} \text{ m}$. $R = 4.00 \text{ m}$.

EXECUTE: The distance between any two successive minima

is $y_{m+1} - y_m = R \frac{\lambda}{d} = (4.00 \text{ m}) \frac{(400 \times 10^{-9} \text{ m})}{(0.200 \times 10^{-3} \text{ m})} = 8.00 \text{ mm}$. Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

EVALUATE: For small angles, when $y_m \ll R$, the interference minima are equally spaced.

- 35.13. IDENTIFY and SET UP:** The dark lines are located by $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$. The distance of each line from the center of the screen is given by $y = R \tan \theta$.

EXECUTE: First dark line is for $m = 0$ and $d \sin \theta_1 = \lambda/2$.

$$\sin \theta_1 = \frac{\lambda}{2d} = \frac{550 \times 10^{-9} \text{ m}}{2(1.80 \times 10^{-6} \text{ m})} = 0.1528 \text{ and } \theta_1 = 8.789^\circ. \text{ Second dark line is for } m = 1 \text{ and } d \sin \theta_2 = 3\lambda/2.$$

$$\sin \theta_2 = \frac{3\lambda}{2d} = 3 \left(\frac{550 \times 10^{-9} \text{ m}}{2(1.80 \times 10^{-6} \text{ m})} \right) = 0.4583 \text{ and } \theta_2 = 27.28^\circ.$$

$$y_1 = R \tan \theta_1 = (0.350 \text{ m}) \tan 8.789^\circ = 0.0541 \text{ m}$$

$$y_2 = R \tan \theta_2 = (0.350 \text{ m}) \tan 27.28^\circ = 0.1805 \text{ m}$$

The distance between the lines is $\Delta y = y_2 - y_1 = 0.1805 \text{ m} - 0.0541 \text{ m} = 0.126 \text{ m} = 12.6 \text{ cm}$.

EVALUATE: $\sin \theta_1 = 0.1528$ and $\tan \theta_1 = 0.1546$. $\sin \theta_2 = 0.4583$ and $\tan \theta_2 = 0.5157$. As the angle increases, $\sin \theta \approx \tan \theta$ becomes a poorer approximation.

- 35.14. IDENTIFY:** Using Eq.(35.6) for small angles: $y_m = R \frac{m\lambda}{d}$.

SET UP: First-order means $m = 1$.

EXECUTE: The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d} \Delta \lambda = \frac{(5.00 \text{ m})(1)}{(0.300 \times 10^{-3} \text{ m})} (660 - 470) \times (10^{-9} \text{ m}) = 3.17 \text{ mm}.$$

EVALUATE: The separation between these fringes for different wavelengths increases when the slit separation decreases.

- 35.15. IDENTIFY and SET UP:** Use the information given about the bright fringe to find the distance d between the two slits. Then use Eq.(35.5) and $y = R \tan \theta$ to calculate λ for which there is a first-order dark fringe at this same place on the screen.

$$\text{EXECUTE: } y_1 = \frac{R\lambda_1}{d}, \text{ so } d = \frac{R\lambda_1}{y_1} = \frac{(3.00 \text{ m})(600 \times 10^{-9} \text{ m})}{4.84 \times 10^{-3} \text{ m}} = 3.72 \times 10^{-4} \text{ m. (} R \text{ is much greater than } d, \text{ so Eq.35.6}$$

is valid.) The dark fringes are located by $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$, $m = 0, \pm 1, \pm 2, \dots$. The first order dark fringe is located by $\sin \theta = \lambda_2/2d$, where λ_2 is the wavelength we are seeking.

$$y = R \tan \theta \approx R \sin \theta = \frac{\lambda_2 R}{2d}$$

We want λ_2 such that $y = y_1$. This gives $\frac{R\lambda_1}{d} = \frac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200 \text{ nm}$.

EVALUATE: For $\lambda = 600 \text{ nm}$ the path difference from the two slits to this point on the screen is 600 nm. For this same path difference (point on the screen) the path difference is $\lambda/2$ when $\lambda = 1200 \text{ nm}$.

- 35.16. IDENTIFY:** Bright fringes are located at $y_m = R \frac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d \sin \theta = (m + \frac{1}{2})\lambda$ and $y = R \tan \theta$.

SET UP: $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.32 \times 10^{14} \text{ Hz}} = 4.75 \times 10^{-7} \text{ m}$. For the third bright fringe (not counting the central bright spot), $m = 3$. For the third dark fringe, $m = 2$.

EXECUTE: (a) $d = \frac{m\lambda R}{y_m} = \frac{3(4.75 \times 10^{-7} \text{ m})(0.850 \text{ m})}{0.0311 \text{ m}} = 3.89 \times 10^{-5} \text{ m} = 0.0389 \text{ mm}$

(b) $\sin \theta = (2 + \frac{1}{2}) \frac{\lambda}{d} = (2.5) \left(\frac{4.75 \times 10^{-7} \text{ m}}{3.89 \times 10^{-5} \text{ m}} \right) = 0.0305$ and $\theta = 1.75^\circ$. $y = R \tan \theta = (85.0 \text{ cm}) \tan 1.75^\circ = 2.60 \text{ cm}$.

EVALUATE: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

35.17. IDENTIFY: Bright fringes are located at angles θ given by $d \sin \theta = m\lambda$.

SET UP: The largest value $\sin \theta$ can have is 1.00.

EXECUTE: (a) $m = \frac{d \sin \theta}{\lambda}$. For $\sin \theta = 1$, $m = \frac{d}{\lambda} = \frac{0.0116 \times 10^{-3} \text{ m}}{5.85 \times 10^{-7} \text{ m}} = 19.8$. Therefore, the largest m for fringes on the screen is $m = 19$. There are $2(19) + 1 = 39$ bright fringes, the central one and 19 above and 19 below it.

(b) The most distant fringe has $m = \pm 19$. $\sin \theta = m \frac{\lambda}{d} = \pm 19 \left(\frac{5.85 \times 10^{-7} \text{ m}}{0.0116 \times 10^{-3} \text{ m}} \right) = \pm 0.958$ and $\theta = \pm 73.3^\circ$.

EVALUATE: For small θ the spacing Δy between adjacent fringes is constant but this is no longer the case for larger angles.

35.18. IDENTIFY: At large distances from the antennas the equation $d \sin \theta = m\lambda$, $m = 0, \pm 1, \pm 2, \dots$ gives the angles where maximum intensity is observed and $d \sin \theta = (m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2, \dots$ gives the angles where minimum intensity is observed.

SET UP: $d = 12.0 \text{ m}$. $\lambda = \frac{c}{f}$.

EXECUTE: (a) $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{107.9 \times 10^6 \text{ Hz}} = 2.78 \text{ m}$. $\sin \theta = \frac{m\lambda}{d} = m \left(\frac{2.78 \text{ m}}{12.0 \text{ m}} \right) = m(0.232)$.

$\theta = \pm 13.4^\circ, \pm 27.6^\circ, \pm 44.1^\circ, \pm 68.1^\circ$.

(b) $\sin \theta = (m + \frac{1}{2}) \frac{\lambda}{d} = (m + \frac{1}{2})(0.232)$. $\theta = \pm 6.66^\circ, \pm 20.4^\circ, \pm 35.5^\circ, \pm 54.3^\circ$.

EVALUATE: The angles for zero intensity are approximately midway between those for maximum intensity.

35.19. IDENTIFY: Eq.(35.10): $I = I_0 \cos^2(\phi/2)$. Eq.(35.11): $\phi = (2\pi/\lambda)(r_2 - r_1)$.

SET UP: ϕ is the phase difference and $(r_2 - r_1)$ is the path difference.

EXECUTE: (a) $I = I_0 (\cos 30.0^\circ)^2 = 0.750 I_0$

(b) $60.0^\circ = (\pi/3) \text{ rad}$. $(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80 \text{ nm}$.

EVALUATE: $\phi = 360^\circ/6$ and $(r_2 - r_1) = \lambda/6$.

35.20. IDENTIFY: $\frac{\Delta\phi}{2\pi} = \frac{\text{path difference}}{\lambda}$ relates the path difference to the phase difference $\Delta\phi$.

SET UP: The sources and point P are shown in Figure 35.20.

EXECUTE: $\Delta\phi = 2\pi \left(\frac{524 \text{ cm} - 486 \text{ cm}}{2 \text{ cm}} \right) = 119 \text{ radians}$

EVALUATE: The distances from B to P and A to P aren't important, only the difference in these distances.

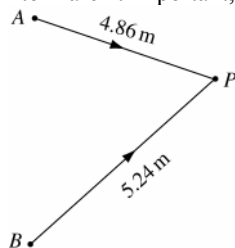


Figure 35.20

35.21. IDENTIFY and SET UP: The phase difference ϕ is given by $\phi = (2\pi d/\lambda) \sin \theta$ (Eq.35.13.)

EXECUTE: $\phi = [2\pi(0.340 \times 10^{-3} \text{ m})/(500 \times 10^{-9} \text{ m})] \sin 23.0^\circ = 1670 \text{ rad}$

EVALUATE: The m th bright fringe occurs when $\phi = 2\pi m$, so there are a large number of bright fringes within 23.0° from the centerline. Note that Eq.(35.13) gives ϕ in radians.

35.22. IDENTIFY: The maximum intensity occurs at all the points of constructive interference. At these points, the path difference between waves from the two transmitters is an integral number of wavelengths.

SET UP: For constructive interference, $\sin \theta = m\lambda/d$.

EXECUTE: (a) First find the wavelength of the UHF waves:

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(1575.42 \text{ MHz}) = 0.1904 \text{ m}$$

For maximum intensity $(\pi d \sin \theta)/\lambda = m\pi$, so

$$\sin \theta = m\lambda/d = m[(0.1904 \text{ m})/(5.18 \text{ m})] = 0.03676m$$

The maximum possible m would be for $\theta = 90^\circ$, or $\sin \theta = 1$, so

$$m_{\max} = d/\lambda = (5.18 \text{ m})/(0.1904 \text{ m}) = 27.2$$

which must be ± 27 since m is an integer. The total number of maxima is 27 on either side of the central fringe, plus the central fringe, for a total of $27 + 27 + 1 = 55$ bright fringes.

(b) Using $\sin \theta = m\lambda/d$, where $m = 0, \pm 1, \pm 2$, and ± 3 , we have

$$\sin \theta = m\lambda/d = m[(0.1904 \text{ m})/(5.18 \text{ m})] = 0.03676m$$

$$m = 0: \sin \theta = 0, \text{ which gives } \theta = 0^\circ$$

$$m = \pm 1: \sin \theta = \pm(0.03676)(1), \text{ which gives } \theta = \pm 2.11^\circ$$

$$m = \pm 2: \sin \theta = \pm(0.03676)(2), \text{ which gives } \theta = \pm 4.22^\circ$$

$$m = \pm 3: \sin \theta = \pm(0.03676)(3), \text{ which gives } \theta = \pm 6.33^\circ$$

$$(c) I = I_0 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right) = (2.00 \text{ W/m}^2) \cos^2 \left[\frac{\pi(5.18 \text{ m}) \sin(4.65^\circ)}{0.1904 \text{ m}} \right] = 1.28 \text{ W/m}^2.$$

EVALUATE: Notice that $\sin \theta$ increases in integer steps, but θ only increases in integer steps for small θ .

35.23. (a) IDENTIFY and SET UP: The minima are located at angles θ given by $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$. The first minimum corresponds to $m = 0$. Solve for θ . Then the distance on the screen is $y = R \tan \theta$.

$$\text{EXECUTE: } \sin \theta = \frac{\lambda}{2d} = \frac{660 \times 10^{-9} \text{ m}}{2(0.260 \times 10^{-3} \text{ m})} = 1.27 \times 10^{-3} \text{ and } \theta = 1.27 \times 10^{-3} \text{ rad}$$

$$y = (0.700 \text{ m}) \tan(1.27 \times 10^{-3} \text{ rad}) = 0.889 \text{ mm}.$$

(b) **IDENTIFY and SET UP:** Eq.(35.15) given the intensity I as a function of the position y on the screen:

$$I = I_0 \cos^2 \left(\frac{\pi dy}{\lambda R} \right). \text{ Set } I = I_0/2 \text{ and solve for } y.$$

$$\text{EXECUTE: } I = \frac{1}{2} I_0 \text{ says } \cos^2 \left(\frac{\pi dy}{\lambda R} \right) = \frac{1}{2}$$

$$\cos \left(\frac{\pi dy}{\lambda R} \right) = \frac{1}{\sqrt{2}} \text{ so } \frac{\pi dy}{\lambda R} = \frac{\pi}{4} \text{ rad}$$

$$y = \frac{\lambda R}{4d} = \frac{(660 \times 10^{-9} \text{ m})(0.700 \text{ m})}{4(0.260 \times 10^{-3} \text{ m})} = 0.444 \text{ mm}$$

EVALUATE: $I = I_0/2$ at a point on the screen midway between where $I = I_0$ and $I = 0$.

35.24. IDENTIFY: Eq. (35.14): $I = I_0 \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta \right)$.

SET UP: The intensity goes to zero when the cosine's argument becomes an odd integer multiple of $\frac{\pi}{2}$

$$\text{EXECUTE: } \frac{\pi d}{\lambda} \sin \theta = (m + 1/2)\pi \text{ gives } d \sin \theta = \lambda(m + 1/2), \text{ which is Eq. (35.5).}$$

EVALUATE: Section 35.3 shows that the maximum-intensity directions from Eq.(35.14) agree with Eq.(35.4).

35.25. IDENTIFY: The intensity decreases as we move away from the central maximum.

$$\text{SET UP: } \text{The intensity is given by } I = I_0 \cos^2 \left(\frac{\pi dy}{\lambda R} \right).$$

EXECUTE: First find the wavelength: $\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(12.5 \text{ MHz}) = 24.00 \text{ m}$
At the farthest the receiver can be placed, $I = I_0/4$, which gives

$$\frac{I_0}{4} = I_0 \cos^2 \left(\frac{\pi dy}{\lambda R} \right) \Rightarrow \cos^2 \left(\frac{\pi dy}{\lambda R} \right) = \frac{1}{4} \Rightarrow \cos \left(\frac{\pi dy}{\lambda R} \right) = \pm \frac{1}{2}$$

The solutions are $\pi dy/\lambda R = \pi/3$ and $2\pi/3$. Using $\pi/3$, we get

$$y = \lambda R/3d = (24.00 \text{ m})(500 \text{ m})/[3(56.0 \text{ m})] = 71.4 \text{ m}$$

It must remain within 71.4 m of point C.

EVALUATE: Using $\pi dy/\lambda R = 2\pi/3$ gives $y = 142.8 \text{ m}$. But to reach this point, the receiver would have to go beyond 71.4 m from C, where the signal would be too weak, so this second point is not possible.

- 35.26. IDENTIFY:** The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \frac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is

given by $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$.

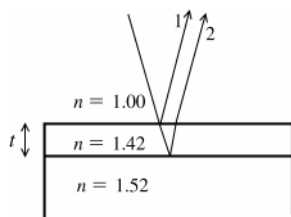
SET UP: $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.20 \times 10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures zero intensity I_0 , $\phi = 0$.

EXECUTE: (a) $\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = \frac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}$.

(b) $I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{4.52 \text{ rad}}{2}\right) = 0.404 I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

- 35.27. IDENTIFY:** Consider interference between rays reflected at the upper and lower surfaces of the film. Consider phase difference due to the path difference of $2t$ and any phase differences due to phase changes upon reflection.
SET UP: Consider Figure 35.27.



Both rays (1) and (2) undergo a 180° phase change on reflection, so there is no net phase difference introduced and the condition for destructive interference is

$$2t = \left(m + \frac{1}{2}\right)\lambda.$$

Figure 35.27

EXECUTE: $t = \frac{\left(m + \frac{1}{2}\right)\lambda}{2}$; thinnest film says $m = 0$ so $t = \frac{\lambda}{4}$

$$\lambda = \frac{\lambda_0}{1.42} \text{ and } t = \frac{\lambda_0}{4(1.42)} = \frac{650 \times 10^{-9} \text{ m}}{4(1.42)} = 1.14 \times 10^{-7} \text{ m} = 114 \text{ nm}$$

EVALUATE: We compared the path difference to the wavelength in the film, since that is where the path difference occurs.

- 35.28. IDENTIFY:** Require destructive interference for light reflected at the front and rear surfaces of the film.
SET UP: At the front surface of the film, light in air ($n = 1.00$) reflects from the film ($n = 2.62$) and there is a 180° phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to reflection. Therefore, there is a net 180° phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \frac{505 \text{ nm}}{2.62}$.

EXECUTE: (a) Since the reflection produces a net 180° phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\frac{505 \text{ nm}}{2[2.62]}\right) = (96.4 \text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2, 3$ and 4 : 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 35.27, where the minimum thickness for destructive interference is $t = \lambda/4n$.

- 35.29. IDENTIFY:** The fringes are produced by interference between light reflected from the top and bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do

have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) is therefore $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.29. At a distance x from the point of contact of the two plates, the thickness of the air wedge is t .

EXECUTE: $\tan \theta = \frac{t}{x}$ so $t = x \tan \theta$. $t_m = (m + \frac{1}{2})\frac{\lambda}{2}$. $x_m = (m + \frac{1}{2})\frac{\lambda}{2 \tan \theta}$ and $x_{m+1} = (m + \frac{3}{2})\frac{\lambda}{2 \tan \theta}$. The

distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \frac{\lambda}{2 \tan \theta}$. $15.0 \text{ fringes/cm} = \frac{1.00}{\Delta x}$ and

$$\Delta x = \frac{1.00}{15.0 \text{ fringes/cm}} = 0.0667 \text{ cm}. \quad \tan \theta = \frac{\lambda}{2 \Delta x} = \frac{546 \times 10^{-9} \text{ m}}{2(0.0667 \times 10^{-2} \text{ m})} = 4.09 \times 10^{-4}.$$

The angle of the wedge is $4.09 \times 10^{-4} \text{ rad} = 0.0234^\circ$.

EVALUATE: The fringes are equally spaced; Δx is independent of m .

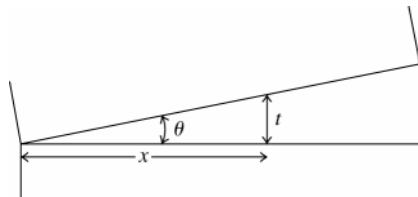


Figure 35.29

- 35.30. IDENTIFY:** The fringes are produced by interference between light reflected from the top and from the bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) therefore is $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.30.

EXECUTE: $\tan \theta = \frac{0.0800 \text{ mm}}{90.0 \text{ mm}} = 8.89 \times 10^{-4}$. $\tan \theta = \frac{t}{x}$ so $t = (8.89 \times 10^{-4})x$. $t_m = (m + \frac{1}{2})\frac{\lambda}{2}$.

$x_m = (m + \frac{1}{2})\frac{\lambda}{2(8.89 \times 10^{-4})}$ and $x_{m+1} = (m + \frac{3}{2})\frac{\lambda}{2(8.89 \times 10^{-4})}$. The distance along the plate between adjacent fringes

is $\Delta x = x_{m+1} - x_m = \frac{\lambda}{2(8.89 \times 10^{-4})} = \frac{656 \times 10^{-9} \text{ m}}{2(8.89 \times 10^{-4})} = 3.69 \times 10^{-4} \text{ m} = 0.369 \text{ mm}$. The number of fringes per cm is

$$\frac{1.00}{\Delta x} = \frac{1.00}{0.0369 \text{ cm}} = 27.1 \text{ fringes/cm}.$$

EVALUATE: As $t \rightarrow 0$ the interference is destructive and there is a dark fringe at the line of contact between the two plates.

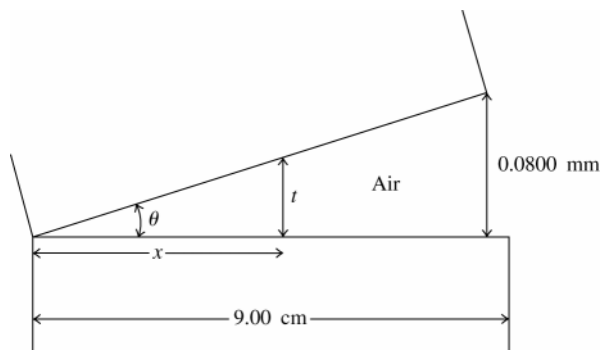


Figure 35.30

- 35.31. IDENTIFY:** The light reflected from the top of the TiO_2 film interferes with the light reflected from the top of the glass surface. These waves are out of phase due to the path difference in the film and the phase differences caused by reflection.

SET UP: There is a π phase change at the TiO_2 surface but none at the glass surface, so for destructive interference the path difference must be $m\lambda$ in the film.

EXECUTE: (a) Calling T the thickness of the film gives $2T = m\lambda_0/n$, which yields $T = m\lambda_0/(2n)$. Substituting the numbers gives

$$T = m(520.0 \text{ nm})/[2(2.62)] = 99.237m$$

T must be greater than 1036 nm, so $m = 11$, which gives $T = 1091.6$ nm, since we want to know the minimum thickness to add.

$$\Delta T = 1091.6 \text{ nm} - 1036 \text{ nm} = 55.6 \text{ nm}$$

(b) (i) Path difference $= 2T = 2(1092 \text{ nm}) = 2184 \text{ nm} = 2180 \text{ nm}$.

(ii) The wavelength in the film is $\lambda = \lambda_0/n = (520.0 \text{ nm})/2.62 = 198.5 \text{ nm}$.

$$\text{Path difference} = (2180 \text{ nm})/[(198.5 \text{ nm})/\text{wavelength}] = 11.0 \text{ wavelengths}$$

EVALUATE: Because the path difference in the film is 11.0 wavelengths, the light reflected off the top of the film will be 180° out of phase with the light that traveled through the film and was reflected off the glass due to the phase change at reflection off the top of the film.

35.32. IDENTIFY: Consider the phase difference produced by the path difference and by the reflections. For destructive interference the total phase difference is an integer number of half cycles.

SET UP: The reflection at the top surface of the film produces a half-cycle phase shift. There is no phase shift at the reflection at the bottom surface.

EXECUTE: (a) Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for

constructive interference is $t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{550 \text{ nm}}{4(1.85)} = 74.3 \text{ nm}$.

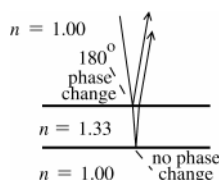
(b) The next smallest thickness for constructive interference is with another half wavelength thickness added:

$$t = \frac{3\lambda}{4} = \frac{3\lambda_0}{4n} = \frac{3(550 \text{ nm})}{4(1.85)} = 223 \text{ nm}.$$

EVALUATE: Note that we must compare the path difference to the wavelength in the film.

35.33. IDENTIFY: Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.

(a) **SET UP:** Consider Figure 35.33.



There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

Figure 35.33

EXECUTE: The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to

the reflections. Hence the condition for constructive interference is $2t = \left(m + \frac{1}{2}\right)(\lambda_0/n)$, $m = 0, 1, 2, \dots$. Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m + \frac{1}{2}} = \frac{2(290 \text{ nm})(1.33)}{m + \frac{1}{2}} = \frac{771.4 \text{ nm}}{m + \frac{1}{2}}$$

for $m = 0$, $\lambda = 1543 \text{ nm}$; for $m = 1$, $\lambda = 514 \text{ nm}$; for $m = 2$, $\lambda = 308 \text{ nm}$;... Only 514 nm is in the visible region; the color for this wavelength is green.

$$(b) \lambda_0 = \frac{2tn}{m + \frac{1}{2}} = \frac{2(340 \text{ nm})(1.33)}{m + \frac{1}{2}} = \frac{904.4 \text{ nm}}{m + \frac{1}{2}}$$

for $m = 0$, $\lambda = 1809 \text{ nm}$; for $m = 1$, $\lambda = 603 \text{ nm}$; for $m = 2$, $\lambda = 362 \text{ nm}$;... Only 603 nm is in the visible region; the color for this wavelength is orange.

EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

35.34. IDENTIFY: The number of waves along the path is the path length divided by the wavelength. The path difference and the reflections determine the phase difference.

SET UP: The path length is $2t = 17.52 \times 10^{-6} \text{ m}$. The wavelength in the film is $\lambda = \frac{\lambda_0}{n}$.

EXECUTE: (a) $\lambda = \frac{648 \text{ nm}}{1.35} = 480 \text{ nm}$. The number of waves is $\frac{2t}{\lambda} = \frac{17.52 \times 10^{-6} \text{ m}}{480 \times 10^{-9} \text{ m}} = 36.5$.

(b) The path difference introduces a $\lambda/2$, or 180° , phase difference. The ray reflected at the top surface of the film undergoes a 180° phase shift upon reflection. The reflection at the lower surface introduces no phase shift. Both rays undergo a 180° phase shift, one due to reflection and one due to reflection. The two effects cancel and the two rays are in phase as they leave the film.

EVALUATE: Note that we must use the wavelength in the film to determine the number of waves in the film.

35.35. IDENTIFY: Require destructive interference between light reflected from the two points on the disc.

SET UP: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is

$$2t = (m + \frac{1}{2})\lambda, \text{ where } t \text{ is the depth of the pit. } \lambda = \frac{\lambda_0}{n}. \text{ The minimum pit depth is for } m = 0.$$

$$\text{EXECUTE: } 2t = \frac{\lambda}{2}. \quad t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{790 \text{ nm}}{4(1.8)} = 110 \text{ nm} = 0.11 \mu\text{m}.$$

EVALUATE: The path difference occurs in the plastic substrate and we must compare the wavelength in the substrate to the path difference.

35.36. IDENTIFY: Consider light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air ($n = 1.00$) reflects from the film ($n = 2.62$) and there is a 180° phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to reflection. Therefore, there is a net 180° phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \frac{505 \text{ nm}}{2.62}$.

EXECUTE: (a) Since the reflection produces a net 180° phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m \left(\frac{505 \text{ nm}}{2[2.62]} \right) = (96.4 \text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2, 3$ and 4 : 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 34.27, where the minimum thickness for destructive interference is $t = \lambda/4n$.

35.37. IDENTIFY and SET UP: Apply Eq.(35.19) and calculate y for $m = 1800$.

$$\text{EXECUTE: Eq.(35.19): } y = m(\lambda/2) = 1800(633 \times 10^{-9} \text{ m})/2 = 5.70 \times 10^{-4} \text{ m} = 0.570 \text{ mm}$$

EVALUATE: A small displacement of the mirror corresponds to many wavelengths and a large number of fringes cross the line.

35.38. IDENTIFY: Apply Eq.(35.19).

SET UP: $m = 818$. Since the fringes move in opposite directions, the two people move the mirror in opposite directions.

$$\text{EXECUTE: (a) For Jan, the total shift was } y_1 = \frac{m\lambda_1}{2} = \frac{818(6.06 \times 10^{-7} \text{ m})}{2} = 2.48 \times 10^{-4} \text{ m. For Linda, the total shift was } y_2 = \frac{m\lambda_2}{2} = \frac{818(5.02 \times 10^{-7} \text{ m})}{2} = 2.05 \times 10^{-4} \text{ m.}$$

(b) The net displacement of the mirror is the difference of the above values:

$$\Delta y = y_1 - y_2 = 0.248 \text{ mm} - 0.205 \text{ mm} = 0.043 \text{ mm}.$$

EVALUATE: The person using the larger wavelength moves the mirror the greater distance.

35.39. IDENTIFY: Consider the interference between light reflected from the top and bottom surfaces of the air film between the lens and the glass plate.

SET UP: For maximum intensity, with a net half-cycle phase shift due to reflections, $2t = \left(m + \frac{1}{2}\right)\lambda$.

$$t = R - \sqrt{R^2 - r^2}.$$

$$\text{EXECUTE: } \frac{(2m+1)\lambda}{4} = R - \sqrt{R^2 - r^2} \Rightarrow \sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4}$$

$$\Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4} \right]^2 - \frac{(2m+1)\lambda R}{2} \Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4} \right]^2}$$

$$\Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}, \text{ for } R \gg \lambda.$$

The second bright ring is when $m = 1$:

$$r \approx \sqrt{\frac{(2(1) + 1)(5.80 \times 10^{-7} \text{ m})(0.952 \text{ m})}{2}} = 9.10 \times 10^{-4} \text{ m} = 0.910 \text{ mm}.$$

So the diameter of the second bright ring is 1.82 mm.

EVALUATE: The diameter of the m^{th} ring is proportional to $\sqrt{2m+1}$, so the rings get closer together as m increases. This agrees with Figure 35.17b in the textbook.

35.40. IDENTIFY: As found in Problem 35.39, the radius of the m^{th} bright ring is $r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}$, for $R \gg \lambda$.

SET UP: Introducing a liquid between the lens and the plate just changes the wavelength from λ to $\frac{\lambda}{n}$, where n is the refractive index of the liquid.

EXECUTE: $r(n) \approx \sqrt{\frac{(2m+1)\lambda R}{2n}} = \frac{r}{\sqrt{n}} = \frac{0.850 \text{ mm}}{\sqrt{1.33}} = 0.737 \text{ mm}.$

EVALUATE: The refractive index of the water is less than that of the glass plate, so the phase changes on reflection are the same as when air is in the space.

35.41. IDENTIFY: The liquid alters the wavelength of the light and that affects the locations of the interference minima.

SET UP: The interference minima are located by $d \sin \theta = (m + \frac{1}{2})\lambda$. For a liquid with refractive index n ,

$$\lambda_{\text{liq}} = \frac{\lambda_{\text{air}}}{n}.$$

EXECUTE: $\frac{\sin \theta}{\lambda} = \frac{(m + \frac{1}{2})}{d} = \text{constant}$, so $\frac{\sin \theta_{\text{air}}}{\lambda_{\text{air}}} = \frac{\sin \theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\frac{\sin \theta_{\text{air}}}{\lambda_{\text{air}}} = \frac{\sin \theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and $n = \frac{\sin \theta_{\text{air}}}{\sin \theta_{\text{liq}}} = \frac{\sin 35.20^\circ}{\sin 19.46^\circ} = 1.730.$

EVALUATE: In the liquid the wavelength is shorter and $\sin \theta = (m + \frac{1}{2})\frac{\lambda}{d}$ gives a smaller θ than in air, for the same m .

35.42. IDENTIFY: As the brass is heated, thermal expansion will cause the two slits to move farther apart.

SET UP: For destructive interference, $d \sin \theta = \lambda/2$. The change in separation due to thermal expansion is $dw = \alpha w_0 dT$, where w is the distance between the slits.

EXECUTE: The first dark fringe is at $d \sin \theta = \lambda/2 \Rightarrow \sin \theta = \lambda/2d$.

Call $d \equiv w$ for these calculations to avoid confusion with the differential. $\sin \theta = \lambda/2w$

Taking differentials gives $d(\sin \theta) = d(\lambda/2w)$ and $\cos \theta d\theta = -\lambda/2 dw/w^2$.

For thermal expansion, $dw = \alpha w_0 dT$, which gives $\cos \theta d\theta = -\frac{\lambda}{2} \frac{\alpha w_0 dT}{w_0^2} = -\frac{\lambda \alpha dT}{2w_0}$. Solving for $d\theta$ gives

$$d\theta = -\frac{\lambda \alpha dT}{2w_0 \cos \theta_0}. \text{ Get } \lambda: w_0 \sin \theta_0 = \lambda/2 \rightarrow \lambda = 2w_0 \sin \theta_0. \text{ Substituting this quantity into the equation for } d\theta \text{ gives}$$

$$d\theta = -\frac{2w_0 \sin \theta_0 \alpha dT}{2w_0 \cos \theta_0} = -\tan \theta_0 \alpha dT.$$

$$d\theta = -\tan(32.5^\circ)(2.0 \times 10^{-5} \text{ K}^{-1})(115 \text{ K}) = -0.001465 \text{ rad} = -0.084^\circ$$

The minus sign tells us that the dark fringes move closer together.

EVALUATE: We can also see that the dark fringes move closer together because $\sin \theta$ is proportional to $1/d$, so as d increases due to expansion, θ decreases.

35.43. IDENTIFY: Both frequencies will interfere constructively when the path difference from both of them is an integral number of wavelengths.

SET UP: Constructive interference occurs when $\sin \theta = m\lambda/d$.

EXECUTE: First find the two wavelengths.

$$\lambda_1 = v/f_1 = (344 \text{ m/s})/(900 \text{ Hz}) = 0.3822 \text{ m}$$

$$\lambda_2 = v/f_2 = (344 \text{ m/s})/(1200 \text{ Hz}) = 0.2867 \text{ m}$$

To interfere constructively at the same angle, the angles must be the same, and hence the sines of the angles must be equal. Each sine is of the form $\sin \theta = m\lambda/d$, so we can equate the sines to get

$$m_1 \lambda_1/d = m_2 \lambda_2/d$$

$$m_1(0.3822 \text{ m}) = m_2(0.2867 \text{ m})$$

$$m_2 = 4/3 m_1$$

Since both m_1 and m_2 must be integers, the allowed pairs of values of m_1 and m_2 are

$$m_1 = m_2 = 0$$

$$m_1 = 3, m_2 = 4$$

$$m_1 = 6, m_2 = 8$$

$$m_1 = 9, m_2 = 12$$

etc.

For $m_1 = m_2 = 0$, we have $\theta = 0$.

For $m_1 = 3, m_2 = 4$, we have $\sin \theta_1 = (3)(0.3822 \text{ m})/(2.50 \text{ m})$, giving $\theta_1 = 27.3^\circ$

For $m_1 = 6, m_2 = 8$, we have $\sin \theta_1 = (6)(0.3822 \text{ m})/(2.50 \text{ m})$, giving $\theta_1 = 66.5^\circ$

For $m_1 = 9, m_2 = 12$, we have $\sin \theta_1 = (9)(0.3822 \text{ m})/(2.50 \text{ m}) = 1.38 > 1$, so no angle is possible.

EVALUATE: At certain other angles, one frequency will interfere constructively, but the other will not.

- 35.44. IDENTIFY:** For destructive interference, $d = r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda$.

SET UP: $r_2 - r_1 = \sqrt{(200 \text{ m})^2 + x^2} - x$

EXECUTE: $(200 \text{ m})^2 + x^2 = x^2 + \left[\left(m + \frac{1}{2}\right)\lambda\right]^2 + 2x\left(m + \frac{1}{2}\right)\lambda$.

$$x = \frac{20,000 \text{ m}^2}{\left(m + \frac{1}{2}\right)\lambda} - \frac{1}{2}\left(m + \frac{1}{2}\right)\lambda. \text{ The wavelength is calculated by } \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.80 \times 10^6 \text{ Hz}} = 51.7 \text{ m}.$$

$$m = 0: x = 761 \text{ m}; m = 1: x = 219 \text{ m}; m = 2: x = 90.1 \text{ m}; m = 3: x = 20.0 \text{ m}.$$

EVALUATE: For $m = 3$, $d = 3.5\lambda = 181 \text{ m}$. The maximum possible path difference is the separation of 200 m between the sources.

- 35.45. IDENTIFY:** The two scratches are parallel slits, so the light that passes through them produces an interference pattern. However the light is traveling through a medium (plastic) that is different from air.

SET UP: The central bright fringe is bordered by a dark fringe on each side of it. At these dark fringes, $d \sin \theta = \frac{1}{2} \lambda/n$, where n is the refractive index of the plastic.

EXECUTE: First use geometry to find the angles at which the two dark fringes occur. At the first dark fringe $\tan \theta = [(5.82 \text{ mm})/2]/(3250 \text{ mm})$, giving $\theta = \pm 0.0513^\circ$

For destructive interference, we have $d \sin \theta = \frac{1}{2} \lambda/n$ and

$$n = \lambda/(2d \sin \theta) = (632.8 \text{ nm})/[2(0.000225 \text{ m})(\sin 0.0513^\circ)] = 1.57$$

EVALUATE: The wavelength of the light in the plastic is reduced compared to what it would be in air.

- 35.46. IDENTIFY:** Interference occurs due to the path difference of light in the thin film.

SET UP: Originally the path difference was an odd number of half-wavelengths for cancellation to occur. If the path difference decreases by $\frac{1}{2}$ wavelength, it will be a multiple of the wavelength, so constructive interference will occur.

EXECUTE: Calling ΔT the thickness that must be removed, we have

$$\text{path difference} = 2\Delta T = \frac{1}{2} \lambda/n \text{ and } \Delta T = \lambda/4n = (525 \text{ nm})/[4(1.40)] = 93.75 \text{ nm},$$

At 4.20 nm/yr, we have $(4.20 \text{ nm/yr})t = 93.75 \text{ nm}$ and $t = 22.3 \text{ yr}$.

EVALUATE: If you were giving a warranty on this film, you certainly could not give it a "lifetime guarantee"!

- 35.47. IDENTIFY and SET UP:** If the total phase difference is an integer number of cycles the interference is constructive and if it is a half-integer number of cycles it is destructive.

EXECUTE: (a) If the two sources are out of phase by one half-cycle, we must add an extra half a wavelength to the path difference equations Eq.(35.1) and Eq.(35.2). This exactly changes one for the other, for $m \rightarrow m + \frac{1}{2}$ and $m + \frac{1}{2} \rightarrow m$, since m in any integer.

(b) If one source leads the other by a phase angle ϕ , the fraction of a cycle difference is $\frac{\phi}{2\pi}$. Thus the path length difference for the two sources must be adjusted for both destructive and constructive interference, by this amount. So for constructive interference: $r_1 - r_2 = (m + \phi/2\pi)\lambda$, and for destructive interference, $r_1 - r_2 = (m + 1/2 + \phi/2\pi)\lambda$, where in each case $m = 0, \pm 1, \pm 2, \dots$

EVALUATE: If $\phi = 0$ these results reduce to Eqs.(35.1) and (35.2).

- 35.48. IDENTIFY:** Follow the steps specified in the problem.

SET UP: Use $\cos(\omega t + \phi/2) = \cos(\omega t)\cos(\phi/2) - \sin(\omega t)\sin(\phi/2)$. Then

$$2\cos(\phi/2)\cos(\omega t + \phi/2) = 2\cos(\omega t)\cos^2(\phi/2) - 2\sin(\omega t)\sin(\phi/2)\cos(\phi/2). \text{ Then use } \cos^2(\phi/2) = \frac{1 + \cos(\phi)}{2} \text{ and}$$

$2\sin(\phi/2)\cos(\phi/2) = \sin\phi$. This gives $\cos(\omega t) + (\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)) = \cos(\omega t) + \cos(\omega t + \phi)$, using again the trig identity for the cosine of the sum of two angles.

EXECUTE: (a) The electric field is the sum of the two fields and can be written as

$$E_p(t) = E_2(t) + E_1(t) = E\cos(\omega t) + E\cos(\omega t + \phi). \quad E_p(t) = 2E\cos(\phi/2)\cos(\omega t + \phi/2).$$

(b) $E_p(t) = A\cos(\omega t + \phi/2)$, so comparing with part (a), we see that the amplitude of the wave (which is always positive) must be $A = 2E|\cos(\phi/2)|$.

(c) To have an interference maximum, $\frac{\phi}{2} = 2\pi m$. So, for example, using $m = 1$, the relative phases are

$$E_2: 0; E_1: \phi = 4\pi; E_p: \frac{\phi}{2} = 2\pi, \text{ and all waves are in phase.}$$

(d) To have an interference minimum, $\frac{\phi}{2} = \pi\left(m + \frac{1}{2}\right)$. So, for example using $m = 0$, relative phases are

$$E_2: 0; E_1: \phi = \pi; E_p: \phi/2 = \pi/2, \text{ and the resulting wave is out of phase by a quarter of a cycle from both of the original waves.}$$

(e) The instantaneous magnitude of the Poynting vector is

$$|\vec{S}| = \epsilon_0 c E_p^2(t) = \epsilon_0 c (4E^2 \cos^2(\phi/2) \cos^2(\omega t + \phi/2)).$$

For a time average, $\cos^2(\omega t + \phi/2) = \frac{1}{2}$, so $|S_{av}| = 2\epsilon_0 c E^2 \cos^2(\phi/2)$.

EVALUATE: The result of part (e) shows that the intensity at a point depends on the phase difference ϕ at that point for the waves from each source.

35.49. IDENTIFY: Follow the steps specified in the problem.

SET UP: The definition of hyperbola is the locus of points such that the difference between P to S_2 and P to S_1 is a constant.

EXECUTE: (a) $\Delta r = m\lambda$. $r_1 = \sqrt{x^2 + (y-d)^2}$ and $r_2 = \sqrt{x^2 + (y+d)^2}$.

$$\Delta r = \sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = m\lambda.$$

(b) For a given m and λ , Δr is a constant and we get a hyperbola. Or, in the case of all m for a given λ , a family of hyperbolas.

$$(c) \sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = (m + \frac{1}{2})\lambda.$$

EVALUATE: The hyperbolas approach straight lines at large distances from the source.

35.50. IDENTIFY: Follow the derivation of Eq.(35.7), but with different amplitudes for the two waves.

SET UP: $\cos(\pi - \phi) = -\cos\phi$

EXECUTE: (a) $E_p^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(\pi - \phi) = E^2 + 4E^2 + 4E^2\cos\phi = 5E^2 + 4E^2\cos\phi$

$$I = \frac{1}{2}\epsilon_0 c E_p^2 = \epsilon_0 c \left[\left(\frac{5}{2} E^2 \right) + \left(\frac{4}{2} E^2 \right) \cos\phi \right]. \quad \phi = 0 \Rightarrow I_0 = \frac{9}{2}\epsilon_0 c E^2. \quad \text{Therefore, } I = I_0 \left[\frac{5}{9} + \frac{4}{9} \cos\phi \right].$$

(b) The graph is shown in Figure 35.50. $I_{\min} = \frac{1}{9}I_0$ which occurs when $\phi = n\pi$ (n odd).

EVALUATE: The maxima and minima occur at the same points on the screen as when the two sources have the same amplitude, but when the amplitudes are different the intensity is no longer zero at the minima.

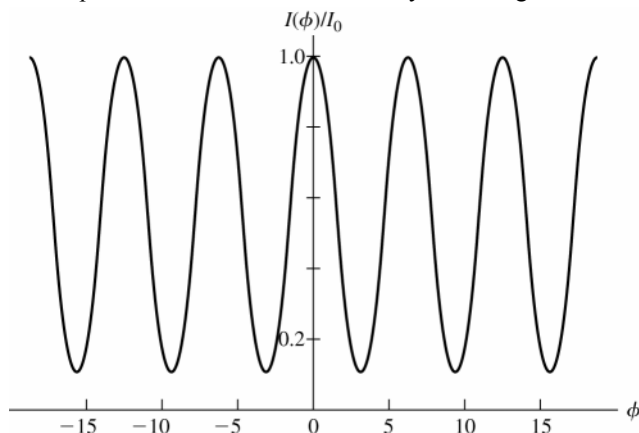


Figure 35.50

- 35.51. IDENTIFY and SET UP:** Consider interference between rays reflected from the upper and lower surfaces of the film to relate the thickness of the film to the wavelengths for which there is destructive interference. The thermal expansion of the film changes the thickness of the film when the temperature changes.

EXECUTE: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.

Smallest nonzero thickness is given by $t = \lambda/2n$.

At 20.0°C , $t_0 = (582.4 \text{ nm})/[(2)(1.750)] = 166.4 \text{ nm}$.

At 170°C , $t_0 = (588.5 \text{ nm})/[(2)(1.750)] = 168.1 \text{ nm}$.

$t = t_0(1 + \alpha\Delta T)$ so

$$\alpha = (t - t_0)/(t_0\Delta T) = (1.7 \text{ nm})/[(166.4 \text{ nm})(150^\circ\text{C})] = 6.8 \times 10^{-5} (\text{C}^\circ)^{-1}$$

EVALUATE: When the film is heated its thickness increases, and it takes a larger wavelength in the film to equal $2t$. The value we calculated for α is the same order of magnitude as those given in Table 17.1.

- 35.52. IDENTIFY and SET UP:** At the $m = 3$ bright fringe for the red light there must be destructive interference at this same θ for the other wavelength.

EXECUTE: For constructive interference: $d \sin \theta = m\lambda_1 \Rightarrow d \sin \theta = 3(700 \text{ nm}) = 2100 \text{ nm}$. For destructive

interference: $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda_2 \Rightarrow \lambda_2 = \frac{d \sin \theta}{m + \frac{1}{2}} = \frac{2100 \text{ nm}}{m + \frac{1}{2}}$. So the possible wavelengths are

$\lambda_2 = 600 \text{ nm}$, for $m = 3$, and $\lambda_2 = 467 \text{ nm}$, for $m = 4$.

EVALUATE: Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

- 35.53. IDENTIFY:** Apply $I = I_0 \cos^2\left(\frac{\pi d}{\lambda} \sin \theta\right)$.

SET UP: $I = I_0/2$ when $\frac{\pi d}{\lambda} \sin \theta$ is $\frac{\pi}{4} \text{ rad}$, $\frac{3\pi}{4} \text{ rad}$, ...

EXECUTE: First we need to find the angles at which the intensity drops by one-half from the value of the m^{th}

bright fringe. $I = I_0 \cos^2\left(\frac{\pi d}{\lambda} \sin \theta\right) = \frac{I_0}{2} \Rightarrow \frac{\pi d}{\lambda} \sin \theta \approx \frac{\pi d \theta_m}{\lambda} = (m + 1/2)\frac{\pi}{2}$.

$$m = 0: \theta = \theta_m^- = \frac{\lambda}{4d}; m = 1: \theta = \theta_m^+ = \frac{3\lambda}{4d} \Rightarrow \Delta\theta_m = \frac{\lambda}{2d}.$$

EVALUATE: There is no dependence on the m -value of the fringe, so all fringes at small angles have the same half-width.

- 34.54. IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections.

SET UP: There is just one half-cycle phase change upon reflection, so for constructive interference

$2t = (m_1 + \frac{1}{2})\lambda_1 = (m_2 + \frac{1}{2})\lambda_2$, where these wavelengths are in the glass. The two different wavelengths differ by just one m -value, $m_2 = m_1 - 1$.

EXECUTE: $\left(m_1 + \frac{1}{2}\right)\lambda_1 = \left(m_1 - \frac{1}{2}\right)\lambda_2 \Rightarrow m_1(\lambda_2 - \lambda_1) = \frac{\lambda_1 + \lambda_2}{2} \Rightarrow m_1 = \frac{\lambda_1 + \lambda_2}{2(\lambda_2 - \lambda_1)}$.

$$m_1 = \frac{477.0 \text{ nm} + 540.6 \text{ nm}}{2(540.6 \text{ nm} - 477.0 \text{ nm})} = 8. \quad 2t = \left(8 + \frac{1}{2}\right)\frac{\lambda_{01}}{n} \Rightarrow t = \frac{17(477.0 \text{ nm})}{4(1.52)} = 1334 \text{ nm}.$$

EVALUATE: Now that we have t we can calculate all the other wavelengths for which there is constructive interference.

- 35.55. IDENTIFY:** Consider the phase difference due to the path difference and due to the reflection of one ray from the glass surface.

(a) SET UP: Consider Figure 35.55

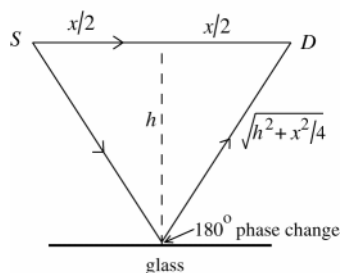


Figure 35.55

$$\begin{aligned} \text{path} \\ \text{difference} = \\ 2\sqrt{h^2 + x^2/4} - x = \\ \sqrt{4h^2 + x^2} - x \end{aligned}$$

Since there is a 180° phase change for the reflected ray, the condition for constructive interference is path

difference $= \left(m + \frac{1}{2}\right)\lambda$ and the condition for destructive interference is path difference $= m\lambda$.

(b) EXECUTE: Constructive interference: $\left(m + \frac{1}{2}\right)\lambda = \sqrt{4h^2 + x^2} - x$ and $\lambda = \frac{\sqrt{4h^2 + x^2} - x}{m + \frac{1}{2}}$. Longest λ is for

$$m = 0 \text{ and then } \lambda = 2\left(\sqrt{4h^2 + x^2} - x\right) = 2\left(\sqrt{4(0.24 \text{ m})^2 + (0.14 \text{ m})^2} - 0.14 \text{ m}\right) = 0.72 \text{ m}$$

EVALUATE: For $\lambda = 0.72 \text{ m}$ the path difference is $\lambda/2$.

35.56. IDENTIFY: Require constructive interference for the reflection from the top and bottom surfaces of each cytoplasm layer and each guanine layer.

SET UP: At the water (or cytoplasm) to guanine interface, there is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams, just due to the reflections.

EXECUTE: For the guanine layers:

$$2t_g = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_g} \Rightarrow \lambda = \frac{2t_g n_g}{\left(m + \frac{1}{2}\right)} = \frac{2(74 \text{ nm})(1.80)}{\left(m + \frac{1}{2}\right)} = \frac{266 \text{ nm}}{\left(m + \frac{1}{2}\right)} \Rightarrow \lambda = 533 \text{ nm } (m = 0).$$

For the cytoplasm layers:

$$2t_c = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_c} \Rightarrow \lambda = \frac{2t_c n_c}{\left(m + \frac{1}{2}\right)} = \frac{2(100 \text{ nm})(1.333)}{\left(m + \frac{1}{2}\right)} = \frac{267 \text{ nm}}{\left(m + \frac{1}{2}\right)} \Rightarrow \lambda = 533 \text{ nm } (m = 0).$$

(b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

(c) At different angles, the path length in the layers changes (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

EVALUATE: The thickness of the guanine and cytoplasm layers are inversely proportional to their refractive indices $\left(\frac{100}{74} = \frac{1.80}{1.333}\right)$, so both kinds of layers produce constructive interference for the same wavelength in air.

35.57. IDENTIFY: The slits will produce an interference pattern, but in the liquid, the wavelength of the light will be less than it was in air.

SET UP: The first bright fringe occurs when $d \sin \theta = \lambda/n$.

EXECUTE: In air: $d \sin 18.0^\circ = \lambda$. In the liquid: $d \sin 12.6^\circ = \lambda/n$. Dividing the equations gives

$$n = (\sin 18.0^\circ)/(\sin 12.6^\circ) = 1.42$$

EVALUATE: It was not necessary to know the spacing of the slits, since it was the same in both air and the liquid.

35.58. IDENTIFY: Consider light reflected at the top and bottom surfaces of the film. Wavelengths that are predominant in the transmitted light are those for which there is destructive interference in the reflected light.

SET UP: For the waves reflected at the top surface of the oil film there is a half-cycle reflection phase shift. For the waves reflected at the bottom surface of the oil film there is no reflection phase shift. The condition for constructive interference is $2t = \left(m + \frac{1}{2}\right)\lambda$. The condition for destructive interference is $2t = m\lambda$. The range of

visible wavelengths is approximately 400 nm to 700 nm. In the oil film, $\lambda = \frac{\lambda_0}{n}$.

EXECUTE: (a) $2t = \left(m + \frac{1}{2}\right)\lambda = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n}$. $\lambda_0 = \frac{2tn}{m + \frac{1}{2}} = \frac{2(380 \text{ nm})(1.45)}{m + \frac{1}{2}} = \frac{1102 \text{ nm}}{m + \frac{1}{2}}$.

$m = 0$: $\lambda_0 = 2200 \text{ nm}$. $m = 1$: $\lambda_0 = 735 \text{ nm}$. $m = 2$: $\lambda_0 = 441 \text{ nm}$. $m = 3$: $\lambda_0 = 315 \text{ nm}$. The visible wavelength for which there is constructive interference in the reflected light is 441 nm.

(b) $2t = m\lambda = m\frac{\lambda_0}{n}$. $\lambda_0 = \frac{2tn}{m} = \frac{1102 \text{ nm}}{m}$. $m = 1$: $\lambda_0 = 1102 \text{ nm}$. $m = 2$: $\lambda_0 = 551 \text{ nm}$. $m = 3$: $\lambda_0 = 367 \text{ nm}$.

The visible wavelength for which there is destructive interference in the reflected light is 551 nm. This is the visible wavelength predominant in the transmitted light.

EVALUATE: At a particular wavelength the sum of the intensities of the reflected and transmitted light equals the intensity of the incident light.

35.59. (a) IDENTIFY: The wavelength in the glass is decreased by a factor of $1/n$, so for light through the upper slit a shorter path is needed to produce the same phase at the screen. Therefore, the interference pattern is shifted downward on the screen.

(b) SET UP: Consider the total phase difference produced by the path length difference and also by the different wavelength in the glass.

EXECUTE: At a point on the screen located by the angle θ the difference in path length is $d \sin \theta$. This introduces a phase difference of $\phi = \left(\frac{2\pi}{\lambda_0} \right) (d \sin \theta)$, where λ_0 is the wavelength of the light in air or vacuum.

In the thickness L of glass the number of wavelengths is $\frac{L}{\lambda} = \frac{nL}{\lambda_0}$. A corresponding length L of the path of the ray through the lower slit, in air, contains L/λ_0 wavelengths. The phase difference this introduces is

$$\phi = 2\pi \left(\frac{nL}{\lambda_0} - \frac{L}{\lambda_0} \right) \text{ and } \phi = 2\pi(n-1)(L/\lambda_0). \text{ The total phase difference is the sum of these two,}$$

$$\left(\frac{2\pi}{\lambda_0} \right) (d \sin \theta) + 2\pi(n-1)(L/\lambda_0) = (2\pi/\lambda_0)(d \sin \theta + L(n-1)). \text{ Eq.(35.10) then gives}$$

$$I = I_0 \cos^2 \left[\left(\frac{\pi}{\lambda_0} \right) (d \sin \theta + L(n-1)) \right].$$

(c) Maxima means $\cos \phi/2 = \pm 1$ and $\phi/2 = m\pi$, $m = 0, \pm 1, \pm 2, \dots$ $(\pi/\lambda_0)(d \sin \theta + L(n-1)) = m\pi$

$$d \sin \theta + L(n-1) = m\lambda_0$$

$$\sin \theta = \frac{m\lambda_0 - L(n-1)}{d}$$

EVALUATE: When $L \rightarrow 0$ or $n \rightarrow 1$ the effect of the plate goes away and the maxima are located by Eq.(35.4).

35.60. IDENTIFY: Dark fringes occur because the path difference is one-half of a wavelength.

SET UP: At the first dark fringe, $d \sin \theta = \lambda/2$. The intensity at any angle θ is given by $I = I_0 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right)$.

(a) At the first dark fringe, we have

$$d \sin \theta = \lambda/2$$

$$d/\lambda = 2/(2 \sin 15.0^\circ) = 1.93$$

$$(b) I = I_0 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right) = \frac{I_0}{10} \Rightarrow \cos \left(\frac{\pi d \sin \theta}{\lambda} \right) = \frac{1}{\sqrt{10}}$$

$$\frac{\pi d \sin \theta}{\lambda} = \arccos \left(\frac{1}{\sqrt{10}} \right) = 71.57^\circ = 1.249 \text{ rad}$$

Using the result from part (a), that $d/\lambda = 1.93$, we have $\pi(1.93) \sin \theta = 1.249$. $\sin \theta = 0.2060$ and

$$\theta = \pm 11.9^\circ$$

EVALUATE: Since the first dark fringes occur at $\pm 15.0^\circ$, it is reasonable that at $\approx 12^\circ$ the intensity is reduced to only 1/10 of its maximum central value.

35.61. IDENTIFY: There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index.

SET UP: $\lambda = \frac{\lambda_0}{n}$ and $\Delta n = n_0 (2.50 \times 10^{-5} \text{ (C}^\circ)^{-1}) \Delta T$. $\Delta L = L_0 (5.00 \times 10^{-6} \text{ (C}^\circ)^{-1}) \Delta T$.

EXECUTE: The extra length of rod replaces a little of the air so that the change in the number of wavelengths due

to this is given by: $\Delta N_1 = \frac{2n_{\text{glass}} \Delta L}{\lambda_0} - \frac{2n_{\text{air}} \Delta L}{\lambda_0} = \frac{2(n_{\text{glass}} - 1)L_0 \alpha \Delta T}{\lambda_0}$ and

$$\Delta N_1 = \frac{2(1.48 - 1)(0.030 \text{ m})(5.00 \times 10^{-6} / \text{C}^\circ)(5.00 \text{ C}^\circ)}{5.89 \times 10^{-7} \text{ m}} = 1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2 = \frac{2\Delta n_{\text{glass}} L_0}{\lambda_0} = \frac{2(2.50 \times 10^{-5} / \text{C}^\circ)(5.00 \text{ C}^\circ / \text{min})(1.00 \text{ min})(0.0300 \text{ m})}{5.89 \times 10^{-7} \text{ m}} = 12.73.$$

So, the total change in the number of wavelengths as the rod expands is $\Delta N = 12.73 + 1.22 = 14.0$ fringes/minute.

EVALUATE: Both effects increase the number of wavelengths along the length of the rod. Both ΔL and Δn_{glass} are very small and the two effects can be considered separately.

35.62. IDENTIFY: Apply Snell's law to the refraction at the two surfaces of the prism. S_1 and S_2 serve as coherent

sources so the fringe spacing is $\Delta y = \frac{R\lambda}{d}$, where d is the distance between S_1 and S_2 .

SET UP: For small angles, $\sin \theta \approx \theta$, with θ expressed in radians.

EXECUTE: (a) Since we can approximate the angles of incidence on the prism as being small, Snell's Law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n , where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\theta/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n - 1)A$. At the plane of the source S_0 , we can calculate the

height of one image above the source: $\frac{d}{2} = \tan(\theta - \theta')a \approx (\theta - \theta')a = (n - 1)Aa \Rightarrow d = 2aA(n - 1)$.

(b) To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n - 1)} = \frac{(2.00 \text{ m} + 0.200 \text{ m})(5.00 \times 10^{-7} \text{ m})}{2(0.200 \text{ m})(3.50 \times 10^{-3} \text{ rad})(1.50 - 1.00)} = 1.57 \times 10^{-3} \text{ m}.$$

EVALUATE: The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.

DIFFRACTION

- 36.1. IDENTIFY:** Use $y = x \tan \theta$ to calculate the angular position θ of the first minimum. The minima are located by Eq.(36.2): $\sin \theta = \frac{m\lambda}{a}$, $m = \pm 1, \pm 2, \dots$. First minimum means $m = 1$ and $\sin \theta_1 = \lambda/a$ and $\lambda = a \sin \theta_1$. Use this equation to calculate λ .

SET UP: The central maximum is sketched in Figure 36.1.

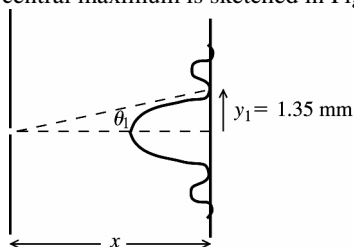


Figure 36.1

EXECUTE: $y_1 = x \tan \theta_1$

$$\begin{aligned}\tan \theta_1 &= \frac{y_1}{x} = \frac{1.35 \times 10^{-3} \text{ m}}{2.00 \text{ m}} = 0.675 \times 10^{-3} \\ \theta_1 &= 0.675 \times 10^{-3} \text{ rad}\end{aligned}$$

$$\lambda = a \sin \theta_1 = (0.750 \times 10^{-3} \text{ m}) \sin(0.675 \times 10^{-3} \text{ rad}) = 506 \text{ nm}$$

EVALUATE: θ_1 is small so the approximation used to obtain Eq.(36.3) is valid and this equation could have been used.

- 36.2. IDENTIFY:** The angle is small, so $y_m = x \frac{m\lambda}{a}$.

SET UP: $y_1 = 10.2 \text{ mm}$

$$\text{EXECUTE: } y_1 = \frac{x\lambda}{a} \Rightarrow a \frac{x\lambda}{y_1} = \frac{(0.600 \text{ m})(5.46 \times 10^{-7} \text{ m})}{10.2 \times 10^{-3} \text{ m}} = 3.21 \times 10^{-5} \text{ m}.$$

EVALUATE: The diffraction pattern is observed at a distance of 60.0 cm from the slit.

- 36.3. IDENTIFY:** The dark fringes are located at angles θ that satisfy $\sin \theta = \frac{m\lambda}{a}$, $m = \pm 1, \pm 2, \dots$

SET UP: The largest value of $|\sin \theta|$ is 1.00.

EXECUTE: (a) Solve for m that corresponds to $\sin \theta = 1$: $m = \frac{a}{\lambda} = \frac{0.0666 \times 10^{-3} \text{ m}}{585 \times 10^{-9} \text{ m}} = 113.8$. The largest value m can have is 113. $m = \pm 1, \pm 2, \dots, \pm 113$ gives 226 dark fringes.

(b) For $m = \pm 113$, $\sin \theta = \pm 113 \left(\frac{585 \times 10^{-9} \text{ m}}{0.0666 \times 10^{-3} \text{ m}} \right) = \pm 0.9926$ and $\theta = \pm 83.0^\circ$.

EVALUATE: When the slit width a is decreased, there are fewer dark fringes. When $a < \lambda$ there are no dark fringes and the central maximum completely fills the screen.

- 36.4. IDENTIFY and SET UP:** λ/a is very small, so the approximate expression $y_m = R \frac{m\lambda}{a}$ is accurate. The distance between the two dark fringes on either side of the central maximum is $2y_1$.

$$\text{EXECUTE: } y_1 = \frac{\lambda R}{a} = \frac{(633 \times 10^{-9} \text{ m})(3.50 \text{ m})}{0.750 \times 10^{-3} \text{ m}} = 2.95 \times 10^{-3} \text{ m} = 2.95 \text{ mm}. \quad 2y_1 = 5.90 \text{ mm}.$$

EVALUATE: When a is decreased, the width $2y_1$ of the central maximum increases.

- 36.5. IDENTIFY:** The minima are located by $\sin \theta = \frac{m\lambda}{a}$

SET UP: $a = 12.0 \text{ cm}$. $x = 40.0 \text{ cm}$.

EXECUTE: The angle to the first minimum is $\theta = \arcsin\left(\frac{\lambda}{a}\right) = \arcsin\left(\frac{9.00 \text{ cm}}{12.00 \text{ cm}}\right) = 48.6^\circ$.

So the distance from the central maximum to the first minimum is just $y_1 = x \tan \theta = (40.0 \text{ cm}) \tan(48.6^\circ) = \pm 45.4 \text{ cm}$.

EVALUATE: $2\lambda/a$ is greater than 1, so only the $m=1$ minimum is seen.

- 36.6. IDENTIFY:** The angle that locates the first diffraction minimum on one side of the central maximum is given by $\sin \theta = \frac{\lambda}{a}$. The time between crests is the period T . $f = \frac{1}{T}$ and $\lambda = \frac{v}{f}$.

SET UP: The time between crests is the period, so $T = 1.0 \text{ h}$.

EXECUTE: (a) $f = \frac{1}{T} = \frac{1}{1.0 \text{ h}} = 1.0 \text{ h}^{-1}$. $\lambda = \frac{v}{f} = \frac{800 \text{ km/h}}{1.0 \text{ h}^{-1}} = 800 \text{ km}$.

(b) Africa-Antarctica: $\sin \theta = \frac{800 \text{ km}}{4500 \text{ km}}$ and $\theta = 10.2^\circ$.

Australia-Antarctica: $\sin \theta = \frac{800 \text{ km}}{3700 \text{ km}}$ and $\theta = 12.5^\circ$.

EVALUATE: Diffraction effects are observed when the wavelength is about the same order of magnitude as the dimensions of the opening through which the wave passes.

- 36.7. IDENTIFY:** We can model the hole in the concrete barrier as a single slit that will produce a single-slit diffraction pattern of the water waves on the shore.

SET UP: For single-slit diffraction, the angles at which destructive interference occurs are given by $\sin \theta_m = m\lambda/a$, where $m = 1, 2, 3, \dots$

EXECUTE: (a) The frequency of the water waves is $f = 75.0 \text{ min}^{-1} = 1.25 \text{ s}^{-1} = 1.25 \text{ Hz}$, so their wavelength is $\lambda = v/f = (15.0 \text{ cm/s})/(1.25 \text{ Hz}) = 12.0 \text{ cm}$.

At the first point for which destructive interference occurs, we have

$\tan \theta = (0.613 \text{ m})/(3.20 \text{ m}) \Rightarrow \theta = 10.84^\circ$. $a \sin \theta = \lambda$ and

$$a = \lambda/\sin \theta = (12.0 \text{ cm})/(\sin 10.84^\circ) = 63.8 \text{ cm}.$$

(b) First find the angles at which destructive interference occurs.

$$\sin \theta_2 = 2\lambda/a = 2(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_2 = \pm 22.1^\circ$$

$$\sin \theta_3 = 3\lambda/a = 3(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_3 = \pm 34.3^\circ$$

$$\sin \theta_4 = 4\lambda/a = 4(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_4 = \pm 48.8^\circ$$

$$\sin \theta_5 = 5\lambda/a = 5(12.0 \text{ cm})/(63.8 \text{ cm}) \rightarrow \theta_5 = \pm 70.1^\circ$$

EVALUATE: These are large angles, so we cannot use the approximation that $\theta_m \approx m\lambda/a$.

- 36.8. IDENTIFY:** The minima are located by $\sin \theta = \frac{m\lambda}{a}$. For part (b) apply Eq.(36.7).

SET UP: For the first minimum, $m=1$. The intensity at $\theta=0$ is I_0 .

EXECUTE: (a) $\sin \theta = \frac{m\lambda}{a} = \sin 90.0^\circ = 1 = \frac{m\lambda}{a} = \frac{\lambda}{a}$. Thus $a = \lambda = 580 \text{ nm} = 5.80 \times 10^{-4} \text{ mm}$.

(b) According to Eq.(36.7),

$$\frac{I}{I_0} = \left\{ \frac{\sin[\pi a(\sin \theta)/\lambda]}{\pi a(\sin \theta)/\lambda} \right\}^2 = \left\{ \frac{\sin[\pi(\sin \pi/4)]}{\pi(\sin \pi/4)} \right\}^2 = 0.128.$$

EVALUATE: If $a = \lambda/2$, for example, then at $\theta = 45^\circ$, $\frac{I}{I_0} = \left\{ \frac{\sin[(\pi/2)(\sin \pi/4)]}{(\pi/2)(\sin \pi/4)} \right\}^2 = 0.81$. As a/λ decreases,

the screen becomes more uniformly illuminated.

- 36.9. IDENTIFY and SET UP:** $v = f\lambda$ gives λ . The person hears no sound at angles corresponding to diffraction minima. The diffraction minima are located by $\sin \theta = m\lambda/a$, $m = \pm 1, \pm 2, \dots$. Solve for θ .

EXECUTE: $\lambda = v/f = (344 \text{ m/s})/(1250 \text{ Hz}) = 0.2752 \text{ m}$; $a = 1.00 \text{ m}$. $m = \pm 1$, $\theta = \pm 16.0^\circ$; $m = \pm 2$, $\theta = \pm 33.4^\circ$; $m = \pm 3$, $\theta = \pm 55.6^\circ$; no solution for larger m

EVALUATE: $\lambda/a = 0.28$ so for the large wavelength sound waves diffraction by the doorway is a large effect. Diffraction would not be observable for visible light because its wavelength is much smaller and $\lambda/a \ll 1$.

36.10. IDENTIFY: Compare E_y to the expression $E_y = E_{\max} \sin(kx - \omega t)$ and determine k , and from that calculate λ .

$$f = c/\lambda. \text{ The dark bands are located by } \sin \theta = \frac{m\lambda}{a}.$$

SET UP: $c = 3.00 \times 10^8$ m/s. The first dark band corresponds to $m = 1$.

EXECUTE: (a) $E = E_{\max} \sin(kx - \omega t)$. $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.20 \times 10^7 \text{ m}^{-1}} = 5.24 \times 10^{-7} \text{ m}$.

$$f\lambda = c \Rightarrow f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.24 \times 10^{-7} \text{ m}} = 5.73 \times 10^{14} \text{ Hz}.$$

(b) $a \sin \theta = \lambda$. $a = \frac{\lambda}{\sin \theta} = \frac{5.24 \times 10^{-7} \text{ m}}{\sin 28.6^\circ} = 1.09 \times 10^{-6} \text{ m}$.

(c) $a \sin \theta = m\lambda$ ($m = 1, 2, 3, \dots$). $\sin \theta_2 = \pm 2 \frac{\lambda}{a} = \pm 2 \frac{5.24 \times 10^{-7} \text{ m}}{1.09 \times 10^{-6} \text{ m}}$ and $\theta_2 = \pm 74^\circ$.

EVALUATE: For $m = 3$, $\frac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

36.11. IDENTIFY and SET UP: $\sin \theta = \lambda/a$ locates the first minimum. $y = x \tan \theta$.

EXECUTE: $\tan \theta = y/x = (36.5 \text{ cm})/(40.0 \text{ cm})$ and $\theta = 42.38^\circ$.

$$a = \lambda/\sin \theta = (620 \times 10^{-9} \text{ m})/(\sin 42.38^\circ) = 0.920 \mu\text{m}$$

EVALUATE: $\theta = 0.74$ rad and $\sin \theta = 0.67$, so the approximation $\sin \theta \approx \theta$ would not be accurate.

36.12. IDENTIFY: The angle is small, so $y_m = x \frac{m\lambda}{a}$ applies.

SET UP: The width of the central maximum is $2y_1$, so $y_1 = 3.00$ mm.

EXECUTE: (a) $y_1 = \frac{x\lambda}{a} \Rightarrow a = \frac{x\lambda}{y_1} = \frac{(2.50 \text{ m})(5.00 \times 10^{-7} \text{ m})}{3.00 \times 10^{-3} \text{ m}} = 4.17 \times 10^{-4} \text{ m}$.

(b) $a = \frac{x\lambda}{y_1} = \frac{(2.50 \text{ m})(5.00 \times 10^{-5} \text{ m})}{3.00 \times 10^{-3} \text{ m}} = 4.17 \times 10^{-2} \text{ m} = 4.2 \text{ cm}$.

(c) $a = \frac{x\lambda}{y_1} = \frac{(2.50 \text{ m})(5.00 \times 10^{-10} \text{ m})}{3.00 \times 10^{-3} \text{ m}} = 4.17 \times 10^{-7} \text{ m}$.

EVALUATE: The ratio a/λ stays constant, so a is smaller when λ is smaller.

36.13. IDENTIFY: Calculate the angular positions of the minima and use $y = x \tan \theta$ to calculate the distance on the screen between them.

(a) **SET UP:** The central bright fringe is shown in Figure 36.13a.

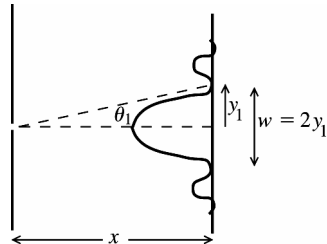


Figure 36.13a

$$y_1 = x \tan \theta_1 = (3.00 \text{ m}) \tan(1.809 \times 10^{-3} \text{ rad}) = 5.427 \times 10^{-3} \text{ m}$$

$$w = 2y_1 = 2(5.427 \times 10^{-3} \text{ m}) = 1.09 \times 10^{-2} \text{ m} = 10.9 \text{ mm}$$

(b) **SET UP:** The first bright fringe on one side of the central maximum is shown in Figure 36.13b.

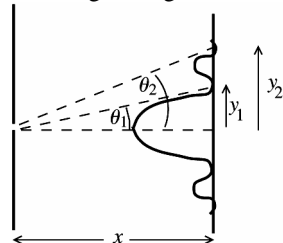


Figure 36.13b

$$w = y_2 - y_1 = 1.085 \times 10^{-2} \text{ m} - 5.427 \times 10^{-3} \text{ m} = 5.4 \text{ mm}$$

EVALUATE: The central bright fringe is twice as wide as the other bright fringes.

EXECUTE: The first minimum is located by

$$\sin \theta_1 = \frac{\lambda}{a} = \frac{633 \times 10^{-9} \text{ m}}{0.350 \times 10^{-3} \text{ m}} = 1.809 \times 10^{-3}$$

$$\theta_1 = 1.809 \times 10^{-3} \text{ rad}$$

EXECUTE: $w = y_2 - y_1$

$$y_1 = 5.427 \times 10^{-3} \text{ m (part (a))}$$

$$\sin \theta_2 = \frac{2\lambda}{a} = 3.618 \times 10^{-3}$$

$$\theta_2 = 3.618 \times 10^{-3} \text{ rad}$$

$$y_2 = x \tan \theta_2 = 1.085 \times 10^{-2} \text{ m}$$

36.14. IDENTIFY: $I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2$. $\beta = \frac{2\pi}{\lambda} a \sin \theta$.

SET UP: The angle θ is small, so $\sin \theta \approx \tan \theta \approx y/x$.

EXECUTE: $\beta = \frac{2\pi a}{\lambda} \sin \theta \approx \frac{2\pi a}{\lambda} \frac{y}{x} = \frac{2\pi(4.50 \times 10^{-4} \text{ m})}{(6.20 \times 10^{-7} \text{ m})(3.00 \text{ m})} y = (1520 \text{ m}^{-1})y$.

(a) $y = 1.00 \times 10^{-3} \text{ m}$: $\frac{\beta}{2} = \frac{(1520 \text{ m}^{-1})(1.00 \times 10^{-3} \text{ m})}{2} = 0.760$.

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(0.760)}{0.760} \right)^2 = 0.822 I_0$$

(b) $y = 3.00 \times 10^{-3} \text{ m}$: $\frac{\beta}{2} = \frac{(1520 \text{ m}^{-1})(3.00 \times 10^{-3} \text{ m})}{2} = 2.28$.

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(2.28)}{2.28} \right)^2 = 0.111 I_0$$

(c) $y = 5.00 \times 10^{-3} \text{ m}$: $\frac{\beta}{2} = \frac{(1520 \text{ m}^{-1})(5.00 \times 10^{-3} \text{ m})}{2} = 3.80$.

$$\Rightarrow I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin(3.80)}{3.80} \right)^2 = 0.0259 I_0$$

EVALUATE: The first minimum occurs at $y_1 = \frac{\lambda x}{a} = \frac{(6.20 \times 10^{-7} \text{ m})(3.00 \text{ m})}{4.50 \times 10^{-4} \text{ m}} = 4.1 \text{ mm}$. The distances in parts (a) and (b) are within the central maximum. $y = 5.00 \text{ mm}$ is within the first secondary maximum.

36.15. (a) IDENTIFY: Use Eq.(36.2) with $m=1$ to locate the angular position of the first minimum and then use $y = x \tan \theta$ to find its distance from the center of the screen.

SET UP: The diffraction pattern is sketched in Figure 36.15.

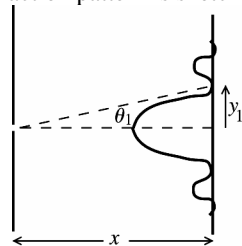


Figure 36.15

$$\begin{aligned} \sin \theta_1 &= \frac{\lambda}{a} = \\ &= \frac{540 \times 10^{-9} \text{ m}}{0.240 \times 10^{-3} \text{ m}} = 2.25 \times 10^{-3} \\ \theta_1 &= 2.25 \times 10^{-3} \text{ rad} \end{aligned}$$

$$y_1 = x \tan \theta_1 = (3.00 \text{ m}) \tan(2.25 \times 10^{-3} \text{ rad}) = 6.75 \times 10^{-3} \text{ m} = 6.75 \text{ mm}$$

(b) IDENTIFY and SET UP: Use Eqs.(36.5) and (36.6) to calculate the intensity at this point.

EXECUTE: Midway between the center of the central maximum and the first minimum implies

$$y = \frac{1}{2}(6.75 \text{ mm}) = 3.375 \times 10^{-3} \text{ m}.$$

$$\tan \theta = \frac{y}{x} = \frac{3.375 \times 10^{-3} \text{ m}}{3.00 \text{ m}} = 1.125 \times 10^{-3}; \theta = 1.125 \times 10^{-3} \text{ rad}$$

The phase angle β at this point on the screen is

$$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta = \frac{2\pi}{540 \times 10^{-9} \text{ m}} (0.240 \times 10^{-3} \text{ m}) \sin(1.125 \times 10^{-3} \text{ rad}) = \pi.$$

$$\text{Then } I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2 = (6.00 \times 10^{-6} \text{ W/m}^2) \left(\frac{\sin \pi/2}{\pi/2} \right)^2$$

$$I = \left(\frac{4}{\pi^2} \right) (6.00 \times 10^{-6} \text{ W/m}^2) = 2.43 \times 10^{-6} \text{ W/m}^2.$$

EVALUATE: The intensity at this point midway between the center of the central maximum and the first minimum is less than half the maximum intensity. Compare this result to the corresponding one for the two-slit pattern, Exercise 35.23.

36.16. IDENTIFY: In the single-slit diffraction pattern, the intensity is a maximum at the center and zero at the dark spots. At other points, it depends on the angle at which one is observing the light.

SET UP: Dark fringes occur when $\sin \theta_m = m\lambda/a$, where $m = 1, 2, 3, \dots$, and the intensity is given by

$$I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ where } \beta/2 = \frac{\pi a \sin \theta}{\lambda}.$$

EXECUTE: (a) At the maximum possible angle, $\theta = 90^\circ$, so

$$m_{\max} = (a \sin 90^\circ)/\lambda = (0.0250 \text{ mm})/(632.8 \text{ nm}) = 39.5$$

Since m must be an integer and $\sin \theta$ must be ≤ 1 , $m_{\max} = 39$. The total number of dark fringes is 39 on each side of the central maximum for a total of 78.

(b) The farthest dark fringe is for $m = 39$, giving

$$\sin \theta_{39} = (39)(632.8 \text{ nm})/(0.0250 \text{ mm}) \Rightarrow \theta_{39} = \pm 80.8^\circ$$

(c) The next closer dark fringe occurs at $\sin \theta_{38} = (38)(632.8 \text{ nm})/(0.0250 \text{ mm}) \Rightarrow \theta_{38} = 74.1^\circ$.

The angle midway these two extreme fringes is $(80.8^\circ + 74.1^\circ)/2 = 77.45^\circ$, and the intensity at this angle is $I =$

$$I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ where } \beta/2 = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(0.0250 \text{ mm}) \sin(77.45^\circ)}{632.8 \text{ nm}} = 121.15 \text{ rad, which}$$

$$\text{gives } I = (8.50 \text{ W/m}^2) \left[\frac{\sin(121.15 \text{ rad})}{121.15 \text{ rad}} \right]^2 = 5.55 \times 10^{-4} \text{ W/m}^2$$

EVALUATE: At the angle in part (c), the intensity is so low that the light would be barely perceptible.

36.17. IDENTIFY and SET UP: Use Eq.(36.6) to calculate λ and use Eq.(36.5) to calculate I . $\theta = 3.25^\circ$,

$$\beta = 56.0 \text{ rad}, a = 0.105 \times 10^{-3} \text{ m}.$$

(a) **EXECUTE:** $\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta$ so

$$\lambda = \frac{2\pi a \sin \theta}{\beta} = \frac{2\pi(0.105 \times 10^{-3} \text{ m}) \sin 3.25^\circ}{56.0 \text{ rad}} = 668 \text{ nm}$$

$$(b) I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2 = I_0 \left(\frac{4}{\beta^2} \right) (\sin(\beta/2))^2 = I_0 \frac{4}{(56.0 \text{ rad})^2} [\sin(28.0 \text{ rad})]^2 = 9.36 \times 10^{-5} I_0$$

EVALUATE: At the first minimum $\beta = 2\pi$ rad and at the point considered in the problem $\beta = 17.8\pi$ rad, so the point is well outside the central maximum. Since β is close to $m\pi$ with $m = 18$, this point is near one of the minima. The intensity here is much less than I_0 .

36.18. IDENTIFY: Use $\beta = \frac{2\pi a}{\lambda} \sin \theta$ to calculate β .

SET UP: The total intensity is given by drawing an arc of a circle that has length E_0 and finding the length of the chord which connects the starting and ending points of the curve.

EXECUTE: (a) $\beta = \frac{2\pi a}{\lambda} \sin \theta = \frac{2\pi a}{\lambda} \frac{\lambda}{2a} = \pi$. From Figure 36.18a, $\pi \frac{E_p}{2} = E_0 \Rightarrow E_p = \frac{2}{\pi} E_0$.

The intensity is $I = \left(\frac{2}{\pi} \right)^2 I_0 = \frac{4I_0}{\pi^2} = 0.405 I_0$. This agrees with Eq.(36.5).

(b) $\beta = \frac{2\pi a}{\lambda} \sin \theta = \frac{2\pi a}{\lambda} \frac{\lambda}{a} = 2\pi$. From Figure 36.18b, it is clear that the total amplitude is zero, as is the intensity.

This also agrees with Eq.(36.5).

(c) $\beta = \frac{2\pi a}{\lambda} \sin \theta = \frac{2\pi a}{\lambda} \frac{3\lambda}{2a} = 3\pi$. From Figure 36.18c, $3\pi \frac{E_p}{2} = E_0 \Rightarrow E_p = \frac{2}{3\pi} E_0$. The intensity is

$$I = \left(\frac{2}{3\pi} \right)^2 I_0 = \frac{4}{9\pi^2} I_0. \text{ This agrees with Eq.(36.5).}$$

EVALUATE: In part (a) the point is midway between the center of the central maximum and the first minimum. In part (b) the point is at the first maximum and in (c) the point is approximately at the location of the first secondary maximum. The phasor diagrams help illustrate the rapid decrease in intensity at successive maxima.

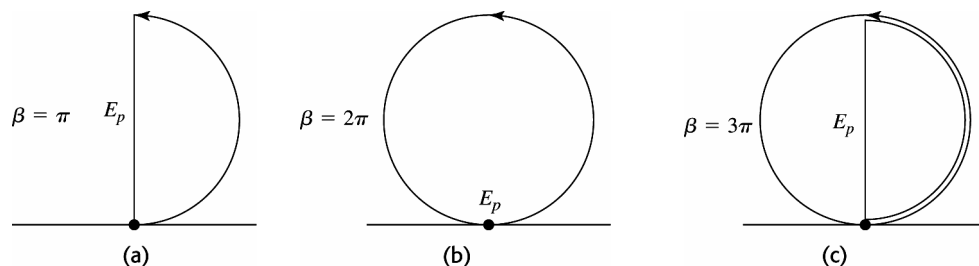


Figure 36.18

36.19. IDENTIFY: The space between the skyscrapers behaves like a single slit and diffracts the radio waves.

SET UP: Cancellation of the waves occurs when $a \sin \theta = m\lambda$, $m = 1, 2, 3, \dots$, and the intensity of the waves is

given by $I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$, where $\beta/2 = \frac{\pi a \sin \theta}{\lambda}$.

EXECUTE: (a) First find the wavelength of the waves:

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(88.9 \text{ MHz}) = 3.375 \text{ m}$$

For no signal, $a \sin \theta = m\lambda$.

$$m = 1: \sin \theta_1 = (1)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_1 = \pm 13.0^\circ$$

$$m = 2: \sin \theta_2 = (2)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_2 = \pm 26.7^\circ$$

$$m = 3: \sin \theta_3 = (3)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_3 = \pm 42.4^\circ$$

$$m = 4: \sin \theta_4 = (4)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_4 = \pm 64.1^\circ$$

$$(b) I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ where } \beta/2 = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(15.0 \text{ m}) \sin(5.00^\circ)}{3.375 \text{ m}} = 1.217 \text{ rad}$$

$$I = (3.50 \text{ W/m}^2) \left[\frac{\sin(1.217 \text{ rad})}{1.217 \text{ rad}} \right]^2 = 2.08 \text{ W/m}^2$$

EVALUATE: The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

36.20. IDENTIFY: The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

SET UP: The double-slit factor is $I_{\text{DS}} = I_0 \left(\cos^2 \frac{\phi}{2} \right)$ and the single-slit factor is $I_{\text{ss}} = \left(\frac{\sin \beta/2}{\beta/2} \right)^2$.

EXECUTE: (a) $d \sin \theta = m\lambda \Rightarrow \sin \theta = m\lambda/d$.

$$\sin \theta_1 = \lambda/d, \sin \theta_2 = 2\lambda/d, \sin \theta_3 = 3\lambda/d, \sin \theta_4 = 4\lambda/d$$

$$(b) \text{ At the interference bright fringes, } \cos^2 \phi/2 = 1 \text{ and } \beta/2 = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi(d/3) \sin \theta}{\lambda}.$$

$$\text{At } \theta_1, \sin \theta_1 = \lambda/d, \text{ so } \beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3. \text{ The intensity is therefore}$$

$$I_1 = I_0 \left(\cos^2 \frac{\phi}{2} \right) \left(\frac{\sin \beta/2}{\beta/2} \right)^2 = I_0 (1) \left(\frac{\sin \pi/3}{\pi/3} \right)^2 = 0.684 I_0$$

$$\text{At } \theta_2, \sin \theta_2 = 2\lambda/d, \text{ so } \beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3. \text{ Using the same procedure as for } \theta_1, \text{ we have } I_2 =$$

$$I_0 (1) \left(\frac{\sin 2\pi/3}{2\pi/3} \right)^2 = 0.171 I_0$$

At θ_3 , we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin \pi = 0$.

$$\text{At } \theta_4, \sin \theta_4 = 4\lambda/d, \text{ so } \beta/2 = 4\pi/3, \text{ which gives } I_4 = I_0 \left(\frac{\sin 4\pi/3}{4\pi/3} \right)^2 = 0.0427 I_0$$

(c) Since $d = 3a$, every third interference maximum is missing.

(d) In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a$.

EVALUATE: The result in this problem is different from that in Figure 36.12c because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the “envelope” of the intensity function decreases more rapidly here than in Figure 36.12c because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4$.

- 36.21. (a) IDENTIFY and SET UP:** The interference fringes (maxima) are located by $d \sin \theta = m\lambda$, with

$$m = 0, \pm 1, \pm 2, \dots \text{ The intensity } I \text{ in the diffraction pattern is given by } I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2, \text{ with } \beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta.$$

We want $m = \pm 3$ in the first equation to give θ that makes $I = 0$ in the second equation.

EXECUTE: $d \sin \theta = m\lambda$ gives $\beta = \left(\frac{2\pi}{\lambda} \right) a \left(\frac{3\lambda}{d} \right) = 2\pi(3a/d)$.

$$I = 0 \text{ says } \frac{\sin \beta/2}{\beta/2} = 0 \text{ so } \beta = 2\pi \text{ and then } 2\pi = 2\pi(3a/d) \text{ and } (d/a) = 3.$$

(b) IDENTIFY and SET UP: Fringes $m = 0, \pm 1, \pm 2$ are within the central diffraction maximum and the $m = \pm 3$ fringes coincide with the first diffraction minimum. Find the value of m for the fringes that coincide with the second diffraction minimum.

EXECUTE: Second minimum implies $\beta = 4\pi$.

$$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta = \left(\frac{2\pi}{\lambda} \right) a \left(\frac{m\lambda}{d} \right) = 2\pi m(a/d) = 2\pi(m/3)$$

Then $\beta = 4\pi$ says $4\pi = 2\pi(m/3)$ and $m = 6$. Therefore the $m = \pm 4$ and $m = \pm 5$ fringes are contained within the first diffraction maximum on one side of the central maximum; two fringes.

EVALUATE: The central maximum is twice as wide as the other maxima so it contains more fringes.

- 36.22. IDENTIFY and SET UP:** Use Figure 36.14b in the textbook. There is totally destructive interference between slits whose phasors are in opposite directions.

EXECUTE: By examining the diagram, we see that every fourth slit cancels each other.

EVALUATE: The total electric field is zero so the phasor diagram corresponds to a point of zero intensity. The first two maxima are at $\phi = 0$ and $\phi = \pi$, so this point is not midway between two maxima.

- 36.23. (a) IDENTIFY and SET UP:** If the slits are very narrow then the central maximum of the diffraction pattern for each slit completely fills the screen and the intensity distribution is given solely by the two-slit interference. The maxima are given by

$$d \sin \theta = m\lambda \text{ so } \sin \theta = m\lambda/d. \text{ Solve for } \theta.$$

EXECUTE: 1st order maximum: $m = 1$, so $\sin \theta = \frac{\lambda}{d} = \frac{580 \times 10^{-9} \text{ m}}{0.530 \times 10^{-3} \text{ m}} = 1.094 \times 10^{-3}$; $\theta = 0.0627^\circ$

2nd order maximum: $m = 2$, so $\sin \theta = \frac{2\lambda}{d} = 2.188 \times 10^{-3}$; $\theta = 0.125^\circ$

(b) IDENTIFY and SET UP: The intensity is given by Eq.(36.12): $I = I_0 \cos^2(\phi/2) \left(\frac{\sin \beta/2}{\beta/2} \right)^2$. Calculate ϕ and β at each θ from part (a).

EXECUTE: $\phi = \left(\frac{2\pi d}{\lambda} \right) \sin \theta = \left(\frac{2\pi d}{\lambda} \right) \left(\frac{m\lambda}{d} \right) = 2\pi m$, so $\cos^2(\phi/2) = \cos^2(m\pi) = 1$

(Since the angular positions in part (a) correspond to interference maxima.)

$$\beta = \left(\frac{2\pi a}{\lambda} \right) \sin \theta = \left(\frac{2\pi a}{\lambda} \right) \left(\frac{m\lambda}{d} \right) = 2\pi m(a/d) = m2\pi \left(\frac{0.320 \text{ mm}}{0.530 \text{ mm}} \right) = m(3.794 \text{ rad})$$

1st order maximum: $m = 1$, so $I = I_0(1) \left(\frac{\sin(3.794/2) \text{ rad}}{(3.794/2) \text{ rad}} \right)^2 = 0.249 I_0$

2nd order maximum: $m = 2$, so $I = I_0(1) \left(\frac{\sin 3.794 \text{ rad}}{3.794 \text{ rad}} \right)^2 = 0.0256 I_0$

EVALUATE: The first diffraction minimum is at an angle θ given by $\sin \theta = \lambda/a$ so $\theta = 0.104^\circ$. The first order fringe is within the central maximum and the second order fringe is inside the first diffraction maximum on one side of the central maximum. The intensity here at this second fringe is much less than I_0 .

- 36.24. IDENTIFY:** A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.
SET UP: Single-slit diffraction dark fringes occur when $a \sin \theta = m\lambda$, and double-slit interference bright fringes occur when $d \sin \theta = m'\lambda$.

EXECUTE: (a) The angles are the same for cancellation, so dividing the equations gives

$$d/a = m'/m \Rightarrow m'/m = 7 \Rightarrow m' = 7m$$

When $m = 1$, $m' = 7$; when $m = 2$, $m' = 14$, and so forth, so every 7th bright fringe is missing from the double-slit interference pattern.

EVALUATE: (b) The result is independent of the wavelength, so every 7th fringe will be cancelled for all wavelengths. But the bright interference fringes occur when $d \sin \theta = m\lambda$, so the *location* of the cancelled fringes *does* depend on the wavelength.

36.25. IDENTIFY and SET UP: The phasor diagrams are similar to those in Fig. 36.14. An interference minimum occurs when the phasors add to zero.

EXECUTE: (a) The phasor diagram is given in Figure 36.25a

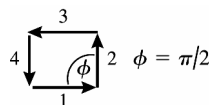


Figure 36.25a

There is destructive interference between the light through slits 1 and 3 and between 2 and 4.

(b) The phasor diagram is given in Figure 36.25b.

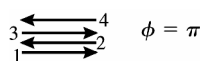


Figure 36.25b

There is destructive interference between the light through slits 1 and 2 and between 3 and 4.

(c) The phasor diagram is given in Figure 36.25c.

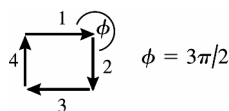


Figure 36.25c

There is destructive interference between light through slits 1 and 3 and between 2 and 4.

EVALUATE: Maxima occur when $\phi = 0, 2\pi, 4\pi$, etc. Our diagrams show that there are three minima between the maxima at $\phi = 0$ and $\phi = 2\pi$. This agrees with the general result that for N slits there are $N - 1$ minima between each pair of principal maxima.

36.26. IDENTIFY: A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.

SET UP: Single-slit diffraction dark fringes occur when $a \sin \theta = m\lambda$, and double-slit interference bright fringes occur when $d \sin \theta = m'\lambda$.

EXECUTE: (a) The angle at which the first bright fringe occurs is given by

$$\tan \theta_1 = (1.53 \text{ mm})/(2500 \text{ mm}) \Rightarrow \theta_1 = 0.03507^\circ. \quad d \sin \theta_1 = \lambda \text{ and}$$

$$d = \lambda/(\sin \theta_1) = (632.8 \text{ nm})/\sin(0.03507^\circ) = 0.00103 \text{ m} = 1.03 \text{ mm}$$

(b) The 7th double-slit interference bright fringe is just cancelled by the 1st diffraction dark fringe, so $\sin \theta_{\text{diff}} = \lambda/a$ and $\sin \theta_{\text{interf}} = 7\lambda/d$

The angles are equal, so $\lambda/a = 7\lambda/d \rightarrow a = d/7 = (1.03 \text{ mm})/7 = 0.148 \text{ mm}$.

EVALUATE: We can generalize that if $d = na$, where n is a positive integer, then every n^{th} double-slit bright fringe will be missing in the pattern.

36.27. IDENTIFY: The diffraction minima are located by $\sin \theta = \frac{m_d \lambda}{a}$ and the two-slit interference maxima are located

by $\sin \theta = \frac{m_i \lambda}{d}$. The third bright band is missing because the first order single slit minimum occurs at the same angle as the third order double slit maximum.

SET UP: The pattern is sketched in Figure 36.27. $\tan \theta = \frac{3 \text{ cm}}{90 \text{ cm}}$, so $\theta = 1.91^\circ$.

EXECUTE: Single-slit dark spot: $a \sin \theta = \lambda$ and $a = \frac{\lambda}{\sin \theta} = \frac{500 \text{ nm}}{\sin 1.91^\circ} = 1.50 \times 10^4 \text{ nm} = 15.0 \mu\text{m}$ (width)

Double-slit bright fringe: $d \sin \theta = 3\lambda$ and $d = \frac{3\lambda}{\sin \theta} = \frac{3(500 \text{ nm})}{\sin 1.91^\circ} = 4.50 \times 10^4 \text{ nm} = 45.0 \mu\text{m}$ (separation).

EVALUATE: Note that $d/a = 3.0$.

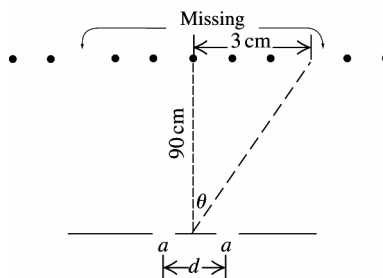


Figure 36.27

36.28. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: The order corresponds to the values of m .

EXECUTE: First-order: $d \sin \theta_1 = \lambda$. Fourth-order: $d \sin \theta_4 = 4\lambda$.

$$\frac{d \sin \theta_4}{d \sin \theta_1} = \frac{4\lambda}{\lambda}, \sin \theta_4 = 4 \sin \theta_1 = 4 \sin 8.94^\circ \text{ and } \theta_4 = 38.4^\circ.$$

EVALUATE: We did not have to solve for d .

36.29. IDENTIFY and SET UP: The bright bands are at angles θ given by $d \sin \theta = m\lambda$. Solve for d and then solve for θ for the specified order.

EXECUTE: (a) $\theta = 78.4^\circ$ for $m = 3$ and $\lambda = 681 \text{ nm}$, so $d = m\lambda / \sin \theta = 2.086 \times 10^{-4} \text{ cm}$

The number of slits per cm is $1/d = 4790 \text{ slits/cm}$

(b) 1st order: $m = 1$, so $\sin \theta = \lambda/d = (681 \times 10^{-9} \text{ m}) / (2.086 \times 10^{-6} \text{ m})$ and $\theta = 19.1^\circ$

2nd order: $m = 2$, so $\sin \theta = 2\lambda/d$ and $\theta = 40.8^\circ$

(c) For $m = 4$, $\sin \theta = 4\lambda/d$ is greater than 1.00, so there is no 4th-order bright band.

EVALUATE: The angular position of the bright bands for a particular wavelength increases as the order increases.

36.30. IDENTIFY: The bright spots are located by $d \sin \theta = m\lambda$.

SET UP: Third-order means $m = 3$ and second-order means $m = 2$.

EXECUTE: $\frac{m\lambda}{\sin \theta} = d = \text{constant}$, so $\frac{m_r \lambda_r}{\sin \theta_r} = \frac{m_v \lambda_v}{\sin \theta_v}$.

$$\sin \theta_v = \sin \theta_r \left(\frac{m_v}{m_r} \right) \left(\frac{\lambda_v}{\lambda_r} \right) = (\sin 65.0^\circ) \left(\frac{2}{3} \right) \left(\frac{400 \text{ nm}}{700 \text{ nm}} \right) = 0.345 \text{ and } \theta_v = 20.2^\circ.$$

EVALUATE: The third-order line for a particular λ occurs at a larger angle than the second-order line. In a given order, the line for violet light (400 nm) occurs at a smaller angle than the line for red light (700 nm).

36.31. IDENTIFY and SET UP: Calculate d for the grating. Use Eq.(36.13) to calculate θ for the longest wavelength in the visible spectrum and verify that θ is small. Then use Eq.(36.3) to relate the linear separation of lines on the screen to the difference in wavelength.

EXECUTE: (a) $d = \left(\frac{1}{900} \right) \text{ cm} = 1.111 \times 10^{-5} \text{ m}$

For $\lambda = 700 \text{ nm}$, $\lambda/d = 6.3 \times 10^{-2}$. The first-order lines are located at $\sin \theta = \lambda/d$; $\sin \theta$ is small enough for $\sin \theta \approx \theta$ to be an excellent approximation.

(b) $y = x\lambda/d$, where $x = 2.50 \text{ m}$.

The distance on the screen between 1st order bright bands for two different wavelengths is $\Delta y = x(\Delta\lambda)/d$, so

$$\Delta\lambda = d(\Delta y)/x = (1.111 \times 10^{-5} \text{ m})(3.00 \times 10^{-3} \text{ m})/(2.50 \text{ m}) = 13.3 \text{ nm}$$

EVALUATE: The smaller d is (greater number of lines per cm) the smaller the $\Delta\lambda$ that can be measured.

36.32. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: $350 \text{ slits/mm} \Rightarrow d = \frac{1}{3.50 \times 10^5 \text{ m}^{-1}} = 2.86 \times 10^{-6} \text{ m}$

EXECUTE: $m = 1$: $\theta_{400} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{4.00 \times 10^{-7} \text{ m}}{2.86 \times 10^{-6} \text{ m}}\right) = 8.05^\circ$.

$\theta_{700} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{7.00 \times 10^{-7} \text{ m}}{2.86 \times 10^{-6} \text{ m}}\right) = 14.18^\circ$. $\Delta\theta_1 = 14.18^\circ - 8.05^\circ = 6.13^\circ$.

$m = 3$: $\theta_{400} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(4.00 \times 10^{-7} \text{ m})}{2.86 \times 10^{-6} \text{ m}}\right) = 24.8^\circ$.

$\theta_{700} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(7.00 \times 10^{-7} \text{ m})}{2.86 \times 10^{-6} \text{ m}}\right) = 47.3^\circ$. $\Delta\theta_1 = 47.3^\circ - 24.8^\circ = 22.5^\circ$.

EVALUATE: $\Delta\theta$ is larger in third order.

36.33. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: $d = 1.60 \times 10^{-6} \text{ m}$

EXECUTE: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m[6.328 \times 10^{-7} \text{ m}]}{1.60 \times 10^{-6} \text{ m}}\right) = \arcsin([0.396]m)$. For $m = 1$, $\theta_1 = 23.3^\circ$. For

$m = 2$, $\theta_2 = 52.3^\circ$. There are no other maxima.

EVALUATE: The reflective surface produces the same interference pattern as a grating with slit separation d .

36.34. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: $5000 \text{ slits/cm} \Rightarrow d = \frac{1}{5.00 \times 10^5 \text{ m}^{-1}} = 2.00 \times 10^{-6} \text{ m}$.

EXECUTE: (a) $\lambda = \frac{d \sin \theta}{m} = \frac{(2.00 \times 10^{-6} \text{ m}) \sin 13.5^\circ}{1} = 4.67 \times 10^{-7} \text{ m}$.

(b) $m = 2$: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{2(4.67 \times 10^{-7} \text{ m})}{2.00 \times 10^{-6} \text{ m}}\right) = 27.8^\circ$.

EVALUATE: Since the angles are fairly small, the second-order deviation is approximately twice the first-order deviation.

36.35. IDENTIFY: The maxima are located by $d \sin \theta = m\lambda$.

SET UP: $350 \text{ slits/mm} \Rightarrow d = \frac{1}{3.50 \times 10^5 \text{ m}^{-1}} = 2.86 \times 10^{-6} \text{ m}$

EXECUTE: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(5.20 \times 10^{-7} \text{ m})}{2.86 \times 10^{-6} \text{ m}}\right) = \arcsin((0.182)m)$.

$m = 1$: $\theta = 10.5^\circ$; $m = 2$: $\theta = 21.3^\circ$; $m = 3$: $\theta = 33.1^\circ$.

EVALUATE: The angles are not precisely proportional to m , and deviate more from being proportional as the angles increase.

36.36. IDENTIFY: The resolution is described by $R = \frac{\lambda}{\Delta\lambda} = Nm$. Maxima are located by $d \sin \theta = m\lambda$.

SET UP: For 500 slits/mm, $d = (500 \text{ slits/mm})^{-1} = (500,000 \text{ slits/m})^{-1}$.

EXECUTE: (a) $N = \frac{\lambda}{m\Delta\lambda} = \frac{6.5645 \times 10^{-7} \text{ m}}{2(6.5645 \times 10^{-7} \text{ m} - 6.5627 \times 10^{-7} \text{ m})} = 1820 \text{ slits}$.

(b) $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \Rightarrow \theta_1 = \sin^{-1}((2)(6.5645 \times 10^{-7} \text{ m})(500,000 \text{ m}^{-1})) = 41.0297^\circ$ and

$\theta_2 = \sin^{-1}((2)(6.5627 \times 10^{-7} \text{ m})(500,000 \text{ m}^{-1})) = 41.0160^\circ$. $\Delta\theta = 0.0137^\circ$

EVALUATE: $d \cos \theta d\theta = \lambda/N$, so for 1820 slits the angular interval $\Delta\theta$ between each of these maxima and the

first adjacent minimum is $\Delta\theta = \frac{\lambda}{Nd \cos \theta} = \frac{6.56 \times 10^{-7} \text{ m}}{(1820)(2.0 \times 10^{-6} \text{ m}) \cos 41^\circ} = 0.0137^\circ$. This is the same as the angular

separation of the maxima for the two wavelengths and 1820 slits is just sufficient to resolve these two wavelengths in second order.

36.37. IDENTIFY: The resolving power depends on the line density and the width of the grating.

SET UP: The resolving power is given by $R = Nm = \lambda/\Delta\lambda$.

EXECUTE: (a) $R = Nm = (5000 \text{ lines/cm})(3.50 \text{ cm})(1) = 17,500$

(b) The resolving power needed to resolve the sodium doublet is

$$R = \lambda/\Delta\lambda = (589 \text{ nm})/(589.59 \text{ nm} - 589.00 \text{ nm}) = 998$$

so this grating can easily resolve the doublet.

(c) (i) $R = \lambda/\Delta\lambda$. Since $R = 17,500$ when $m = 1$, $R = 2 \times 17,500 = 35,000$ for $m = 2$. Therefore

$$\Delta\lambda = \lambda/R = (587.8 \text{ nm})/35,000 = 0.0168 \text{ nm}$$

$$\lambda_{\min} = \lambda + \Delta\lambda = 587.8002 \text{ nm} + 0.0168 \text{ nm} = 587.8170 \text{ nm}$$

(ii) $\lambda_{\max} = \lambda - \Delta\lambda = 587.8002 \text{ nm} - 0.0168 \text{ nm} = 587.7834 \text{ nm}$

EVALUATE: (iii) Therefore the range of resolvable wavelengths is $587.7834 \text{ nm} < \lambda < 587.8170 \text{ nm}$.

36.38. IDENTIFY and SET UP: $\frac{\lambda}{\Delta\lambda} = Nm$

$$\text{EXECUTE: } N = \frac{\lambda}{m\Delta\lambda} = \frac{587.8002 \text{ nm}}{(587.9782 \text{ nm} - 587.8002 \text{ nm})} = \frac{587.8002}{0.178} = 3302 \text{ slits. } \frac{N}{1.20 \text{ cm}} = \frac{3302}{1.20 \text{ cm}} = 2752 \frac{\text{slits}}{\text{cm}}.$$

EVALUATE: A smaller number of slits would be needed to resolve these two lines in higher order.

36.39. IDENTIFY and SET UP: The maxima occur at angles θ given by Eq.(36.16), $2d \sin \theta = m\lambda$, where d is the spacing between adjacent atomic planes. Solve for d .

EXECUTE: second order says $m = 2$.

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{2(0.0850 \times 10^{-9} \text{ m})}{2 \sin 21.5^\circ} = 2.32 \times 10^{-10} \text{ m} = 0.232 \text{ nm}$$

EVALUATE: Our result is similar to d calculated in Example 36.5.

36.40. IDENTIFY: The maxima are given by $2d \sin \theta = m\lambda$, $m = 1, 2, \dots$

SET UP: $d = 3.50 \times 10^{-10} \text{ m}$.

EXECUTE: (a) $m = 1$ and $\lambda = \frac{2d \sin \theta}{m} = 2(3.50 \times 10^{-10} \text{ m}) \sin 15.0^\circ = 1.81 \times 10^{-10} \text{ m} = 0.181 \text{ nm}$. This is an x ray.

(b) $\sin \theta = m \left(\frac{\lambda}{2d} \right) = m \left(\frac{1.81 \times 10^{-10} \text{ m}}{2[3.50 \times 10^{-10} \text{ m}]} \right) = m(0.2586)$. $m = 2$: $\theta = 31.1^\circ$. $m = 3$: $\theta = 50.9^\circ$. The equation

doesn't have any solutions for $m > 3$.

EVALUATE: In this problem $\lambda/d = 0.52$.

36.41. IDENTIFY: Rayleigh's criterion says $\sin \theta = 1.22 \frac{\lambda}{D}$

SET UP: The best resolution is 0.3 arcseconds, which is about $(8.33 \times 10^{-5})^\circ$.

$$\text{EXECUTE: (a) } D = \frac{1.22\lambda}{\sin \theta} = \frac{1.22(5.5 \times 10^{-7} \text{ m})}{\sin(8.33 \times 10^{-5}^\circ)} = 0.46 \text{ m}$$

EVALUATE: (b) The Keck telescopes are able to gather more light than the Hale telescope, and hence they can detect fainter objects. However, their larger size does not allow them to have greater resolution—atmospheric conditions limit the resolution.

36.42. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: $\theta = (1/60)^\circ$

$$\text{EXECUTE: } D = \frac{1.22\lambda}{\sin \theta} = \frac{1.22(5.5 \times 10^{-7} \text{ m})}{\sin(1/60)^\circ} = 2.31 \times 10^{-3} \text{ m} = 2.3 \text{ mm}$$

EVALUATE: The larger the diameter the smaller the angle that can be resolved.

36.43. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: $\theta = \frac{W}{h}$, where $W = 28 \text{ km}$ and $h = 1200 \text{ km}$. θ is small, so $\sin \theta \approx \theta$.

$$\text{EXECUTE: } D = \frac{1.22\lambda}{\sin \theta} = 1.22\lambda \frac{h}{W} = 1.22(0.036 \text{ m}) \frac{1.2 \times 10^6 \text{ m}}{2.8 \times 10^4 \text{ m}} = 1.88 \text{ m}$$

EVALUATE: D must be significantly larger than the wavelength, so a much larger diameter is needed for microwaves than for visible wavelengths.

36.44. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: θ is small, so $\sin \theta \approx \theta = 1.00 \times 10^{-8} \text{ rad}$.

$$\text{EXECUTE: } \lambda = \frac{D \sin \theta}{1.22} \approx \frac{D\theta}{1.22} = \frac{(8.00 \times 10^6 \text{ m})(1.00 \times 10^{-8})}{1.22} = 0.0656 \text{ m} = 6.56 \text{ cm}$$

EVALUATE: λ corresponds to microwaves.

- 36.45. IDENTIFY and SET UP:** The angular size of the first dark ring is given by $\sin \theta_1 = 1.22\lambda/D$ (Eq. 36.17). Calculate θ_1 , and then the diameter of the ring on the screen is $2(4.5 \text{ m})\tan \theta_1$.

EXECUTE: $\sin \theta_1 = 1.22 \left(\frac{620 \times 10^{-9} \text{ m}}{7.4 \times 10^{-6} \text{ m}} \right) = 0.1022$; $\theta_1 = 0.1024 \text{ rad}$

The radius of the Airy disk (central bright spot) is $r = (4.5 \text{ m})\tan \theta_1 = 0.462 \text{ m}$. The diameter is $2r = 0.92 \text{ m} = 92 \text{ cm}$.

EVALUATE: $\lambda/D = 0.084$. For this small D the central diffraction maximum is broad.

- 36.46. IDENTIFY:** Rayleigh's criterion limits the angular resolution.

SET UP: Rayleigh's criterion is $\sin \theta \approx \theta = 1.22 \lambda/D$.

EXECUTE: (a) Using Rayleigh's criterion

$$\sin \theta \approx \theta = 1.22 \lambda/D = (1.22)(550 \text{ nm})/(135/4 \text{ mm}) = 1.99 \times 10^{-5} \text{ rad}$$

On the bear this angle subtends a distance x . $\theta = x/R$ and

$$x = R\theta = (11.5 \text{ m})(1.99 \times 10^{-5} \text{ rad}) = 2.29 \times 10^{-4} \text{ m} = 0.23 \text{ mm}$$

(b) At $f/22$, D is $4/22$ times as large as at $f/4$. Since θ is proportional to $1/D$, and x is proportional to θ , x is $1/(4/22) = 22/4$ times as large as it was at $f/4$. $x = (0.229 \text{ mm})(22/4) = 1.3 \text{ mm}$

EVALUATE: A wide-angle lens, such as one having a focal length of 28 mm, would have a much smaller opening at $f/22$ and hence would have an even less resolving ability.

- 36.47. IDENTIFY and SET UP:** Resolved by Rayleigh's criterion means angular separation θ of the objects equals $1.22\lambda/D$. The angular separation θ of the objects is their linear separation divided by their distance from the telescope.

EXECUTE: $\theta = \frac{250 \times 10^3 \text{ m}}{5.93 \times 10^{11} \text{ m}}$, where $5.93 \times 10^{11} \text{ m}$ is the distance from earth to Jupiter. Thus $\theta = 4.216 \times 10^{-7}$.

Then $\theta = 1.22 \frac{\lambda}{D}$ and $D = \frac{1.22\lambda}{\theta} = \frac{1.22(500 \times 10^{-9} \text{ m})}{4.216 \times 10^{-7}} = 1.45 \text{ m}$

EVALUATE: This is a very large telescope mirror. The greater the angular resolution the greater the diameter the lens or mirror must be.

- 36.48. IDENTIFY:** Rayleigh's criterion says $\theta_{\text{res}} = 1.22 \frac{\lambda}{D}$.

SET UP: $D = 7.20 \text{ cm}$. $\theta_{\text{res}} = \frac{y}{s}$, where s is the distance of the object from the lens and $y = 4.00 \text{ mm}$.

EXECUTE: $\frac{y}{s} = 1.22 \frac{\lambda}{D}$. $s = \frac{yD}{1.22\lambda} = \frac{(4.00 \times 10^{-3} \text{ m})(7.20 \times 10^{-2} \text{ m})}{1.22(550 \times 10^{-9} \text{ m})} = 429 \text{ m}$.

EVALUATE: The focal length of the lens doesn't enter into the calculation. In practice, it is difficult to achieve resolution that is at the diffraction limit.

- 36.49. IDENTIFY and SET UP:** Let y be the separation between the two points being resolved and let s be their distance from the telescope. Then the limit of resolution corresponds to $1.22 \frac{\lambda}{D} = \frac{y}{s}$.

EXECUTE: (a) Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8 \times 10^8 \text{ m}$. $y = 1.22\lambda s/D$.

Hubble: $D = 2.4 \text{ m}$ and $\lambda = 400 \text{ nm}$ gives the maximum resolution, so $y = 77 \text{ m}$

Arecibo: $D = 305 \text{ m}$ and $\lambda = 0.75 \text{ m}$; $y = 1.1 \times 10^6 \text{ m}$

(b) $s = \frac{yD}{1.22\lambda}$. Let $y \approx 0.30$ (the size of a license plate). $s = (0.30 \text{ m})(2.4 \text{ m})/[(1.22)(400 \times 10^{-9} \text{ m})] = 1500 \text{ km}$.

EVALUATE: D/λ is much larger for the optical telescope and it has a much larger resolution even though the diameter of the radio telescope is much larger.

- 36.50. IDENTIFY:** Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: θ is small, so $\sin \theta \approx \theta$. Smallest resolving angle is for short-wavelength light (400 nm).

EXECUTE: $\theta \approx 1.22 \frac{\lambda}{D} = (1.22) \frac{400 \times 10^{-9} \text{ m}}{5.08 \text{ m}} = 9.61 \times 10^{-8} \text{ rad}$. $\theta = \frac{10,000 \text{ mi}}{R}$, where R is the distance to the star.

$$R = \frac{10,000 \text{ mi}}{\theta} = \frac{16,000 \text{ km}}{9.6 \times 10^{-8} \text{ rad}} = 1.7 \times 10^{11} \text{ km}.$$

EVALUATE: This is less than a light year, so there are no stars this close.

36.51. IDENTIFY: Let y be the separation between the two points being resolved and let s be their distance from the telescope. The limit of resolution corresponds to $1.22 \lambda/D = y/s$.

SET UP: $s = 4.28 \text{ ly} = 4.05 \times 10^{16} \text{ m}$. Assume visible light, with $\lambda = 400 \text{ nm}$.

EXECUTE: $y = 1.22 \lambda s/D = 1.22(400 \times 10^{-9} \text{ m})(4.05 \times 10^{16} \text{ m})/(10.0 \text{ m}) = 2.0 \times 10^9 \text{ m}$

EVALUATE: The diameter of Jupiter is $1.38 \times 10^8 \text{ m}$, so the resolution is insufficient, by about one order of magnitude.

36.52. IDENTIFY: If the apparatus of Exercise 36.4 is placed in water, then all that changes is the wavelength

$$\lambda \rightarrow \lambda' = \frac{\lambda}{n}$$

SET UP: For $y \ll x$, the distance between the two dark fringes on either side of the central maximum is

$D' = 2y'$. Let $D = 2y$ be the separation of $5.91 \times 10^{-3} \text{ m}$ found in Exercise 36.4.

EXECUTE: $2y'_1 = \frac{2x\lambda'}{a} = \frac{2x\lambda}{an} = \frac{D}{n} = \frac{5.91 \times 10^{-3} \text{ m}}{1.33} = 4.44 \times 10^{-3} \text{ m} = 4.44 \text{ mm}$.

EVALUATE: The water shortens the wavelength and this decreases the width of the central maximum.

36.53. (a) IDENTIFY and SET UP: The intensity in the diffraction pattern is given by Eq.(36.5): $I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$, where

$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta$. Solve for θ that gives $I = \frac{1}{2} I_0$. The angles θ_+ and θ_- are shown in Figure 36.53.

EXECUTE: $I = \frac{1}{2} I_0$ so $\frac{\sin \beta/2}{\beta/2} = \frac{1}{\sqrt{2}}$

Let $x = \beta/2$; the equation for x is $\frac{\sin x}{x} = \frac{1}{\sqrt{2}} = 0.7071$.

Use trial and error to find the value of x that is a solution to this equation.

x	$(\sin x)/x$
1.0 rad	0.841
1.5 rad	0.665
1.2 rad	0.777
1.4 rad	0.7039
1.39 rad	0.7077; thus $x = 1.39 \text{ rad}$ and $\beta = 2x = 2.78 \text{ rad}$

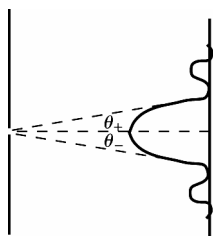


Figure 36.53

$$\Delta \theta = |\theta_+ - \theta_-| = 2\theta_+$$

$$\sin \theta_+ = \frac{\lambda \beta}{2\pi a} =$$

$$\frac{\lambda}{a} \left(\frac{2.78 \text{ rad}}{2\pi \text{ rad}} \right) = 0.4425 \left(\frac{\lambda}{a} \right)$$

(i) For $\frac{a}{\lambda} = 2$, $\sin \theta_+ = 0.4425 \left(\frac{1}{2} \right) = 0.2212$; $\theta_+ = 12.78^\circ$; $\Delta \theta = 2\theta_+ = 25.6^\circ$

(ii) For $\frac{a}{\lambda} = 5$, $\sin \theta_+ = 0.4425 \left(\frac{1}{5} \right) = 0.0885$; $\theta_+ = 5.077^\circ$; $\Delta \theta = 2\theta_+ = 10.2^\circ$

(iii) For $\frac{a}{\lambda} = 10$, $\sin \theta_+ = 0.4425 \left(\frac{1}{10} \right) = 0.04425$; $\theta_+ = 2.536^\circ$; $\Delta \theta = 2\theta_+ = 5.1^\circ$

(b) IDENTIFY and SET UP: $\sin \theta_0 = \frac{\lambda}{a}$ locates the first minimum. Solve for θ_0 .

EXECUTE: (i) For $\frac{a}{\lambda} = 2$, $\sin \theta_0 = \frac{1}{2}$; $\theta_0 = 30.0^\circ$; $2\theta_0 = 60.0^\circ$

(ii) For $\frac{a}{\lambda} = 5$, $\sin \theta_0 = \frac{1}{5}$; $\theta_0 = 11.54^\circ$; $2\theta_0 = 23.1^\circ$

(iii) For $\frac{a}{\lambda} = 10$, $\sin \theta_0 = \left(\frac{1}{10}\right)$; $\theta_0 = 5.74^\circ$; $2\theta_0 = 11.5^\circ$

EVALUATE: Either definition of the width shows that the central maximum gets narrower as the slit gets wider.

36.54. IDENTIFY: The two holes behave like double slits and cause the sound waves to interfere after they pass through the holes. The motion of the speakers causes a Doppler shift in the wavelength of the sound.

SET UP: The wavelength of the sound that strikes the wall is $\lambda = \lambda_0 - v_s T_s$, and destructive interference first occurs where $\sin \theta = \lambda/2$.

EXECUTE: (a) First find the wavelength of the sound that strikes the openings in the wall.

$$\lambda = \lambda_0 - v_s T_s = v/f_s - v_s/f_s = (v - v_s)/f_s = (344 \text{ m/s} - 80.0 \text{ m/s})/(1250 \text{ Hz}) = 0.211 \text{ m}$$

Destructive interference first occurs where $d \sin \theta = \lambda/2$, which gives

$$d = \lambda/(2 \sin \theta) = (0.211 \text{ m})/(2 \sin 12.7^\circ) = 0.480 \text{ m}$$

(b) $\lambda = v/f = (344 \text{ m/s})/(1250 \text{ Hz}) = 0.275 \text{ m}$

$$\sin \theta = \lambda/2d = (0.275 \text{ m})/[2(0.480 \text{ m})] \rightarrow \theta = \pm 16.7^\circ$$

EVALUATE: The moving source produces sound of shorter wavelength than the stationary source, so the angles at which destructive interference occurs are smaller for the moving source than for the stationary source.

36.55. IDENTIFY and SET UP: $\sin \theta = \lambda/a$ locates the first dark band. In the liquid the wavelength changes and this changes the angular position of the first diffraction minimum.

EXECUTE: $\sin \theta_{\text{air}} = \frac{\lambda_{\text{air}}}{a}$; $\sin \theta_{\text{liquid}} = \frac{\lambda_{\text{liquid}}}{a}$

$$\lambda_{\text{liquid}} = \lambda_{\text{air}} \left(\frac{\sin \theta_{\text{liquid}}}{\sin \theta_{\text{air}}} \right) = 0.4836$$

$$\lambda = \lambda_{\text{air}}/n \text{ (Eq.33.5), so } n = \lambda_{\text{air}}/\lambda_{\text{liquid}} = 1/0.4836 = 2.07$$

EVALUATE: Light travels faster in air and n must be > 1.00 . The smaller λ in the liquid reduces θ that locates the first dark band.

36.56. IDENTIFY: $d = \frac{1}{N}$, so the bright fringes are located by $\frac{1}{N} \sin \theta = \lambda$

SET UP: Red: $\frac{1}{N} \sin \lambda_R = 700 \text{ nm}$. Violet: $\frac{1}{N} \sin \lambda_V = 400 \text{ nm}$.

EXECUTE: $\frac{\sin \theta_R}{\sin \theta_V} = \frac{7}{4}$. $\theta_R - \theta_V = 15^\circ \rightarrow \theta_R = \theta_V + 15^\circ$. $\frac{\sin(\theta_V + 15^\circ)}{\sin \theta_V} = \frac{7}{4}$. Using a trig identity from Appendix B,

$$\frac{\sin \theta_V \cos 15^\circ + \cos \theta_V \sin 15^\circ}{\sin \theta_V} = 7/4. \quad \cos 15^\circ + \cot \theta_V \sin 15^\circ = 7/4.$$

$$\tan \theta_V = 0.330 \Rightarrow \theta_V = 18.3^\circ \text{ and } \theta_R = \theta_V + 15^\circ = 18.3^\circ + 15^\circ = 33.3^\circ. \text{ Then } \frac{1}{N} \sin \theta_R = 700 \text{ nm gives}$$

$$N = \frac{\sin \theta_R}{700 \text{ nm}} = \frac{\sin 33.3^\circ}{700 \times 10^{-9} \text{ m}} = 7.84 \times 10^5 \text{ lines/m} = 7840 \text{ lines/cm. The spectrum begins at } 18.3^\circ \text{ and ends at } 33.3^\circ.$$

EVALUATE: As N is increased, the angular range of the visible spectrum increases.

36.57. (a) IDENTIFY and SET UP: The angular position of the first minimum is given by $a \sin \theta = m\lambda$ (Eq.36.2), with $m = 1$. The distance of the minimum from the center of the pattern is given by $y = x \tan \theta$.

$$\sin \theta = \frac{\lambda}{a} = \frac{540 \times 10^{-9} \text{ m}}{0.360 \times 10^{-3} \text{ m}} = 1.50 \times 10^{-3}; \quad \theta = 1.50 \times 10^{-3} \text{ rad}$$

$$y_1 = x \tan \theta = (1.20 \text{ m}) \tan(1.50 \times 10^{-3} \text{ rad}) = 1.80 \times 10^{-3} \text{ m} = 1.80 \text{ mm.}$$

(Note that θ is small enough for $\theta \approx \sin \theta \approx \tan \theta$, and Eq.(36.3) applies.)

(b) **IDENTIFY and SET UP:** Find the phase angle β where $I = I_0/2$. Then use Eq.(36.6) to solve for θ and $y = x \tan \theta$ to find the distance.

EXECUTE: From part (a) of Problem 36.53, $I = \frac{1}{2} I_0$ when $\beta = 2.78 \text{ rad}$.

$$\beta = \left(\frac{2\pi}{\lambda} \right) a \sin \theta \text{ (Eq.(36.6)), so } \sin \theta = \frac{\beta \lambda}{2\pi a}.$$

$$y = x \tan \theta \approx x \sin \theta \approx \frac{\beta \lambda x}{2\pi a} = \frac{(2.78 \text{ rad})(540 \times 10^{-9} \text{ m})(1.20 \text{ m})}{2\pi(0.360 \times 10^{-3} \text{ m})} = 7.96 \times 10^{-4} \text{ m} = 0.796 \text{ mm}$$

EVALUATE: The point where $I = I_0/2$ is not midway between the center of the central maximum and the first minimum; see Exercise 36.15.

36.58. IDENTIFY: $I = I_0 \left(\frac{\sin \gamma}{\gamma} \right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

SET UP: $\frac{d \sin \gamma}{d \gamma} = \cos \gamma$. $\frac{d}{d \gamma} \left(\frac{1}{\gamma} \right) = -\frac{1}{\gamma^2}$.

EXECUTE: $\frac{dI}{d\gamma} = I_0 \frac{d}{d\gamma} \left(\frac{\sin \gamma}{\gamma} \right)^2 = 2 \left(\frac{\sin \gamma}{\gamma} \right) \left(\frac{\cos \gamma}{\gamma} - \frac{\sin \gamma}{\gamma^2} \right) = 0$. $\frac{\cos \gamma}{\gamma} - \frac{\sin \gamma}{\gamma^2} = 0 \Rightarrow \gamma \cos \gamma = \sin \gamma \Rightarrow \gamma = \tan \gamma$.

(b) The graph in Figure 36.58 is a plot of $f(\gamma) = \gamma - \tan \gamma$. When $f(\gamma)$ equals zero, there is an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the three smallest γ -values are $\gamma = 4.49$ rad, 7.73 rad, and 10.9 rad.

EVALUATE: $\gamma = 0$ is the central maximum. The three values of γ we found are the locations of the first three secondary maxima. The first four minima are at $\gamma = 3.14$ rad, 6.28 rad, 9.42 rad, and 12.6 rad. The maxima are between adjacent minima, but not precisely midway between them.

Gamma minus
tangent gamma

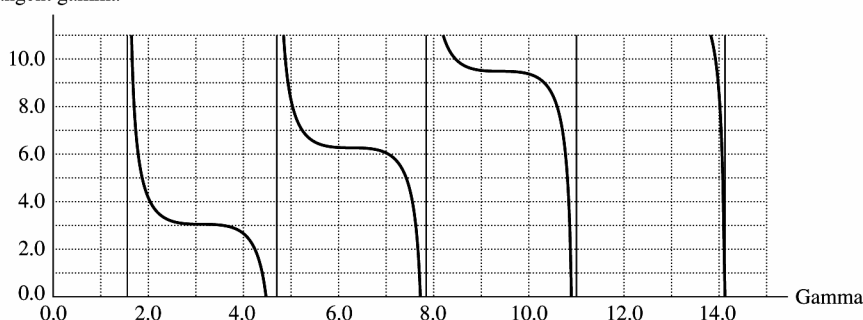


Figure 36.58

36.59. IDENTIFY and SET UP: Relate the phase difference between adjacent slits to the sum of the phasors for all slits. The phase difference between adjacent slits is $\phi = \frac{2\pi d}{\lambda} \sin \theta \approx \frac{2\pi d \theta}{\lambda}$ when θ is small and $\sin \theta \approx \theta$. Thus $\theta = \frac{\lambda \phi}{2\pi d}$.

EXECUTE: A principal maximum occurs when $\phi = \phi_{\max} = m2\pi$, where m is an integer, since then all the phasors add. The first minima on either side of the m^{th} principal maximum occur when $\phi = \phi_{\min}^{\pm} = m2\pi \pm (2\pi/N)$ and the phasor diagram for N slits forms a closed loop and the resultant phasor is zero. The angular position of a principal maximum is $\theta = \left(\frac{\lambda}{2\pi d} \right) \phi_{\max}$. The angular position of the adjacent minimum is $\theta_{\min}^{\pm} = \left(\frac{\lambda}{2\pi d} \right) \phi_{\min}^{\pm}$.

$$\theta_{\min}^{+} = \left(\frac{\lambda}{2\pi d} \right) \left(\phi_{\max} + \frac{2\pi}{N} \right) = \theta + \left(\frac{\lambda}{2\pi d} \right) \left(\frac{2\pi}{N} \right) = \theta + \frac{\lambda}{Nd}$$

$$\theta_{\min}^{-} = \left(\frac{\lambda}{2\pi d} \right) \left(\phi_{\max} - \frac{2\pi}{N} \right) = \theta - \frac{\lambda}{Nd}$$

The angular width of the principal maximum is $\theta_{\min}^{+} - \theta_{\min}^{-} = \frac{2\lambda}{Nd}$, as was to be shown.

EVALUATE: The angular width of the principal maximum decreases like $1/N$ as N increases.

36.60. IDENTIFY: The change in wavelength of the H_{α} line is due to a Doppler shift in the wavelength due to the motion of the galaxy.

SET UP: From Equation 16.30, the Doppler effect formula for light is $f_R = \sqrt{\frac{c-v}{c+v}} f_S$.

EXECUTE: First find the wavelength of the light using the grating information.

$$\lambda = d \sin \theta_1 = [1/(575,800 \text{ lines/m})] \sin 23.41^{\circ} = 6.900 \times 10^{-7} \text{ m} = 690.0 \text{ nm}$$

Using Equation 16.30, we have $f_R = \sqrt{\frac{c-v}{c+v}} f_S$. In this case, f_R is the frequency of the 690.0-nm light that the cosmologist measures, and f_S is the frequency of the 656.3-nm light of the H_{α} line obtained in the laboratory.

Solving for v gives $v = \frac{1 - (f_R/f_S)^2}{1 + (f_R/f_S)^2} c$. Since $f\lambda = c$, $f = c/\lambda$, which gives $f_R/f_S = \lambda_S/\lambda_R$. Substituting this into the equation for v , we get

$$v = \frac{1 - \left(\frac{\lambda_S}{\lambda_R}\right)^2}{1 + \left(\frac{\lambda_S}{\lambda_R}\right)^2} c = \frac{1 - \left(\frac{656.3 \text{ nm}}{690.0 \text{ nm}}\right)^2}{1 + \left(\frac{656.3 \text{ nm}}{690.0 \text{ nm}}\right)^2} (3.00 \times 10^8 \text{ m/s}) = 1.501 \times 10^7 \text{ m/s},$$

which is 5.00% the speed of light.

EVALUATE: Since v is positive, the galaxy is moving *away from* us. We can also see this because the wavelength has increased due to the motion.

36.61. IDENTIFY and SET UP: Draw the specified phasor diagrams. There is totally destructive interference between two slits when their phasors are in opposite directions.

EXECUTE: (a) For eight slits, the phasor diagrams must have eight vectors. The diagrams for each specified value of ϕ are sketched in Figure 36.61a. In each case the phasors all sum to zero.

(b) The additional phasor diagrams for $\phi = 3\pi/2$ and $3\pi/4$ are sketched in Figure 36.61b.

For $\phi = \frac{3\pi}{4}$, $\phi = \frac{5\pi}{4}$, and $\phi = \frac{7\pi}{4}$, totally destructive interference occurs between slits four apart. For $\phi = \frac{3\pi}{2}$,

totally destructive interference occurs with every second slit.

EVALUATE: At a minimum the phasors for all slits sum to zero.

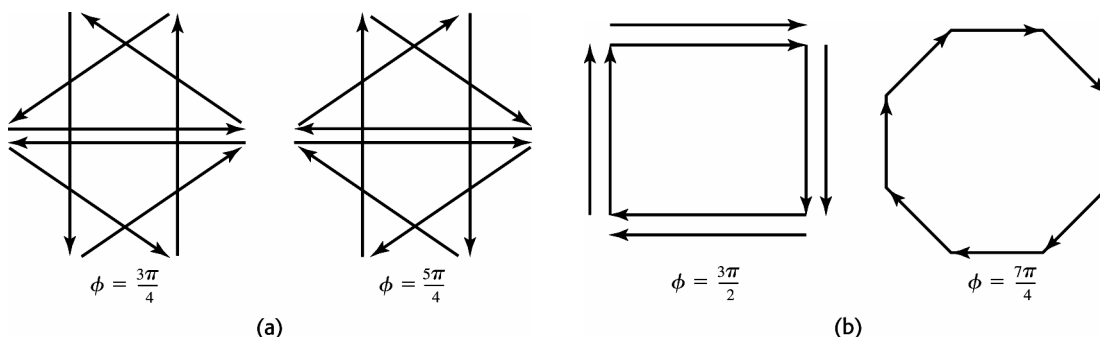


Figure 36.61

36.62. IDENTIFY: Maxima are given by $2d \sin \theta = m\lambda$.

SET UP: d is the separation between crystal planes.

EXECUTE: (a) $\theta = \arcsin\left(\frac{m\lambda}{2d}\right) = \arcsin\left(m \frac{0.125 \text{ nm}}{2(0.282 \text{ nm})}\right) = \arcsin(0.2216m)$.

For $m=1$: $\theta = 12.8^\circ$, $m=2$: $\theta = 26.3^\circ$, $m=3$: $\theta = 41.7^\circ$, and $m=4$: $\theta = 62.4^\circ$. No larger m values yield answers.

(b) If the separation $d = \frac{a}{\sqrt{2}}$, then $\theta = \arcsin\left(\frac{\sqrt{2}m\lambda}{2a}\right) = \arcsin(0.3134m)$.

So for $m=1$: $\theta = 18.3^\circ$, $m=2$: $\theta = 38.8^\circ$, and $m=3$: $\theta = 70.1^\circ$. No larger m values yield answers.

EVALUATE: In part (b), where d is smaller, the maxima for each m are at larger θ .

36.63. IDENTIFY and SET UP: In each case consider the relevant phasor diagram.

EXECUTE: (a) For the maxima to occur for N slits, the sum of all the phase differences between the slits must add to zero (the phasor diagram closes on itself). This requires that, adding up all the relative phase shifts,

$N\phi = 2\pi m$, for some integer m . Therefore $\phi = \frac{2\pi m}{N}$, for m not an integer multiple of N , which would give a maximum.

(b) The sum of N phase shifts $\phi = \frac{2\pi m}{N}$ brings you full circle back to the maximum, so only the $N-1$ previous phases yield minima between each pair of principal maxima.

EVALUATE: The $N-1$ minima between each pair of principal maxima cause the maxima to become sharper as N increases.

36.64. IDENTIFY: Set $d = a$ in the expressions for ϕ and β and use the results in Eq.(36.12).

SET UP: Figure 36.64 shows a pair of slits whose width and separation are equal

EXECUTE: Figure 36.64 shows that the two slits are equivalent to a single slit of width $2a$.

$\phi = \frac{2\pi d}{\lambda} \sin \theta$, so $\beta = \frac{2\pi a}{\lambda} \sin \theta = \phi$. So then the intensity is

$$I = I_0 \cos^2(\beta/2) \left(\frac{\sin^2(\beta/2)}{(\beta/2)^2} \right) = I_0 \frac{(2 \sin(\beta/2) \cos(\beta/2))^2}{\beta^2} = I_0 \frac{\sin^2 \beta}{\beta^2} = I_0 \frac{\sin^2(\beta'/2)}{(\beta'/2)^2}, \text{ where } \beta' = \frac{2\pi(2a)}{\lambda} \sin \theta,$$

which is Eq. (35.5) with double the slit width.

EVALUATE: In Chapter 35 we considered the limit where $a \ll d$. $a > d$ is not possible.

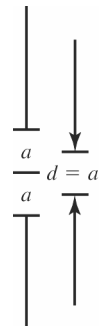


Figure 36.64

- 36.65. IDENTIFY and SET UP:** The condition for an intensity maximum is $d \sin \theta = m\lambda$, $m = 0, \pm 1, \pm 2, \dots$. Third order means $m = 3$. The longest observable wavelength is the one that gives $\theta = 90^\circ$ and hence $\sin \theta = 1$.

EXECUTE: 6500 lines/cm so 6.50×10^5 lines/m and $d = \frac{1}{6.50 \times 10^5} \text{ m} = 1.538 \times 10^{-6} \text{ m}$

$$\lambda = \frac{d \sin \theta}{m} = \frac{(1.538 \times 10^{-6} \text{ m})(1)}{3} = 5.13 \times 10^{-7} \text{ m} = 513 \text{ nm}$$

EVALUATE: The longest wavelength that can be obtained decreases as the order increases.

- 36.66. IDENTIFY and SET UP:** As the rays first reach the slits there is already a phase difference between adjacent slits of $\frac{2\pi d \sin \theta'}{\lambda}$. This, added to the usual phase difference introduced after passing through the slits, yields the condition for an intensity maximum. For a maximum the total phase difference must equal $2\pi m$.

EXECUTE: $\frac{2\pi d \sin \theta}{\lambda} + \frac{2\pi d \sin \theta'}{\lambda} = 2\pi m \Rightarrow d(\sin \theta + \sin \theta') = m\lambda$

$$(b) 600 \text{ slits/mm} \Rightarrow d = \frac{1}{6.00 \times 10^5 \text{ m}^{-1}} = 1.67 \times 10^{-6} \text{ m}.$$

For $\theta' = 0^\circ$,

$$m = 0: \theta = \arcsin(0) = 0.$$

$$m = 1: \theta = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{6.50 \times 10^{-7} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right) = 22.9^\circ.$$

$$m = -1: \theta = \arcsin\left(-\frac{\lambda}{d}\right) = \arcsin\left(-\frac{6.50 \times 10^{-7} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right) = -22.9^\circ.$$

For $\theta' = 20.0^\circ$,

$$m = 0: \theta = \arcsin(-\sin 20.0^\circ) = -20.0^\circ.$$

$$m = 1: \theta = \arcsin\left(\frac{6.50 \times 10^{-7} \text{ m}}{1.67 \times 10^{-6} \text{ m}} - \sin 20.0^\circ\right) = 2.71^\circ.$$

$$m = -1: \theta = \arcsin\left(-\frac{6.50 \times 10^{-7} \text{ m}}{1.67 \times 10^{-6} \text{ m}} - \sin 20.0^\circ\right) = -47.0^\circ.$$

EVALUATE: When $\theta' > 0$, the maxima are shifted downward on the screen, toward more negative angles.

- 36.67. IDENTIFY:** The maxima are given by $d \sin \theta = m\lambda$. We need $\sin \theta = \frac{m\lambda}{d} \leq 1$ in order for all the visible wavelengths are to be seen.

SET UP: For 650 slits/mm $\Rightarrow d = \frac{1}{6.50 \times 10^5 \text{ m}^{-1}} = 1.53 \times 10^{-6} \text{ m}.$

EXECUTE: $\lambda_1 = 4.00 \times 10^{-7} \text{ m}$; $m = 1$: $\frac{\lambda_1}{d} = 0.26$; $m = 2$: $\frac{2\lambda_1}{d} = 0.52$; $m = 3$: $\frac{3\lambda_1}{d} = 0.78$.

$\lambda_2 = 7.00 \times 10^{-7} \text{ m}$; $m = 1$: $\frac{\lambda_2}{d} = 0.46$; $m = 2$: $\frac{2\lambda_2}{d} = 0.92$; $m = 3$: $\frac{3\lambda_2}{d} = 1.37$. So, the third order does not contain the violet

end of the spectrum, and therefore only the first and second order diffraction patterns contain all colors of the spectrum.

EVALUATE: θ for each maximum is larger for longer wavelengths.

36.68. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: θ is small, so $\sin \theta \approx \frac{\Delta x}{R}$, where Δx is the size of the detail and $R = 7.2 \times 10^8 \text{ ly}$. $1 \text{ ly} = 9.41 \times 10^{12} \text{ km}$. $\lambda = c/f$

EXECUTE: $\sin \theta = 1.22 \frac{\lambda}{D} \approx \frac{\Delta x}{R} \Rightarrow \Delta x = \frac{1.22 \lambda R}{D} = \frac{(1.22)cR}{Df} = \frac{(1.22)(3.00 \times 10^5 \text{ km/s})(7.2 \times 10^8 \text{ ly})}{(77.000 \times 10^3 \text{ km})(1.665 \times 10^9 \text{ Hz})} = 2.06 \text{ ly}$.

$(9.41 \times 10^{12} \text{ km/ly})(2.06 \text{ ly}) = 1.94 \times 10^{13} \text{ km}$.

EVALUATE: $\lambda = 18 \text{ cm}$. λ/D is very small, so $\frac{\Delta x}{R}$ is very small. Still, R is very large and Δx is many orders of magnitude larger than the diameter of the sun.

36.69. IDENTIFY and SET UP: Add the phases between adjacent sources.

EXECUTE: (a) $d \sin \theta = m\lambda$. Place 1st maximum at ∞ or $\theta = 90^\circ$. $d = \lambda$. If $d < \lambda$, this puts the first maximum "beyond ∞ ." Thus, if $d < \lambda$ there is only a single principal maximum.

(b) At a principal maximum when $\delta = 0$, the phase difference due to the path difference between adjacent slits is $\Phi_{\text{path}} = 2\pi \left(\frac{d \sin \theta}{\lambda} \right)$. This just scales 2π radians by the fraction the wavelength is of the path difference between

adjacent sources. If we add a relative phase δ between sources, we still must maintain a total phase difference of zero to keep our principal maximum.

$$\Phi_{\text{path}} \pm \delta = 0 \Rightarrow \frac{2\pi d \sin \theta}{\lambda} = \pm \delta \text{ or } \theta = \sin^{-1} \left(\frac{\delta \lambda}{2\pi d} \right)$$

(c) $d = \frac{0.280 \text{ m}}{14} = 0.0200 \text{ m}$ (count the number of spaces between 15 points). Let $\theta = 45^\circ$. Also recall $f\lambda = c$, so

$$\delta_{\text{max}} = \pm \frac{2\pi(0.0200 \text{ m})(8.800 \times 10^9 \text{ Hz}) \sin 45^\circ}{(3.00 \times 10^8 \text{ m/s})} = \pm 2.61 \text{ radians.}$$

EVALUATE: δ must vary over a wider range in order to sweep the beam through a greater angle.

36.70. IDENTIFY: The wavelength of the light is smaller under water than it is in air, which will affect the resolving power of the lens, by Rayleigh's criterion.

SET UP: The wavelength under water is $\lambda = \lambda_0/n$, and for small angles Rayleigh's criterion is $\theta = 1.22\lambda/D$.

EXECUTE: (a) In air the wavelength is $\lambda_0 = c/f = (3.00 \times 10^8 \text{ m/s})/(6.00 \times 10^{14} \text{ Hz}) = 5.00 \times 10^{-7} \text{ m}$. In water the wavelength is $\lambda = \lambda_0/n = (5.00 \times 10^{-7} \text{ m})/1.33 = 3.76 \times 10^{-7} \text{ m}$. With the lens open all the way, we have $D = f/2.8 = (35.0 \text{ mm})/2.80 = (0.0350 \text{ m})/2.80$. In the water, we have

$$\sin \theta \approx \theta = 1.22 \lambda/D = (1.22)(3.76 \times 10^{-7} \text{ m})/[(0.0350 \text{ m})/2.80] = 3.67 \times 10^{-5} \text{ rad}$$

Calling w the width of the resolvable detail, we have

$$\theta = w/R \rightarrow w = R\theta = (2750 \text{ mm})(3.67 \times 10^{-5} \text{ rad}) = 0.101 \text{ mm}$$

(b) $\theta = 1.22 \lambda/D = (1.22)(5.00 \times 10^{-7} \text{ m})/[(0.0350 \text{ m})/2.80] = 4.88 \times 10^{-5} \text{ rad}$

$$w = R\theta = (2750 \text{ mm})(4.88 \times 10^{-5} \text{ rad}) = 0.134 \text{ mm}$$

EVALUATE: Due to the reduced wavelength underwater, the resolution of the lens is better under water than in air.

36.71. IDENTIFY and SET UP: Resolved by Rayleigh's criterion means the angular separation θ of the objects is given by $\theta = 1.22\lambda/D$. $\theta = y/s$, where $y = 75.0 \text{ m}$ is the distance between the two objects and s is their distance from the astronaut (her altitude).

EXECUTE: $\frac{y}{s} = 1.22 \frac{\lambda}{D}$

$$s = \frac{yD}{1.22\lambda} = \frac{(75.0 \text{ m})(4.00 \times 10^{-3} \text{ m})}{1.22(500 \times 10^{-9} \text{ m})} = 4.92 \times 10^5 \text{ m} = 492 \text{ km}$$

EVALUATE: In practice, this diffraction limit of resolution is not achieved. Defects of vision and distortion by the earth's atmosphere limit the resolution more than diffraction does.

36.72. IDENTIFY: Apply $\sin \theta = 1.22 \frac{\lambda}{D}$.

SET UP: θ is small, so $\sin \theta \approx \frac{\Delta x}{R}$, where Δx is the size of the details and R is the distance to the earth.

$$1 \text{ ly} = 9.41 \times 10^{15} \text{ m}.$$

EXECUTE: (a) $R = \frac{D \Delta x}{1.22 \lambda} = \frac{(6.00 \times 10^6 \text{ m})(2.50 \times 10^5 \text{ m})}{(1.22)(1.0 \times 10^{-5} \text{ m})} = 1.23 \times 10^{17} \text{ m} = 13.1 \text{ ly}$

(b) $\Delta x = \frac{1.22 \lambda R}{D} = \frac{(1.22)(1.0 \times 10^{-5} \text{ m})(4.22 \text{ ly})(9.41 \times 10^{15} \text{ m/ly})}{1.0 \text{ m}} = 4.84 \times 10^8 \text{ km}$. This is about 10,000 times the

diameter of the earth! Not enough resolution to see an earth-like planet! Δx is about 3 times the distance from the earth to the sun.

(c) $\Delta x = \frac{(1.22)(1.0 \times 10^{-5} \text{ m})(59 \text{ ly})(9.41 \times 10^{15} \text{ m/ly})}{6.00 \times 10^6 \text{ m}} = 1.13 \times 10^6 \text{ m} = 1130 \text{ km}$.

$$\frac{\Delta x}{D_{\text{planet}}} = \frac{1130 \text{ km}}{1.38 \times 10^5 \text{ km}} = 8.19 \times 10^{-3}; \Delta x \text{ is small compared to the size of the planet.}$$

EVALUATE: The very large diameter of *Planet Imager* allows it to resolve planet-sized detail at great distances.

36.73. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) From the segment dy' , the fraction of the amplitude of E_0 that gets through is

$$E_0 \left(\frac{dy'}{a} \right) \Rightarrow dE = E_0 \left(\frac{dy'}{a} \right) \sin(kx - \omega t).$$

(b) The path difference between each little piece is

$$y' \sin \theta \Rightarrow kx = k(D - y' \sin \theta) \Rightarrow dE = \frac{E_0 dy'}{a} \sin(k(D - y' \sin \theta) - \omega t). \text{ This can be rewritten as}$$

$$dE = \frac{E_0 dy'}{a} (\sin(kD - \omega t) \cos(ky' \sin \theta) + \sin(ky' \sin \theta) \cos(kD - \omega t)).$$

(c) So the total amplitude is given by the integral over the slit of the above.

$$\Rightarrow E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a} \int_{-a/2}^{a/2} dy' (\sin(kD - \omega t) \cos(ky' \sin \theta) + \sin(ky' \sin \theta) \cos(kD - \omega t)).$$

But the second term integrates to zero, so we have:

$$\begin{aligned} E &= \frac{E_0}{a} \sin(kD - \omega t) \int_{-a/2}^{a/2} dy' (\cos(ky' \sin \theta)) = E_0 \sin(kD - \omega t) \left[\frac{\sin(ky' \sin \theta)}{ka \sin \theta/2} \right]_{-a/2}^{a/2} \\ &\Rightarrow E = E_0 \sin(kD - \omega t) \left(\frac{\sin(ka(\sin \theta)/2)}{ka(\sin \theta)/2} \right) = E_0 \sin(kD - \omega t) \left(\frac{\sin(\pi a(\sin \theta)/\lambda)}{\pi a(\sin \theta)/\lambda} \right). \end{aligned}$$

$$\text{At } \theta = 0, \frac{\sin[\dots]}{[\dots]} = 1 \Rightarrow E = E_0 \sin(kD - \omega t).$$

(d) Since $I \propto E^2 \Rightarrow I = I_0 \left(\frac{\sin(ka(\sin \theta)/2)}{ka(\sin \theta)/2} \right)^2 = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2$, where we have used $I_0 = E_0^2 \sin^2(kx - \omega t)$.

EVALUATE: The same result for $I(\theta)$ is obtained as was obtained using phasors.

36.74. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) Each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0 . However, each successive source will pick up an extra phase from its respective pathlength to point

$$P. \phi = 2\pi \left(\frac{d \sin \theta}{\lambda} \right) \text{ which is just } 2\pi, \text{ the maximum phase, scaled by whatever fraction the path difference,}$$

$d \sin \theta$, is of the wavelength, λ . By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

(b) $e^{i(kR - \omega t + n\phi)} = \cos(kR - \omega t + n\phi) + i \sin(kR - \omega t + n\phi)$. The real part is just $\cos(kR - \omega t + n\phi)$. So,

$$\text{Re} \left[\sum_{n=0}^{N-1} E_0 e^{i(kR - \omega t + n\phi)} \right] = \sum_{n=0}^{N-1} E_0 \cos(kR - \omega t + n\phi). \text{ (Note: Re means "the real part of ..."). But this is just}$$

$$E_0 \cos(kR - \omega t) + E_0 \cos(kR - \omega t + \phi) + E_0 \cos(kR - \omega t + 2\phi) + \dots + E_0 \cos(kR - \omega t + (N-1)\phi)$$

$$(c) \sum_{n=0}^{N-1} E_0 e^{i(kR - \alpha x + n\phi)} = E_0 \sum_{n=0}^{N-1} e^{-i\alpha x} e^{+ikR} e^{in\phi} = E_0 e^{i(kR - \alpha x)} \sum_{n=0}^{N-1} e^{in\phi}. \quad \sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1} (e^{i\phi})^n. \text{ But recall } \sum_{n=0}^{N-1} x^n = \frac{x^N - 1}{x - 1}.$$

$$\text{Let } x = e^{i\phi} \text{ so } \sum_{n=0}^{N-1} (e^{i\phi})^n = \frac{e^{iN\phi} - 1}{e^{i\phi} - 1} \text{ (nice trick!). But } \frac{e^{iN\phi} - 1}{e^{i\phi} - 1} = \frac{e^{iN\phi/2} (e^{iN\phi/2} - e^{-iN\phi/2})}{e^{i\phi/2} (e^{i\phi/2} - e^{-i\phi/2})} = e^{i(N-1)\phi/2} \frac{(e^{iN\phi/2} - e^{-iN\phi/2})}{(e^{i\phi/2} - e^{-i\phi/2})}.$$

Putting everything together:

$$\sum_{n=0}^{N-1} E_0 e^{i(kR - \alpha x + n\phi)} = E_0 e^{i(kR - \alpha x + (N-1)\phi/2)} \frac{(e^{iN\phi/2} - e^{-iN\phi/2})}{(e^{i\phi/2} - e^{-i\phi/2})}$$

$$= E_0 [\cos(kR - \alpha x + (N-1)\phi/2) + i \sin(kR - \alpha x + (N-1)\phi/2)] \left[\frac{\cos N\phi/2 + i \sin N\phi/2 - \cos N\phi/2 + i \sin N\phi/2}{\cos \phi/2 + i \sin \phi/2 - \cos \phi/2 + i \sin \phi/2} \right]$$

$$\text{Taking only the real part gives } \Rightarrow E_0 \cos(kR - \alpha x + (N-1)\phi/2) \frac{\sin(N\phi/2)}{\sin \phi/2} = E.$$

$$(d) I = |E|_{\text{av}}^2 = I_0 \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}. \text{ (The } \cos^2 \text{ term goes to } \frac{1}{2} \text{ in the time average and is included in the definition of } I_0.)$$

$$I_0 \propto \frac{E_0^2}{2}.$$

$$\text{EVALUATE: (e) } N = 2. \quad I = I_0 \frac{\sin^2(2\phi/2)}{\sin^2 \phi/2} = \frac{I_0 (2 \sin \phi/2 \cos \phi/2)^2}{\sin^2 \phi/2} = 4 I_0 \cos^2 \frac{\phi}{2}. \text{ Looking at Eq.(35.9),}$$

$$I'_0 \propto 2 E_0^2 \text{ but for us } I_0 \propto \frac{E_0^2}{2} = \frac{I'_0}{4}.$$

36.75. IDENTIFY and SET UP: From Problem 36.74, $I = I_0 \frac{\sin^2(N\phi/2)}{\sin^2 \phi/2}$. Use this result to obtain each result specified in the problem.

$$\text{EXECUTE: (a) } \lim_{\phi \rightarrow 0} I \rightarrow \frac{0}{0}. \text{ Use l'Hôpital's rule: } \lim_{\phi \rightarrow 0} \frac{\sin(N\phi/2)}{\sin \phi/2} = \lim_{\phi \rightarrow 0} \left(\frac{N/2}{1/2} \right) \frac{\cos(N\phi/2)}{\cos(\phi/2)} = N. \text{ So } \lim_{\phi \rightarrow 0} I = N^2 I_0.$$

(b) The location of the first minimum is when the numerator first goes to zero at $\frac{N}{2} \phi_{\min} = \pi$ or $\phi_{\min} = \frac{2\pi}{N}$. The width of the central maximum goes like $2\phi_{\min}$, so it is proportional to $\frac{1}{N}$.

(c) Whenever $\frac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity. That is, I is a minimum wherever $\phi = \frac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which

occurs when $\frac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a

maximum. That is, if $\frac{n}{N}$ is an integer, there will be a maximum.

(d) From part (c), if $\frac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\frac{n}{N}$ is not an integer for fixed N and integer n .) For example, $n=0$ will be a maximum, but $n=1, 2, \dots, N-1$ will be minima with another maximum at $n=N$.

(e) Between maxima $\frac{\phi}{2}$ is a half-integer multiple of π (i.e. $\frac{\pi}{2}, \frac{3\pi}{2}$, etc.) and if N is odd then

$$\frac{\sin^2(N\phi/2)}{\sin^2 \phi/2} \rightarrow 1, \text{ so } I \rightarrow I_0.$$

EVALUATE: These results show that the principal maxima become sharper as the number of slits is increased.

RELATIVITY

- 37.1. IDENTIFY and SET UP:** Consider the distance A to O' and B to O' as observed by an observer on the ground (Figure 37.1).

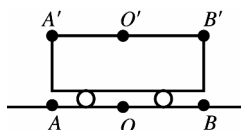


Figure 37.1

EXECUTE: Simultaneous to observer on train means light pulses from A' and B' arrive at O' at the same time. To observer at O light from A' has a longer distance to travel than light from B' so O will conclude that the pulse from $A(A')$ started before the pulse at $B(B')$. To observer at O bolt A appeared to strike first.

EVALUATE: Section 37.2 shows that if they are simultaneous to the observer on the ground then an observer on the train measures that the bolt at B' struck first.

37.2. (a) $\gamma = \frac{1}{\sqrt{1-(0.9)^2}} = 2.29$. $t = \gamma\tau = (2.29)(2.20 \times 10^{-6} \text{ s}) = 5.05 \times 10^{-6} \text{ s}$.

(b) $d = vt = (0.900)(3.00 \times 10^8 \text{ m/s})(5.05 \times 10^{-6} \text{ s}) = 1.36 \times 10^3 \text{ m} = 1.36 \text{ km}$.

- 37.3. IDENTIFY and SET UP:** The problem asks for u such that $\Delta t_0 / \Delta t = \frac{1}{2}$.

EXECUTE: $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = (3.00 \times 10^8 \text{ m/s})\sqrt{1-\left(\frac{1}{2}\right)^2} = 2.60 \times 10^8 \text{ m/s}$; $\frac{u}{c} = 0.867$

Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for u .

- 37.4. IDENTIFY:** Time dilation occurs because the rocket is moving relative to Mars.

SET UP: The time dilation equation is $\Delta t = \gamma\Delta t_0$, where t_0 is the proper time.

EXECUTE: **(a)** The two time measurements are made at the same place on Mars by an observer at rest there, so the observer on Mars measures the proper time.

(b) $\Delta t = \gamma\Delta t_0 = \frac{1}{\sqrt{1-(0.985)^2}}(75.0 \mu\text{s}) = 435 \mu\text{s}$

EVALUATE: The pulse lasts for a shorter time relative to the rocket than it does relative to the Mars observer.

- 37.5. (a) IDENTIFY and SET UP:** $\Delta t_0 = 2.60 \times 10^{-8} \text{ s}$; $\Delta t = 4.20 \times 10^{-7} \text{ s}$. In the lab frame the pion is created and decays at different points, so this time is not the proper time.

EXECUTE: $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ says $1 - \frac{u^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$

$$\frac{u}{c} = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1 - \left(\frac{2.60 \times 10^{-8} \text{ s}}{4.20 \times 10^{-7} \text{ s}}\right)^2} = 0.998; u = 0.998c$$

EVALUATE: $u < c$, as it must be, but u/c is close to unity and the time dilation effects are large.

(b) IDENTIFY and SET UP: The speed in the laboratory frame is $u = 0.998c$; the time measured in this frame is Δt , so the distance as measured in this frame is $d = u\Delta t$

EXECUTE: $d = (0.998)(2.998 \times 10^8 \text{ m/s})(4.20 \times 10^{-7} \text{ s}) = 126 \text{ m}$

EVALUATE: The distance measured in the pion's frame will be different because the time measured in the pion's frame is different (shorter).

37.6. $\gamma = 1.667$

(a) $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{1.20 \times 10^8 \text{ m}}{\gamma(0.800c)} = 0.300 \text{ s}.$

(b) $(0.300 \text{ s})(0.800c) = 7.20 \times 10^7 \text{ m}.$

(c) $\Delta t_0 = 0.300 \text{ s}/\gamma = 0.180 \text{ s}.$ (This is what the *racer* measures *your* clock to read at that instant.) At *your* origin you read the original $\frac{1.20 \times 10^8 \text{ m}}{(0.800)(3 \times 10^8 \text{ m/s})} = 0.5 \text{ s}.$ Clearly the observers (you and the racer) will not agree on the order of events!

37.7. **IDENTIFY and SET UP:** A clock moving with respect to an observer appears to run more slowly than a clock at rest in the observer's frame. The clock in the spacecraft measures the proper time Δt_0 . $\Delta t = 365 \text{ days} = 8760 \text{ hours}.$

EXECUTE: The clock on the moving spacecraft runs slow and shows the smaller elapsed time.

$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = (8760 \text{ h}) \sqrt{1 - (4.80 \times 10^6 / 3.00 \times 10^8)^2} = 8758.88 \text{ h}.$ The difference in elapsed times is $8760 \text{ h} - 8758.88 \text{ h} = 1.12 \text{ h}.$

37.8. **IDENTIFY and SET UP:** The proper time is measured in the frame where the two events occur at the same point.

EXECUTE: (a) The time of 12.0 ms measured by the first officer on the craft is the proper time.

(b) $\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$ gives $u = c \sqrt{1 - (\Delta t_0 / \Delta t)^2} = c \sqrt{1 - (12.0 \times 10^{-3} / 0.190)^2} = 0.998c.$

EVALUATE: The observer at rest with respect to the searchlight measures a much shorter duration for the event.

37.9. **IDENTIFY and SET UP:** $l = l_0 \sqrt{1 - u^2/c^2}$. The length measured when the spacecraft is moving is $l = 74.0 \text{ m}; l_0$ is the length measured in a frame at rest relative to the spacecraft.

EXECUTE: $l_0 = \frac{l}{\sqrt{1 - u^2/c^2}} = \frac{74.0 \text{ m}}{\sqrt{1 - (0.600c/c)^2}} = 92.5 \text{ m}.$

EVALUATE: $l_0 > l$. The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

37.10. **IDENTIFY and SET UP:** When the meterstick is at rest with respect to you, you measure its length to be 1.000 m, and that is its proper length, l_0 . $l = 0.3048 \text{ m}.$

EXECUTE: $l = l_0 \sqrt{1 - u^2/c^2}$ gives $u = c \sqrt{1 - (l/l_0)^2} = c \sqrt{1 - (0.3048/1.00)^2} = 0.9524c = 2.86 \times 10^8 \text{ m/s}.$

37.11. **IDENTIFY and SET UP:** The 2.2 μs lifetime is Δt_0 and the observer on earth measures Δt . The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is l and l_0 is 10 km.

EXECUTE: (a) The greatest speed the muon can have is c , so the greatest distance it can travel in $2.2 \times 10^{-6} \text{ s}$ is $d = vt = (3.00 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 660 \text{ m} = 0.66 \text{ km}.$

(b) $\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1 - (0.999)^2}} = 4.9 \times 10^{-5} \text{ s}$

$d = vt = (0.999)(3.00 \times 10^8 \text{ m/s})(4.9 \times 10^{-5} \text{ s}) = 15 \text{ km}$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c) $l = l_0 \sqrt{1 - u^2/c^2} = (10 \text{ km}) \sqrt{1 - (0.999)^2} = 0.45 \text{ km}$

In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

37.12. **IDENTIFY and SET UP:** The scientist at rest on the earth's surface measures the proper length of the separation between the point where the particle is created and the surface of the earth, so $l_0 = 45.0 \text{ km}.$ The transit time measured in the particle's frame is the proper time, Δt_0 .

EXECUTE: (a) $t = \frac{l_0}{v} = \frac{45.0 \times 10^3 \text{ m}}{(0.99540)(3.00 \times 10^8 \text{ m/s})} = 1.51 \times 10^{-4} \text{ s}$

(b) $l = l_0 \sqrt{1 - u^2/c^2} = (45.0 \text{ km}) \sqrt{1 - (0.99540)^2} = 4.31 \text{ km}$

(c) time dilation formula: $\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = (1.51 \times 10^{-4} \text{ s}) \sqrt{1 - (0.99540)^2} = 1.44 \times 10^{-5} \text{ s}$

from Δl : $t = \frac{l}{v} = \frac{4.31 \times 10^3 \text{ m}}{(0.99540)(3.00 \times 10^8 \text{ m/s})} = 1.44 \times 10^{-5} \text{ s}$

The two results agree.

37.13. (a) $l_0 = 3600 \text{ m}.$

$l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = l_0 (3600 \text{ m}) \sqrt{1 - \frac{(4.00 \times 10^7 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}} = (3600 \text{ m})(0.991) = 3568 \text{ m}.$

$$(b) \Delta t_0 = \frac{l_0}{u} = \frac{3600 \text{ m}}{4.00 \times 10^7 \text{ m/s}} = 9.00 \times 10^{-5} \text{ s}.$$

$$(c) \Delta t = \frac{l}{u} = \frac{3568 \text{ m}}{4.00 \times 10^7 \text{ m/s}} = 8.92 \times 10^{-5} \text{ s}.$$

37.14. Multiplying the last equation of (37.21) by u and adding to the first to eliminate t gives

$$x' + ut' = \gamma x \left(1 - \frac{u^2}{c^2} \right) = \frac{1}{\gamma} x,$$

and multiplying the first by $\frac{u}{c^2}$ and adding to the last to eliminate x gives

$$t' + \frac{u}{c^2} x' = \gamma t \left(1 - \frac{u^2}{c^2} \right) = \frac{1}{\gamma} t,$$

so $x = \gamma(x' + ut')$ and $t = \gamma(t' + ux'/c^2)$, which is indeed the same as Eq. (37.21) with the primed coordinates replacing the unprimed, and a change of sign of u .

$$\mathbf{37.15.} \quad (a) v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.400c + 0.600c}{1 + (0.400)(0.600)} = 0.806c$$

$$(b) v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.900c + 0.600c}{1 + (0.900)(0.600)} = 0.974c$$

$$(c) v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.990c + 0.600c}{1 + (0.990)(0.600)} = 0.997c.$$

37.16. $\gamma = 1.667$ ($\gamma = 5/3$ if $u = (4/5)c$).

(a) In Mavis's frame the event "light on" has space-time coordinates $x' = 0$ and $t' = 5.00$ s, so from the result of Exercise 37.14 or Example 37.7, $x = \gamma(x' + ut')$ and $t = \gamma \left(t' + \frac{ux'}{c^2} \right) \Rightarrow x = \gamma ut' = 2.00 \times 10^9 \text{ m}$, $t = \gamma t' = 8.33$ s.

(b) The 5.00-s interval in Mavis's frame is the proper time Δt_0 in Eq. (37.6), so $\Delta t = \gamma \Delta t_0 = 8.33$ s, as in part (a).

(c) $(8.33 \text{ s})(0.800c) = 2.00 \times 10^9 \text{ m}$, which is the distance x found in part (a).

37.17. IDENTIFY: The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$.

EXECUTE: (a) For the pursuit ship to catch the cruiser, the distance between them must be decreasing, so the velocity of the cruiser relative to the pursuit ship must be directed toward the pursuit ship.

(b) Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine).

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} = \frac{0.600c - 0.800c}{1 - (0.600)(0.800)} = -0.385c$$

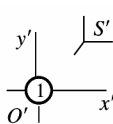
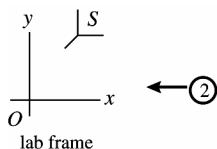
The result implies that the cruiser is moving toward the pursuit ship at $0.385c$.

EVALUATE: The nonrelativistic formula would have given $-0.200c$, which is considerably different from the correct result.

37.18. Let u_y be the y -component of the velocity of S' relative to S . Following the steps used in the derivation of

$$\text{Eq. (37.23) we get } v_y = \frac{v'_y + u_y}{1 + u_y v'_y / c^2}.$$

37.19. IDENTIFY and SET UP: Reference frames S and S' are shown in Figure 37.19.



Frame S is at rest in the laboratory. Frame S' is attached to particle 1.

Figure 37.19

u is the speed of S' relative to S ; this is the speed of particle 1 as measured in the laboratory. Thus $u = +0.650c$. The speed of particle 2 in S' is $0.950c$. Also, since the two particles move in opposite directions, 2 moves in the $-x'$ direction and $v'_x = -0.950c$. We want to calculate v_x , the speed of particle 2 in frame S ; use Eq. (37.23).

EXECUTE: $v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.950c + 0.650c}{1 + (0.950c)(-0.650c)/c^2} = \frac{-0.300c}{1 - 0.6175} = -0.784c$. The speed of the second particle, as measured in the laboratory, is $0.784c$.

EVALUATE: The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is $0.300c$. The correct relativistic calculation gives a result more than twice this.

- 37.20. IDENTIFY and SET UP:** Let S be the laboratory frame and let S' be the frame of one of the particles, as shown in Figure 37.20. Let the positive x direction for both frames be from particle 1 to particle 2. In the lab frame particle 1 is moving in the $+x$ direction and particle 2 is moving in the $-x$ direction. Then $u = 0.9520c$ and $v = -0.9520c$. v' is the velocity of particle 2 relative to particle 1.

EXECUTE: $v' = \frac{v - u}{1 - uv/c^2} = \frac{-0.9520c - 0.9520c}{1 - (0.9520c)(-0.9520c)/c^2} = -0.9988c$. The speed of particle 2 relative to particle 1 is $0.9988c$. $v' < 0$ shows particle 2 is moving toward particle 1.

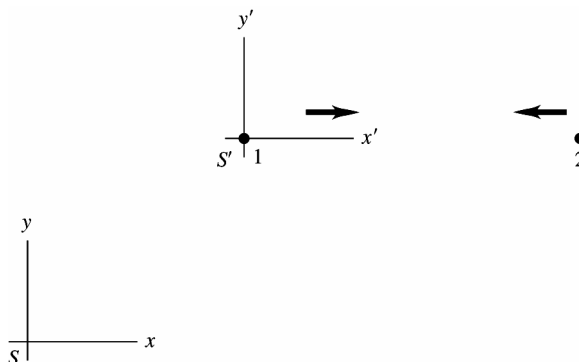


Figure 37.20

- 37.21. IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$.

EXECUTE: In the relativistic velocity addition formula for this case, v'_x is the relative speed of particle 1 with respect to particle 2, v is the speed of particle 2 measured in the laboratory, and u is the speed of particle 1 measured in the laboratory, $u = -v$.

$$v'_x = \frac{v - (-v)}{1 - (-v)v/c^2} = \frac{2v}{1 + v^2/c^2}. \quad \frac{v'_x}{c^2}v^2 - 2v + v'_x = 0 \text{ and } (0.890c)v^2 - 2c^2v + (0.890c^3) = 0.$$

This is a quadratic equation with solution $v = 0.611c$ (v must be less than c).

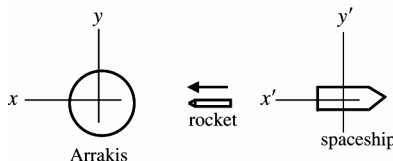
EVALUATE: The nonrelativistic result would be $0.445c$, which is considerably different from this result.

- 37.22. IDENTIFY and SET UP:** Let the starfighter's frame be S and let the enemy spaceship's frame be S' . Let the positive x direction for both frames be from the enemy spaceship toward the starfighter. Then $u = +0.400c$. $v' = +0.700c$. v is the velocity of the missile relative to you.

EXECUTE: (a) $v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.700c + 0.400c}{1 + (0.400)(0.700)} = 0.859c$

(b) Use the distance it moves as measured in your frame and the speed it has in your frame to calculate the time it takes in your frame. $t = \frac{8.00 \times 10^9 \text{ m}}{(0.859)(3.00 \times 10^8 \text{ m/s})} = 31.0 \text{ s}$.

- 37.23. IDENTIFY and SET UP:** The reference frames are shown in Figure 37.23.



S = Arrakis frame
 S' = spaceship frame
 The object is the rocket.

Figure 37.23

u is the velocity of the spaceship relative to Arrakis.

$$v_x = +0.360c; \quad v'_x = +0.920c$$

(In each frame the rocket is moving in the positive coordinate direction.)

Use the Lorentz velocity transformation equation, Eq.(37.22): $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$.

EXECUTE: $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ so $v'_x - u \left(\frac{v_x v'_x}{c^2} \right) = v_x - u$ and $u \left(1 - \frac{v_x v'_x}{c^2} \right) = v_x - v'_x$

$$u = \frac{v_x - v'_x}{1 - v_x v'_x / c^2} = \frac{0.360c - 0.920c}{1 - (0.360c)(0.920c)/c^2} = -\frac{0.560c}{0.6688} = -0.837c$$

The speed of the spacecraft relative to Arrakis is $0.837c = 2.51 \times 10^8$ m/s. The minus sign in our result for u means that the spacecraft is moving in the $-x$ -direction, so it is moving away from Arrakis.

EVALUATE: The incorrect Galilean expression also says that the spacecraft is moving away from Arrakis, but with speed $0.920c - 0.360c = 0.560c$.

37.24. IDENTIFY: We need to use the relativistic Doppler shift formula.

SET UP: The relativistic Doppler shift formula, Eq.(37.25), is $f = \sqrt{\frac{c+u}{c-u}} f_0$.

EXECUTE: $f^2 = \frac{c+u}{c-u} f_0^2$. $(c-u)f^2 = (c+u)f_0^2$. $cf^2 - uf^2 = cf_0^2 + uf_0^2$. $cf^2 - cf_0^2 = uf^2 + uf_0^2$ and

$$u = \frac{c(f^2 - f_0^2)}{f^2 + f_0^2} = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c.$$

(a) For $f/f_0 = 0.95$, $u = -0.051c$ moving away from the source.

(b) For $f/f_0 = 5.0$, $u = 0.923c$ moving towards the source.

EVALUATE: Note that the speed required to achieve a 10 times greater Doppler shift is not 10 times the original speed.

37.25. IDENTIFY and SET UP: Source and observer are approaching, so use Eq.(37.25): $f = \sqrt{\frac{c+u}{c-u}} f_0$. Solve for u , the speed of the light source relative to the observer.

(a) **EXECUTE:** $f^2 = \left(\frac{c+u}{c-u} \right) f_0^2$

$$(c-u)f^2 = (c+u)f_0^2 \text{ and } u = \frac{c(f^2 - f_0^2)}{f^2 + f_0^2} = c \left(\frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} \right)$$

$$\lambda_0 = 675 \text{ nm}, \quad \lambda = 575 \text{ nm}$$

$$u = \left(\frac{(675 \text{ nm}/575 \text{ nm})^2 - 1}{(675 \text{ nm}/575 \text{ nm})^2 + 1} \right) c = 0.159c = (0.159)(2.998 \times 10^8 \text{ m/s}) = 4.77 \times 10^7 \text{ m/s}; \text{ definitely speeding}$$

(b) $4.77 \times 10^7 \text{ m/s} = (4.77 \times 10^7 \text{ m/s})(1 \text{ km}/1000 \text{ m})(3600 \text{ s}/1 \text{ h}) = 1.72 \times 10^8 \text{ km/h}$. Your fine would be $\$1.72 \times 10^8$ (172 million dollars).

EVALUATE: The source and observer are approaching, so $f > f_0$ and $\lambda < \lambda_0$. Our result gives $u < c$, as it must.

37.26. Using $u = -0.600c = -(3/5)c$ in Eq.(37.25) gives

$$f = \sqrt{\frac{1-(3/5)}{1+(3/5)}} f_0 = \sqrt{\frac{2/5}{8/5}} f_0 = f_0/2.$$

37.27. IDENTIFY and SET UP: If $\vec{F}$ is parallel to $\vec{v}$ then $\vec{F}$ changes the magnitude of $\vec{v}$ and not its direction.

$$F = \frac{dp}{dt} = \frac{d}{dt} \left(\frac{mv}{\sqrt{1-v^2/c^2}} \right)$$

Use the chain rule to evaluate the derivative: $\frac{d}{dt} f(v(t)) = \frac{df}{dv} \frac{dv}{dt}$.

EXECUTE: (a) $F = \frac{m}{(1-v^2/c^2)^{1/2}} \left(\frac{dv}{dt} \right) + \frac{mv}{(1-v^2/c^2)^{3/2}} \left(-\frac{1}{2} \right) \left(-\frac{2v}{c^2} \right) \left(\frac{dv}{dt} \right)$

$$F = \frac{dv}{dt} \frac{m}{(1-v^2/c^2)^{3/2}} \left(1 - \frac{v^2}{c^2} + \frac{v^2}{c^2} \right) = \frac{dv}{dt} \frac{m}{(1-v^2/c^2)^{3/2}}$$

But $\frac{dv}{dt} = a$, so $a = (F/m)(1-v^2/c^2)^{3/2}$.

EVALUATE: Our result agrees with Eq.(37.30).

(b) **IDENTIFY and SET UP:** If $\vec{F}$ is perpendicular to $\vec{v}$ then $\vec{F}$ changes the direction of $\vec{v}$ and not its magnitude.

$$\vec{F} = \frac{d}{dt} \left(\frac{m\vec{v}}{\sqrt{1-v^2/c^2}} \right).$$

$\vec{a} = d\vec{v}/dt$ but the magnitude of v in the denominator of Eq.(37.29) is constant.

EXECUTE: $F = \frac{ma}{\sqrt{1-v^2/c^2}}$ and $a = (F/m)(1-v^2/c^2)^{1/2}$.

EVALUATE: This result agrees with Eq.(37.33).

37.28. IDENTIFY and SET UP: $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$. If γ is 1.0% greater than 1 then $\gamma = 1.010$, if γ is 10% greater than 1

then $\gamma = 1.10$ and if γ is 100% greater than 1 then $\gamma = 2.00$.

EXECUTE: $v = c\sqrt{1-1/\gamma^2}$

(a) $v = c\sqrt{1-1/(1.010)^2} = 0.140c$

(b) $v = c\sqrt{1-1/(1.10)^2} = 0.417c$

(c) $v = c\sqrt{1-1/(2.00)^2} = 0.866c$

37.29. (a) $p = \frac{mv}{\sqrt{1-v^2/c^2}} = 2mv$.

$$\Rightarrow 1 = 2\sqrt{1-v^2/c^2} \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.866c.$$

(b) $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$ so $\frac{1}{1-v^2/c^2} = 2^{2/3} \Rightarrow \frac{v}{c} = \sqrt{1-2^{-2/3}} = 0.608$

37.30. The force is found from Eq.(37.32) or Eq.(37.33).

(a) Indistinguishable from $F = ma = 0.145$ N.

(b) $\gamma^3 ma = 1.75$ N.

(c) $\gamma^3 ma = 51.7$ N.

(d) $\gamma ma = 0.145$ N, 0.333 N, 1.03 N.

37.31. (a) $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}}c = 0.866c.$$

(b) $K = 5mc^2 \Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \frac{1}{36} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{35}{36}}c = 0.986c.$

37.32. $E = 2mc^2 = 2(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 3.01 \times 10^{-10} \text{ J} = 1.88 \times 10^9 \text{ eV}.$

37.33. IDENTIFY and SET UP: Use Eqs.(37.38) and (37.39).

EXECUTE: (a) $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50 \times 10^{-10} \text{ J}$

(b) $E^2 = (mc^2)^2 + (pc)^2$; $E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$

$$p = \sqrt{15}mc = 1.94 \times 10^{-18} \text{ kg} \cdot \text{m/s}$$

(c) $E = mc^2 / \sqrt{1-v^2/c^2}$

$$E = 4.00mc^2 \text{ gives } 1-v^2/c^2 = 1/16 \text{ and } v = \sqrt{15/16}c = 0.968c$$

EVALUATE: The speed is close to c since the kinetic energy is greater than the rest energy. Nonrelativistic expressions relating E , K , p and v will be very inaccurate.

37.34. (a) $W = \Delta K = (\gamma_f - 1)mc^2 = (4.07 \times 10^{-3})mc^2.$

(b) $(\gamma_f - \gamma_i)mc^2 = 4.79mc^2.$

(c) The result of part (b) is far larger than that of part (a).

37.35. IDENTIFY: Use $E = mc^2$ to relate the mass increase to the energy increase.

(a) **SET UP:** Your total energy E increases because your gravitational potential energy mgy increases.

EXECUTE: $\Delta E = mg\Delta y$

$$\Delta E = (\Delta m)c^2 \text{ so } \Delta m = \Delta E / c^2 = mg(\Delta y) / c^2$$

$$\Delta m / m = (g\Delta y) / c^2 = (9.80 \text{ m/s}^2)(30 \text{ m}) / (2.998 \times 10^8 \text{ m/s})^2 = 3.3 \times 10^{-13} \%$$

This increase is much, much too small to be noticed.

(b) SET UP: The energy increases because potential energy is stored in the compressed spring.

$$\text{EXECUTE: } \Delta E = \Delta U = \frac{1}{2}kx^2 = \frac{1}{2}(2.00 \times 10^4 \text{ N/m})(0.060 \text{ m})^2 = 36.0 \text{ J}$$

$$\Delta m = (\Delta E) / c^2 = 4.0 \times 10^{-16} \text{ kg}$$

Energy increases so mass increases. The mass increase is much, much too small to be noticed.

EVALUATE: In both cases the energy increase corresponds to a mass increase. But since c^2 is a very large number the mass increase is very small.

37.36. (a) $E_0 = m_0 c^2$. $2E = mc^2 = 2m_0 c^2$. Therefore, $m = 2m_0 \Rightarrow \frac{m_0}{\sqrt{1-v^2/c^2}} = 2m_0$.

$$\frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = c\sqrt{3/4} = 0.866c = 2.60 \times 10^8 \text{ m/s}$$

(b) $10 m_0 c^2 = mc^2 = \frac{m_0}{\sqrt{1-v^2/c^2}} c^2$.

$$1 - \frac{v^2}{c^2} = \frac{1}{100} \Rightarrow \frac{v^2}{c^2} = \frac{99}{100}. \quad v = c\sqrt{\frac{99}{100}} = 0.995c = 2.98 \times 10^8 \text{ m/s}.$$

37.37. IDENTIFY and SET UP: The energy equivalent of mass is $E = mc^2$. $\rho = 7.86 \text{ g/cm}^3 = 7.86 \times 10^3 \text{ kg/m}^3$. For a cube, $V = L^3$.

EXECUTE: (a) $m = \frac{E}{c^2} = \frac{1.0 \times 10^{20} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 1.11 \times 10^3 \text{ kg}$

(b) $\rho = \frac{m}{V}$ so $V = \frac{m}{\rho} = \frac{1.11 \times 10^3 \text{ kg}}{7.86 \times 10^3 \text{ kg/m}^3} = 0.141 \text{ m}^3$. $L = V^{1/3} = 0.521 \text{ m} = 52.1 \text{ cm}$

EVALUATE: Particle/antiparticle annihilation has been observed in the laboratory, but only with small quantities of antimatter.

37.38. $(5.52 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 4.97 \times 10^{-10} \text{ J} = 3105 \text{ MeV}$.

37.39. IDENTIFY and SET UP: The total energy is given in terms of the momentum by Eq.(37.39). In terms of the total energy E , the kinetic energy K is $K = E - mc^2$ (from Eq.37.38). The rest energy is mc^2 .

EXECUTE: (a) $E = \sqrt{(mc^2)^2 + (pc)^2} =$

$$\sqrt{[(6.64 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2]^2 + [(2.10 \times 10^{-18} \text{ kg})(2.998 \times 10^8 \text{ m/s})]^2} \text{ J}$$

$$E = 8.67 \times 10^{-10} \text{ J}$$

(b) $mc^2 = (6.64 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 5.97 \times 10^{-10} \text{ J}$

$$K = E - mc^2 = 8.67 \times 10^{-10} \text{ J} - 5.97 \times 10^{-10} \text{ J} = 2.70 \times 10^{-10} \text{ J}$$

(c) $\frac{K}{mc^2} = \frac{2.70 \times 10^{-10} \text{ J}}{5.97 \times 10^{-10} \text{ J}} = 0.452$

EVALUATE: The incorrect nonrelativistic expressions for K and p give $K = p^2 / 2m = 3.3 \times 10^{-10} \text{ J}$; the correct relativistic value is less than this.

37.40. $E = (m^2 c^4 + p^2 c^2)^{1/2} = mc^2 \left(1 + \left(\frac{p}{mc} \right)^2 \right)^{1/2}$

$$E \approx mc^2 \left(1 + \frac{1}{2} \frac{p^2}{m^2 c^2} \right) = mc^2 + \frac{p^2}{2m} = mc^2 + \frac{1}{2} mv^2$$
, the sum of the rest mass energy and the classical kinetic energy.

37.41. (a) $v = 8 \times 10^7 \text{ m/s} \Rightarrow \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 1.0376$. For $m = m_p$, $K_{\text{nonrel}} = \frac{1}{2} mv^2 = 5.34 \times 10^{-12} \text{ J}$.

$$K_{\text{rel}} = (\gamma - 1)mc^2 = 5.65 \times 10^{-12} \text{ J}. \quad \frac{K_{\text{rel}}}{K_{\text{nonrel}}} = 1.06.$$

(b) $v = 2.85 \times 10^8 \text{ m/s}$; $\gamma = 3.203$.

$$K_{\text{rel}} = \frac{1}{2} mv^2 = 6.78 \times 10^{-11} \text{ J}; \quad K_{\text{rel}} = (\gamma - 1)mc^2 = 3.31 \times 10^{-10} \text{ J}; \quad K_{\text{rel}} / K_{\text{nonrel}} = 4.88.$$

37.42. IDENTIFY: Since the speeds involved are close to that of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic kinetic energy is $K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) mc^2$.

$$(a) K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) mc^2 = (1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1}{\sqrt{1 - (0.100c/c)^2}} - 1 \right)$$

$$K = (1.50 \times 10^{-10} \text{ J}) \left(\frac{1}{\sqrt{1 - 0.0100}} - 1 \right) = 7.56 \times 10^{-13} \text{ J} = 4.73 \text{ MeV}$$

$$(b) K = (1.50 \times 10^{-10} \text{ J}) \left(\frac{1}{\sqrt{1 - (0.500)^2}} - 1 \right) = 2.32 \times 10^{-11} \text{ J} = 145 \text{ MeV}$$

$$(c) K = (1.50 \times 10^{-10} \text{ J}) \left(\frac{1}{\sqrt{1 - (0.900)^2}} - 1 \right) = 1.94 \times 10^{-10} \text{ J} = 1210 \text{ MeV}$$

$$(d) \Delta E = 2.32 \times 10^{-11} \text{ J} - 7.56 \times 10^{-13} \text{ J} = 2.24 \times 10^{-11} \text{ J} = 140 \text{ MeV}$$

$$(e) \Delta E = 1.94 \times 10^{-10} \text{ J} - 2.32 \times 10^{-11} \text{ J} = 1.71 \times 10^{-10} \text{ J} = 1070 \text{ MeV}$$

(f) Without relativity, $K = \frac{1}{2}mv^2$. The work done in accelerating a proton from $0.100c$ to $0.500c$ in the

nonrelativistic limit is $\Delta E = \frac{1}{2}m(0.500c)^2 - \frac{1}{2}m(0.100c)^2 = 1.81 \times 10^{-11} \text{ J} = 113 \text{ MeV}$.

The work done in accelerating a proton from $0.500c$ to $0.900c$ in the nonrelativistic limit is

$$\Delta E = \frac{1}{2}m(0.900c)^2 - \frac{1}{2}m(0.500c)^2 = 4.21 \times 10^{-11} \text{ J} = 263 \text{ MeV}.$$

EVALUATE: We see in the first case the nonrelativistic result is within 20% of the relativistic result. In the second case, the nonrelativistic result is very different from the relativistic result since the velocities are closer to c .

37.43. IDENTIFY and SET UP: Use Eq.(23.12) and conservation of energy to relate the potential difference to the kinetic energy gained by the electron. Use Eq.(37.36) to calculate the kinetic energy from the speed.

EXECUTE: (a) $K = q\Delta V = e\Delta V$

$$K = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) = 4.025mc^2 = 3.295 \times 10^{-13} \text{ J} = 2.06 \text{ MeV}$$

$$\Delta V = K/e = 2.06 \times 10^6 \text{ V}$$

(b) From part (a), $K = 3.30 \times 10^{-13} \text{ J} = 2.06 \text{ MeV}$

EVALUATE: The speed is close to c and the kinetic energy is four times the rest mass.

37.44. (a) According to Eq.(37.38) and conservation of mass-energy

$$2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1 + \frac{m}{2M} = 1 + \frac{9.75}{2(16.7)} = 1.292.$$

Note that since $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$, we have that $\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.292)^2}} = 0.6331$.

(b) According to Eq.(37.36), the kinetic energy of each proton is

$$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1.00 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}} \right) = 274 \text{ MeV}.$$

(c) The rest energy of η^0 is $mc^2 = (9.75 \times 10^{-28} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1.00 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}} \right) = 548 \text{ MeV}$.

(d) The kinetic energy lost by the protons is the energy that produces the η^0 ,

$$548 \text{ MeV} = 2(274 \text{ MeV}).$$

37.45. IDENTIFY: The relativistic expression for the kinetic energy is $K = (\gamma - 1)mc^2$, where $\gamma = \frac{1}{\sqrt{1 - x}}$ and $x = v^2/c^2$.

The Newtonian expression for the kinetic energy is $K_N = \frac{1}{2}mv^2$.

SET UP: Solve for v such that $K = \frac{3}{2}K_N$.

EXECUTE: $(\gamma-1)mc^2 = \frac{3}{4}mv^2$. $\frac{1}{\sqrt{1-x}} - 1 = \frac{3}{4}x$. $\frac{1}{1-x} = \left(1 + \frac{3}{4}x\right)^2$. After a little algebra this becomes

$$9x^2 + 15x - 8 = 0. \quad x = \frac{1}{18}(-15 \pm \sqrt{(15)^2 + 4(9)(8)}). \quad \text{The positive root is } x = 0.425. \quad x = v^2/c^2, \text{ so}$$

$$v = \sqrt{x}c = 0.652c.$$

EVALUATE: The fractional increase of the relativistic expression above the nonrelativistic one increases as v increases.

- 37.46.** The fraction of the initial mass (a) that becomes energy is $1 - \frac{(4.0015 \text{ u})}{2(2.0136 \text{ u})} = 6.382 \times 10^{-3}$, and so the energy released per kilogram is $(6.382 \times 10^{-3})(1.00 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 5.74 \times 10^{14} \text{ J}$.

(b) $\frac{1.0 \times 10^{19} \text{ J}}{5.74 \times 10^{14} \text{ J/kg}} = 1.7 \times 10^4 \text{ kg}$.

- 37.47.** (a) $E = mc^2$, $m = E/c^2 = (3.8 \times 10^{26} \text{ J})/(2.998 \times 10^8 \text{ m/s})^2 = 4.2 \times 10^9 \text{ kg}$.

1 kg is equivalent to 2.2 lbs, so $m = 4.6 \times 10^6$ tons

(b) The current mass of the sun is $1.99 \times 10^{30} \text{ kg}$, so it would take it

$$(1.99 \times 10^{30} \text{ kg})/(4.2 \times 10^9 \text{ kg/s}) = 4.7 \times 10^{20} \text{ s} = 1.5 \times 10^{13} \text{ years} \text{ to use up all its mass.}$$

- 37.48.** **IDENTIFY:** Since the final speed is close to the speed of light, there will be a considerable difference between the relativistic and nonrelativistic results.

SET UP: The nonrelativistic work-energy theorem is $F\Delta x = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$, and the relativistic formula for a constant force is $F\Delta x = (\gamma-1)mc^2$.

(a) Using the classical work-energy theorem and solving for Δx , we obtain

$$\Delta x = \frac{m(v^2 - v_0^2)}{2F} = \frac{(0.100 \times 10^{-9} \text{ kg})[(0.900)(3.00 \times 10^8 \text{ m/s})]^2}{2(1.00 \times 10^6 \text{ N})} = 3.65 \text{ m}.$$

(b) Using the relativistic work-energy theorem for a constant force, we obtain

$$\Delta x = \frac{(\gamma-1)mc^2}{F}.$$

For the given speed, $\gamma = \frac{1}{\sqrt{1-0.900^2}} = 2.29$, thus

$$\Delta x = \frac{(2.29-1)(0.100 \times 10^{-9} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2}{(1.00 \times 10^6 \text{ N})} = 11.6 \text{ m}.$$

EVALUATE: (c) The distance obtained from the relativistic treatment is greater. As we have seen, more energy is required to accelerate an object to speeds close to c , so that force must act over a greater distance.

- 37.49.** (a) **IDENTIFY and SET UP:** $\Delta t_0 = 2.60 \times 10^{-8} \text{ s}$ is the proper time, measured in the pion's frame. The time measured in the lab must satisfy $d = c\Delta t$, where $u \approx c$. Calculate Δt and then use Eq.(37.6) to calculate u .

EXECUTE: $\Delta t = \frac{d}{c} = \frac{1.20 \times 10^3 \text{ m}}{2.998 \times 10^8 \text{ m/s}} = 4.003 \times 10^{-6} \text{ s}$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \text{ so } (1-u^2/c^2)^{1/2} = \frac{\Delta t_0}{\Delta t} \text{ and } (1-u^2/c^2) = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$

Write $u = (1-\Delta)c$ so that $(u/c)^2 = (1-\Delta)^2 = 1-2\Delta + \Delta^2 \approx 1-2\Delta$ since Δ is small.

$$\text{Using this in the above gives } 1-(1-2\Delta) = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Delta = \frac{1}{2} \left(\frac{\Delta t_0}{\Delta t}\right)^2 = \frac{1}{2} \left(\frac{2.60 \times 10^{-8} \text{ s}}{4.003 \times 10^{-6} \text{ s}}\right)^2 = 2.11 \times 10^{-5}$$

EVALUATE: An alternative calculation is to say that the length of the tube must contract relative to the moving pion so that the pion travels that length before decaying. The contracted length must be

$$l = c\Delta t_0 = (2.998 \times 10^8 \text{ m/s})(2.60 \times 10^{-8} \text{ s}) = 7.79 \text{ m}.$$

$$l = l_0 \sqrt{1-u^2/c^2} \text{ so } 1-u^2/c^2 = \left(\frac{l}{l_0}\right)^2$$

Then $u = (1 - \Delta)c$ gives $\Delta = \frac{1}{2} \left(\frac{l}{l_0} \right)^2 = \frac{1}{2} \left(\frac{7.79 \text{ m}}{1.20 \times 10^3 \text{ m}} \right)^2 = 2.11 \times 10^{-5}$, which checks.

(b) IDENTIFY and SET UP: $E = \gamma mc^2$ (Eq.(37.38)).

EXECUTE: $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{2\Delta}} = \frac{1}{\sqrt{2(2.11 \times 10^{-5})}} = 154$

$E = 154(139.6 \text{ MeV}) = 2.15 \times 10^4 \text{ MeV} = 21.5 \text{ GeV}$

EVALUATE: The total energy is 154 times the rest energy.

- 37.50. IDENTIFY and SET UP:** The proper length of a side is $l_0 = a$. The side along the direction of motion is shortened to $l = l_0 \sqrt{1 - v^2/c^2}$. The sides in the two directions perpendicular to the motion are unaffected by the motion and still have a length a .

EXECUTE: $V = a^2 l = a^3 \sqrt{1 - v^2/c^2}$

- 37.51. IDENTIFY and SET UP:** There must be a length contraction such that the length a becomes the same as b ; $l_0 = a$, $l = b$. l_0 is the distance measured by an observer at rest relative to the spacecraft. Use Eq.(37.16) and solve for u .

EXECUTE: $\frac{l}{l_0} = \sqrt{1 - u^2/c^2}$ so $\frac{b}{a} = \sqrt{1 - u^2/c^2}$;

$a = 1.40b$ gives $b/1.40b = \sqrt{1 - u^2/c^2}$ and thus $1 - u^2/c^2 = 1/(1.40)^2$

$u = \sqrt{1 - 1/(1.40)^2} c = 0.700c = 2.10 \times 10^8 \text{ m/s}$

EVALUATE: A length on the spacecraft in the direction of the motion is shortened. A length perpendicular to the motion is unchanged.

- 37.52. IDENTIFY and SET UP:** The proper time Δt_0 is the time that elapses in the frame of the space probe. Δt is the time that elapses in the frame of the earth. The distance traveled is 42.2 light years, as measured in the earth frame.

EXECUTE: (a) Light travels 42.2 light years in 42.2 yr, so $\Delta t = \left(\frac{c}{0.9910c} \right) (42.2 \text{ yr}) = 42.6 \text{ yr}$.

$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = (42.6 \text{ yr}) \sqrt{1 - (0.9910)^2} = 5.7 \text{ yr}$. She measures her biological age to be $19 \text{ yr} + 5.7 \text{ yr} = 24.7 \text{ yr}$.

(b) Her age measured by someone on earth is $19 \text{ yr} + 42.6 \text{ yr} = 61.6 \text{ yr}$.

- 37.53. (a)** $E = \gamma mc^2$ and $\gamma = 10 = \frac{1}{\sqrt{1 - (v/c)^2}} \Rightarrow \frac{v}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \Rightarrow \frac{v}{c} = \sqrt{\frac{99}{100}} = 0.995$.

(b) $(pc)^2 = m^2 v^2 \gamma^2 c^2$, $E^2 = m^2 c^4 \left(\left(\frac{v}{c} \right)^2 \gamma^2 + 1 \right)$

$$\Rightarrow \frac{E^2 - (pc)^2}{E^2} = \frac{1}{1 + \gamma^2 \left(\frac{v}{c} \right)^2} = \frac{1}{1 + (10/(0.995))^2} = 0.01 = 1\%.$$

- 37.54. IDENTIFY and SET UP:** The clock on the plane measures the proper time Δt_0 .

$\Delta t = 4.00 \text{ h} = 4.00 \text{ h} (3600 \text{ s/1 h}) = 1.44 \times 10^4 \text{ s}$.

$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$ and $\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2}$

EXECUTE: $\frac{u}{c}$ small so $\sqrt{1 - u^2/c^2} = (1 - u^2/c^2)^{1/2} \approx 1 - \frac{1}{2} \frac{u^2}{c^2}$; thus $\Delta t_0 = \Delta t \left(1 - \frac{1}{2} \frac{u^2}{c^2} \right)$

The difference in the clock readings is $\Delta t - \Delta t_0 = \frac{1}{2} \frac{u^2}{c^2} \Delta t = \frac{1}{2} \left(\frac{250 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}} \right)^2 (1.44 \times 10^4 \text{ s}) = 5.01 \times 10^{-9} \text{ s}$. The clock on the plane has the shorter elapsed time.

EVALUATE: Δt_0 is always less than Δt ; our results agree with this. The speed of the plane is much less than the speed of light, so the difference in the reading of the two clocks is very small.

- 37.55. IDENTIFY:** Since the speed is very close to the speed of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic formula for kinetic energy is $K = mc^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right)$ and the relativistic mass is

$$m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}}.$$

EXECUTE: (a) $K = 7 \times 10^{12} \text{ eV} = 1.12 \times 10^{-6} \text{ J}$. Using this value in the relativistic kinetic energy formula and

substituting the mass of the proton for m , we get $K = mc^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right)$

which gives $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45 \times 10^3$ and $1 - \frac{v^2}{c^2} = \frac{1}{(7.45 \times 10^3)^2}$. Solving for v gives

$$1 - \frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c}, \text{ since } c+v \approx 2c. \text{ Substituting } v = (1-\Delta)c, \text{ we have.}$$

$$1 - \frac{v^2}{c^2} = \frac{2(c-v)}{c} = \frac{2[c-(1-\Delta)c]}{c} = 2\Delta. \text{ Solving for } \Delta \text{ gives } \Delta = \frac{1-v^2/c^2}{2} = \frac{\left(\frac{1}{7.45 \times 10^3}\right)^2}{2} = 9 \times 10^{-9}, \text{ to one significant digit.}$$

(b) Using the relativistic mass formula and the result that $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45 \times 10^3$, we have

$$m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}} = m \left(\frac{1}{\sqrt{1-v^2/c^2}} \right) = (7 \times 10^3)m, \text{ to one significant digit.}$$

EVALUATE: At such high speeds, the proton's mass is over 7000 times as great as its rest mass.

37.56. IDENTIFY and SET UP: The energy released is $E = (\Delta m)c^2$. $\Delta m = \left(\frac{1}{10^4} \right) (8.00 \text{ kg})$. $P_{\text{av}} = \frac{E}{t}$. The change in gravitational potential energy is $mg\Delta y$.

$$\text{EXECUTE: (a) } E = (\Delta m)c^2 = \left(\frac{1}{10^4} \right) (8.00 \text{ kg}) (3.00 \times 10^8 \text{ m/s})^2 = 7.20 \times 10^{13} \text{ J}$$

$$\text{(b) } P_{\text{av}} = \frac{E}{t} = \frac{7.20 \times 10^{13} \text{ J}}{4.00 \times 10^{-6} \text{ s}} = 1.80 \times 10^{19} \text{ W}$$

$$\text{(c) } E = \Delta U = mg\Delta y. \quad m = \frac{E}{g\Delta y} = \frac{7.20 \times 10^{13} \text{ J}}{(9.80 \text{ m/s}^2)(1.00 \times 10^3 \text{ m})} = 7.35 \times 10^9 \text{ kg}$$

37.57. IDENTIFY and SET UP: In crown glass the speed of light is $v = \frac{c}{n}$. Calculate the kinetic energy of an electron that has this speed.

$$\text{EXECUTE: } v = \frac{2.998 \times 10^8 \text{ m/s}}{1.52} = 1.972 \times 10^8 \text{ m/s.}$$

$$K = mc^2(\gamma - 1)$$

$$mc^2 = (9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 8.187 \times 10^{-14} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 0.5111 \text{ MeV}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-((1.972 \times 10^8 \text{ m/s})/(2.998 \times 10^8 \text{ m/s}))^2}} = 1.328$$

$$K = mc^2(\gamma - 1) = (0.5111 \text{ MeV})(1.328 - 1) = 0.168 \text{ MeV}$$

EVALUATE: No object can travel faster than the speed of light in vacuum but there is nothing that prohibits an object from traveling faster than the speed of light in some material.

37.58. (a) $v = \frac{p}{m} = \frac{(E/c)}{m} = \frac{E}{mc}$, where the atom and the photon have the same magnitude of momentum, E/c .

$$\text{(b) } v = \frac{E}{mc} \ll c, \text{ so } E \ll mc^2.$$

37.59. IDENTIFY and SET UP: Let S be the lab frame and S' be the frame of the proton that is moving in the $+x$ direction, so $u = +c/2$. The reference frames and moving particles are shown in Figure 37.59. The other proton moves in

the $-x$ direction in the lab frame, so $v = -c/2$. A proton has rest mass $m_p = 1.67 \times 10^{-27}$ kg and rest energy $m_p c^2 = 938$ MeV.

EXECUTE: (a) $v' = \frac{v-u}{1-uv/c^2} = \frac{-c/2 - c/2}{1-(c/2)(-c/2)/c^2} = -\frac{4c}{5}$

The speed of each proton relative to the other is $\frac{4}{5}c$.

(b) In nonrelativistic mechanics the speeds just add and the speed of each relative to the other is c .

(c) $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$

(i) Relative to the lab frame each proton has speed $v = c/2$. The total kinetic energy of each proton is

$$K = \frac{938 \text{ MeV}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} - (938 \text{ MeV}) = 145 \text{ MeV}.$$

(ii) In its rest frame one proton has zero speed and zero kinetic energy and the other has speed $\frac{4}{5}c$. In this frame

the kinetic energy of the moving proton is $K = \frac{938 \text{ MeV}}{\sqrt{1-\left(\frac{4}{5}\right)^2}} - (938 \text{ MeV}) = 625 \text{ MeV}$

(d) (i) Each proton has speed $v = c/2$ and kinetic energy

$$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}m\right)(c/2)^2 = \frac{mc^2}{8} = \frac{938 \text{ MeV}}{8} = 117 \text{ MeV}$$

(ii) One proton has speed $v = 0$ and the other has speed c . The kinetic energy of the moving proton

$$\text{is } K = \frac{1}{2}mc^2 = \frac{938 \text{ MeV}}{2} = 469 \text{ MeV}$$

EVALUATE: The relativistic expression for K gives a larger value than the nonrelativistic expression. The kinetic energy of the system is different in different frames.

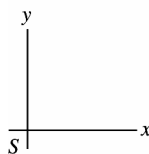
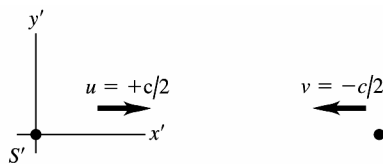


Figure 37.59

37.60. IDENTIFY and SET UP: Let S be the lab frame and let S' the frame of the proton that is moving in the $+x$ direction in the lab frame, as shown in Figure 37.60. In S' the other proton moves in the $-x'$ direction with speed $c/2$, so $v' = -c/2$. In the lab frame each proton has speed αc , where α is a constant that we need to solve for.

EXECUTE: (a) $v = \frac{v' + u}{1 + uv'/c^2}$ with $v = -\alpha c$, $u = +\alpha c$ and $v' = -0.50c$ gives $-\alpha c = \frac{-0.50c + \alpha c}{1 + (\alpha c)(-0.50c)/c^2}$ and

$$-\alpha = \frac{-0.50 + \alpha}{1 - 0.50\alpha}. \quad \alpha^2 - 4\alpha + 1 = 0 \text{ and } \alpha = 0.268 \text{ or } \alpha = 3.73. \text{ Can't have } v > c, \text{ so only } \alpha = 0.268 \text{ is physically}$$

allowed. The speed measured by the observer in the lab is $0.268c$.

(b) (i) $v = 0.269c$. $\gamma = 1.0380$. $K = (\gamma - 1)mc^2 = 35.6 \text{ MeV}$.

(ii) $v = 0.500c$. $\gamma = 1.1547$. $K = (\gamma - 1)mc^2 = 145 \text{ MeV}$.

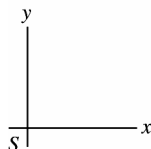
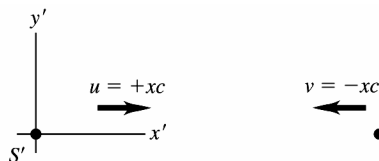


Figure 37.60

37.61. $x'^2 = c^2 t'^2 \Rightarrow (x - ut)^2 \gamma^2 = c^2 \gamma^2 (t - ux/c^2)^2$

$$\Rightarrow x - ut = c(t - ux/c^2) \Rightarrow x \left(1 + \frac{u}{c}\right) = \frac{1}{c} x(u + c) = t(u + c) \Rightarrow x = ct \Rightarrow x^2 = c^2 t^2.$$

37.62. **IDENTIFY and SET UP:** Let S be the lab frame and let S' be the frame of the nucleus. Let the $+x$ direction be the direction the nucleus is moving. $u = 0.7500c$.

EXECUTE: (a) $v' = +0.9995c$. $v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.9995c + 0.7500c}{1 + (0.7500)(0.9995)} = 0.999929c$

(b) $v' = -0.9995c$. $v = \frac{-0.9995c + 0.7500c}{1 + (0.7500)(-0.9995)} = -0.9965c$

(c) emitted in same direction:

(i) $K = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) mc^2 = (0.511 \text{ MeV}) \left(\frac{1}{\sqrt{1 - (0.999929)^2}} - 1 \right) = 42.4 \text{ MeV}$

(ii) $K' = \left(\frac{1}{\sqrt{1 - v'^2/c^2}} - 1 \right) mc^2 = (0.511 \text{ MeV}) \left(\frac{1}{\sqrt{1 - (0.9995)^2}} - 1 \right) = 15.7 \text{ MeV}$

(d) emitted in opposite direction:

(i) $K = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) mc^2 = (0.511 \text{ MeV}) \left(\frac{1}{\sqrt{1 - (0.9965)^2}} - 1 \right) = 5.60 \text{ MeV}$

(ii) $K' = \left(\frac{1}{\sqrt{1 - v'^2/c^2}} - 1 \right) mc^2 = (0.511 \text{ MeV}) \left(\frac{1}{\sqrt{1 - (0.9995)^2}} - 1 \right) = 15.7 \text{ MeV}$

37.63. **IDENTIFY and SET UP:** Use Eq.(37.30), with $a = dv/dt$, to obtain an expression for dv/dt . Separate the variables v and t and integrate to obtain an expression for $v(t)$. In this expression, let $t \rightarrow \infty$.

EXECUTE: $a = \frac{dv}{dt} = \frac{F}{m} (1 - v^2/c^2)^{3/2}$. (One-dimensional motion is assumed, and all the F , v , and a refer to x -components.)

$$\frac{dv}{(1 - v^2/c^2)^{3/2}} = \left(\frac{F}{m} \right) dt$$

Integrate from $t = 0$, when $v = 0$, to time t , when the velocity is v .

$$\int_0^v \frac{dv}{(1 - v^2/c^2)^{3/2}} = \int_0^t \left(\frac{F}{m} \right) dt$$

Since F is constant, $\int_0^t \left(\frac{F}{m} \right) dt = \frac{Ft}{m}$. In the velocity integral make the change of variable $y = v/c$; then $dy = dv/c$.

$$\int_0^v \frac{dv}{(1 - v^2/c^2)^{3/2}} = c \int_0^{v/c} \frac{dy}{(1 - y^2)^{3/2}} = c \left[\frac{y}{(1 - y^2)^{1/2}} \right]_0^{v/c} = \frac{v}{\sqrt{1 - v^2/c^2}}$$

Thus $\frac{v}{\sqrt{1 - v^2/c^2}} = \frac{Ft}{m}$.

Solve this equation for v :

$$\frac{v^2}{1-v^2/c^2} = \left(\frac{Ft}{m}\right)^2 \text{ and } v^2 = \left(\frac{Ft}{m}\right)^2 (1-v^2/c^2)$$

$$v^2 \left(1 + \left(\frac{Ft}{mc}\right)^2\right) = \left(\frac{Ft}{m}\right)^2 \text{ so } v = \frac{(Ft/m)}{\sqrt{1 + (Ft/mc)^2}} = c \frac{Ft}{\sqrt{m^2 c^2 + F^2 t^2}}$$

$$\text{As } t \rightarrow \infty, \frac{Ft}{\sqrt{m^2 c^2 + F^2 t^2}} \rightarrow \frac{Ft}{\sqrt{F^2 t^2}} \rightarrow 1, \text{ so } v \rightarrow c.$$

EVALUATE: Note that $\frac{Ft}{\sqrt{m^2 c^2 + F^2 t^2}}$ is always less than 1, so $v < c$ always and v approaches c only when $t \rightarrow \infty$.

37.64. Setting $x = 0$ in Eq.(37.21), the first equation becomes $x' = -\gamma ut$ and the last, upon multiplication by c , becomes $ct' = \gamma ct$. Squaring and subtracting gives $c^2 t'^2 - x'^2 = \gamma^2 (c^2 t^2 - u^2 t^2) = c^2 t^2$, or $x' = c\sqrt{t'^2 - t^2} = 4.53 \times 10^8 \text{ m}$.

37.65. (a) IDENTIFY and SET UP: Use the Lorentz coordinate transformation (Eq.37.21) for (x_1, t_1) and (x_2, t_2) :

$$x'_1 = \frac{x_1 - ut_1}{\sqrt{1 - u^2/c^2}}, \quad x'_2 = \frac{x_2 - ut_2}{\sqrt{1 - u^2/c^2}}$$

$$t'_1 = \frac{t_1 - ux_1/c^2}{\sqrt{1 - u^2/c^2}}, \quad t'_2 = \frac{t_2 - ux_2/c^2}{\sqrt{1 - u^2/c^2}}$$

Same point in S' implies $x'_1 = x'_2$. What then is $\Delta t' = t'_2 - t'_1$?

EXECUTE: $x'_1 = x'_2$ implies $x_1 - ut_1 = x_2 - ut_2$

$$u(t_2 - t_1) = x_2 - x_1 \text{ and } u = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

From the time transformation equations,

$$\Delta t' = t'_2 - t'_1 = \frac{1}{\sqrt{1 - u^2/c^2}} (\Delta t - u\Delta x/c^2)$$

Using the result that $u = \frac{\Delta x}{\Delta t}$ gives

$$\Delta t' = \frac{1}{\sqrt{1 - (\Delta x)^2/((\Delta t)^2 c^2)}} (\Delta t - (\Delta x)^2/((\Delta t) c^2))$$

$$\Delta t' = \frac{\Delta t}{\sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}} (\Delta t - (\Delta x)^2/((\Delta t) c^2))$$

$$\Delta t' = \frac{(\Delta t)^2 - (\Delta x)^2/c^2}{\sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}} = \sqrt{(\Delta t)^2 - (\Delta x/c)^2}, \text{ as was to be shown.}$$

This equation doesn't have a physical solution (because of a negative square root) if $(\Delta x/c)^2 > (\Delta t)^2$ or $\Delta x \geq c\Delta t$.

(b) IDENTIFY and SET UP: Now require that $t'_2 = t'_1$ (the two events are simultaneous in S') and use the Lorentz coordinate transformation equations.

EXECUTE: $t'_2 = t'_1$ implies $t_1 - ux_1/c^2 = t_2 - ux_2/c^2$

$$t_2 - t_1 = \left(\frac{x_2 - x_1}{c^2}\right)u \text{ so } \Delta t = \left(\frac{\Delta x}{c^2}\right)u \text{ and } u = \frac{c^2 \Delta t}{\Delta x}$$

From the Lorentz transformation equations,

$$\Delta x' = x'_2 - x'_1 = \left(\frac{1}{\sqrt{1 - u^2/c^2}}\right)(\Delta x - u\Delta t).$$

Using the result that $u = c^2 \Delta t / \Delta x$ gives

$$\Delta x' = \frac{1}{\sqrt{1 - c^2(\Delta t)^2/(\Delta x)^2}} (\Delta x - c^2(\Delta t)^2/\Delta x)$$

$$\Delta x' = \frac{\Delta x}{\sqrt{(\Delta x)^2 - c^2(\Delta t)^2}} (\Delta x - c^2(\Delta t)^2/\Delta x)$$

$$\Delta x' = \frac{(\Delta x)^2 - c^2(\Delta t)^2}{\sqrt{(\Delta x)^2 - c^2(\Delta t)^2}} = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$$

(c) IDENTIFY and SET UP: The result from part (b) is $\Delta x' = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$

Solve for Δt : $(\Delta x')^2 = (\Delta x)^2 - c^2(\Delta t)^2$

$$\text{EXECUTE: } \Delta t = \frac{\sqrt{(\Delta x)^2 - (\Delta x')^2}}{c} = \frac{\sqrt{(5.00 \text{ m})^2 - (2.50 \text{ m})^2}}{2.998 \times 10^8 \text{ m/s}} = 1.44 \times 10^{-8} \text{ s}$$

EVALUATE: This provides another illustration of the concept of simultaneity (Section 37.2): events observed to be simultaneous in one frame are not simultaneous in another frame that is moving relative to the first.

37.66. (a) 80.0 m/s is non-relativistic, and $K = \frac{1}{2}mv^2 = 186 \text{ J}$.

(b) $(\gamma - 1)mc^2 = 1.31 \times 10^{15} \text{ J}$.

(c) In Eq. (37.23), c $v' = 2.20 \times 10^8 \text{ m/s}$, $u = -1.80 \times 10^8 \text{ m/s}$, and so $v = 7.14 \times 10^7 \text{ m/s}$.

(d) $\frac{20.0 \text{ m}}{\gamma} = 13.6 \text{ m}$.

(e) $\frac{20.0 \text{ m}}{2.20 \times 10^8 \text{ m/s}} = 9.09 \times 10^{-8} \text{ s}$.

(f) $t' = \frac{t}{\gamma} = 6.18 \times 10^{-8} \text{ s}$, or $t' = \frac{13.6 \text{ m}}{2.20 \times 10^8 \text{ m/s}} = 6.18 \times 10^{-8} \text{ s}$.

37.67. IDENTIFY and SET UP: An increase in wavelength corresponds to a decrease in frequency ($f = c/\lambda$), so the

atoms are moving away from the earth. Receding, so use Eq.(37.26): $f = \sqrt{\frac{c-u}{c+u}} f_0$

EXECUTE: Solve for u : $(f/f_0)^2(c+u) = c-u$ and $u = c \left(\frac{1-(f/f_0)^2}{1+(f/f_0)^2} \right)$

$f = c/\lambda$, $f_0 = c/\lambda_0$ so $f/f_0 = \lambda_0/\lambda$

$u = c \left(\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2} \right) = c \left(\frac{1-(656.3/953.4)^2}{1+(656.3/953.4)^2} \right) = 0.357c = 1.07 \times 10^8 \text{ m/s}$

EVALUATE: The relative speed is large, 36% of c . The cosmological implication of such observations will be discussed in Section 44.6.

37.68. The baseball had better be moving non-relativistically, so the Doppler shift formula (Eq.(37.25)) becomes $f \equiv f_0(1-(u/c))$. In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at f . But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is $f(1-(u/c)) = f_0(1-(u/c))^2 \approx f_0(1-2(u/c))$, so $\Delta f = 2f_0(u/c)$ and the fractional frequency shift is

$\frac{\Delta f}{f_0} = 2(u/c)$. In this case,

$u = \frac{\Delta f}{2f_0} c = \frac{(2.86 \times 10^{-7})}{2} (3.00 \times 10^8 \text{ m/s}) = 42.9 \text{ m/s} = 154 \text{ km/h} = 92.5 \text{ mi/h}$.

37.69. IDENTIFY and SET UP: 500 light years $= 4.73 \times 10^{18} \text{ m}$. The proper distance l_0 to the star is 500 light years. The energy needed is the kinetic energy of the rocket at its final speed.

EXECUTE: (a) $u = 0.50c$. $\Delta t = \frac{d}{u} = \frac{4.73 \times 10^{18} \text{ m}}{(0.50)(3.00 \times 10^8 \text{ m/s})} = 3.2 \times 10^{10} \text{ s} = 1000 \text{ yr}$

The proper time is measured by the astronauts. $\Delta t_0 = \Delta t \sqrt{1-u^2/c^2} = 866 \text{ yr}$

$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (1000 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1}{\sqrt{1-(0.500)^2}} - 1 \right) = 1.4 \times 10^{19} \text{ J}$

This is 140% of the U.S. yearly use of energy.

(b) $u = 0.99c$. $\Delta t = \frac{d}{u} = \frac{4.73 \times 10^{18} \text{ m}}{(0.99)(3.00 \times 10^8 \text{ m/s})} = 1.6 \times 10^{10} \text{ s} = 505 \text{ yr}$, $\Delta t_0 = 71 \text{ yr}$

$K = (9.00 \times 10^{19} \text{ J}) \left(\frac{1}{\sqrt{1-(0.99)^2}} - 1 \right) = 5.5 \times 10^{20} \text{ J}$

This is 55 times the U.S. yearly use.

$$(c) \quad u = 0.9999c. \quad \Delta t = \frac{d}{u} = \frac{4.73 \times 10^{18} \text{ m}}{(0.9999)(3.00 \times 10^8 \text{ m/s})} = 1.58 \times 10^{10} \text{ s} = 501 \text{ yr}, \quad \Delta t_0 = 7.1 \text{ yr}$$

$$K = (9.00 \times 10^{19} \text{ J}) \left(\frac{1}{\sqrt{1 - (0.9999)^2}} - 1 \right) = 6.3 \times 10^{21} \text{ J}$$

This is 630 times the U.S. yearly use.

The energy cost of accelerating a rocket to these speeds is immense.

- 37.70.** (a) As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time $T = 1/f$ as the proper time, with the result that $f = \gamma f_0 > f_0$.

$$(b) \text{ Toward: } f = f_0 \sqrt{\frac{c+u}{c-u}} = 345 \text{ MHz} \left(\frac{1+0.758}{1-0.758} \right)^{1/2} = 930 \text{ MHz}$$

$$f - f_0 = 930 \text{ MHz} - 345 \text{ MHz} = 585 \text{ MHz.}$$

$$\text{Away: } f = f_0 \sqrt{\frac{c-u}{c+u}} = 345 \text{ MHz} \left(\frac{1-0.758}{1+0.758} \right)^{1/2} = 128 \text{ MHz and } f - f_0 = -217 \text{ MHz.}$$

- (c) $\gamma f_0 = 1.53 f_0 = 528 \text{ MHz}$, $f - f_0 = 183 \text{ MHz}$. The shift is still bigger than f_0 , but not as large as the approaching frequency.

- 37.71.** The crux of this problem is the question of simultaneity. To be “in the barn at one time” for the runner is different than for a stationary observer in the barn. The diagram in Figure 37.71a shows the rod fitting into the barn at time $t = 0$, according to the stationary observer. The diagram in Figure 37.71b is in the runner’s frame of reference. The front of the rod enters the barn at time t_1 and leaves the back of the barn at time t_2 . However, the back of the rod does not enter the front of the barn until the later time t_3 .

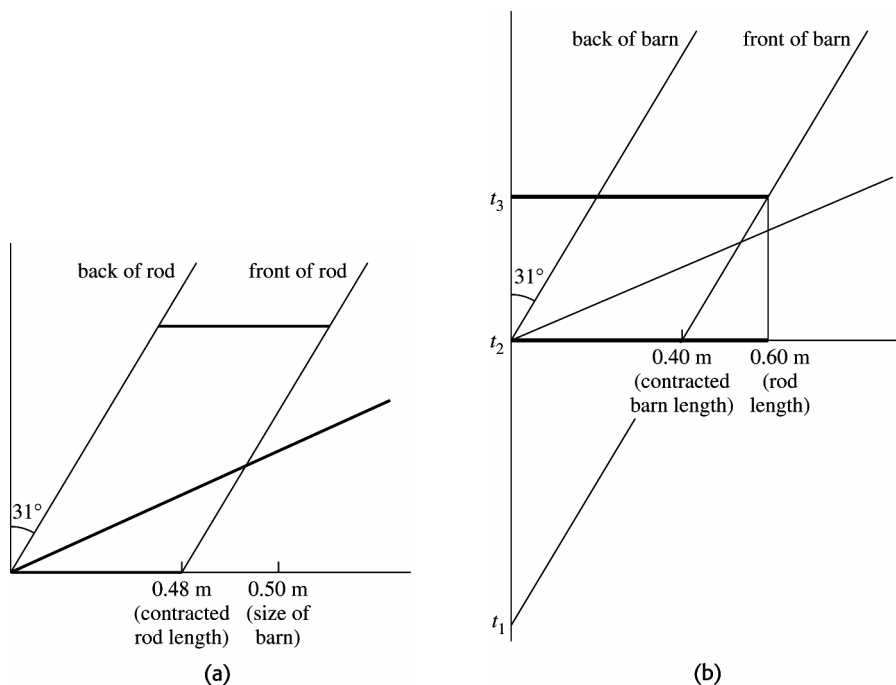


Figure 37.71

- 37.72.** In Eq.(37.23), $u = V$, $v' = (c/n)$, and so $v = \frac{(c/n) + V}{1 + \frac{cV}{nc^2}} = \frac{(c/n) + V}{1 + (V/nc)}$. For V non-relativistic, this is

$$v \approx ((cn) + V)(1 - (V/nc)) = (nc/n) + V - (V/n^2) - (V^2/nc) \approx \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)V, \text{ so } k = \left(1 - \frac{1}{n^2}\right). \text{ For water, } n = 1.333 \text{ and } k = 0.437.$$

$$37.73. \quad (\text{a}) \quad a' = \frac{dv}{dt'}. \quad dt' = \gamma(dt - udx/c^2). \quad dv' = \frac{dv}{(1-uv/c^2)} + \frac{v-u}{(1-uv/c^2)^2} \frac{u}{c^2} dv$$

$$\frac{dv'}{dv} = \frac{1}{1-uv/c^2} + \frac{v-u}{(1-uv/c^2)^2} \left(\frac{u}{c^2} \right).$$

$$dv' = dv \left(\frac{1}{1-uv/c^2} + \frac{(v-u)u/c^2}{(1-uv/c^2)^2} \right) = dv \left(\frac{1-u^2/c^2}{(1-uv/c^2)^2} \right)$$

$$a' = \frac{dv \frac{(1-u^2/c^2)}{(1-uv/c^2)^2}}{\gamma dt - u\gamma dx/c^2} = \frac{dv}{dt} \frac{(1-u^2/c^2)}{(1-uv/c^2)^2} \frac{1}{\gamma(1-uv/c^2)}$$

$$= a(1-u^2/c^2)^{3/2} (1-uv/c^2)^{-3}.$$

$$(\text{b}) \text{ Changing frames from } S' \rightarrow S \text{ just involves changing } a \rightarrow a', v \rightarrow -v' \Rightarrow a = a'(1-u^2/c^2)^{3/2} \left(1 + \frac{uv'}{c^2} \right)^{-3}.$$

37.74. (a) The speed v' is measured relative to the rocket, and so for the rocket and its occupant, $v' = 0$. The acceleration as seen in the rocket is given to be $a' = g$, and so the acceleration as measured on the earth is

$$a = \frac{du}{dt} = g \left(1 - \frac{u^2}{c^2} \right)^{3/2}.$$

(b) With $v_1 = 0$ when $t = 0$,

$$dt = \frac{1}{g} \frac{du}{(1-u^2/c^2)^{3/2}}. \quad \int_0^{t_1} dt = \frac{1}{g} \int_0^{v_1} \frac{du}{(1-u^2/c^2)^{3/2}}. \quad t_1 = \frac{v_1}{g\sqrt{1-v_1^2/c^2}}.$$

(c) $dt' = \gamma dt = dt / \sqrt{1-u^2/c^2}$, so the relation in part (b) between dt and du , expressed in terms of dt' and du , is

$$dt' = \gamma dt = \frac{1}{\sqrt{1-u^2/c^2}} \frac{du}{g(1-u^2/c^2)^{3/2}} = \frac{1}{g} \frac{du}{(1-u^2/c^2)^2}.$$

Integrating as above (perhaps using the substitution $z = u/c$) gives $t'_1 = \frac{c}{g} \operatorname{arctanh} \left(\frac{v_1}{c} \right)$. For those who wish to avoid inverse hyperbolic functions, the above integral may be done by the method of partial fractions;

$$g dt' = \frac{du}{(1+u/c)(1-u/c)} = \frac{1}{2} \left[\frac{du}{1+u/c} + \frac{du}{1-uc} \right], \text{ which integrates to } t'_1 = \frac{c}{2g} \ln \left(\frac{c+v_1}{c-v_1} \right).$$

(d) Solving the expression from part (c) for v_1 in terms of t_1 , $(v_1/c) = \tanh(gt'_1/c)$, so that

$$\sqrt{1-(v_1/c)^2} = 1/\cosh(gt'_1/c), \text{ using the appropriate identities for hyperbolic functions. Using this in the expression}$$

$$\text{found in part (b), } t_1 = \frac{c}{g} \frac{\tanh(gt'_1/c)}{1/\cosh(gt'_1/c)} = \frac{c}{g} \sinh(gt'_1/c), \text{ which may be rearranged slightly as } \frac{gt_1}{c} = \sinh \left(\frac{gt'_1}{c} \right). \text{ If}$$

hyperbolic functions are not used, v_1 in terms of t'_1 is found to be $\frac{v_1}{c} = \frac{e^{gt'_1/c} - e^{-gt'_1/c}}{e^{gt'_1/c} + e^{-gt'_1/c}}$ which is the same as

$$\tanh(gt'_1/c). \text{ Inserting this expression into the result of part (b) gives, after much algebra, } t_1 = \frac{c}{2g} (e^{gt'_1/c} - e^{-gt'_1/c}),$$

which is equivalent to the expression found using hyperbolic functions.

(e) After the first acceleration period (of 5 years by Stella's clock), the elapsed time on earth is

$$t'_1 = \frac{c}{g} \sinh(gt'_1/c) = 2.65 \times 10^9 \text{ s} = 84.0 \text{ yr}.$$

The elapsed time will be the same for each of the four parts of the voyage, so when Stella has returned, Terra has aged 336 yr and the year is 2436. (Keeping more precision than is given in the problem gives February 7 of that year.)

$$37.75. \quad (\text{a}) \quad f_0 = 4.568110 \times 10^{14} \text{ Hz}; f_+ = 4.568910 \times 10^{14} \text{ Hz}; f_- = 4.567710 \times 10^{14} \text{ Hz}$$

$$\left. \begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}} f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}} f_0 \end{aligned} \right\} \Rightarrow \begin{aligned} f_+^2(c-(u+v)) &= f_0^2(c+(u+v)) \\ f_-^2(c-(u-v)) &= f_0^2(c+(u-v)) \end{aligned}$$

where u is the velocity of the center of mass and v is the orbital velocity.

$$\Rightarrow (u+v) = \frac{(f_+/f_0)^2 - 1}{(f_+/f_0)^2 + 1} c \quad \text{and} \quad (u-v) = \frac{(f_-^2/f_0^2) - 1}{(f_-^2/f_0^2) + 1} c$$

$$\Rightarrow u+v = 5.25 \times 10^4 \text{ m/s} \quad \text{and} \quad u-v = -2.63 \times 10^4 \text{ m/s}.$$

This gives $u = +1.31 \times 10^4 \text{ m/s}$ (moving toward at 13.1 km/s) and $v = 3.94 \times 10^4 \text{ m/s}$.

(b) $v = 3.94 \times 10^4 \text{ m/s}$; $T = 11.0 \text{ days}$. $2\pi R = vt \Rightarrow$

$$R = \frac{(3.94 \times 10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} = 5.96 \times 10^9 \text{ m}.$$

This is about 0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} = \frac{4(5.96 \times 10^9 \text{ m})(3.94 \times 10^4 \text{ m/s})^2}{6.672 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 5.55 \times 10^{29} \text{ kg} = 0.279 m_{\text{sun}}.$$

37.76. For any function $f = f(x, t)$ and $x = x(x', t')$, $t = t(x', t')$, let $F(x', t') = f(x(x', t'), t(x', t'))$ and use the standard (but mathematically improper) notation $F(x', t') = f(x', t')$. The chain rule is then

$$\begin{aligned} \frac{\partial f(x', t')}{\partial x} &= \frac{\partial f(x, t)}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial f(x', t')}{\partial t'} \frac{\partial t'}{\partial x}, \\ \frac{\partial f(x', t')}{\partial t} &= \frac{\partial f(x, t)}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial f(x', t')}{\partial t'} \frac{\partial t'}{\partial t}. \end{aligned}$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is suppressed, and the

above relations are then $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'} \frac{\partial t'}{\partial x}$, $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial f}{\partial t'} \frac{\partial t'}{\partial t}$.

(a) $\frac{\partial x'}{\partial x} = 1$, $\frac{\partial x'}{\partial t} = -v$, $\frac{\partial t'}{\partial x} = 0$ and $\frac{\partial t'}{\partial t} = 1$. Then, $\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}$, and $\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$. For the time derivative,

$\frac{\partial E}{\partial t} = -v \frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}$. To find the second time derivative, the chain rule must be applied to both terms; that is,

$$\begin{aligned} \frac{\partial}{\partial t} \frac{\partial E}{\partial x'} &= -v \frac{\partial^2 E}{\partial x'^2} + \frac{\partial^2 E}{\partial t' \partial x'}, \\ \frac{\partial}{\partial t} \frac{\partial E}{\partial t'} &= -v \frac{\partial^2 E}{\partial x' \partial t'} + \frac{\partial^2 E}{\partial t'^2}. \end{aligned}$$

Using these in $\frac{\partial^2 E}{\partial t^2}$, collecting terms and equating the mixed partial derivatives gives

$$\frac{\partial^2 E}{\partial t^2} = v^2 \frac{\partial^2 E}{\partial x'^2} - 2v \frac{\partial^2 E}{\partial x' \partial t'} + \frac{\partial^2 E}{\partial t'^2}, \quad \text{and using this and the above expression for } \frac{\partial^2 E}{\partial x'^2} \text{ gives the result.}$$

(b) For the Lorentz transformation, $\frac{\partial x'}{\partial x} = \gamma$, $\frac{\partial x'}{\partial t} = \gamma v$, $\frac{\partial t'}{\partial x} = \gamma v/c^2$ and $\frac{\partial t'}{\partial t} = \gamma$.

The first partials are then

$$\frac{\partial E}{\partial x} = \gamma \frac{\partial E}{\partial x'} - \gamma \frac{v}{c^2} \frac{\partial E}{\partial t'}, \quad \frac{\partial E}{\partial t} = -\gamma v \frac{\partial E}{\partial x'} + \gamma \frac{\partial E}{\partial t'}$$

and the second partials are (again equating the mixed partials)

$$\begin{aligned} \frac{\partial^2 E}{\partial x^2} &= \gamma^2 \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \frac{v^2}{c^4} \frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 \frac{v}{c^2} \frac{\partial^2 E}{\partial x' \partial t'}, \\ \frac{\partial^2 E}{\partial t^2} &= \gamma^2 v^2 \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v \frac{\partial^2 E}{\partial x' \partial t'}. \end{aligned}$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \gamma^2 \left(1 - \frac{v^2}{c^2} \right) \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \left(\frac{v^2}{c^4} - \frac{1}{c^2} \right) \frac{\partial^2 E}{\partial t'^2} = \frac{\partial^2 E}{\partial x'^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t'^2} = 0.$$

- 37.77.** (a) In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons $\Rightarrow 2(\gamma_{\text{cm}} - 1)m_p c^2 =$

$$2m_k c^2 \Rightarrow \gamma_{\text{cm}} = 1 + \frac{m_k}{m_p} = 1.526. \text{ The velocity of a proton in the center of momentum frame is then}$$

$$v_{\text{cm}} = c \sqrt{\frac{\gamma_{\text{cm}}^2 - 1}{\gamma_{\text{cm}}^2}} = 0.7554c.$$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as “hopping” into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v' + u}{1 + \frac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1 + \frac{v_{\text{cm}}^2}{c^2}} = 0.9619c \Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1 - \frac{v_{\text{lab}}^2}{c^2}}} = 3.658 \Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}} - 1)m_p c^2 = 2494 \text{ MeV}.$$

(b) $\frac{K_{\text{lab}}}{2m_k} = \frac{2494 \text{ MeV}}{2(493.7 \text{ MeV})} = 2.526.$

(c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required is just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7 \text{ MeV}) = 987.4 \text{ MeV}$. This offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.

PHOTONS, ELECTRONS, AND ATOMS

38.1. IDENTIFY and SET UP: The stopping potential V_0 is related to the frequency of the light by $V_0 = \frac{h}{e}f - \frac{\phi}{e}$. The slope of V_0 versus f is h/e . The value f_{th} of f when $V_0 = 0$ is related to ϕ by $\phi = hf_{th}$.

EXECUTE: (a) From the graph, $f_{th} = 1.25 \times 10^{15}$ Hz. Therefore, with the value of h from part (b), $\phi = hf_{th} = 4.8$ eV.

(b) From the graph, the slope is 3.8×10^{-15} V · s. $h = (e)(\text{slope}) = (1.60 \times 10^{-16} \text{ C})(3.8 \times 10^{-15} \text{ V} \cdot \text{s}) = 6.1 \times 10^{-34} \text{ J} \cdot \text{s}$

(c) No photoelectrons are produced for $f < f_{th}$.

(d) For a different metal f_{th} and ϕ are different. The slope is h/e so would be the same, but the graph would be shifted right or left so it has a different intercept with the horizontal axis.

EVALUATE: As the frequency f of the light is increased above f_{th} the energy of the photons in the light increases and more energetic photons are produced. The work function we calculated is similar to that for gold or nickel.

38.2. IDENTIFY and SET UP: $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00 \times 10^8$ m/s. $1 \text{ eV} = 1.60 \times 10^{-16} \text{ J}$.

EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{505 \times 10^{-9} \text{ m}} = 5.94 \times 10^{14} \text{ Hz}$

(b) $E = hf = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(5.94 \times 10^{14} \text{ Hz}) = 3.94 \times 10^{-19} \text{ J} = 2.46 \text{ eV}$

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.94 \times 10^{-19} \text{ J})}{9.5 \times 10^{-15} \text{ kg}}} = 9.1 \text{ mm/s}$

38.3. $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{5.20 \times 10^{-7} \text{ m}} = 5.77 \times 10^{14} \text{ Hz}$

$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{5.20 \times 10^{-7} \text{ m}} = 1.28 \times 10^{-27} \text{ kg} \cdot \text{m/s}$

$E = pc = (1.28 \times 10^{-27} \text{ kg} \cdot \text{m/s})(3.00 \times 10^8 \text{ m/s}) = 3.84 \times 10^{-19} \text{ J} = 2.40 \text{ eV}$.

38.4. IDENTIFY and SET UP: $P_{av} = \frac{\text{energy}}{t}$. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. For a photon, $E = hf = \frac{hc}{\lambda}$. $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$.

EXECUTE: (a) energy = $P_{av}t = (0.600 \text{ W})(20.0 \times 10^{-3} \text{ s}) = 1.20 \times 10^{-2} \text{ J} = 7.5 \times 10^{16} \text{ eV}$

(b) $E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{652 \times 10^{-9} \text{ m}} = 3.05 \times 10^{-19} \text{ J} = 1.91 \text{ eV}$

(c) The number of photons is the total energy in a pulse divided by the energy of one photon:

$$\frac{1.20 \times 10^{-2} \text{ J}}{3.05 \times 10^{-19} \text{ J/photon}} = 3.93 \times 10^{16} \text{ photons}.$$

EVALUATE: The number of photons in each pulse is very large.

38.5. IDENTIFY and SET UP: Eq.(38.2) relates the photon energy and wavelength. $c = f\lambda$ relates speed, frequency and wavelength for an electromagnetic wave.

EXECUTE: (a) $E = hf$ so $f = \frac{E}{h} = \frac{(2.45 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/1 eV})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = 5.92 \times 10^{20} \text{ Hz}$

(b) $c = f\lambda$ so $\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{5.92 \times 10^{20} \text{ Hz}} = 5.06 \times 10^{-13} \text{ m}$

(c) **EVALUATE:** λ is comparable to a nuclear radius. Note that in doing the calculation the energy in MeV was converted to the SI unit of Joules.

38.6. IDENTIFY and SET UP: $\lambda_{\text{th}} = 272 \text{ nm}$. $c = f\lambda$. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$. At the threshold frequency, f_{th} , $v_{\text{max}} \rightarrow 0$.
 $h = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$.

EXECUTE: (a) $f_{\text{th}} = \frac{c}{\lambda_{\text{th}}} = \frac{3.00 \times 10^8 \text{ m/s}}{272 \times 10^{-9} \text{ m}} = 1.10 \times 10^{15} \text{ Hz}$.

(b) $\phi = hf_{\text{th}} = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(1.10 \times 10^{15} \text{ Hz}) = 4.55 \text{ eV}$.

(c) $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(1.45 \times 10^{15} \text{ Hz}) - 4.55 \text{ eV} = 6.00 \text{ eV} - 4.55 \text{ eV} = 1.45 \text{ eV}$

EVALUATE: The threshold wavelength depends on the work function for the surface.

38.7. IDENTIFY and SET UP: Eq.(38.3): $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \frac{hc}{\lambda} - \phi$. Take the work function ϕ from Table 38.1. Solve for v_{max} . Note that we wrote f as c/λ .

EXECUTE: $\frac{1}{2}mv_{\text{max}}^2 = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{235 \times 10^{-9} \text{ m}} - (5.1 \text{ eV})(1.602 \times 10^{-19} \text{ J/1 eV})$

$\frac{1}{2}mv_{\text{max}}^2 = 8.453 \times 10^{-19} \text{ J} - 8.170 \times 10^{-19} \text{ J} = 2.83 \times 10^{-20} \text{ J}$

$v_{\text{max}} = \sqrt{\frac{2(2.83 \times 10^{-20} \text{ J})}{9.109 \times 10^{-31} \text{ kg}}} = 2.49 \times 10^5 \text{ m/s}$

EVALUATE: The work function in eV was converted to joules for use in Eq.(38.3). A photon with $\lambda = 235 \text{ nm}$ has energy greater than the work function for the surface.

38.8. IDENTIFY and SET UP: $\phi = hf_{\text{th}} = \frac{hc}{\lambda_{\text{th}}}$. The minimum ϕ corresponds to the minimum λ .

EXECUTE: $\phi = \frac{hc}{\lambda_{\text{th}}} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{700 \times 10^{-9} \text{ m}} = 1.77 \text{ eV}$

38.9. IDENTIFY and SET UP: $c = f\lambda$. The source emits $(0.05)(75 \text{ J}) = 3.75 \text{ J}$ of energy as visible light each second. $E = hf$, with $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$.

EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{600 \times 10^{-9} \text{ m}} = 5.00 \times 10^{14} \text{ Hz}$

(b) $E = hf = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(5.00 \times 10^{14} \text{ Hz}) = 3.32 \times 10^{-19} \text{ J}$. The number of photons emitted per second is
 $\frac{3.75 \text{ J}}{3.32 \times 10^{-19} \text{ J/photon}} = 1.13 \times 10^{19} \text{ photons}$.

(c) No. The frequency of the light depends on the energy of each photon. The number of photons emitted per second is proportional to the power output of the source.

38.10. IDENTIFY: In the photoelectric effect, the energy of the photon is used to eject an electron from the surface, and any excess energy goes into kinetic energy of the electron.

SET UP: The energy of a photon is $E = hf$, and the work function is given by $\phi = hf_0$, where f_0 is the threshold frequency.

EXECUTE: (a) From the graph, we see that $K_{\text{max}} = 0$ when $\lambda = 250 \text{ nm}$, so the threshold wavelength is 250 nm. Calling f_0 the threshold frequency, we have

$$f_0 = c/\lambda_0 = (3.00 \times 10^8 \text{ m/s})/(250 \text{ nm}) = 1.2 \times 10^{15} \text{ Hz}$$

(b) $\phi = hf_0 = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(1.2 \times 10^{15} \text{ Hz}) = 4.96 \text{ eV} = 5.0 \text{ eV}$

(c) The graph (see Figure 38.10) is linear for $\lambda < \lambda_0$ ($1/\lambda > 1/\lambda_0$), and linear graphs are easier to interpret than curves.

EVALUATE: If the wavelength of the light is longer than the threshold wavelength (that is, if $1/\lambda < 1/\lambda_0$), the kinetic energy of the electrons is really not defined since no photoelectrons are ejected from the metal.

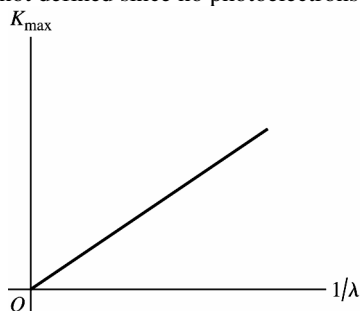


Figure 38.10

38.11. IDENTIFY: Protons have mass and photons are massless.

(a) SET UP: For a particle with mass, $K = p^2 / 2m$.

EXECUTE: $p_2 = 2p_1$ means $K_2 = 4K_1$.

(b) SET UP: For a photon, $E = pc$.

EXECUTE: $p_2 = 2p_1$ means $E_2 = 2E_1$.

EVALUATE: The relation between E and p is different for particles with mass and particles without mass.

38.12. IDENTIFY and SET UP: $eV_0 = \frac{1}{2}mv_{\max}^2$, where V_0 is the stopping potential. The stopping potential in volts equals

$$eV_0 \text{ in electron volts. } \frac{1}{2}mv_{\max}^2 = hf - \phi.$$

EXECUTE: (a) $eV_0 = \frac{1}{2}mv_{\max}^2$ so

$$eV_0 = hf - \phi = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{250 \times 10^{-9} \text{ m}} - 2.3 \text{ eV} = 4.96 \text{ eV} - 2.3 \text{ eV} = 2.7 \text{ eV}.$$

The stopping potential is 2.7 electron volts.

(b) $\frac{1}{2}mv_{\max}^2 = 2.7 \text{ eV}$

(c) $v_{\max} = \sqrt{\frac{2(2.7 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}} = 9.7 \times 10^5 \text{ m/s}$

38.13. (a) IDENTIFY: First use Eq.(38.4) to find the work function ϕ .

SET UP: $eV_0 = hf - \phi$ so $\phi = hf - eV_0 = \frac{hc}{\lambda} - eV_0$

EXECUTE: $\phi = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{254 \times 10^{-9} \text{ m}} - (1.602 \times 10^{-19} \text{ C})(0.181 \text{ V})$

$$\phi = 7.821 \times 10^{-19} \text{ J} - 2.900 \times 10^{-20} \text{ J} = 7.531 \times 10^{-19} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 4.70 \text{ eV}$$

IDENTIFY and SET UP: The threshold frequency f_{th} is the smallest frequency that still produces photoelectrons. It corresponds to $K_{\max} = 0$ in Eq.(38.3), so $hf_{\text{th}} = \phi$.

EXECUTE: $f = \frac{c}{\lambda}$ says $\frac{hc}{\lambda_{\text{th}}} = \phi$

$$\lambda_{\text{th}} = \frac{hc}{\phi} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{7.531 \times 10^{-19} \text{ J}} = 2.64 \times 10^{-7} \text{ m} = 264 \text{ nm}$$

(b) EVALUATE: As calculated in part (a), $\phi = 4.70 \text{ eV}$. This is the value given in Table 38.1 for copper.

38.14. IDENTIFY and SET UP: A photon has zero rest mass, so its energy and momentum are related by Eq.(37.40). Eq.(38.5) then relates its momentum and wavelength.

EXECUTE: (a) $E = pc = (8.24 \times 10^{-28} \text{ kg} \cdot \text{m/s})(2.998 \times 10^8 \text{ m/s}) = 2.47 \times 10^{-19} \text{ J} = (2.47 \times 10^{-19} \text{ J})(1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 1.54 \text{ eV}$

(b) $p = \frac{h}{\lambda}$ so $\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{8.24 \times 10^{-28} \text{ kg} \cdot \text{m/s}} = 8.04 \times 10^{-7} \text{ m} = 804 \text{ nm}$

EVALUATE: This wavelength is longer than visible wavelengths; it is in the infrared region of the electromagnetic spectrum. To check our result we could verify that the same E is given by Eq.(38.2), using the λ we have calculated.

38.15. IDENTIFY and SET UP: Balmer's formula is $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$. For the H_γ spectral line $n = 5$. Once we have λ , calculate f from $f = c/\lambda$ and E from Eq.(38.2).

EXECUTE: (a) $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = R \left(\frac{25-4}{100} \right) = R \left(\frac{21}{100} \right)$.

$$\text{Thus } \lambda = \frac{100}{21R} = \frac{100}{21(1.097 \times 10^7)} \text{ m} = 4.341 \times 10^{-7} \text{ m} = 434.1 \text{ nm}.$$

(b) $f = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{4.341 \times 10^{-7} \text{ m}} = 6.906 \times 10^{14} \text{ Hz}$

$$(c) E = hf = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(6.906 \times 10^{14} \text{ Hz}) = 4.576 \times 10^{-19} \text{ J} = 2.856 \text{ eV}$$

EVALUATE: Section 38.3 shows that the longest wavelength in the Balmer series (H_α) is 656 nm and the shortest is 365 nm. Our result for H_γ falls within this range. The photon energies for hydrogen atom transitions are in the eV range, and our result is of this order.

38.16. IDENTIFY and SET UP: For the Lyman series the final state is $n = 1$ and the wavelengths are given by

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right), \quad n = 2, 3, \dots$$

For the Paschen series the final state is $n = 3$ and the wavelengths are given by

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right), \quad n = 4, 5, \dots \quad R = 1.097 \times 10^7 \text{ m}^{-1}.$$

The longest wavelength is for the smallest n and the shortest wavelength is for $n \rightarrow \infty$.

EXECUTE: Lyman Longest: $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3R}{4}$. $\lambda = \frac{4}{3(1.097 \times 10^7 \text{ m}^{-1})} = 121.5 \text{ nm}$.

Shortest: $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = R$. $\lambda = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} = 91.16 \text{ nm}$

Paschen Longest: $\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{7R}{144}$. $\lambda = \frac{144}{7(1.097 \times 10^7 \text{ m}^{-1})} = 1875 \text{ nm}$.

Shortest: $\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{\infty^2} \right) = \frac{R}{9}$.

38.17. (a) $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{8.60 \times 10^{-7} \text{ m}} = 2.31 \times 10^{-19} \text{ J} = 1.44 \text{ eV}$.

So the internal energy of the atom increases by 1.44 eV to $E = -6.52 \text{ eV} + 1.44 \text{ eV} = -5.08 \text{ eV}$.

(b) $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{4.20 \times 10^{-7} \text{ m}} = 4.74 \times 10^{-19} \text{ J} = 2.96 \text{ eV}$.

So the final internal energy of the atom decreases to $E = -2.68 \text{ eV} - 2.96 \text{ eV} = -5.64 \text{ eV}$.

38.18. IDENTIFY and SET UP: The ionization threshold is at $E = 0$. The energy of an absorbed photon equals the energy gained by the atom and the energy of an emitted photon equals the energy lost by the atom.

EXECUTE: (a) $\Delta E = 0 - (-20 \text{ eV}) = 20 \text{ eV}$

(b) When the atom in the $n = 1$ level absorbs a 18 eV photon, the final level of the atom is $n = 4$. The possible transitions from $n = 4$ and corresponding photon energies are $n = 4 \rightarrow n = 3$, 3 eV; $n = 4 \rightarrow n = 2$, 8 eV; $n = 4 \rightarrow n = 1$, 18 eV. Once the atom has gone to the $n = 3$ level, the following transitions can occur: $n = 3 \rightarrow n = 2$, 5 eV; $n = 3 \rightarrow n = 1$, 15 eV. Once the atom has gone to the $n = 2$ level, the following transition can occur: $n = 2 \rightarrow n = 1$, 10 eV. The possible energies of emitted photons are: 3 eV, 5 eV, 8 eV, 10 eV, 15 eV, and 18 eV.

(c) There is no energy level 8 eV higher in energy than the ground state, so the photon cannot be absorbed.

(d) The photon energies for $n = 3 \rightarrow n = 2$ and for $n = 3 \rightarrow n = 1$ are 5 eV and 15 eV. The photon energy for $n = 4 \rightarrow n = 3$ is 3 eV. The work function must have a value between 3 eV and 5 eV.

38.19. IDENTIFY and SET UP: The wavelength of the photon is related to the transition energy $E_i - E_f$ of the atom by

$$E_i - E_f = \frac{hc}{\lambda} \quad \text{where } hc = 1.240 \times 10^{-6} \text{ eV} \cdot \text{m}.$$

EXECUTE: (a) The minimum energy to ionize an atom is when the upper state in the transition has $E = 0$, so

$$E_i = -17.50 \text{ eV}. \quad \text{For } n = 5 \rightarrow n = 1, \lambda = 73.86 \text{ nm} \quad \text{and} \quad E_5 - E_1 = \frac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{73.86 \times 10^{-9} \text{ m}} = 16.79 \text{ eV}.$$

$$E_5 = -17.50 \text{ eV} + 16.79 \text{ eV} = -0.71 \text{ eV}. \quad \text{For } n = 4 \rightarrow n = 1, \lambda = 75.63 \text{ nm} \quad \text{and} \quad E_4 = -1.10 \text{ eV}. \quad \text{For } n = 3 \rightarrow n = 1, \lambda = 79.76 \text{ nm} \quad \text{and} \quad E_3 = -1.95 \text{ eV}. \quad \text{For } n = 2 \rightarrow n = 1, \lambda = 94.54 \text{ nm} \quad \text{and} \quad E_2 = -4.38 \text{ eV}.$$

(b) $E_i - E_f = E_4 - E_2 = -1.10 \text{ eV} - (-4.38 \text{ eV}) = 3.28 \text{ eV}$ and $\lambda = \frac{hc}{E_i - E_f} = \frac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{3.28 \text{ eV}} = 378 \text{ nm}$

EVALUATE: The $n = 4 \rightarrow n = 2$ transition energy is smaller than the $n = 4 \rightarrow n = 1$ transition energy so the wavelength is longer. In fact, this wavelength is longer than for any transition that ends in the $n = 1$ state.

- 38.20.** (a) Equating initial kinetic energy and final potential energy and solving for the separation radius r ,

$$r = \frac{1}{4\pi\epsilon_0} \frac{(92e)(2e)}{K} = \frac{1}{4\pi\epsilon_0} \frac{(184)(1.60 \times 10^{-19} \text{ C})}{(4.78 \times 10^6 \text{ J/C})} = 5.54 \times 10^{-14} \text{ m}.$$

(b) The above result may be substituted into Coulomb's law, or, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulombic field is

$$F = \frac{K}{r} = \frac{(4.78 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(5.54 \times 10^{-14} \text{ m})} = 13.8 \text{ N}.$$

- 38.21.** (a) **IDENTIFY:** If the particles are treated as point charges, $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

SET UP: $q_1 = 2e$ (alpha particle); $q_2 = 82e$ (gold nucleus); r is given so we can solve for U .

$$\text{EXECUTE: } U = (8.987 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2)(82)(1.602 \times 10^{-19} \text{ C})^2}{6.50 \times 10^{-14} \text{ m}} = 5.82 \times 10^{-13} \text{ J}$$

$$U = 5.82 \times 10^{-13} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 3.63 \times 10^6 \text{ eV} = 3.63 \text{ MeV}$$

(b) **IDENTIFY:** Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$.

SET UP: Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$.

EXECUTE: Conservation of energy thus says $K_1 = U_2 = 5.82 \times 10^{-13} \text{ J} = 3.63 \text{ MeV}$.

$$(c) K = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.82 \times 10^{-13} \text{ J})}{6.64 \times 10^{-27} \text{ kg}}} = 1.32 \times 10^7 \text{ m/s}$$

EVALUATE: $v/c = 0.044$, so it is ok to use the nonrelativistic expression to relate K and v . When the alpha particle stops, all its initial kinetic energy has been converted to electrostatic potential energy.

- 38.22.** (a), (b) For either atom, the magnitude of the angular momentum is $\frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s}$.

- 38.23.** **IDENTIFY and SET UP:** Use the energy to calculate n for this state. Then use the Bohr equation, Eq.(38.10), to calculate L .

EXECUTE: $E_n = -(13.6 \text{ eV})/n^2$, so this state has $n = \sqrt{13.6/1.51} = 3$. In the Bohr model, $L = n\hbar$ so for this state $L = 3\hbar = 3.16 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s}$.

EVALUATE: We will find in Section 41.1 that the modern quantum mechanical description gives a different result.

- 38.24.** **IDENTIFY and SET UP:** For a hydrogen atom $E_n = -\frac{13.6 \text{ eV}}{n^2}$. $\Delta E = \frac{hc}{\lambda}$, where ΔE is the magnitude of the energy change for the atom and λ is the wavelength of the photon that is absorbed or emitted.

$$\text{EXECUTE: } \Delta E = E_4 - E_1 = -(13.6 \text{ eV}) \left(\frac{1}{4^2} - \frac{1}{1^2} \right) = +12.75 \text{ eV}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{12.75 \text{ eV}} = 97.3 \text{ nm}. \quad f = \frac{c}{\lambda} = 3.08 \times 10^{15} \text{ Hz}.$$

- 38.25.** **IDENTIFY:** The force between the electron and the nucleus in Be^{3+} is $F = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2}$, where $Z = 4$ is the nuclear charge. All the equations for the hydrogen atom apply to Be^{3+} if we replace e^2 by Ze^2 .

(a) **SET UP:** Modify Eq.(38.18).

$$\text{EXECUTE: } E_n = -\frac{1}{\epsilon_0} \frac{me^4}{8n^2h^2} \text{ (hydrogen) becomes}$$

$$E_n = -\frac{1}{\epsilon_0} \frac{m(Ze^2)^2}{8n^2h^2} = Z^2 \left(-\frac{1}{\epsilon_0} \frac{me^4}{8n^2h^2} \right) = Z^2 \left(-\frac{13.60 \text{ eV}}{n^2} \right) \text{ (for } \text{Be}^{3+})$$

$$\text{The ground-level energy of } \text{Be}^{3+} \text{ is } E_1 = 16 \left(-\frac{13.60 \text{ eV}}{1^2} \right) = -218 \text{ eV}.$$

EVALUATE: The ground-level energy of Be^{3+} is $Z^2 = 16$ times the ground-level energy of H.

(b) **SET UP:** The ionization energy is the energy difference between the $n \rightarrow \infty$ level energy and the $n = 1$ level energy.

EXECUTE: The $n \rightarrow \infty$ level energy is zero, so the ionization energy of Be^{3+} is 218 eV.

EVALUATE: This is 16 times the ionization energy of hydrogen.

(c) **SET UP:** $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ just as for hydrogen but now R has a different value.

EXECUTE: $R_{\text{H}} = \frac{me^4}{8\epsilon_0 h^3 c} = 1.097 \times 10^7 \text{ m}^{-1}$ for hydrogen becomes

$$R_{\text{Be}} = Z^2 \frac{me^4}{8\epsilon_0 h^3 c} = 16(1.097 \times 10^7 \text{ m}^{-1}) = 1.755 \times 10^8 \text{ m}^{-1} \text{ for } \text{Be}^{3+}.$$

For $n = 2$ to $n = 1$, $\frac{1}{\lambda} = R_{\text{Be}} \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 3R/4$.

$$\lambda = 4/(3R) = 4/(3(1.755 \times 10^8 \text{ m}^{-1})) = 7.60 \times 10^{-9} \text{ m} = 7.60 \text{ nm}.$$

EVALUATE: This wavelength is smaller by a factor of 16 compared to the wavelength for the corresponding transition in the hydrogen atom.

(d) **SET UP:** Modify Eq.(38.12): $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$ (hydrogen).

EXECUTE: $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m (Ze^2)}$ (Be^{3+}).

EVALUATE: For a given n the orbit radius for Be^{3+} is smaller by a factor of $Z = 4$ compared to the corresponding radius for hydrogen.

38.26. (a) We can find the photon's energy from Eq. 38.8

$$E = hcR \left(\frac{1}{2^2} - \frac{1}{n^2} \right) = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (3.00 \times 10^8 \text{ m/s}) (1.097 \times 10^7 \text{ m}^{-1}) \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = 4.58 \times 10^{-19} \text{ J. The}$$

corresponding wavelength is $\lambda = \frac{E}{hc} = 434 \text{ nm}$.

(b) In the Bohr model, the angular momentum of an electron with principal quantum number n is given by

Eq. 38.10: $L = n \frac{h}{2\pi}$. Thus, when an electron makes a transition from $n = 5$ to $n = 2$ orbital, there is the following loss in angular momentum (which we would assume is transferred to the photon):

$$\Delta L = (2 - 5) \frac{h}{2\pi} = -\frac{3(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{2\pi} = -3.17 \times 10^{-34} \text{ J} \cdot \text{s}.$$

However, this prediction of the Bohr model is wrong (as shown in Chapter 41).

38.27. (a) $v_n = \frac{1}{\epsilon_0} \frac{e^2}{2nh} : n = 1 \Rightarrow v_1 = \frac{(1.60 \times 10^{-19} \text{ C})^2}{\epsilon_0 2 (6.63 \times 10^{-34} \text{ J} \cdot \text{s})} = 2.18 \times 10^6 \text{ m/s}$

$$h = 2 \Rightarrow v_2 = \frac{v_1}{2} = 1.09 \times 10^6 \text{ m/s. } h = 3 \Rightarrow v_3 = \frac{v_1}{3} = 7.27 \times 10^5 \text{ m/s.}$$

(b) Orbital period $= \frac{2\pi r_n}{v_n} = \frac{2\epsilon_0 n^2 h^2 / me^2}{1/\epsilon_0 \cdot e^2 / 2nh} = \frac{4\epsilon_0^2 n^3 h^3}{me^4}$

$$n = 1 \Rightarrow T_1 = \frac{4\epsilon_0^2 (6.63 \times 10^{-34} \text{ J} \cdot \text{s})^3}{(9.11 \times 10^{-31} \text{ kg}) (1.60 \times 10^{-19} \text{ C})^4} = 1.53 \times 10^{-16} \text{ s}$$

$$n = 2 : T_2 = T_1(2)^3 = 1.22 \times 10^{-15} \text{ s. } n = 3 : T_3 = T_1(3)^3 = 4.13 \times 10^{-15} \text{ s.}$$

(c) number of orbits $= \frac{1.0 \times 10^{-8} \text{ s}}{1.22 \times 10^{-15} \text{ s}} = 8.2 \times 10^6$.

38.28. IDENTIFY and SET UP: $E_n = -\frac{13.6 \text{ eV}}{n^2}$

EXECUTE: (a) $E_n = -\frac{13.6 \text{ eV}}{n^2}$ and $E_{n+1} = -\frac{13.6 \text{ eV}}{(n+1)^2}$

$$\Delta E = E_{n+1} - E_n = (-13.6 \text{ eV}) \left[\frac{1}{(n+1)^2} - \frac{1}{n^2} \right] = -(13.6 \text{ eV}) \frac{n^2 - (n+1)^2}{(n^2)(n+1)^2}$$

$$\Delta E = (13.6 \text{ eV}) \frac{2n+1}{(n^2)(n+1)^2} \text{ As } n \text{ becomes large, } \Delta E \rightarrow (13.6 \text{ eV}) \frac{2n}{n^4} = (13.6 \text{ eV}) \frac{2}{n^3}$$

Thus ΔE becomes small as n becomes large.

(b) $r_n = n^2 r_1$ so the orbits get farther apart in space as n increases.

- 38.29. IDENTIFY and SET UP:** The number of photons emitted each second is the total energy emitted divided by the energy of one photon. The energy of one photon is given by Eq.(38.2). $E = Pt$ gives the energy emitted by the laser in time t .

EXECUTE: In 1.00 s the energy emitted by the laser is $(7.50 \times 10^{-3} \text{ W})(1.00 \text{ s}) = 7.50 \times 10^{-3} \text{ J}$.

The energy of each photon is $E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{10.6 \times 10^{-6} \text{ m}} = 1.874 \times 10^{-20} \text{ J}$.

Therefore $\frac{7.50 \times 10^{-3} \text{ J/s}}{1.874 \times 10^{-20} \text{ J/photon}} = 4.00 \times 10^{17} \text{ photons/s}$

EVALUATE: The number of photons emitted per second is extremely large.

- 38.30. IDENTIFY and SET UP:** Visible light has wavelengths from about 400 nm to about 700 nm. The energy of each photon is $E = hf = \frac{hc}{\lambda} = \frac{1.99 \times 10^{-25} \text{ J} \cdot \text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: (a) 193 nm is shorter than visible light so is in the ultraviolet.

(b) $E = \frac{hc}{\lambda} = 1.03 \times 10^{-18} \text{ J} = 6.44 \text{ eV}$

(c) $P = \frac{E_{\text{tot}}}{t} = \frac{NE}{t}$ so $N = \frac{Pt}{E} = \frac{(1.50 \times 10^{-3} \text{ W})(12.0 \times 10^{-9} \text{ s})}{1.03 \times 10^{-18} \text{ J}} = 1.75 \times 10^7 \text{ photons}$

EVALUATE: A very small amount of energy is delivered to the lens in each pulse, but this still corresponds to a large number of photons.

- 38.31. IDENTIFY:** Apply Eq.(38.21): $\frac{n_{5s}}{n_{3p}} = e^{-(E_{5s} - E_{3p})/kT}$

SET UP: From Fig.38.24a in the textbook, $E_{5s} = 20.66 \text{ eV}$ and $E_{3p} = 18.70 \text{ eV}$

EXECUTE: $E_{5s} - E_{3p} = 20.66 \text{ eV} - 18.70 \text{ eV} = 1.96 \text{ eV} (1.602 \times 10^{-19} \text{ J/eV}) = 3.140 \times 10^{-19} \text{ J}$

(a) $\frac{n_{5s}}{n_{3p}} = e^{-(3.140 \times 10^{-19} \text{ J}) / [(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})]} = e^{-75.79} = 1.2 \times 10^{-33}$

(b) $\frac{n_{5s}}{n_{3p}} = e^{-(3.140 \times 10^{-19} \text{ J}) / [(1.38 \times 10^{-23} \text{ J/K})(600 \text{ K})]} = e^{-37.90} = 3.5 \times 10^{-17}$

(c) $\frac{n_{5s}}{n_{3p}} = e^{-(3.140 \times 10^{-19} \text{ J}) / [(1.38 \times 10^{-23} \text{ J/K})(1200 \text{ K})]} = e^{-18.95} = 5.9 \times 10^{-9}$

(d) **EVALUATE:** At each of these temperatures the number of atoms in the 5s excited state, the initial state for the transition that emits 632.8 nm radiation, is quite small. The ratio increases as the temperature increases.

- 38.32.** $\frac{n_{2p_{3/2}}}{n_{2p_{1/2}}} = e^{-(E_{2p_{3/2}} - E_{2p_{1/2}})/kT}$

From the diagram $\Delta E_{3/2-g} = \frac{hc}{\lambda_1} = \frac{(6.626 \times 10^{-34} \text{ J})(3.000 \times 10^8 \text{ m/s})}{5.890 \times 10^{-7} \text{ m}} = 3.375 \times 10^{-19} \text{ J}$.

$\Delta E_{1/2-g} = \frac{hc}{\lambda_2} = \frac{(6.626 \times 10^{-34} \text{ J})(3.000 \times 10^8 \text{ m/s})}{5.896 \times 10^{-7} \text{ m}} = 3.371 \times 10^{-19} \text{ J}$. so $\Delta E_{3/2-1/2} = 3.375 \times 10^{-19} \text{ J} - 3.371 \times 10^{-19} \text{ J} =$

$4.00 \times 10^{-22} \text{ J}$. $\frac{n_{2p_{3/2}}}{n_{2p_{1/2}}} = e^{-(4.00 \times 10^{-22} \text{ J}) / (1.38 \times 10^{-23} \text{ J/K} \cdot 500 \text{ K})} = 0.944$. So more atoms are in the $2p_{1/2}$ state.

- 38.33.** $eV_{\text{AC}} = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$

$$\Rightarrow \lambda_{\text{min}} = \frac{hc}{eV_{\text{AC}}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(4000 \text{ V})} = 3.11 \times 10^{-10} \text{ m}$$

This is the same answer as would be obtained if electrons of this energy were used. Electron beams are much more easily produced and accelerated than proton beams.

38.34. IDENTIFY and SET UP: $\frac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: (a) $V = \frac{hc}{e\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.150 \times 10^{-9} \text{ m})} = 8.29 \text{ kV}$

(b) $\lambda = \frac{hc}{eV} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(30.0 \times 10^3 \text{ V})} = 4.14 \times 10^{-11} \text{ m} = 0.0414 \text{ nm}$

(c) No. A proton has the same magnitude of charge as an electron and therefore gains the same amount of kinetic energy when accelerated by the same magnitude of potential difference.

38.35. IDENTIFY: The initial electrical potential energy of the accelerated electrons is converted to kinetic energy which is then given to a photon.

SET UP: The electrical potential energy of an electron is eV_{AC} , where V_{AC} is the accelerating potential, and the energy of a photon is hf . Since the energy of the electron is all given to a photon, we have $eV_{AC} = hf$. For any wave, $f\lambda = v$.

EXECUTE: (a) $eV_{AC} = hf_{\min}$ gives

$$f_{\min} = eV_{AC}/h = (1.60 \times 10^{-19} \text{ C})(25,000 \text{ V})/(6.626 \times 10^{-34} \text{ J} \cdot \text{s}) = 6.037 \times 10^{18} \text{ Hz}$$

$$= 6.04 \times 10^{18} \text{ Hz, rounded to three digits}$$

(b) $\lambda_{\min} = c/f_{\max} = (3.00 \times 10^8 \text{ m/s})/(6.037 \times 10^{18} \text{ Hz}) = 4.97 \times 10^{-11} \text{ m} = 0.0497 \text{ nm}$

(c) We assume that all the energy of the electron produces only one photon on impact with the screen.

EVALUATE: These photons are in the x-ray and γ -ray part of the electromagnetic spectrum (see Figure 32.4 in the textbook) and would be harmful to the eyes without protective glass on the screen to absorb them.

38.36. IDENTIFY and SET UP: The wavelength of the x rays produced by the tube is give by $\frac{hc}{\lambda} = eV$.

$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi)$. $\frac{h}{mc} = 2.426 \times 10^{-12} \text{ m}$. The energy of the scattered x ray is $\frac{hc}{\lambda'}$.

EXECUTE: (a) $\lambda = \frac{hc}{eV} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(18.0 \times 10^3 \text{ V})} = 6.91 \times 10^{-11} \text{ m} = 0.0691 \text{ nm}$

(b) $\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi) = 6.91 \times 10^{-11} \text{ m} + (2.426 \times 10^{-12} \text{ m})(1 - \cos 45.0^\circ)$.

$\lambda' = 6.98 \times 10^{-11} \text{ m} = 0.0698 \text{ nm}$.

(c) $E = \frac{hc}{\lambda'} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{6.98 \times 10^{-11} \text{ m}} = 17.8 \text{ keV}$

EVALUATE: The incident x ray has energy 18.0 keV. In the scattering event, the photon loses energy and its wavelength increases.

38.37. IDENTIFY: Apply Eq.(38.23): $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) = \lambda_c(1 - \cos\phi)$

SET UP: Solve for λ' : $\lambda' = \lambda + \lambda_c(1 - \cos\phi)$

The largest λ' corresponds to $\phi = 180^\circ$, so $\cos\phi = -1$.

EXECUTE: $\lambda' = \lambda + 2\lambda_c = 0.0665 \times 10^{-9} \text{ m} + 2(2.426 \times 10^{-12} \text{ m}) = 7.135 \times 10^{-11} \text{ m} = 0.0714 \text{ nm}$. This wavelength occurs at a scattering angle of $\phi = 180^\circ$.

EVALUATE: The incident photon transfers some of its energy and momentum to the electron from which it scatters. Since the photon loses energy its wavelength increases, $\lambda' > \lambda$.

38.38. (a) From Eq. (38.23), $\cos\phi = 1 - \frac{\Delta\lambda}{(h/mc)}$, and so $\Delta\lambda = 0.0542 \text{ nm} - 0.0500 \text{ nm}$,

$\cos\phi = 1 - \frac{0.0042 \text{ nm}}{0.002426 \text{ nm}} = -0.731$, and $\phi = 137^\circ$.

(b) $\Delta\lambda = 0.0521 \text{ nm} - 0.0500 \text{ nm}$. $\cos\phi = 1 - \frac{0.0021 \text{ nm}}{0.002426 \text{ nm}} = 0.134$. $\phi = 82.3^\circ$.

(c) $\Delta\lambda = 0$, the photon is undeflected, $\cos\phi = 1$ and $\phi = 0$.

38.39. IDENTIFY and SET UP: The shift in wavelength of the photon is $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$ where λ' is the

wavelength after the scattering and $\frac{h}{mc} = \lambda_c = 2.426 \times 10^{-12} \text{ m}$. The energy of a photon of wavelength λ is

$E = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-6} \text{ eV} \cdot \text{m}}{\lambda}$. Conservation of energy applies to the collision, so the energy lost by the photon equals the energy gained by the electron.

EXECUTE: (a) $\lambda' - \lambda = \lambda_c(1 - \cos \phi) = (2.426 \times 10^{-12} \text{ m})(1 - \cos 35.0^\circ) = 4.39 \times 10^{-13} \text{ m} = 4.39 \times 10^{-4} \text{ nm}$

(b) $\lambda' = \lambda + 4.39 \times 10^{-4} \text{ nm} = 0.04250 \text{ nm} + 4.39 \times 10^{-4} \text{ nm} = 0.04294 \text{ nm}$

(c) $E_\lambda = \frac{hc}{\lambda} = 2.918 \times 10^4 \text{ eV}$ and $E_{\lambda'} = \frac{hc}{\lambda'} = 2.888 \times 10^4 \text{ eV}$ so the photon loses 300 eV of energy.

(d) Energy conservation says the electron gains 300 eV of energy.

38.40. The change in wavelength of the scattered photon is given by Eq. 38.23

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{mc\lambda}(1 - \cos \phi) \Rightarrow \lambda = \frac{h}{mc\left(\frac{\Delta\lambda}{\lambda}\right)}(1 - \cos \phi).$$

Thus, $\lambda = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})(0.100)}(1 + 1) = 2.65 \times 10^{-14} \text{ m}$.

38.41. The derivation of Eq.(38.23) is explicitly shown in Equations (38.24) through (38.27) with the final substitution of $p' = h/\lambda'$ and $p = h/\lambda$ yielding $\lambda' - \lambda = \frac{h}{mc}(1 - \cos \phi)$.

38.42. From Eq. (38.30), (a) $\lambda_m = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{3.00 \text{ K}} = 0.966 \text{ mm}$, and $f = \frac{c}{\lambda_m} = 3.10 \times 10^{11} \text{ Hz}$. Note that a more precise value of the Wien displacement law constant has been used.

(b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to $9.66 \mu\text{m}$ and raises the frequency by the same factor, to $3.10 \times 10^{13} \text{ Hz}$.

(c) Similarly, $\lambda_m = 966 \text{ nm}$ and $f = 3.10 \times 10^{14} \text{ Hz}$.

38.43. (a) $H = Ae\sigma T^4$; $A = \pi r^2 l$

$$T = \left(\frac{H}{Ae\sigma} \right)^{1/4} = \left(\frac{100 \text{ W}}{2\pi(0.20 \times 10^{-3} \text{ m})(0.30 \text{ m})(0.26)(5.671 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)} \right)^{1/4}$$

$$T = 2.06 \times 10^3 \text{ K}$$

(b) $\lambda_m T = 2.90 \times 10^{-3} \text{ m} \cdot \text{K}$; $\lambda_m = 1410 \text{ nm}$

Much of the emitted radiation is in the infrared.

38.44. $T = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{\lambda_m} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{400 \times 10^{-9} \text{ m}} = 7.25 \times 10^3 \text{ K}$.

38.45. IDENTIFY and SET UP: The wavelength λ_m where the Planck distribution peaks is given by Eq.(38.30).

$$\text{EXECUTE: } \lambda_m = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{2.728 \text{ K}} = 1.06 \times 10^{-3} \text{ m} = 1.06 \text{ mm}.$$

EVALUATE: This wavelength is in the microwave portion of the electromagnetic spectrum. This radiation is often referred to as the “microwave background” (Section 44.7). Note that in Eq.(38.30), T must be in kelvins.

38.46. IDENTIFY: Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law and Wien’s displacement law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$. Wien’s displacement law tells us that the peak-intensity wavelength is $\lambda_m = (\text{constant})/T$.

EXECUTE: (a) The hot and cool stars radiate the same total power, so the Stefan-Boltzmann law gives $\sigma A_h T_h^4 = \sigma A_c T_c^4 \Rightarrow 4\pi R_h^2 T_h^4 = 4\pi R_c^2 T_c^4 = 4\pi (3R_h)^2 T_c^4 \Rightarrow T_h^4 = 9T_c^4 \Rightarrow T_h = T\sqrt{3} = 1.7T$, rounded to two significant digits.

(b) Using Wien’s law, we take the ratio of the wavelengths, giving

$$\frac{\lambda_m(\text{hot})}{\lambda_m(\text{cool})} = \frac{T_c}{T_h} = \frac{T}{T\sqrt{3}} = \frac{1}{\sqrt{3}} = 0.58, \text{ rounded to two significant digits.}$$

EVALUATE: Although the hot star has only 1/9 the surface area of the cool star, its absolute temperature has to be only 1.7 times as great to radiate the same amount of energy.

38.47. (a) Let $\alpha = hc/kT$. To find the maximum in the Planck distribution:

$$\frac{dI}{d\lambda} = \frac{d}{d\lambda} \left(\frac{2\pi hc^2}{\lambda^5 (e^{\alpha/\lambda} - 1)} \right) = 0 = -5 \frac{(2\pi hc^2)}{\lambda^5 (e^{\alpha/\lambda} - 1)} - \frac{2\pi hc^2 (-\alpha/\lambda^2)}{\lambda^5 (e^{\alpha/\lambda} - 1)^2}$$

$$\Rightarrow -5(e^{\alpha/\lambda} - 1)\lambda = \alpha \Rightarrow -5e^{\alpha/\lambda} + 5 = \alpha/\lambda \Rightarrow \text{Solve } 5 - x = 5e^x \text{ where } x = \frac{\alpha}{\lambda} = \frac{hc}{\lambda kT}.$$

Its root is 4.965, so $\frac{\alpha}{\lambda} = 4.965 \Rightarrow \lambda = \frac{hc}{(4.965)kT}$.

$$(b) \lambda_m T = \frac{hc}{(4.965)k} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(4.965)(1.38 \times 10^{-23} \text{ J/K})} = 2.90 \times 10^{-3} \text{ m} \cdot \text{K}.$$

38.48. IDENTIFY: Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma AT^4$.

EXECUTE: (a) $I = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(24,000 \text{ K})^4 = 1.9 \times 10^{10} \text{ W/m}^2$

(b) Wien's law gives $\lambda_m = (0.00290 \text{ m} \cdot \text{K})/(24,000 \text{ K}) = 1.2 \times 10^{-7} \text{ m} = 120 \text{ nm}$

This is not visible since the wavelength is less than 400 nm.

(c) $P = AI \Rightarrow 4\pi R^2 = P/I = (1.00 \times 10^{25} \text{ W})/(1.9 \times 10^{10} \text{ W/m}^2)$

which gives $R_{\text{Sirius}} = 6.51 \times 10^6 \text{ m} = 6510 \text{ km}$.

$R_{\text{Sirius}}/R_{\text{sun}} = (6.51 \times 10^6 \text{ m})/(6.96 \times 10^9 \text{ m}) = 0.0093$, which gives

$$R_{\text{Sirius}} = 0.0093 R_{\text{sun}} \approx 1\% R_{\text{sun}}$$

(d) Using the Stefan-Boltzmann law, we have

$$\frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \frac{\sigma A_{\text{sun}} T_{\text{sun}}^4}{\sigma A_{\text{Sirius}} T_{\text{Sirius}}^4} = \frac{4\pi R_{\text{sun}}^2 T_{\text{sun}}^4}{4\pi R_{\text{Sirius}}^2 T_{\text{Sirius}}^4} = \left(\frac{R_{\text{sun}}}{R_{\text{Sirius}}}\right)^2 \left(\frac{T_{\text{sun}}}{T_{\text{Sirius}}}\right)^4 \cdot \frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \left(\frac{R_{\text{sun}}}{0.00935 R_{\text{sun}}}\right)^2 \left(\frac{5800 \text{ K}}{24,000 \text{ K}}\right)^4 = 39$$

EVALUATE: Even though the absolute surface temperature of Sirius B is about 4 times that of our sun, it radiates only 1/39 times as much energy per second as our sun because it is so small.

38.49. Eq. (38.32): $I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$ but $e^x = 1 + x + \frac{x^2}{2} + \dots \approx 1 + x$ for

$$x \ll 1 \Rightarrow I(\lambda) \approx \frac{2\pi hc^2}{\lambda^5 (hc/\lambda kT)} = \frac{2\pi ckT}{\lambda^4} = \text{Eq. (38.31), which is Rayleigh's distribution.}$$

38.50. (a) Wien's law: $\lambda_m = \frac{k}{T}$. $\lambda_m = \frac{2.90 \times 10^{-3} \text{ K} \cdot \text{m}}{30,000 \text{ K}} = 9.7 \times 10^{-8} \text{ m} = 97 \text{ nm}$

This peak is in the ultraviolet region, which is *not* visible. The star is blue because the largest part of the visible light radiated is in the blue/violet part of the visible spectrum

(b) $P = \sigma AT^4$ (Stefan-Boltzmann law)

$$(100,000)(3.86 \times 10^{26} \text{ W}) = \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}\right) (4\pi R^2)(30,000 \text{ K})^4$$

$$R = 8.2 \times 10^9 \text{ m}$$

$$R_{\text{star}}/R_{\text{sun}} = \frac{8.2 \times 10^9 \text{ m}}{6.96 \times 10^8 \text{ m}} = 12$$

(c) The visual luminosity is proportional to the power radiated at visible wavelengths. Much of the power is radiated nonvisible wavelengths, which does not contribute to the visible luminosity.

38.51. IDENTIFY and SET UP: Use $c = f\lambda$ to relate frequency and wavelength and use $E = hf$ to relate photon energy and frequency.

EXECUTE: (a) One photon dissociates one AgBr molecule, so we need to find the energy required to dissociate a single molecule. The problem states that it requires $1.00 \times 10^5 \text{ J}$ to dissociate one mole of AgBr, and one mole contains Avogadro's number (6.02×10^{23}) of molecules, so the energy required to dissociate one AgBr is

$$\frac{1.00 \times 10^5 \text{ J/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 1.66 \times 10^{-19} \text{ J/molecule.}$$

The photon is to have this energy, so $E = 1.66 \times 10^{-19} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 1.04 \text{ eV}$.

(b) $E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.66 \times 10^{-19} \text{ J}} = 1.20 \times 10^{-6} \text{ m} = 1200 \text{ nm}$

(c) $c = f\lambda$ so $f = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{1.20 \times 10^{-6} \text{ m}} = 2.50 \times 10^{14} \text{ Hz}$

(d) $E = hf = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(100 \times 10^6 \text{ Hz}) = 6.63 \times 10^{-26} \text{ J}$

$$E = 6.63 \times 10^{-26} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 4.14 \times 10^{-7} \text{ eV}$$

(e) **EVALUATE:** A photon with frequency $f = 100$ MHz has too little energy, by a large factor, to dissociate a AgBr molecule. The photons in the visible light from a firefly do individually have enough energy to dissociate AgBr. The huge number of 100 MHz photons can't compensate for the fact that individually they have too little energy.

38.52. (a) Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda}$.

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2 = \frac{h^2}{2m\lambda^2}$.

(c) $\frac{K}{E} = \frac{h^2}{2m\lambda^2} \cdot \frac{\lambda}{hc} = \frac{h}{2mc\lambda}$. Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

(d) $E = 10.2 \text{ eV} \Rightarrow \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(10.2 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 1.22 \times 10^{-7} \text{ m} = 122 \text{ nm}$.

$$K = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(1.67 \times 10^{-27} \text{ kg})(1.22 \times 10^{-7} \text{ m})^2} = 8.84 \times 10^{-27} \text{ J} = 5.53 \times 10^{-8} \text{ eV}.$$

$$\frac{K}{E} = \frac{5.53 \times 10^{-8} \text{ eV}}{10.2 \text{ eV}} = 5.42 \times 10^{-9}. \text{ This is quite small so recoil can be neglected.}$$

38.53. **IDENTIFY and SET UP:** $f = \frac{c}{\lambda}$. The (f, V_0) values are: $(8.20 \times 10^{14} \text{ Hz}, 1.48 \text{ V})$, $(7.41 \times 10^{14} \text{ Hz}, 1.15 \text{ V})$, $(6.88 \times 10^{14} \text{ Hz}, 0.93 \text{ V})$, $(6.10 \times 10^{14} \text{ Hz}, 0.62 \text{ V})$, $(5.49 \times 10^{14} \text{ Hz}, 0.36 \text{ V})$, $(5.18 \times 10^{14} \text{ Hz}, 0.24 \text{ V})$. The graph of V_0 versus f is given in Figure 38.53.

EXECUTE: (a) The threshold frequency, f_{th} , is f where $V_0 = 0$. From the graph this is $f_{\text{th}} = 4.56 \times 10^{14} \text{ Hz}$.

(b) $\lambda_{\text{th}} = \frac{c}{f_{\text{th}}} = \frac{3.00 \times 10^8 \text{ m/s}}{4.56 \times 10^{14} \text{ Hz}} = 658 \text{ nm}$

(c) $\phi = hf_{\text{th}} = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(4.56 \times 10^{14} \text{ Hz}) = 1.89 \text{ eV}$

(d) $eV_0 = hf - \phi$ so $V_0 = \left(\frac{h}{e}\right)f - \frac{\phi}{e}$. The slope of the graph is $\frac{h}{e}$.

$$\frac{h}{e} = \left(\frac{1.48 \text{ V} - 0.24 \text{ V}}{8.20 \times 10^{14} \text{ Hz} - 5.18 \times 10^{14} \text{ Hz}} \right) = 4.11 \times 10^{-15} \text{ V/Hz and}$$

$$h = (4.11 \times 10^{-15} \text{ V/Hz})(1.60 \times 10^{-19} \text{ C}) = 6.58 \times 10^{-34} \text{ J} \cdot \text{s}.$$

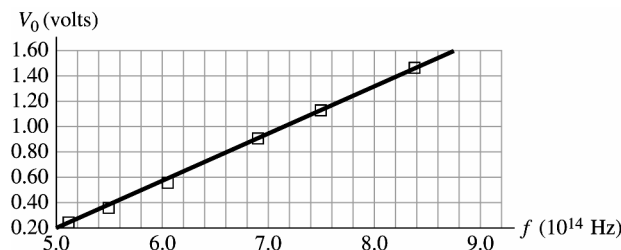


Figure 38.53

38.54. (a) $\frac{dN}{dt} = \frac{(dE/dt)}{(dE/dN)} = \frac{P}{hf} = \frac{(200 \text{ W})(0.10)}{h(5.00 \times 10^{14} \text{ Hz})} = 6.03 \times 10^{19} \text{ photons/sec}$.

(b) Demand $\frac{(dN/dt)}{4\pi r^2} = 1.00 \times 10^{11} \text{ photons/sec} \cdot \text{cm}^2$.

Therefore, $r = \left(\frac{6.03 \times 10^{19} \text{ photons/sec}}{4\pi(1.00 \times 10^{11} \text{ photons/sec} \cdot \text{cm}^2)} \right)^{1/2} = 6930 \text{ cm} = 69.3 \text{ m}$.

38.55. (a) **IDENTIFY:** Apply the photoelectric effect equation, Eq.(38.4).

SET UP: $eV_0 = hf - \phi = (hc/\lambda) - \phi$. Call the stopping potential V_{01} for λ_1 and V_{02} for λ_2 . Thus

$eV_{01} = (hc/\lambda_1) - \phi$ and $eV_{02} = (hc/\lambda_2) - \phi$. Note that the work function ϕ is a property of the material and is independent of the wavelength of the light.

EXECUTE: Subtracting one equation from the other gives $e(V_{02} - V_{01}) = hc \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right)$.

$$(b) \Delta V_0 = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.602 \times 10^{-19} \text{ C}} \left(\frac{295 \times 10^{-9} \text{ m} - 265 \times 10^{-9} \text{ m}}{(295 \times 10^{-9} \text{ m})(265 \times 10^{-9} \text{ m})} \right) = 0.476 \text{ V}.$$

EVALUATE: $e\Delta V_0$, which is 0.476 eV, is the increase in photon energy from 295 nm to 265 nm. The stopping potential increases when λ decreases because the photon energy increases when the wavelength decreases.

- 38.56. IDENTIFY:** The photoelectric effect occurs, so the energy of the photon is used to eject an electron, with any excess energy going into kinetic energy of the electron.

SET UP: Conservation of energy gives $hf = hc/\lambda = K_{\max} + \phi$.

EXECUTE: (a) Using $hc/\lambda = K_{\max} + \phi$, we solve for the work function:

$$\phi = hc/\lambda - K_{\max} = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(124 \text{ nm}) - 4.16 \text{ eV} = 5.85 \text{ eV}$$

(b) The number N of photoelectrons per second is equal to the number of photons per second that strike the metal per second. $N \times (\text{energy of a photon}) = 2.50 \text{ W}$. $N(hc/\lambda) = 2.50 \text{ W}$.

$$N = (2.50 \text{ W})(124 \text{ nm})/[(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})] = 1.56 \times 10^{18} \text{ electrons/s}$$

(c) N is proportional to the power, so if the power is cut in half, so is N , which gives

$$N = (1.56 \times 10^{18} \text{ el/s})/2 = 7.80 \times 10^{17} \text{ el/s}$$

(d) If we cut the wavelength by half, the energy of each photon is doubled since $E = hc/\lambda$. To maintain the same power, the number of photons must be half of what they were in part (b), so N is cut in half to $7.80 \times 10^{17} \text{ el/s}$. We could also see this from part (b), where N is proportional to λ . So if the wavelength is cut in half, so is N .

EVALUATE: In part (c), reducing the power does not reduce the maximum kinetic energy of the photons; it only reduces the number of ejected electrons. In part (d), reducing the wavelength *does* change the maximum kinetic energy of the photoelectrons because we have increased the energy of each photon.

- 38.57. IDENTIFY and SET UP:** The energy added to mass m of the blood to heat it to $T_f = 100^\circ\text{C}$ and to vaporize it is

$$Q = mc(T_f - T_i) + mL_v, \text{ with } c = 4190 \text{ J/kg} \cdot \text{K} \text{ and } L_v = 2.256 \times 10^6 \text{ J/kg}. \text{ The energy of one photon is}$$

$$E = \frac{hc}{\lambda} = \frac{1.99 \times 10^{-25} \text{ J} \cdot \text{m}}{\lambda}.$$

EXECUTE: (a) $Q = (2.0 \times 10^{-9} \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(100^\circ\text{C} - 33^\circ\text{C}) + (2.0 \times 10^{-9} \text{ kg})(2.256 \times 10^6 \text{ J/kg}) = 5.07 \times 10^{-3} \text{ J}$
The pulse must deliver 5.07 mJ of energy.

$$(b) P = \frac{\text{energy}}{t} = \frac{5.07 \times 10^{-3} \text{ J}}{450 \times 10^{-6} \text{ s}} = 11.3 \text{ W}$$

$$(c) \text{ One photon has energy } E = \frac{hc}{\lambda} = \frac{1.99 \times 10^{-25} \text{ J} \cdot \text{m}}{585 \times 10^{-9} \text{ m}} = 3.40 \times 10^{-19} \text{ J}. \text{ The number } N \text{ of photons per pulse is the}$$

$$\text{energy per pulse divided by the energy of one photon: } N = \frac{5.07 \times 10^{-3} \text{ J}}{3.40 \times 10^{-19} \text{ J/photon}} = 1.49 \times 10^{16} \text{ photons}$$

- 38.58. (a)** $\lambda_0 = \frac{hc}{E}$, and the wavelengths are: cesium: 590 nm, copper: 264 nm, potassium: 539 nm, zinc: 288 nm.

(b) The wavelengths of copper and zinc are in the ultraviolet, and visible light is not energetic enough to overcome the threshold energy of these metals.

- 38.59. (a) IDENTIFY and SET UP:** Apply Eq.(38.20): $m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{207 m_e m_p}{207 m_e + m_p}$

$$\text{EXECUTE: } m_r = \frac{207(9.109 \times 10^{-31} \text{ kg})(1.673 \times 10^{-27} \text{ kg})}{207(9.109 \times 10^{-31} \text{ kg}) + 1.673 \times 10^{-27} \text{ kg}} = 1.69 \times 10^{-28} \text{ kg}$$

We have used m_e to denote the electron mass.

$$(b) \text{ IDENTIFY: In Eq.(38.18) replace } m = m_e \text{ by } m_r: E_n = -\frac{1}{\epsilon_0^2} \frac{m_r e^4}{8n^2 h^2}.$$

SET UP: Write as $E_n = \left(\frac{m_r}{m_H} \right) \left(-\frac{1}{\epsilon_0^2} \frac{m_H e^4}{8n^2 h^2} \right)$, since we know that $\frac{1}{\epsilon_0^2} \frac{m_H e^4}{8h^2} = 13.60 \text{ eV}$. Here m_H denotes the reduced mass for the hydrogen atom; $m_H = 0.99946(9.109 \times 10^{-31} \text{ kg}) = 9.104 \times 10^{-31} \text{ kg}$.

$$\text{EXECUTE: } E_n = \left(\frac{m_r}{m_H} \right) \left(-\frac{13.60 \text{ eV}}{n^2} \right)$$

$$E_1 = \frac{1.69 \times 10^{-28} \text{ kg}}{9.104 \times 10^{-31} \text{ kg}} (-13.60 \text{ eV}) = 186(-13.60 \text{ eV}) = -2.53 \text{ keV}$$

(c) **SET UP:** From part (b), $E_n = \left(\frac{m_r}{m_H} \right) \left(-\frac{R_H ch}{n^2} \right)$, where $R_H = 1.097 \times 10^7 \text{ m}^{-1}$ is the Rydberg constant for the hydrogen atom. Use this result in $\frac{hc}{\lambda} = E_i - E_f$ to find an expression for $1/\lambda$. The initial level for the transition is the $n_i = 2$ level and the final level is the $n_f = 1$ level.

EXECUTE:
$$\frac{hc}{\lambda} = \frac{m_r}{m_H} \left(-\frac{R_H ch}{n_i^2} - \left(-\frac{R_H ch}{n_f^2} \right) \right)$$

$$\frac{1}{\lambda} = \frac{m_r}{m_H} R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{1}{\lambda} = \frac{1.69 \times 10^{-28} \text{ kg}}{9.104 \times 10^{-31} \text{ kg}} (1.097 \times 10^7 \text{ m}^{-1}) \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 1.527 \times 10^9 \text{ m}^{-1}$$

$$\lambda = 0.655 \text{ nm}$$

EVALUATE: From Example 38.6 the wavelength of the radiation emitted in this transition in hydrogen is 122 nm.

The wavelength for muonium is $\frac{m_H}{m_\mu} = 5.39 \times 10^{-3}$ times this. The reduced mass for hydrogen is very close to the electron mass because the electron mass is much less than the proton mass: $m_p/m_e = 1836$. The muon mass is $207m_e = 1.886 \times 10^{-28} \text{ kg}$. The proton is only about 10 times more massive than the muon, so the reduced mass is somewhat smaller than the muon mass. The muon-proton atom has much more strongly bound energy levels and much shorter wavelengths in its spectrum than for hydrogen.

38.60. (a) The change in wavelength of the scattered photon is given by Eq. 38.23

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi) \Rightarrow \lambda = \lambda' - \frac{h}{mc} (1 - \cos \phi) =$$

$$(0.0830 \times 10^{-9} \text{ m}) - \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} (1 + 1) = 0.0781 \text{ nm}.$$

(b) Since the collision is one-dimensional, the magnitude of the electron's momentum must be equal to the magnitude of the change in the photon's momentum. Thus,

$$p_e = h \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(\frac{1}{0.0781} + \frac{1}{0.0830} \right) (10^9 \text{ m}^{-1})$$

$$= 1.65 \times 10^{-23} \text{ kg} \cdot \text{m/s} \approx 2 \times 10^{-23} \text{ kg} \cdot \text{m/s}.$$

(c) Since the electron is non relativistic ($\beta = 0.06$), $K_e = \frac{p_e^2}{2m} = 1.49 \times 10^{-16} \text{ J} \approx 10^{-16} \text{ J}$.

38.61. IDENTIFY and SET UP: $\lambda' = \lambda + \frac{h}{mc} (1 - \cos \phi)$

$\phi = 180^\circ$ so $\lambda' = \lambda + \frac{2h}{mc} = 0.09485 \text{ m}$. Use Eq.(38.5) to calculate the momentum of the scattered photon. Apply conservation of energy to the collision to calculate the kinetic energy of the electron after the scattering. The energy of the photon is given by Eq.(38.2),

EXECUTE: (a) $p' = h/\lambda' = 6.99 \times 10^{-24} \text{ kg} \cdot \text{m/s}$.

(b) $E = E' + E_e$; $hc/\lambda = hc/\lambda' + E_e$

$$E_e = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = (hc) \frac{\lambda' - \lambda}{\lambda \lambda'} = 1.129 \times 10^{-16} \text{ J} = 705 \text{ eV}$$

EVALUATE: The energy of the incident photon is 13.8 keV, so only about 5% of its energy is transferred to the electron. This corresponds to a fractional shift in the photon's wavelength that is also 5%.

38.62. (a) $\phi = 180^\circ$ so $(1 - \cos \phi) = 2 \Rightarrow \Delta \lambda = \frac{2h}{mc} = 0.0049 \text{ nm}$, so $\lambda' = 0.1849 \text{ nm}$.

(b) $\Delta E = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = 2.93 \times 10^{-17} \text{ J} = 183 \text{ eV}$. This will be the kinetic energy of the electron.

(c) The kinetic energy is far less than the rest mass energy, so a non-relativistic calculation is adequate;

$$v = \sqrt{2K/m} = 8.02 \times 10^6 \text{ m/s}.$$

38.63. IDENTIFY and SET UP: The H_α line in the Balmer series corresponds to the $n = 3$ to $n = 2$ transition.

$$E_n = -\frac{13.6 \text{ eV}}{n^2} \cdot \frac{hc}{\lambda} = \Delta E.$$

EXECUTE: (a) The atom must be given an amount of energy $E_3 - E_1 = -(13.6 \text{ eV})\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = 12.1 \text{ eV}$.

(b) There are three possible transitions. $n = 3 \rightarrow n = 1$: $\Delta E = 12.1 \text{ eV}$ and $\lambda = \frac{hc}{\Delta E} = 103 \text{ nm}$;

$n = 3 \rightarrow n = 2$: $\Delta E = -(13.6 \text{ eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.89 \text{ eV}$ and $\lambda = 657 \text{ nm}$; $n = 2 \rightarrow n = 1$:

$\Delta E = -(13.6 \text{ eV})\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2 \text{ eV}$ and $\lambda = 122 \text{ nm}$.

38.64. $\frac{n_2}{n_1} = e^{-(E_{\text{ex}} - E_g)/kT} \Rightarrow T = \frac{-(E_{\text{ex}} - E_g)}{k \ln(n_2/n_1)}.$

$$E_{\text{ex}} = E_2 = \frac{-13.6 \text{ eV}}{4} = -3.4 \text{ eV}. \quad E_g = -13.6 \text{ eV}. \quad E_{\text{ex}} - E_g = 10.2 \text{ eV} = 1.63 \times 10^{-18} \text{ J}.$$

(a) $\frac{n_2}{n_1} = 10^{-12}$. $T = \frac{-(1.63 \times 10^{-18} \text{ J})}{(1.38 \times 10^{-23} \text{ J/K}) \ln(10^{-12})} = 4275 \text{ K}.$

(b) $\frac{n_2}{n_1} = 10^{-8}$. $T = \frac{-(1.63 \times 10^{-18} \text{ J})}{(1.38 \times 10^{-23} \text{ J/K}) \ln(10^{-8})} = 6412 \text{ K}.$

(c) $\frac{n_2}{n_1} = 10^{-4}$. $T = \frac{-(1.63 \times 10^{-18} \text{ J})}{(1.38 \times 10^{-23} \text{ J/K}) \ln(10^{-4})} = 12824 \text{ K}.$

(d) For absorption to take place in the Balmer series, hydrogen must *start* in the $n = 2$ state. From part (a), colder stars have fewer atoms in this state leading to weaker absorption lines.

38.65. (a) IDENTIFY and SET UP: The photon energy is given to the electron in the atom. Some of this energy overcomes the binding energy of the atom and what is left appears as kinetic energy of the free electron. Apply $hf = E_f - E_i$, the energy given to the electron in the atom when a photon is absorbed.

EXECUTE: The energy of one photon is $\frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{85.5 \times 10^{-9} \text{ m}}$

$$\frac{hc}{\lambda} = 2.323 \times 10^{-18} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 14.50 \text{ eV}.$$

The final energy of the electron is $E_f = E_i + hf$. In the ground state of the hydrogen atom the energy of the electron is $E_i = -13.60 \text{ eV}$. Thus $E_f = -13.60 \text{ eV} + 14.50 \text{ eV} = 0.90 \text{ eV}$.

(b) **EVALUATE:** At thermal equilibrium a few atoms will be in the $n = 2$ excited levels, which have an energy of $-13.6 \text{ eV}/4 = -3.40 \text{ eV}$, 10.2 eV greater than the energy of the ground state. If an electron with $E = -3.40 \text{ eV}$ gains 14.5 eV from the absorbed photon, it will end up with $14.5 \text{ eV} - 3.4 \text{ eV} = 11.1 \text{ eV}$ of kinetic energy.

38.66. IDENTIFY: The diffraction grating allows us to determine the peak-intensity wavelength of the light. Then Wien's displacement law allows us to calculate the temperature of the blackbody, and the Stefan-Boltzmann law allows us to calculate the rate at which it radiates energy.

SET UP: The bright spots for a diffraction grating occur when $d \sin \theta = m\lambda$. Wien's displacement law is

$$\lambda_{\text{peak}} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}, \text{ and the Stefan-Boltzmann law says that the intensity of the radiation is } I = \sigma T^4, \text{ so the}$$

total radiated power is $P = \sigma AT^4$.

EXECUTE: (a) First find the wavelength of the light:

$$\lambda = d \sin \theta = [1/(385,000 \text{ lines/m})] \sin(11.6^\circ) = 5.22 \times 10^{-7} \text{ m}$$

Now use Wien's law to find the temperature: $T = (2.90 \times 10^{-3} \text{ m} \cdot \text{K})/(5.22 \times 10^{-7} \text{ m}) = 5550 \text{ K}.$

(b) The energy radiated by the blackbody is equal to the power times the time, giving

$$U = Pt = IAt = \sigma AT^4 t, \text{ which gives}$$

$$t = U/(\sigma AT^4) = (12.0 \times 10^6 \text{ J})/[(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(4\pi)(0.0750 \text{ m})^2(5550 \text{ K})^4] = 3.16 \text{ s}.$$

EVALUATE: By ordinary standards, this blackbody is very hot, so it does not take long to radiate 12.0 MJ of energy.

- 38.67. IDENTIFY:** Assuming that Betelgeuse radiates like a perfect blackbody, Wien's displacement and the Stefan-Boltzmann law apply to its radiation.

SET UP: Wien's displacement law is $\lambda_{\text{peak}} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$, and the Stefan-Boltzmann law says that the

intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: (a) First use Wien's law to find the peak wavelength:

$$\lambda_m = (2.90 \times 10^{-3} \text{ m} \cdot \text{K}) / (3000 \text{ K}) = 9.667 \times 10^{-7} \text{ m}$$

Call N the number of photons/second radiated. $N \times (\text{energy per photon}) = IA = \sigma A T^4$.

$$N(hc/\lambda_m) = \sigma A T^4. \quad N = \frac{\lambda_m \sigma A T^4}{hc}$$

$$N = \frac{(9.667 \times 10^{-7} \text{ m})(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(4\pi)(600 \times 6.96 \times 10^8 \text{ m})^2(3000 \text{ K})^4}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}$$

$$N = 5 \times 10^{49} \text{ photons/s.}$$

$$(b) \frac{I_B A_B}{I_S A_S} = \frac{\sigma A_B T_B^4}{\sigma A_S T_S^4} = \frac{4\pi R_B^2 T_B^4}{4\pi R_S^2 T_S^4} = \left(\frac{600 R_S}{R_S}\right)^2 \left(\frac{3000 \text{ K}}{5800 \text{ K}}\right)^4 = 3 \times 10^4$$

EVALUATE: Betelgeuse radiates 30,000 times as much energy per second as does our sun!

- 38.68. IDENTIFY:** The blackbody radiates heat into the water, but the water also radiates heat back into the blackbody. The net heat entering the water causes evaporation. Wien's law tells us the peak wavelength radiated, but a thermophile in the water measures the wavelength and frequency of the light in the water.

SET UP: By the Stefan-Boltzmann law, the net power radiated by the blackbody is $\frac{dQ}{dt} = \sigma A(T_{\text{sphere}}^4 - T_{\text{water}}^4)$. Since

this heat evaporates water, the rate at which water evaporates is $\frac{dQ}{dt} = L_v \frac{dm}{dt}$. Wien's displacement law is

$$\lambda_m = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}, \text{ and the wavelength in the water is } \lambda_w = \lambda_0/n.$$

EXECUTE: (a) The net radiated heat is $\frac{dQ}{dt} = \sigma A(T_{\text{sphere}}^4 - T_{\text{water}}^4)$ and the evaporation rate is $\frac{dQ}{dt} = L_v \frac{dm}{dt}$, where

dm is the mass of water that evaporates in time dt . Equating these two rates gives $L_v \frac{dm}{dt} = \sigma A(T_{\text{sphere}}^4 - T_{\text{water}}^4)$.

$$\frac{dm}{dt} = \frac{\sigma(4\pi R^2)(T_{\text{sphere}}^4 - T_{\text{water}}^4)}{L_v}$$

$$\frac{dm}{dt} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(4\pi)(0.120 \text{ m})^2[(498 \text{ K})^4 - (373 \text{ K})^4]}{2256 \times 10^3 \text{ J/Kg}} = 1.92 \times 10^{-4} \text{ kg/s} = 0.193 \text{ g/s}$$

(b) (i) Wien's law gives $\lambda_m = (0.00290 \text{ m} \cdot \text{K}) / (498 \text{ K}) = 5.82 \times 10^{-6} \text{ m}$

But this would be the wavelength in vacuum. In the water the thermophile organism would measure $\lambda_w = \lambda_0/n = (5.82 \times 10^{-6} \text{ m}) / 1.333 = 4.37 \times 10^{-6} \text{ m} = 4.37 \mu\text{m}$

(ii) The frequency is the same as if the wave were in air, so

$$f = c/\lambda_0 = (3.00 \times 10^8 \text{ m/s}) / (5.82 \times 10^{-6} \text{ m}) = 5.15 \times 10^{13} \text{ Hz}$$

EVALUATE: An alternative way is to use the quantities in the water: $f = \frac{c/n}{\lambda_0/n} = c/\lambda_0$, which gives the same

answer for the frequency. An organism in the water would measure the light coming to it through the water, so the wavelength it would measure would be reduced by a factor of $1/n$.

- 38.69. IDENTIFY:** The energy of the peak-intensity photons must be equal to the energy difference between the $n = 1$ and the $n = 4$ states. Wien's law allows us to calculate what the temperature of the blackbody must be for it to radiate with its peak intensity at this wavelength.

SET UP: In the Bohr model, the energy of an electron in shell n is $E_n = -\frac{13.6 \text{ eV}}{n^2}$, and Wien's displacement law

$$\text{is } \lambda_m = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}. \text{ The energy of a photon is } E = hf = hc/\lambda.$$

EXECUTE: First find the energy (ΔE) that a photon would need to excite the atom. The ground state of the atom is $n = 1$ and the third excited state is $n = 4$. This energy is the *difference* between the two energy levels. Therefore

$\Delta E = (-13.6 \text{ eV})\left(\frac{1}{4^2} - \frac{1}{1^2}\right) = 12.8 \text{ eV}$. Now find the wavelength of the photon having this amount of energy.

$hc/\lambda = 12.8 \text{ eV}$ and

$$\lambda = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(12.8 \text{ eV}) = 9.73 \times 10^{-8} \text{ m}$$

Now use Wien's law to find the temperature. $T = (0.00290 \text{ m} \cdot \text{K})/(9.73 \times 10^{-8} \text{ m}) = 2.98 \times 10^4 \text{ K}$.

EVALUATE: This temperature is well above ordinary room temperatures, which is why hydrogen atoms are not in excited states during everyday conditions.

38.70. IDENTIFY and SET UP: Electrical power is VI . $Q = mc\Delta T$.

EXECUTE: (a) $(0.010)VI = (0.010)(18.0 \times 10^3 \text{ V})(60.0 \times 10^{-3} \text{ A}) = 10.8 \text{ W} = 10.8 \text{ J/s}$

(b) The energy in the electron beam that isn't converted to x rays stays in the target and appears as thermal energy.

For $t = 1.00 \text{ s}$, $Q = (0.990)VI(1.00 \text{ s}) = 1.07 \times 10^3 \text{ J}$ and $\Delta T = \frac{Q}{mc} = \frac{1.07 \times 10^3 \text{ J}}{(0.250 \text{ kg})(130 \text{ J/kg} \cdot \text{K})} = 32.9 \text{ K}$. The

temperature rises at a rate of 32.9 K/s .

EVALUATE: The target must be made of a material that has a high melting point.

38.71. IDENTIFY: Apply conservation of energy and conservation of linear momentum to the system of atom plus photon.

(a) **SET UP:** Let E_{tr} be the transition energy, E_{ph} be the energy of the photon with wavelength λ' , and E_r be the kinetic energy of the recoiling atom. Conservation of energy gives $E_{ph} + E_r = E_{tr}$.

$$E_{ph} = \frac{hc}{\lambda'} \text{ so } \frac{hc}{\lambda'} = E_{tr} - E_r \text{ and } \lambda' = \frac{hc}{E_{tr} - E_r}.$$

EXECUTE: If the recoil energy is neglected then the photon wavelength is $\lambda = hc/E_{tr}$.

$$\Delta\lambda = \lambda' - \lambda = hc\left(\frac{1}{E_{tr} - E_r} - \frac{1}{E_{tr}}\right) = \left(\frac{hc}{E_{tr}}\right)\left(\frac{1}{1 - E_r/E_{tr}} - 1\right)$$

$$\frac{1}{1 - E_r/E_{tr}} = \left(1 - \frac{E_r}{E_{tr}}\right)^{-1} \approx 1 + \frac{E_r}{E_{tr}} \text{ since } \frac{E_r}{E_{tr}} \ll 1$$

(We have used the binomial theorem, Appendix B.)

$$\text{Thus } \Delta\lambda = \frac{hc}{E_{tr}}\left(\frac{E_r}{E_{tr}}\right), \text{ or since } E_{tr} = hc/\lambda, \Delta\lambda = \left(\frac{E_r}{hc}\right)\lambda^2.$$

SET UP: Use conservation of linear momentum to find E_r : Assuming that the atom is initially at rest, the momentum p_r of the recoiling atom must be equal in magnitude and opposite in direction to the momentum $p_{ph} = h/\lambda$ of the emitted photon: $h/\lambda = p_r$.

$$\text{EXECUTE: } E_r = \frac{p_r^2}{2m}, \text{ where } m \text{ is the mass of the atom, so } E_r = \frac{h^2}{2m\lambda^2}.$$

$$\text{Use this result in the above equation: } \Delta\lambda = \left(\frac{E_r}{hc}\right)\lambda^2 = \left(\frac{h^2}{2m\lambda^2}\right)\left(\frac{\lambda^2}{hc}\right) = \frac{h}{2mc};$$

note that this result for $\Delta\lambda$ is independent of the atomic transition energy.

$$\text{(b) For a hydrogen atom } m = m_p \text{ and } \Delta\lambda = \frac{h}{2m_p c} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2(1.673 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})} = 6.61 \times 10^{-16} \text{ m}$$

EVALUATE: The correction is independent of n . The wavelengths of photons emitted in hydrogen atom transitions are on the order of $100 \text{ nm} = 10^{-7} \text{ m}$, so the recoil correction is exceedingly small.

38.72. (a) $\Delta\lambda_1 = (h/mc)(1 - \cos\theta_1)$, $\Delta\lambda_2 = (h/mc)(1 - \cos\theta_2)$, and so the overall wavelength shift is

$$\Delta\lambda = (h/mc)(2 - \cos\theta_1 - \cos\theta_2).$$

(b) For a single scattering through angle θ , $\Delta\lambda_s = (h/mc)(1 - \cos\theta)$. For two successive scatterings through an angle of $\theta/2$ for each scattering,

$$\Delta\lambda_t = 2(h/mc)(1 - \cos\theta/2).$$

$$1 - \cos\theta = 2(1 - \cos^2(\theta/2)) \text{ and } \Delta\lambda_s = (h/mc)2(1 - \cos^2(\theta/2))$$

$$\cos(\theta/2) \leq 1 \text{ so } 1 - \cos^2(\theta/2) \geq (1 - \cos(\theta/2)) \text{ and } \Delta\lambda_s \geq \Delta\lambda_t$$

Equality holds only when $\theta = 180^\circ$.

(c) $(h/mc)2(1 - \cos 30.0^\circ) = 0.268(h/mc)$.

(d) $(h/mc)(1 - \cos 60^\circ) = 0.500(h/mc)$, which is indeed greater than the shift found in part (c).

- 38.73. IDENTIFY and SET UP:** Find the average change in wavelength for one scattering and use that in $\Delta\lambda$ in Eq.(38.23) to calculate the average scattering angle ϕ .

EXECUTE: (a) The wavelength of a 1 MeV photon is

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1 \times 10^6 \text{ eV}} = 1 \times 10^{-12} \text{ m}$$

The total change in wavelength therefore is $500 \times 10^{-9} \text{ m} - 1 \times 10^{-12} \text{ m} = 500 \times 10^{-9} \text{ m}$.

If this shift is produced in 10^{26} Compton scattering events, the wavelength shift in each scattering event is

$$\Delta\lambda = \frac{500 \times 10^{-9} \text{ m}}{1 \times 10^{26}} = 5 \times 10^{-33} \text{ m}.$$

(b) Use this $\Delta\lambda$ in $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$ and solve for ϕ . We anticipate that ϕ will be very small, since $\Delta\lambda$ is much less than h/mc , so we can use $\cos\phi \approx 1 - \phi^2/2$.

$$\Delta\lambda = \frac{h}{mc}(1 - (1 - \phi^2/2)) = \frac{h}{2mc}\phi^2$$

$$\phi = \sqrt{\frac{2\Delta\lambda}{h/mc}} = \sqrt{\frac{2(5 \times 10^{-33} \text{ m})}{2.426 \times 10^{-12} \text{ m}}} = 6.4 \times 10^{-11} \text{ rad} = (4 \times 10^{-9})^\circ$$

ϕ in radians is much less than 1 so the approximation we used is valid.

(c) **IDENTIFY and SET UP:** We know the total transit time and the total number of scatterings, so we can calculate the average time between scatterings.

EXECUTE: The total time to travel from the core to the surface is $(10^6 \text{ y})(3.156 \times 10^7 \text{ s/y}) = 3.2 \times 10^{13} \text{ s}$. There are

10^{26} scatterings during this time, so the average time between scatterings is $t = \frac{3.2 \times 10^{13} \text{ s}}{10^{26}} = 3.2 \times 10^{-13} \text{ s}$.

The distance light travels in this time is $d = ct = (3.0 \times 10^8 \text{ m/s})(3.2 \times 10^{-13} \text{ s}) = 0.1 \text{ mm}$

EVALUATE: The photons are on the average scattered through a very small angle in each scattering event. The average distance a photon travels between scatterings is very small.

- 38.74. (a)** The final energy of the photon is $E' = \frac{hc}{\lambda'}$, and $E = E' + K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda = \frac{hc}{E' + K} = \frac{hc}{(hc/\lambda') + K} = \frac{hc}{(hc/\lambda') + (\gamma - 1)mc^2} = \frac{\lambda'}{1 + \frac{\lambda' mc}{h} \left[\frac{1}{(1 - v^2/c^2)^{1/2}} - 1 \right]}.$$

($K = mc^2(\gamma - 1)$ since the relativistic expression must be used for three-figure accuracy).

(b) $\phi = \arccos(1 - \Delta\lambda/(h/mc))$.

(c) $\gamma - 1 = \frac{1}{\left(1 - \left(\frac{1.80}{3.00}\right)^2\right)^{1/2}} - 1 = 1.25 - 1 = 0.250$, $\frac{h}{mc} = 2.43 \times 10^{-12} \text{ m}$

$$\Rightarrow \lambda = \frac{5.10 \times 10^{-3} \text{ mm}}{1 + \frac{(5.10 \times 10^{-12} \text{ m})(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})(0.250)}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}} = 3.34 \times 10^{-3} \text{ nm}.$$

$$\phi = \arccos\left(1 - \frac{(5.10 \times 10^{-12} \text{ m} - 3.34 \times 10^{-12} \text{ m})}{2.43 \times 10^{-12} \text{ m}}\right) = 74.0^\circ.$$

- 38.75. (a) IDENTIFY and SET UP:** Conservation of energy applied to the collision gives $E_\lambda = E_{\lambda'} + E_e$, where E_e is the kinetic energy of the electron after the collision and E_λ and $E_{\lambda'}$ are the energies of the photon before and after the collision. The energy of a photon is related to its wavelength according to Eq.(38.2).

EXECUTE: $E_e = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = hc \left(\frac{\lambda' - \lambda}{\lambda \lambda'} \right)$

$$E_e = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s}) \left(\frac{0.0032 \times 10^{-9} \text{ m}}{(0.1100 \times 10^{-9} \text{ m})(0.1132 \times 10^{-9} \text{ m})} \right)$$

$$E_e = 5.105 \times 10^{-17} \text{ J} = 319 \text{ eV}$$

$$E_e = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{2(5.105 \times 10^{-17} \text{ J})}{9.109 \times 10^{-31} \text{ kg}}} = 1.06 \times 10^7 \text{ m/s}$$

(b) The wavelength λ of a photon with energy E_e is given by $E_e = hc/\lambda$ so

$$\lambda = \frac{hc}{E_e} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{5.105 \times 10^{-17} \text{ J}} = 3.89 \text{ nm}$$

EVALUATE: Only a small portion of the incident photon's energy is transferred to the struck electron; this is why the wavelength calculated in part (b) is much larger than the wavelength of the incident photon in the Compton scattering.

38.76. IDENTIFY: Apply the Compton scattering formula $\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1 - \cos\phi) = \lambda_c(1 - \cos\phi)$

(a) SET UP: Largest $\Delta\lambda$ is for $\phi = 180^\circ$.

EXECUTE: For $\phi = 180^\circ$, $\Delta\lambda = 2\lambda_c = 2(2.426 \text{ pm}) = 4.85 \text{ pm}$.

(b) SET UP: $\lambda' - \lambda = \lambda_c(1 - \cos\phi)$

Wavelength doubles implies $\lambda' = 2\lambda$ so $\lambda' - \lambda = \lambda$. Thus $\lambda = \lambda_c(1 - \cos\phi)$. λ is related to E by Eq.(38.2).

EXECUTE: $E = hc/\lambda$, so smallest energy photon means largest wavelength photon, so $\phi = 180^\circ$ and

$$\lambda = 2\lambda_c = 4.85 \text{ pm. Then } E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{4.85 \times 10^{-12} \text{ m}} = 4.096 \times 10^{-14} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 0.256 \text{ MeV.}$$

EVALUATE: Any photon Compton scattered at $\phi = 180^\circ$ has a wavelength increase of $2\lambda_c = 4.85 \text{ pm}$. 4.85 pm is near the short-wavelength end of the range of x-ray wavelengths.

38.77. (a) $I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$ but $\lambda = \frac{c}{f}$

$$\Rightarrow I(f) = \frac{2\pi hc^2}{(c/f)^5 (e^{hf/kT} - 1)} = \frac{2\pi hf^5}{c^3 (e^{hf/kT} - 1)}$$

(b) $\int_0^\infty I(\lambda) d\lambda = \int_0^\infty I(f) df \left(\frac{-c}{f^2} \right)$

$$= \int_0^\infty \frac{2\pi hf^3 df}{c^2 (e^{hf/kT} - 1)} = \frac{2\pi (kT)^4}{c^2 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{2\pi (kT)^4}{c^2 h^3} \frac{1}{240} (2\pi)^4 = \frac{(2\pi)^5 (kT)^4}{240 h^3 c^2} = \frac{2\pi^5 k^4 T^4}{15 c^2 h^3}$$

(c) The expression $\frac{2\pi^5 k^4 T^4}{15 h^3 c^2} = \sigma$ as shown in Eq. (38.36). Plugging in the values for the constants we get

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4.$$

38.78. $I = \sigma T^4$, $P = IA$, and $\Delta E = Pt$; combining,

$$t = \frac{\Delta E}{A\sigma T^4} = \frac{(100 \text{ J})}{(4.00 \times 10^{-6} \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(473 \text{ K})^4} = 8.81 \times 10^3 \text{ s} = 2.45 \text{ hrs.}$$

38.79. (a) The period was found in Exercise 38.27b: $T = \frac{4\epsilon_0^2 n^3 h^3}{me^4}$ and frequency is just $f = \frac{1}{T} = \frac{me^4}{4\epsilon_0^2 n^3 h^3}$.

(b) Eq. (38.6) tells us that $f = \frac{1}{h}(E_2 - E_1)$. So $f = \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$ (from Eq. (38.18)).

If $n_2 = n$ and $n_1 = n+1$, then $\frac{1}{n_2^2} - \frac{1}{n_1^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$$= \frac{1}{n^2} \left(1 - \frac{1}{(1+1/n)^2} \right) \approx \frac{1}{n^2} \left(1 - \left(1 - \frac{2}{n} + \dots \right) \right) = \frac{2}{n^3} \text{ for large } n \Rightarrow f \approx \frac{me^4}{4\epsilon_0^2 n^3 h^3}.$$

38.80. Each photon has momentum $p = \frac{h}{\lambda}$, and if the rate at which the photons strike the surface is (dN/dt) , the force on the surface is $(h/\lambda)(dN/dt)$, and the pressure is $(h/\lambda)(dN/dt)/A$. The intensity is $I = (dN/dt)(E)/A = (dN/dt)(hc/\lambda)/A$, and comparison of the two expressions gives the pressure as (I/c) .

38.81. Momentum: $\vec{p} + \vec{P} = \vec{p}' + \vec{P}' \Rightarrow p - P = -p' - P' \Rightarrow p' = P - (p + P')$

$$\text{energy: } pc + E = p'c + E' = p'c + \sqrt{(P'c)^2 + (mc^2)^2}$$

$$\Rightarrow (pc - p'c + E)^2 = (P'c)^2 + (mc^2)^2 = (Pc)^2 + ((p + p')c)^2 - 2P(p + p')c^2 + (mc^2)^2$$

$$(pc - p'c)^2 + E^2 = E^2 + (pc + p'c)^2 - 2(Pc^2)(p + p') + 2Ec(p - p') - 4pp'c^2 + 2Ec(p - p')$$

$$+2(Pc^2)(p + p') = 0$$

$$\Rightarrow p'(Pc^2 - 2pc^2 - Ec) = p(-Ec - Pc^2)$$

$$\Rightarrow p' = p \frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p \frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda \left(\frac{2hc/\lambda + (E - Pc)}{E + Pc} \right) = \lambda \left(\frac{E - Pc}{E + Pc} \right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{(\lambda(E - Pc) + 2hc)}{E + Pc}$$

$$\text{If } E \gg mc^2, Pc = \sqrt{E^2 - (mc^2)^2} = E \sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \approx E \left(1 - \frac{1}{2} \left(\frac{mc^2}{E}\right)^2 + \dots \right)$$

$$\Rightarrow E - Pc \approx \frac{1}{2} \frac{(mc^2)^2}{E} \Rightarrow \lambda_1 \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E} \left(1 + \frac{m^2 c^4 \lambda}{4hcE} \right)$$

(b) If $\lambda = 10.6 \times 10^{-6} \text{ m}$, $E = 1.00 \times 10^{10} \text{ eV} = 1.60 \times 10^{-9} \text{ J}$

$$\begin{aligned} \Rightarrow \lambda' &\approx \frac{hc}{1.60 \times 10^{-9} \text{ J}} \left(1 + \frac{(9.11 \times 10^{-31} \text{ kg})^2 c^4 (10.6 \times 10^{-6} \text{ m})}{4hc (1.6 \times 10^{-9} \text{ J})} \right) \\ &= (1.24 \times 10^{-16} \text{ m})(1 + 56.0) = 7.08 \times 10^{-15} \text{ m}. \end{aligned}$$

(c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.

THE WAVE NATURE OF PARTICLES

39.1. IDENTIFY and SET UP: $\lambda = \frac{h}{p} = \frac{h}{mv}$. For an electron, $m = 9.11 \times 10^{-31}$ kg. For a proton, $m = 1.67 \times 10^{-27}$ kg.

EXECUTE: (a) $\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(4.70 \times 10^6 \text{ m/s})} = 1.55 \times 10^{-10} \text{ m} = 0.155 \text{ nm}$

(b) λ is proportional to $\frac{1}{m}$, so $\lambda_p = \lambda_e \left(\frac{m_e}{m_p} \right) = (1.55 \times 10^{-10} \text{ m}) \left(\frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} \right) = 8.46 \times 10^{-14} \text{ m}$.

39.2. IDENTIFY and SET UP: For a photon, $E = \frac{hc}{\lambda}$. For an electron or proton, $p = \frac{h}{\lambda}$ and $E = \frac{p^2}{2m}$, so $E = \frac{h^2}{2m\lambda^2}$.

EXECUTE: (a) $E = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{0.20 \times 10^{-9} \text{ m}} = 6.2 \text{ keV}$

(b) $E = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(0.20 \times 10^{-9} \text{ m})^2} = 6.03 \times 10^{-18} \text{ J} = 38 \text{ eV}$

(c) $E_p = E_e \left(\frac{m_e}{m_p} \right) = (38 \text{ eV}) \left(\frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} \right) = 0.021 \text{ eV}$

EVALUATE: For a given wavelength a photon has much more energy than an electron, which in turn has more energy than a proton.

39.3. (a) $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(2.80 \times 10^{-10} \text{ m})} = 2.37 \times 10^{-24} \text{ kg} \cdot \text{m/s}$.

(b) $K = \frac{p^2}{2m} = \frac{(2.37 \times 10^{-24} \text{ kg} \cdot \text{m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})} = 3.08 \times 10^{-18} \text{ J} = 19.3 \text{ eV}$.

39.4. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$
 $= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{2(6.64 \times 10^{-27} \text{ kg})(4.20 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}} = 7.02 \times 10^{-15} \text{ m}$.

39.5. IDENTIFY and SET UP: The de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$,

so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n . Then

$$\lambda = h \left(\frac{2\pi r_n}{nh} \right) = \frac{2\pi r_n}{n}.$$

EXECUTE: (a) For $n = 1$, $\lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529 \times 10^{-10} \text{ m}$, so $\lambda = 2\pi(0.529 \times 10^{-10} \text{ m}) = 3.32 \times 10^{-10} \text{ m}$

$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.

(b) For $n = 4$, $\lambda = 2\pi r_4/4$.

$$r_n = n^2 a_0 \text{ so } r_4 = 16a_0.$$

$$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32 \times 10^{-10} \text{ m}) = 1.33 \times 10^{-9} \text{ m}$$

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\frac{1}{n} = \frac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n , the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.6. (a) For a nonrelativistic particle, $K = \frac{p^2}{2m}$, so $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$.

(b) $(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) / \sqrt{2(800 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})(9.11 \times 10^{-31} \text{ kg})} = 4.34 \times 10^{-11} \text{ m}$.

39.7. **IDENTIFY:** A person walking through a door is like a particle going through a slit and hence should exhibit wave properties.

SET UP: The de Broglie wavelength of the person is $\lambda = h/mv$.

EXECUTE: (a) Assume $m = 75 \text{ kg}$ and $v = 1.0 \text{ m/s}$.

$$\lambda = h/mv = (6.626 \times 10^{-34} \text{ J} \cdot \text{s}) / [(75 \text{ kg})(1.0 \text{ m/s})] = 8.8 \times 10^{-36} \text{ m}$$

EVALUATE: (b) A typical doorway is about 1 m wide, so the person's de Broglie wavelength is much too small to show wave behavior through a "slit" that is about 10^{35} times as wide as the wavelength. Hence ordinary objects do not show wave behavior in everyday life.

39.8. Combining Equations 37.38 and 37.39 gives $p = mc\sqrt{\gamma^2 - 1}$.

(a) $\lambda = \frac{h}{p} = (h/mc) / \sqrt{\gamma^2 - 1} = 4.43 \times 10^{-12} \text{ m}$. (The incorrect nonrelativistic calculation gives $5.05 \times 10^{-12} \text{ m}$.)

(b) $(h/mc) / \sqrt{\gamma^2 - 1} = 7.07 \times 10^{-13} \text{ m}$.

39.9. **IDENTIFY and SET UP:** A photon has zero mass and its energy and wavelength are related by Eq.(38.2). An electron has mass. Its energy is related to its momentum by $E = p^2/2m$ and its wavelength is related to its momentum by Eq.(39.1).

EXECUTE: (a) photon

$$E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{(20.0 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})} = 62.0 \text{ nm}$$

electron

$$E = p^2/(2m) \text{ so } p = \sqrt{2mE} = \sqrt{2(9.109 \times 10^{-31} \text{ kg})(20.0 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})} = 2.416 \times 10^{-24} \text{ kg} \cdot \text{m/s}$$

$$\lambda = h/p = 0.274 \text{ nm}$$

(b) photon $E = hc/\lambda = 7.946 \times 10^{-19} \text{ J} = 4.96 \text{ eV}$

electron $\lambda = h/p$ so $p = h/\lambda = 2.650 \times 10^{-27} \text{ kg} \cdot \text{m/s}$

$$E = p^2/(2m) = 3.856 \times 10^{-24} \text{ J} = 2.41 \times 10^{-5} \text{ eV}$$

(c) **EVALUATE:** You should use a probe of wavelength approximately 250 nm. An electron with $\lambda = 250 \text{ nm}$ has much less energy than a photon with $\lambda = 250 \text{ nm}$, so is less likely to damage the molecule. Note that $\lambda = h/p$ applies to all particles, those with mass and those with zero mass. $E = hf = hc/\lambda$ applies only to photons and

$E = p^2/2m$ applies only to particles with mass.

39.10. **IDENTIFY:** Any moving particle has a de Broglie wavelength. The speed of a molecule, and hence its de Broglie wavelength, depends on the temperature of the gas.

SET UP: The average kinetic energy of the molecule is $K_{\text{av}} = 3/2 kT$, and the de Broglie wavelength is $\lambda = h/mv = h/p$.

EXECUTE: (a) Combining $K_{\text{av}} = 3/2 kT$ and $K = p^2/2m$ gives $3/2 kT = p_{\text{av}}^2/2m$ and $p_{\text{av}} = \sqrt{3mkT}$. The de Broglie

wavelength is $\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{3(2 \times 1.67 \times 10^{-27} \text{ kg})(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})}} = 1.08 \times 10^{-10} \text{ m}$.

(b) For an electron, $\lambda = h/p = h/mv$ gives

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.08 \times 10^{-10} \text{ m})} = 6.75 \times 10^6 \text{ m/s}$$

This is about 2% the speed of light, so we do *not* need to use relativity.

(c) For photon:

$$E = hc/\lambda = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s}) / (1.08 \times 10^{-10} \text{ m}) = 1.84 \times 10^{-15} \text{ J}$$

For the H_2 molecule: $K_{\text{av}} = (3/2)kT = 3/2 (1.38 \times 10^{-23} \text{ J/K})(273 \text{ K}) = 5.65 \times 10^{-21} \text{ J}$

For the electron: $K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(6.73 \times 10^6 \text{ m/s})^2 = 2.06 \times 10^{-17} \text{ J}$

EVALUATE: The photon has about 100 times more energy than the electron and 300,000 times more energy than the H_2 molecule. This shows that photons of a given wavelength will have much more energy than particles of the same wavelength.

39.11. IDENTIFY and SET UP: Use Eq.(39.1).

EXECUTE: $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(5.00 \times 10^{-3} \text{ kg})(340 \text{ m/s})} = 3.90 \times 10^{-34} \text{ m}$

EVALUATE: This wavelength is extremely short; the bullet will not exhibit wavelike properties.

39.12. (a) $\lambda = h/mv \rightarrow v = h/m\lambda$

Energy conservation: $e\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2e} = \frac{m \left(\frac{h}{m\lambda} \right)^2}{2e} = \frac{h^2}{2em\lambda^2} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(1.60 \times 10^{-19} \text{ C})(9.11 \times 10^{-31} \text{ kg})(0.15 \times 10^{-9} \text{ m})^2} = 66.9 \text{ V}$$

(b) $E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{0.15 \times 10^{-9} \text{ m}} = 1.33 \times 10^{-15} \text{ J}$

$e\Delta V = K = E_{\text{photon}}$ and $\Delta V = \frac{E_{\text{photon}}}{e} = \frac{1.33 \times 10^{-15} \text{ J}}{1.6 \times 10^{-19} \text{ C}} = 8310 \text{ V}$

39.13. (a) $\lambda = 0.10 \text{ nm}$. $p = mv = h/\lambda$ so $v = h/(m\lambda) = 7.3 \times 10^6 \text{ m/s}$.

(b) $E = \frac{1}{2}mv^2 = 150 \text{ eV}$

(c) $E = hc/\lambda = 12 \text{ KeV}$

(d) The electron is a better probe because for the same λ it has less energy and is less damaging to the structure being probed.

39.14. IDENTIFY: The electrons behave like waves and are diffracted by the slit.

SET UP: We use conservation of energy to find the speed of the electrons, and then use this speed to find their de Broglie wavelength, which is $\lambda = h/mv$. Finally we know that the first dark fringe for single-slit diffraction occurs when $a \sin \theta = \lambda$.

EXECUTE: **(a)** Use energy conservation to find the speed of the electron: $\frac{1}{2}mv^2 = eV$.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(100 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 5.93 \times 10^6 \text{ m/s}$$

which is about 2% the speed of light, so we can ignore relativity.

(b) First find the de Broglie wavelength:

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(5.93 \times 10^6 \text{ m/s})} = 1.23 \times 10^{-10} \text{ m} = 0.123 \text{ nm}$$

For the first single slit dark fringe, we have $a \sin \theta = \lambda$, which gives

$$a = \frac{\lambda}{\sin \theta} = \frac{1.23 \times 10^{-10} \text{ m}}{\sin(11.5^\circ)} = 6.16 \times 10^{-10} \text{ m} = 0.616 \text{ nm}$$

EVALUATE: The slit width is around 5 times the de Broglie wavelength of the electron, and both are much smaller than the wavelength of visible light.

39.15. For $m=1$, $\lambda = d \sin \theta = \frac{h}{\sqrt{2mE}}$.

$$E = \frac{h^2}{2md^2 \sin^2 \theta} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(1.675 \times 10^{-27} \text{ kg})(9.10 \times 10^{-11} \text{ m})^2 \sin^2(28.6^\circ)} = 6.91 \times 10^{-20} \text{ J} = 0.432 \text{ eV}.$$

39.16. Intensity maxima occur when $d \sin \theta = m\lambda$. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2ME}}$ so $d \sin \theta = \frac{mh}{\sqrt{2ME}}$. (Careful! Here, m is the order of the maxima, whereas M is the mass of the incoming particle.)

(a) $d = \frac{mh}{\sqrt{2ME \sin \theta}} = \frac{(2)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(188 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) \sin(60.6^\circ)}}$
 $= 2.06 \times 10^{-10} \text{ m} = 0.206 \text{ nm}.$

(b) $m = 1$ also gives a maximum.

$$\theta = \arcsin \left(\frac{(1)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(188 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})(2.06 \times 10^{-10} \text{ m})}} \right) = 25.8^\circ.$$

This is the only other one. If we let $m \geq 3$, then there are no more maxima.

$$(c) E = \frac{m^2 h^2}{2Md^2 \sin^2 \theta} = \frac{(1)^2 (6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(2.60 \times 10^{-10} \text{ m})^2 \sin^2 (60.6^\circ)}$$

$$= 7.49 \times 10^{-18} \text{ J} = 46.8 \text{ eV}.$$

Using this energy, if we let $m = 2$, then $\sin \theta > 1$. Thus, there is no $m = 2$ maximum in this case.

- 39.17.** The condition for a maximum is $d \sin \theta = m\lambda$. $\lambda = \frac{h}{p} = \frac{h}{Mv}$, so $\theta = \arcsin\left(\frac{mh}{dMv}\right)$. (Careful! Here, m is the order of the maximum, whereas M is the incoming particle mass.)

$$(a) m = 1 \Rightarrow \theta_1 = \arcsin\left(\frac{h}{dMv}\right)$$

$$= \arcsin\left(\frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.60 \times 10^{-6} \text{ m})(9.11 \times 10^{-31} \text{ kg})(1.26 \times 10^4 \text{ m/s})}\right) = 2.07^\circ.$$

$$m = 2 \Rightarrow \theta_2 = \arcsin\left(\frac{(2)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.60 \times 10^{-6} \text{ m})(9.11 \times 10^{-31} \text{ kg})(1.26 \times 10^4 \text{ m/s})}\right) = 4.14^\circ.$$

$$(b) \text{ For small angles (in radians!) } y \cong D\theta, \text{ so } y_1 \approx (50.0 \text{ cm})(2.07^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right) = 1.81 \text{ cm},$$

$$y_2 \approx (50.0 \text{ cm})(4.14^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right) = 3.61 \text{ cm} \text{ and } y_2 - y_1 = 3.61 \text{ cm} - 1.81 \text{ cm} = 1.81 \text{ cm}.$$

- 39.18. IDENTIFY:** Since we know only that the mosquito is somewhere in the room, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x \Delta p_x \geq \hbar$.

EXECUTE: (a) You know the mosquito is somewhere in the room, so the maximum uncertainty in its horizontal position is $\Delta x = 5.0 \text{ m}$.

(b) The uncertainty principle gives $\Delta x \Delta p_x \geq \hbar$, and $\Delta p_x = m \Delta v_x$ since we know the mosquito's mass. This gives $\Delta x m \Delta v_x \geq \hbar$, which we can solve for Δv_x to get the minimum uncertainty in v_x .

$$\Delta v_x = \frac{\hbar}{m \Delta x} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.5 \times 10^{-6} \text{ kg})(5.0 \text{ m})} = 1.4 \times 10^{-29} \text{ m/s}$$

which is hardly a serious impediment!

EVALUATE: For something as "large" as a mosquito, the uncertainty principle places a negligible limitation on our ability to measure its speed.

- 39.19. (a) IDENTIFY and SET UP:** Use $\Delta x \Delta p_x \geq \hbar/2\pi$ to calculate Δx and obtain Δv_x from this.

$$\text{EXECUTE: } \Delta p_x \geq \frac{\hbar}{2\pi \Delta x} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(1.00 \times 10^{-6} \text{ m})} = 1.055 \times 10^{-28} \text{ kg} \cdot \text{m/s}$$

$$\Delta v_x = \frac{\Delta p_x}{m} = \frac{1.055 \times 10^{-28} \text{ kg} \cdot \text{m/s}}{1200 \text{ kg}} = 8.79 \times 10^{-32} \text{ m/s}$$

(b) **EVALUATE:** Even for this very small Δx the minimum Δv_x required by the Heisenberg uncertainty principle is very small. The uncertainty principle does not impose any practical limit on the simultaneous measurements of the positions and velocities of ordinary objects.

- 39.20. IDENTIFY:** Since we know that the marble is somewhere on the table, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is therefore an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x \Delta p_x \geq \hbar$.

EXECUTE: (a) Since the marble is somewhere on the table, the maximum uncertainty in its horizontal position is $\Delta x = 1.75 \text{ m}$.

(b) Following the same procedure as in part (b) of problem 39.18, the minimum uncertainty in the horizontal velocity of the marble is

$$\Delta v_x = \frac{\hbar}{m \Delta x} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.0100 \text{ kg})(1.75 \text{ m})} = 6.03 \times 10^{-33} \text{ m/s}$$

(c) The uncertainty principle tells us that we cannot know that the marble's horizontal velocity is *exactly* zero, so the smallest we could measure it to be is $6.03 \times 10^{-33} \text{ m/s}$, from part (b). The longest time it could remain on the

table is the time to travel the full width of the table (1.75 m), so $t = x/v_x = (1.75 \text{ m})/(6.03 \times 10^{-33} \text{ m/s}) = 2.90 \times 10^{32} \text{ s} = 9.20 \times 10^{24} \text{ years}$

Since the universe is about 14×10^9 years old, this time is about

$$\frac{9.0 \times 10^{24} \text{ yr}}{14 \times 10^9 \text{ yr}} \approx 6 \times 10^{14} \text{ times the age of the universe! Don't hold your breath!}$$

EVALUATE: For household objects, the uncertainty principle places a negligible limitation on our ability to measure their speed.

- 39.21.** Heisenberg's Uncertainty Principles tells us that $\Delta x \Delta p_x \geq \frac{h}{2\pi}$. We can treat the standard deviation as a direct measure of uncertainty. Here $\Delta x \Delta p_x = (1.2 \times 10^{-10} \text{ m})(3.0 \times 10^{-25} \text{ kg} \cdot \text{m/s}) = 3.6 \times 10^{-35} \text{ J} \cdot \text{s}$ but $\frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}$

Therefore $\Delta x \Delta p_x < \frac{h}{2\pi}$ so the claim is *not valid*.

- 39.22.** (a) $(\Delta x)(m\Delta v_x) \geq h/2\pi$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $h/2\pi$ (for the minimum uncertainty) gives $v_x = h/(2\pi m(0.010)\Delta x) = 57.9 \text{ m/s}$.

(b) Repeating with the proton mass gives 31.6 mm/s.

- 39.23.** $\Delta E > \frac{h}{2\pi\Delta t} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{2\pi(5.2 \times 10^{-3} \text{ s})} = 2.03 \times 10^{-32} \text{ J} = 1.27 \times 10^{-13} \text{ eV}$.

- 39.24.** **IDENTIFY and SET UP:** The Heisenberg Uncertainty Principle says $\Delta x \Delta p_x \geq \frac{h}{2\pi}$. The minimum allowed $\Delta x \Delta p_x$ is $h/2\pi$. $\Delta p_x = m\Delta v_x$.

EXECUTE: (a) $m\Delta x \Delta v_x = \frac{h}{2\pi}$. $\Delta v_x = \frac{h}{2\pi m \Delta x} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(1.67 \times 10^{-27} \text{ kg})(2.0 \times 10^{-12} \text{ m})} = 3.2 \times 10^4 \text{ m/s}$

(b) $\Delta x = \frac{h}{2\pi m \Delta v_x} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(9.11 \times 10^{-31} \text{ kg})(0.250 \text{ m/s})} = 4.6 \times 10^{-4} \text{ m}$

- 39.25.** $\Delta E \Delta t = \frac{h}{2\pi}$. $\Delta E = \frac{h}{2\pi \Delta t} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{2\pi(7.6 \times 10^{-21} \text{ s})} = 1.39 \times 10^{-14} \text{ J} = 8.69 \times 10^4 \text{ eV} = 0.0869 \text{ MeV}$.

$$\frac{\Delta E}{E} = \frac{0.0869 \text{ MeV}/c^2}{3097 \text{ MeV}/c^2} = 2.81 \times 10^{-5}.$$

- 39.26.** $\Delta E \Delta t = \frac{h}{2\pi}$. $\Delta E = \Delta mc^2$. $\Delta m = 2.06 \times 10^9 \text{ eV}/c^2 = 3.30 \times 10^{-10} \text{ J}/c^2$.

$$\Delta t = \frac{h}{2\pi \Delta mc^2} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(3.30 \times 10^{-10} \text{ J})} = 3.20 \times 10^{-25} \text{ s}.$$

- 39.27.** **IDENTIFY and SET UP:** For a photon $E_{\text{ph}} = \frac{hc}{\lambda} = \frac{1.99 \times 10^{-25} \text{ J} \cdot \text{m}}{\lambda}$. For an electron $E_e = \frac{p^2}{2m} = \frac{1}{2m} \left(\frac{h}{\lambda} \right)^2 = \frac{h^2}{2m\lambda^2}$.

EXECUTE: (a) photon $E_{\text{ph}} = \frac{1.99 \times 10^{-25} \text{ J} \cdot \text{m}}{10.0 \times 10^{-9} \text{ m}} = 1.99 \times 10^{-17} \text{ J}$

electron $E_e = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(10.0 \times 10^{-9} \text{ m})^2} = 2.41 \times 10^{-21} \text{ J}$

$$\frac{E_{\text{ph}}}{E_e} = \frac{1.99 \times 10^{-17} \text{ J}}{2.41 \times 10^{-21} \text{ J}} = 8.26 \times 10^3$$

(b) The electron has much less energy so would be less damaging.

EVALUATE: For a particle with mass, such as an electron, $E \sim \lambda^{-2}$. For a massless photon $E \sim \lambda^{-1}$.

- 39.28.** (a) $eV = K = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$, so $V = \frac{(h/\lambda)^2}{2me} = 419 \text{ V}$.

(b) The voltage is reduced by the ratio of the particle masses, $(419 \text{ V}) \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 0.229 \text{ V}$.

- 39.29.** **IDENTIFY and SET UP:** $\psi(x) = A \sin kx$. The position probability density is given by $|\psi(x)|^2 = A^2 \sin^2 kx$.

EXECUTE: (a) The probability is highest where $\sin kx = 1$ so $kx = 2\pi x/\lambda = n\pi/2$, $n = 1, 3, 5, \dots$
 $x = n\lambda/4$, $n = 1, 3, 5, \dots$ so $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$

(b) The probability of finding the particle is zero where $|\psi|^2 = 0$, which occurs where $\sin kx = 0$ and $kx = 2\pi x/\lambda = n\pi$, $n = 0, 1, 2, \dots$
 $x = n\lambda/2$, $n = 0, 1, 2, \dots$ so $x = 0, \lambda/2, \lambda, 3\lambda/2, \dots$

EVALUATE: The situation is analogous to a standing wave, with the probability analogous to the square of the amplitude of the standing wave.

39.30. $\Psi^* = \psi^* \sin \omega t$, so $|\Psi|^2 = |\Psi^* \Psi| = \psi^* \psi \sin^2 \omega t = |\psi|^2 \sin^2 \omega t$. $|\Psi|^2$ is not time-independent, so Ψ is not the wavefunction for a stationary state.

39.31. IDENTIFY: To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2 dV$ is the probability that the particle is found in volume dV . Since the particle must be *somewhere*, ψ must have the property that $\int |\psi|^2 dV = 1$ when the integral is taken over all space.

EXECUTE: (a) In one dimension, as we have here, the integral discussed above is of the form $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$.

(b) Using the result from part (a), we have $\int_{-\infty}^{\infty} (e^{ax})^2 dx = \int_{-\infty}^{\infty} e^{2ax} dx = \frac{e^{2ax}}{2a} \Big|_{-\infty}^{\infty} = \infty$. Hence this wave function cannot be normalized and therefore cannot be a valid wave function.

(c) We only need to integrate this wave function of 0 to ∞ because it is zero for $x < 0$. For normalization we have

$$1 = \int_{-\infty}^{\infty} |\psi|^2 dx = \int_0^{\infty} (Ae^{-bx})^2 dx = \int_0^{\infty} A^2 e^{-2bx} dx = \frac{A^2 e^{-2bx}}{-2b} \Big|_0^{\infty} = \frac{A^2}{2b}, \text{ which gives } \frac{A^2}{2b} = 1, \text{ so } A = \sqrt{2b}.$$

EVALUATE: If b were positive, the given wave function could not be normalized, so it would not be allowable.

39.32. (a) The uncertainty in the particle position is proportional to the width of $\psi(x)$, and is inversely proportional to $\sqrt{\alpha}$. This can be seen by either plotting the function for different values of α , finding the expectation value $\langle x^2 \rangle = \int \psi^2 x^2 dx$ for the normalized wave function or by finding the full width at half-maximum. The particle's uncertainty in position decreases with increasing α . The dependence of the expectation value $\langle x^2 \rangle$ on α may be found by considering

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{\infty} x^2 e^{-2\alpha x^2} dx}{\int_{-\infty}^{\infty} e^{-2\alpha x^2} dx} = -\frac{1}{2} \frac{\partial}{\partial \alpha} \ln \left[\int_{-\infty}^{\infty} e^{-2\alpha x^2} dx \right] = -\frac{1}{2} \frac{\partial}{\partial \alpha} \ln \left[\frac{1}{\sqrt{2\alpha}} \int_{-\infty}^{\infty} e^{-u^2} du \right] = \frac{1}{4\alpha},$$

where the substitution $u = \sqrt{\alpha}x$ has been made.

(b) Since the uncertainty in position decreases, the uncertainty in momentum must increase.

39.33. $f(x, y) = \left(\frac{x-iy}{x+iy} \right)$ and $f^*(x, y) = \left(\frac{x+iy}{x-iy} \right) \Rightarrow |f|^2 = f f^* = \left(\frac{x-iy}{x+iy} \right) \cdot \left(\frac{x+iy}{x-iy} \right) = 1$.

39.34. The same. $|\psi(x, y, z)|^2 = \psi^*(x, y, z)\psi(x, y, z)$

$$|\psi(x, y, z)e^{i\phi}|^2 = (\psi^*(x, y, z)e^{-i\phi})(\psi(x, y, z)e^{i\phi}) = \psi^*(x, y, z)\psi(x, y, z).$$

The complex conjugate means convert all i 's to $-i$'s and vice-versa. $e^{i\phi} \cdot e^{-i\phi} = 1$.

39.35. IDENTIFY: To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2 dV$ is the probability that the particle is found in volume dV . Since the particle must be *somewhere*, ψ must have the property that $\int |\psi|^2 dV = 1$ when the integral is taken over all space.

EXECUTE: (a) For normalization of the one-dimensional wave function, we have

$$1 = \int_{-\infty}^{\infty} |\psi|^2 dx = \int_{-\infty}^0 (Ae^{bx})^2 dx + \int_0^{\infty} (Ae^{-bx})^2 dx = \int_{-\infty}^0 A^2 e^{2bx} dx + \int_0^{\infty} A^2 e^{-2bx} dx.$$

$$1 = A^2 \left\{ \frac{e^{2bx}}{2b} \Big|_{-\infty}^0 + \frac{e^{-2bx}}{-2b} \Big|_0^{\infty} \right\} = \frac{A^2}{b}, \text{ which gives } A = \sqrt{b} = \sqrt{2.00 \text{ m}^{-1}} = 1.41 \text{ m}^{-1/2}$$

(b) The graph of the wavefunction versus x is given in Figure 39.35.

(c) (i) $P = \int_{-0.500 \text{ m}}^{+5.00 \text{ m}} |\psi|^2 dx = 2 \int_0^{+5.00 \text{ m}} A^2 e^{-2bx} dx$, where we have used the fact that the wave function is an even function of x . Evaluating the integral gives

$$P = \frac{-A^2}{b} (e^{-2b(0.500 \text{ m})} - 1) = \frac{-(2.00 \text{ m}^{-1})}{2.00 \text{ m}^{-1}} (e^{-2.00} - 1) = 0.865$$

There is a little more than an 86% probability that the particle will be found within 50 cm of the origin.

$$(ii) P = \int_{-\infty}^0 (Ae^{bx})^2 dx = \int_{-\infty}^0 A^2 e^{2bx} dx = \frac{A^2}{2b} = \frac{2.00 \text{ m}^{-1}}{2(2.00 \text{ m}^{-1})} = \frac{1}{2} = 0.500$$

There is a 50-50 chance that the particle will be found to the left of the origin, which agrees with the fact that the wave function is symmetric about the y-axis.

$$(iii) P = \int_{0.500 \text{ m}}^{1.00 \text{ m}} A^2 e^{-2bx} dx = \frac{A^2}{-2b} (e^{-2(2.00 \text{ m}^{-1})(1.00 \text{ m})} - e^{-2(2.00 \text{ m}^{-1})(0.500 \text{ m})}) = -\frac{1}{2} (e^{-4} - e^{-2}) = 0.0585$$

EVALUATE: There is little chance of finding the particle in regions where the wave function is small.

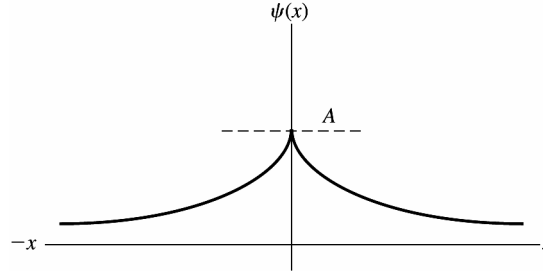


Figure 39.35

$$\begin{aligned} 39.36. \quad \text{Eq. (39.18): } \frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi &= E\psi. \text{ Let } \psi = A\psi_1 + B\psi_2 \\ \Rightarrow \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} (A\psi_1 + B\psi_2) + U(A\psi_1 + B\psi_2) &= E(A\psi_1 + B\psi_2) \\ \Rightarrow A \left(-\frac{\hbar^2}{2m} \frac{d^2\psi_1}{dx^2} + U\psi_1 - E\psi_1 \right) + B \left(-\frac{\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} + U\psi_2 - E\psi_2 \right) &= 0. \end{aligned}$$

But each of ψ_1 and ψ_2 satisfy Schrödinger's equation separately so the equation still holds true, for any A or B .

$$\begin{aligned} 39.37. \quad \frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi &= BE_1\psi_1 + CE_2\psi_2. \text{ If } \psi \text{ were a solution with energy } E, \text{ then } BE_1\psi_1 + CE_2\psi_2 = BE\psi_1 + CE\psi_2 \text{ or} \\ B(E_1 - E)\psi_1 &= C(E - E_2)\psi_2. \text{ This would mean that } \psi_1 \text{ is a constant multiple of } \psi_2, \text{ and } \psi_1 \text{ and } \psi_2 \text{ would be wave} \\ \text{functions with the same energy. However, } E_1 &\neq E_2, \text{ so this is not possible, and } \psi \text{ cannot be a solution to Eq. (39.18).} \end{aligned}$$

$$39.38. \quad (a) \lambda = \frac{h}{\sqrt{2mK}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(40 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}} = 1.94 \times 10^{-10} \text{ m}.$$

$$(b) \frac{R}{v} = \frac{R}{\sqrt{2E/m}} = \frac{(2.5 \text{ m})(9.11 \times 10^{-31} \text{ kg})^{1/2}}{\sqrt{2(40 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}} = 6.67 \times 10^{-7} \text{ s}.$$

$$(c) \text{ The width } w \text{ is } w = 2R \frac{\lambda}{a}, \text{ and } w = \Delta v_y t = \Delta p_y t / m, \text{ where } t \text{ is the time found in part (b) and } a \text{ is the slit width.}$$

$$\text{Combining the expressions for } w, \Delta p_y = \frac{2m\lambda R}{at} = 2.65 \times 10^{-28} \text{ kg} \cdot \text{m/s}.$$

$$(d) \Delta y = \frac{h}{2\pi\Delta p_y} = 0.40 \text{ } \mu\text{m}, \text{ which is the same order of magnitude.}$$

$$39.39. \quad (a) E = hc/\lambda = 12 \text{ eV}$$

$$(b) \text{ Find } E \text{ for an electron with } \lambda = 0.10 \times 10^{-6} \text{ m. } \lambda = h/p \text{ so } p = h/\lambda = 6.626 \times 10^{-27} \text{ kg} \cdot \text{m/s}.$$

$$E = p^2/(2m) = 1.5 \times 10^{-4} \text{ eV. } E = q\Delta V \text{ so } \Delta V = 1.5 \times 10^{-4} \text{ V}$$

$$v = p/m = (6.626 \times 10^{-27} \text{ kg} \cdot \text{m/s})/(9.109 \times 10^{-31} \text{ kg}) = 7.3 \times 10^3 \text{ m/s}$$

$$(c) \text{ Same } \lambda \text{ so same } p. E = p^2/(2m) \text{ but now } m = 1.673 \times 10^{-27} \text{ kg so } E = 8.2 \times 10^{-8} \text{ eV and } \Delta V = 8.2 \times 10^{-8} \text{ V.}$$

$$v = p/m = (6.626 \times 10^{-27} \text{ kg} \cdot \text{m/s})/(1.673 \times 10^{-27} \text{ kg}) = 4.0 \text{ m/s}$$

$$39.40. \quad (a) \text{ Single slit diffraction: } a \sin \theta = m\lambda. \lambda = a \sin \theta = (150 \times 10^{-9} \text{ m}) \sin 20^\circ = 5.13 \times 10^{-8} \text{ m}$$

$$\lambda = h/mv \rightarrow v = h/m\lambda. v = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(5.13 \times 10^{-8} \text{ m})} = 1.42 \times 10^4 \text{ m/s}$$

$$(b) a \sin \theta_2 = 2\lambda. \sin \theta_2 = \pm 2 \frac{\lambda}{a} = \pm 2 \left(\frac{5.13 \times 10^{-8} \text{ m}}{150 \times 10^{-9} \text{ m}} \right) = \pm 0.684. \theta_2 = \pm 43.2^\circ$$

- 39.41. IDENTIFY:** The electrons behave like waves and produce a double-slit interference pattern after passing through the slits.

SET UP: The first angle at which destructive interference occurs is given by $d \sin \theta = \lambda/2$. The de Broglie wavelength of each of the electrons is $\lambda = h/mv$.

EXECUTE: (a) First find the wavelength of the electrons. For the first dark fringe, we have $d \sin \theta = \lambda/2$, which gives $(1.25 \text{ nm})(\sin 18.0^\circ) = \lambda/2$, and $\lambda = 0.7725 \text{ nm}$. Now solve the de Broglie wavelength equation for the speed of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(0.7725 \times 10^{-9} \text{ m})} = 9.42 \times 10^5 \text{ m/s}$$

which is about 0.3% the speed of light, so they are *nonrelativistic*.

(b) Energy conservation gives $eV = \frac{1}{2}mv^2$ and

$$V = mv^2/2e = (9.11 \times 10^{-31} \text{ kg})(9.42 \times 10^5 \text{ m/s})^2/[2(1.60 \times 10^{-19} \text{ C})] = 2.52 \text{ V}$$

EVALUATE: The hole must be much smaller than the wavelength of visible light for the electrons to show diffraction.

- 39.42. IDENTIFY:** The alpha particles and protons behave as waves and exhibit circular-aperture diffraction after passing through the hole.

SET UP: For a round hole, the first dark ring occurs at the angle θ for which $\sin \theta = 1.22\lambda/D$, where D is the diameter of the hole. The de Broglie wavelength for a particle is $\lambda = h/p = h/mv$.

EXECUTE: Taking the ratio of the sines for the alpha particle and proton gives

$$\frac{\sin \theta_\alpha}{\sin \theta_p} = \frac{1.22\lambda_\alpha}{1.22\lambda_p} = \frac{\lambda_\alpha}{\lambda_p}$$

The de Broglie wavelength gives $\lambda_p = h/p_p$ and $\lambda_\alpha = h/p_\alpha$, so $\frac{\sin \theta_\alpha}{\sin \theta_p} = \frac{h/p_\alpha}{h/p_p} = \frac{p_p}{p_\alpha}$. Using $K = p^2/2m$, we have

$p = \sqrt{2mK}$. Since the alpha particle has twice the charge of the proton and both are accelerated through the same potential difference, $K_\alpha = 2K_p$. Therefore $p_p = \sqrt{2m_p K_p}$ and $p_\alpha = \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_\alpha (2K_p)} = \sqrt{4m_\alpha K_p}$.

Substituting these quantities into the ratio of the sines gives

$$\frac{\sin \theta_\alpha}{\sin \theta_p} = \frac{p_p}{p_\alpha} = \frac{\sqrt{2m_p K_p}}{\sqrt{4m_\alpha K_p}} = \sqrt{\frac{m_p}{2m_\alpha}}$$

Solving for $\sin \theta_\alpha$ gives $\sin \theta_\alpha = \sqrt{\frac{1.67 \times 10^{-27} \text{ kg}}{2(6.64 \times 10^{-27} \text{ kg})}} \sin 15.0^\circ$ and $\theta_\alpha = 5.3^\circ$.

EVALUATE: Since $\sin \theta$ is inversely proportional to the mass of the particle, the larger-mass alpha particles form their first dark ring at a smaller angle than the ring for the lighter protons.

- 39.43. IDENTIFY:** Both the electrons and photons behave like waves and exhibit single-slit diffraction after passing through their respective slits.

SET UP: The energy of the photon is $E = hc/\lambda$ and the de Broglie wavelength of the electron is $\lambda = h/mv = h/p$. Destructive interference for a single slit first occurs when $a \sin \theta = \lambda$.

EXECUTE: (a) For the photon: $\lambda = hc/E$ and $a \sin \theta = \lambda$. Since the a and θ are the same for the photons and electrons, they must both have the same wavelength. Equating these two expressions for λ gives $a \sin \theta = hc/E$.

For the electron, $\lambda = h/p = \frac{h}{\sqrt{2mK}}$ and $a \sin \theta = \lambda$. Equating these two expressions for λ gives $a \sin \theta = \frac{h}{\sqrt{2mK}}$.

Equating the two expressions for $a \sin \theta$ gives $hc/E = \frac{h}{\sqrt{2mK}}$, which gives $E = c\sqrt{2mK} = (4.05 \times 10^{-7} \text{ J}^{1/2})\sqrt{K}$

(b) $\frac{E}{K} = \frac{c\sqrt{2mK}}{K} = \sqrt{\frac{2mc^2}{K}}$. Since $v \ll c$, $mc^2 > K$, so the square root is >1 . Therefore $E/K > 1$, meaning that the photon has more energy than the electron.

EVALUATE: As we have seen in Problem 39.10, when a photon and a particle have the same wavelength, the photon has more energy than the particle.

- 39.44.** According to Eq.(35.4) $\lambda = \frac{d \sin \theta}{m} = \frac{(40.0 \times 10^{-6} \text{ m})(\sin(0.0300 \text{ rad}))}{2} = 600 \text{ nm}$. The velocity of an electron with this wavelength is given by Eq.(39.1)

$$v = \frac{p}{m} = \frac{h}{m\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(9.11 \times 10^{-31} \text{ kg})(600 \times 10^{-9} \text{ m})} = 1.21 \times 10^3 \text{ m/s}.$$

Since this velocity is much smaller than c we can calculate the energy of the electron classically

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(1.21 \times 10^3 \text{ m/s})^2 = 6.70 \times 10^{-25} \text{ J} = 4.19 \text{ } \mu\text{eV}.$$

- 39.45.** The de Broglie wavelength of the blood cell is

$$\lambda = \frac{h}{mv} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.00 \times 10^{-14} \text{ kg})(4.00 \times 10^{-3} \text{ m/s})} = 1.66 \times 10^{-17} \text{ m}.$$

We need not be concerned about wave behavior.

$$\begin{aligned} \text{39.46. (a)} \quad \lambda &= \frac{h}{p} = \frac{h \left(1 - \frac{v^2}{c^2}\right)^{1/2}}{mv} \Rightarrow \lambda^2 m^2 v^2 = h^2 \left(1 - \frac{v^2}{c^2}\right) = h^2 - \frac{h^2 v^2}{c^2} \Rightarrow \lambda^2 m^2 v^2 + h^2 \frac{v^2}{c^2} = h^2 \\ &\Rightarrow v^2 = \frac{h^2}{\left(\lambda^2 m^2 + \frac{h^2}{c^2}\right)} = \frac{c^2}{\left(\frac{\lambda^2 m^2 c^2}{h^2} + 1\right)} \Rightarrow v = \frac{c}{\left(1 + \left(\frac{mc\lambda}{h}\right)^2\right)^{1/2}}. \end{aligned}$$

$$\text{(b)} \quad v = \frac{c}{\left(1 + \left(\frac{\lambda}{h/mc}\right)^2\right)^{1/2}} \approx c \left(1 - \frac{1}{2} \left(\frac{mc\lambda}{h}\right)^2\right) = (1 - \Delta)c. \quad \Delta = \frac{m^2 c^2 \lambda^2}{2h^2}.$$

$$\text{(c)} \quad \lambda = 1.00 \times 10^{-15} \text{ m} \ll \frac{h}{mc}.$$

$$\Delta = \frac{(9.11 \times 10^{-31} \text{ kg})^2 (3.00 \times 10^8 \text{ m/s})^2 (1.00 \times 10^{-15} \text{ m})^2}{2(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2} = 8.50 \times 10^{-8}$$

$$\Rightarrow v = (1 - \Delta)c = (1 - 8.50 \times 10^{-8})c.$$

- 39.47. (a)** Recall $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mq\Delta V}}$. So for an electron:

$$\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ C})(125 \text{ V})}} \Rightarrow \lambda = 1.10 \times 10^{-10} \text{ m}.$$

$$\text{(b)} \text{ For an alpha particle: } \lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(6.64 \times 10^{-27} \text{ kg})2(1.60 \times 10^{-19} \text{ C})(125 \text{ V})}} = 9.10 \times 10^{-13} \text{ m}.$$

- 39.48. IDENTIFY and SET UP:** The minimum uncertainty product is $\Delta x \Delta p_x = \frac{h}{2\pi}$. $\Delta x = r_1$, where r_1 is the radius of the $n = 1$ Bohr orbit. In the $n = 1$ Bohr orbit, $mv_1 r_1 = \frac{h}{2\pi}$ and $p_1 = mv_1 = \frac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \frac{h}{2\pi \Delta x} = \frac{h}{2\pi r_1} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.529 \times 10^{-10} \text{ m})} = 2.0 \times 10^{-24} \text{ kg} \cdot \text{m/s}$. This is the same as the magnitude of the momentum of the electron in the $n = 1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

- 39.49. IDENTIFY and SET UP:** Combining the two equations in the hint gives $PC = \sqrt{K(K + 2mc^2)}$ and $\lambda = \frac{hc}{\sqrt{K(K + 2mc^2)}}$.

$$\text{EXECUTE: (a)} \text{ With } K = 3mc^2 \text{ this becomes } \lambda = \frac{hc}{\sqrt{3mc^2(3mc^2 + 2mc^2)}} = \frac{h}{\sqrt{15}mc}.$$

$$\text{(b) (i)} \quad K = 3mc^2 = 3(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 2.456 \times 10^{-13} \text{ J} = 1.53 \text{ MeV}$$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{15}(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})} = 6.26 \times 10^{-13} \text{ m}$$

$$\text{(ii)} \quad K \text{ is proportional to } m, \text{ so for a proton } K = (m_p/m_e)(1.53 \text{ MeV}) = 1836(1.53 \text{ MeV}) = 2810 \text{ MeV}$$

$$\lambda \text{ is proportional to } 1/m, \text{ so for a proton } \lambda = (m_e/m_p)(6.26 \times 10^{-13} \text{ m}) = (1/1836)(6.26 \times 10^{-13} \text{ m}) = 3.41 \times 10^{-16} \text{ m}$$

EVALUATE: The proton has a larger rest mass energy so its kinetic energy is larger when $K = 3mc^2$. The proton also has larger momentum so has a smaller λ .

39.50. (a) $\frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{2\pi(5.0 \times 10^{-15} \text{ m})} = 2.1 \times 10^{-20} \text{ kg} \cdot \text{m/s}.$

(b) $K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 = 1.3 \times 10^{-13} \text{ J} = 0.82 \text{ MeV}.$

(c) The result of part (b), about $1 \text{ MeV} = 1 \times 10^6 \text{ eV}$, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.51. (a) **IDENTIFY and SET UP:** $\Delta x \Delta p_x \geq h/2\pi$

Estimate Δx as $\Delta x \approx 5.0 \times 10^{-15} \text{ m}.$

EXECUTE: Then the minimum allowed Δp_x is $\Delta p_x \approx \frac{h}{2\pi\Delta x} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(5.0 \times 10^{-15} \text{ m})} = 2.1 \times 10^{-20} \text{ kg} \cdot \text{m/s}$

(b) **IDENTIFY and SET UP:** Assume $p \approx 2.1 \times 10^{-20} \text{ kg} \cdot \text{m/s}$. Use Eq.(37.39) to calculate E , and then $K = E - mc^2$.

EXECUTE: $E = \sqrt{(mc^2)^2 + (pc)^2}$

$mc^2 = (9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 8.187 \times 10^{-14} \text{ J}$

$pc = (2.1 \times 10^{-20} \text{ kg} \cdot \text{m/s})(2.998 \times 10^8 \text{ m/s}) = 6.296 \times 10^{-12} \text{ J}$

$E = \sqrt{(8.187 \times 10^{-14} \text{ J})^2 + (6.296 \times 10^{-12} \text{ J})^2} = 6.297 \times 10^{-12} \text{ J}$

$K = E - mc^2 = 6.297 \times 10^{-12} \text{ J} - 8.187 \times 10^{-14} \text{ J} = 6.215 \times 10^{-12} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 39 \text{ MeV}$

(c) **IDENTIFY and SET UP:** The Coulomb potential energy for a pair of point charges is given by Eq.(23.9). The proton has charge $+e$ and the electron has charge $-e$.

EXECUTE: $U = -\frac{ke^2}{r} = -\frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{5.0 \times 10^{-15} \text{ m}} = -4.6 \times 10^{-14} \text{ J} = -0.29 \text{ MeV}$

EVALUATE: The kinetic energy of the electron required by the uncertainty principle would be much larger than the magnitude of the negative Coulomb potential energy. The total energy of the electron would be large and positive and the electron could not be bound within the nucleus.

39.52. (a) Take the direction of the electron beam to be the x -direction and the direction of motion perpendicular to the

beam to be the y -direction. $\Delta v_y = \frac{\Delta p_y}{m} = \frac{h}{2\pi m \Delta y} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(9.11 \times 10^{-31} \text{ kg})(0.50 \times 10^{-3} \text{ m})} = 0.23 \text{ m/s}$

(b) The uncertainty Δr in the position of the point where the electrons strike the screen is

$$\Delta r = \Delta v_y t = \frac{\Delta p_y}{m} \frac{x}{v_x} = \frac{h}{2\pi m \Delta y} \frac{x}{\sqrt{2K/m}} = 9.56 \times 10^{-10} \text{ m},$$

(c) This is far too small to affect the clarity of the picture.

39.53. **IDENTIFY and SET UP:** $\Delta E \Delta t \geq \frac{h}{2\pi}$. Take the minimum uncertainty product, so $\Delta E = \frac{h}{2\pi \Delta t}$, with

$\Delta t = 8.4 \times 10^{-17} \text{ s}.$ $m = 264m_e.$ $\Delta m = \frac{\Delta E}{c^2}.$

EXECUTE: $\Delta E = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(8.4 \times 10^{-17} \text{ s})} = 1.26 \times 10^{-18} \text{ J}.$ $\Delta m = \frac{1.26 \times 10^{-18} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 1.4 \times 10^{-35} \text{ kg}.$

$\frac{\Delta m}{m} = \frac{1.4 \times 10^{-35} \text{ kg}}{(264)(9.11 \times 10^{-31} \text{ kg})} = 5.8 \times 10^{-8}$

39.54. **IDENTIFY:** The insect behaves like a wave as it passes through the hole in the screen.

SET UP: (a) For wave behavior to show up, the wavelength of the insect must be of the order of the diameter of the hole. The de Broglie wavelength is $\lambda = h/mv$.

EXECUTE: The de Broglie wavelength of the insect must be of the order of the diameter of the hole in the screen, so $\lambda \approx 5.00 \text{ mm}$. The de Broglie wavelength gives

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.25 \times 10^{-6} \text{ kg})(0.00400 \text{ m})} = 1.33 \times 10^{-25} \text{ m/s}$$

(b) $t = x/v = (0.000500 \text{ m})/(1.33 \times 10^{-25} \text{ m/s}) = 3.77 \times 10^{21} \text{ s} = 1.4 \times 10^{10} \text{ yr}$

The universe is about 14 billion years old ($1.4 \times 10^{10} \text{ yr}$), so this time would be about 85,000 times the age of the universe.

EVALUATE: Don't expect to see a diffracting insect! Wave behavior of particles occurs only at the very small scale.

39.55. **IDENTIFY and SET UP:** Use Eq.(39.1) to relate your wavelength and speed.

EXECUTE: (a) $\lambda = \frac{h}{mv}$, so $v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(60.0 \text{ kg})(1.0 \text{ m})} = 1.1 \times 10^{-35} \text{ m/s}$

$$(b) t = \frac{\text{distance}}{\text{velocity}} = \frac{0.80 \text{ m}}{1.1 \times 10^{-35} \text{ m/s}} = 7.3 \times 10^{34} \text{ s} (1 \text{ y} / 3.156 \times 10^7 \text{ s}) = 2.3 \times 10^{27} \text{ y}$$

Since you walk through doorways much more quickly than this, you will not experience diffraction effects.

EVALUATE: A 1 kg object moving at 1 m/s has a de Broglie wavelength $\lambda = 6.6 \times 10^{-34} \text{ m}$, which is exceedingly small. An object like you has a very, very small λ at ordinary speeds and does not exhibit wavelike properties.

$$39.56. (a) E = 2.58 \text{ eV} = 4.13 \times 10^{-19} \text{ J}, \text{ with a wavelength of } \lambda = \frac{hc}{E} = 4.82 \times 10^{-7} \text{ m} = 482 \text{ nm}$$

$$(b) \Delta E = \frac{h}{2\pi\Delta t} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{2\pi(1.64 \times 10^{-7} \text{ s})} = 6.43 \times 10^{-28} \text{ J} = 4.02 \times 10^{-9} \text{ eV}.$$

$$(c) \lambda E = hc, \text{ so } (\Delta\lambda)E + \lambda\Delta E = 0, \text{ and } |\Delta E/E| = |\Delta\lambda/\lambda|, \text{ so}$$

$$\Delta\lambda = \lambda|\Delta E/E| = (4.82 \times 10^{-7} \text{ m}) \left(\frac{6.43 \times 10^{-28} \text{ J}}{4.13 \times 10^{-19} \text{ J}} \right) = 7.50 \times 10^{-16} \text{ m} = 7.50 \times 10^{-7} \text{ nm}.$$

39.57. **IDENTIFY:** The electrons behave as waves whose wavelength is equal to the de Broglie wavelength.

SET UP: The de Broglie wavelength is $\lambda = h/mv$, and the energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: (a) Use the de Broglie wavelength to find the speed of the electron.

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^{-9} \text{ m})} = 7.27 \times 10^5 \text{ m/s}$$

which is much less than the speed of light, so it is nonrelativistic.

(b) Energy conservation gives $eV = \frac{1}{2}mv^2$.

$$V = mv^2/2e = (9.11 \times 10^{-31} \text{ kg})(7.27 \times 10^5 \text{ m/s})^2/[2(1.60 \times 10^{-19} \text{ C})] = 1.51 \text{ V}$$

(c) $K = eV = e(1.51 \text{ V}) = 1.51 \text{ eV}$, which is about $\frac{1}{4}$ the potential energy of the NaCl crystal, so the electron would not be too damaging.

(d) $E = hc/\lambda = (4.136 \times 10^{-15} \text{ eV}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})/(1.00 \times 10^{-9} \text{ m}) = 1240 \text{ eV}$ which would certainly destroy the molecules under study.

EVALUATE: As we have seen in Problems 39.10 and 39.43, when a particle and a photon have the same wavelength, the photon has much more energy.

$$39.58. \sin \theta' = \frac{\lambda'}{\lambda} \sin \theta, \text{ and } \lambda' = (h/p') = (h/\sqrt{2mE'}), \text{ and so } \theta' = \arcsin \left(\frac{h}{\lambda\sqrt{2mE'}} \sin \theta \right).$$

$$\theta' = \arcsin \left(\frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s}) \sin 35.8^\circ}{(3.00 \times 10^{-11} \text{ m}) \sqrt{2(9.11 \times 10^{-31} \text{ kg})(4.50 \times 10^{-19} \text{ J/eV})}} \right) = 20.9^\circ$$

39.59. (a) The maxima occur when $2d \sin \theta = m\lambda$ as described in Section 38.7.

$$(b) \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}. \quad \lambda = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(71.0 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}} = 1.46 \times 10^{-10} \text{ m} = 0.146 \text{ nm}.$$

$$\theta = \sin^{-1} \left(\frac{m\lambda}{2d} \right) \text{ (Note: This } m \text{ is the order of the maximum, not the mass.)}$$

$$\Rightarrow \sin^{-1} \left(\frac{(1)(1.46 \times 10^{-10} \text{ m})}{2(9.10 \times 10^{-11} \text{ m})} \right) = 53.3^\circ.$$

(c) The work function of the metal acts like an attractive potential increasing the kinetic energy of incoming electrons by $e\phi$. An increase in kinetic energy is an increase in momentum that leads to a smaller wavelength. A smaller wavelength gives a smaller angle θ (see part (b)).

39.60. (a) Using the given approximation, $E = \frac{1}{2} \left((h/x)^2/m + kx^2 \right)$, $(dE/dx) = kx - (h^2/mx^3)$, and the minimum energy

occurs when $kx = (h^2/mx^3)$, or $x^2 = \frac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

(b) They are the same.

39.61. (a) **IDENTIFY and SET UP:** $U = A|x|$. Eq.(7.17) relates force and potential. The slope of the function $A|x|$ is not continuous at $x = 0$ so we must consider the regions $x > 0$ and $x < 0$ separately.

EXECUTE: For $x > 0$, $|x| = x$ so $U = Ax$ and $F = -\frac{d(Ax)}{dx} = -A$. For $x < 0$, $|x| = -x$ so $U = -Ax$ and

$$F = -\frac{d(-Ax)}{dx} = +A. \text{ We can write this result as } F = -A|x|/x, \text{ valid for all } x \text{ except for } x = 0.$$

(b) IDENTIFY and SET UP: Use the uncertainty principle, expressed as $\Delta p \Delta x \approx h$, and as in Problem 39.50 estimate Δp by p and Δx by x . Use this to write the energy E of the particle as a function of x . Find the value of x that gives the minimum E and then find the minimum E .

EXECUTE: $E = K + U = \frac{p^2}{2m} + A|x|$

$px \approx h$, so $p \approx h/x$

Then $E \approx \frac{h^2}{2mx^2} + A|x|$.

For $x > 0$, $E = \frac{h^2}{2mx^2} + Ax$.

To find the value of x that gives minimum E set $\frac{dE}{dx} = 0$.

$$0 = \frac{-2h^2}{2mx^3} + A$$

$$x^3 = \frac{h^2}{mA} \text{ and } x = \left(\frac{h^2}{mA} \right)^{1/3}$$

With this x the minimum E is

$$E = \frac{h^2}{2m} \left(\frac{mA}{h^2} \right)^{2/3} + A \left(\frac{h^2}{mA} \right)^{1/3} = \frac{1}{2} h^{2/3} m^{-1/3} A^{2/3} + h^{2/3} m^{-1/3} A^{2/3}$$

$$E = \frac{3}{2} \left(\frac{h^2 A^2}{m} \right)^{1/3}$$

EVALUATE: The potential well is shaped like a V. The larger A is the steeper the slope of U and the smaller the region to which the particle is confined and the greater is its energy. Note that for the x that minimizes E , $2K = U$.

39.62. For this wave function, $\Psi^* = \psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t}$, so

$$\Psi^2 = \Psi^* \Psi = (\psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t})(\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}) = \psi_1^* \psi_1 + \psi_2^* \psi_2 + \psi_1^* \psi_2 e^{i(\omega_1 - \omega_2)t} + \psi_2^* \psi_1 e^{i(\omega_2 - \omega_1)t}$$

The frequencies ω_1 and ω_2 are given as not being the same, so $|\Psi|^2$ is not time-independent, and Ψ is not the wave function for a stationary state.

39.63. The time-dependent equation, with the separated form for $\Psi(x, t)$ as given becomes

$$i\hbar \psi(-i\omega) = \left(-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + U(x) \psi \right)$$

Since ψ is a solution of the time-independent solution with energy E , the term in parenthesis is $E\psi$, and so $\omega\hbar = E$, and $\omega = (E/\hbar)$.

39.64. (a) $\omega = 2\pi f = \frac{2\pi E}{h} = \frac{E}{\hbar}$. $k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p = \frac{p}{\hbar}$. $\hbar\omega = E = K = \frac{p^2}{2m} = \frac{(\hbar k)^2}{2m} \Rightarrow \omega = \frac{\hbar k^2}{2m}$.

(b) From Problem 39.63 the time-dependent Schrödinger's equation is $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} +$

$$U(x)\psi(x, t) = i\hbar \frac{\partial \psi(x, t)}{\partial t}. U(x) = 0 \text{ for a free particle, so } \frac{\partial^2 \psi(x, t)}{\partial x^2} = -\frac{2mi}{\hbar} \frac{\partial \psi(x, t)}{\partial t}.$$

Try $\psi(x, t) = \cos(kx - \omega t)$:

$$\frac{\partial \psi}{\partial t}(x, t) = A\omega \sin(kx - \omega t)$$

$$\frac{\partial \psi(x, t)}{\partial x} = -Ak \sin(kx - \omega t) \text{ and } \frac{\partial^2 \psi}{\partial x^2} = Ak^2 \cos(kx - \omega t).$$

Putting this into the Schrödinger's equation, $Ak^2 \cos(kx - \omega t) = -\left(\frac{2mi}{\hbar} \right) A\omega \sin(kx - \omega t)$.

This is not generally true for all x and t so is not a solution.

(c) Try $\psi(x, t) = A \sin(kx - \omega t)$:

$$\frac{\partial \psi(x, t)}{\partial t} = -A\omega \cos(kx - \omega t)$$

$$\frac{\partial \psi(x, t)}{\partial x} = Ak \cos(kx - \omega t) \text{ and } \frac{\partial^2 \psi(x, t)}{\partial x^2} = -Ak^2 \sin(kx - \omega t).$$

Again, $-Ak^2 \sin(kx - \omega t) = -\left(\frac{2mi}{\hbar}\right)A\omega \cos(kx - \omega t)$ is not generally true for *all* x and t so is not a solution.

(d) Try $\psi(x, t) = A \cos(kx - \omega t) + B \sin(kx - \omega t)$:

$$\frac{\partial \psi(x, t)}{\partial t} = +A\omega \sin(kx - \omega t) - B\omega \cos(kx - \omega t)$$

$$\frac{\partial \psi(x, t)}{\partial x} = -Ak \sin(kx - \omega t) + Bk \cos(kx - \omega t) \text{ and } \frac{\partial^2 \psi(x, t)}{\partial x^2} = -Ak^2 \cos(kx - \omega t) - Bk^2 \sin(kx - \omega t).$$

Putting this into the Schrödinger's equation,

$$-Ak^2 \cos(kx - \omega t) - Bk^2 \sin(kx - \omega t) = -\frac{2mi}{\hbar}(+A\omega \sin(kx - \omega t) - B\omega \cos(kx - \omega t)).$$

Recall that $\omega = \frac{\hbar k^2}{2m}$. Collect sin and cos terms.

$$(A + iB)k^2 \cos(kx - \omega t) + (iA - B)k^2 \sin(kx - \omega t) = 0. \text{ This is only true if } B = iA.$$

- 39.65. (a) IDENTIFY and SET UP:** Let the y -direction be from the thrower to the catcher, and let the x -direction be horizontal and perpendicular to the y -direction. A cube with volume $V = 125 \text{ cm}^3 = 0.125 \times 10^{-3} \text{ m}^3$ has side length $l = V^{1/3} = (0.125 \times 10^{-3} \text{ m}^3)^{1/3} = 0.050 \text{ m}$. Thus estimate Δx as $\Delta x \approx 0.050 \text{ m}$. Use the uncertainty principle to estimate Δp_x .

EXECUTE: $\Delta x \Delta p_x \geq \hbar / 2\pi$ then gives $\Delta p_x \approx \frac{\hbar}{2\pi \Delta x} = \frac{0.0663 \text{ J} \cdot \text{s}}{2\pi(0.050 \text{ m})} = 0.21 \text{ kg} \cdot \text{m/s}$

(The value of $\hbar$ in this other universe has been used.)

(b) IDENTIFY and SET UP: $\Delta x = (\Delta v_x)t$ is the uncertainty in the x -coordinate of the ball when it reaches the catcher, where t is the time it takes the ball to reach the second student. Obtain Δv_x from Δp_x .

EXECUTE: The uncertainty in the ball's horizontal velocity is $\Delta v_x = \frac{\Delta p_x}{m} = \frac{0.21 \text{ kg} \cdot \text{m/s}}{0.25 \text{ kg}} = 0.84 \text{ m/s}$

The time it takes the ball to travel to the second student is $t = \frac{12 \text{ m}}{6.0 \text{ m/s}} = 2.0 \text{ s}$. The uncertainty in the x -coordinate

of the ball when it reaches the second student that is introduced by Δv_x is $\Delta x = (\Delta v_x)t = (0.84 \text{ m/s})(2.0 \text{ s}) = 1.7 \text{ m}$.

The ball could miss the second student by about 1.7 m.

EVALUATE: A game of catch would be very different in this universe. We don't notice the effects of the uncertainty principle in everyday life because $\hbar$ is so small.

- 39.66. (a)** $|\psi|^2 = A^2 x^2 e^{-2(\alpha x^2 + \beta y^2 + \gamma z^2)}$. To save some algebra, let $u = x^2$, so that $|\psi|^2 = u e^{-2\alpha u} f(y, z)$.

$$\frac{\partial}{\partial u} |\psi|^2 = (1 - 2\alpha u) |\psi|^2; \text{ the maximum occurs at } u_0 = \frac{1}{2\alpha}, \quad x_0 = \pm \frac{1}{\sqrt{2\alpha}}.$$

(b) ψ vanishes at $x = 0$, so the probability of finding the particle in the $x = 0$ plane is zero. The wave function vanishes for $x = \pm\infty$.

- 39.67. (a) IDENTIFY and SET UP:** The probability is $P = |\psi|^2 dV$ with $dV = 4\pi r^2 dr$

EXECUTE: $|\psi|^2 = A^2 e^{-2\alpha r^2}$ so $P = 4\pi A^2 r^2 e^{-2\alpha r^2} dr$

(b) IDENTIFY and SET UP: P is maximum where $\frac{dP}{dr} = 0$

EXECUTE: $\frac{d}{dr}(r^2 e^{-2\alpha r^2}) = 0$

$$2r e^{-2\alpha r^2} - 4\alpha r^3 e^{-2\alpha r^2} = 0 \text{ and this reduces to } 2r - 4\alpha r^3 = 0$$

$r = 0$ is a solution of the equation but corresponds to a minimum not a maximum. Seek r not equal to 0 so divide by r and get $2 - 4\alpha r^2 = 0$

This gives $r = \frac{1}{\sqrt{2\alpha}}$ (We took the positive square root since r must be positive.)

EVALUATE: This is different from the value of r , $r = 0$, where $|\psi|^2$ is a maximum. At $r = 0$, $|\psi|^2$ has a maximum but the volume element $dV = 4\pi r^2 dr$ is zero here so P does not have a maximum at $r = 0$.

39.68. (a) $B(k) = e^{-\alpha^2 k^2}$ $B(0) = B_{\max} = 1$

$$B(k_h) = \frac{1}{2} = e^{-\alpha^2 k_h^2} \Rightarrow \ln(1/2) = -\alpha^2 k_h^2 \Rightarrow k_h = \frac{1}{\alpha} \sqrt{\ln(2)} = \omega_k.$$

(b) Using integral tables: $\psi(x) = \int_0^\infty e^{-\alpha^2 k^2} \cos kx dk = \frac{\sqrt{\pi}}{2\alpha} (e^{-x^2/4\alpha^2})$. $\psi(x)$ is a maximum when $x = 0$.

(c) $\psi(x_h) = \frac{\sqrt{\pi}}{4\alpha}$ when $e^{-x_h^2/4\alpha^2} = \frac{1}{2} \Rightarrow \frac{-x_h^2}{4\alpha^2} = \ln(1/2) \Rightarrow x_h = 2\alpha\sqrt{\ln 2} = \omega_x$

(d) $\omega_p \omega_x = \left(\frac{h\omega_k}{2\pi}\right) \omega_x = \frac{h}{2\pi} \left(\frac{1}{\alpha} \sqrt{\ln 2}\right) (2\alpha\sqrt{\ln 2}) = \frac{h}{2\pi} (2\ln 2) = \frac{h \ln 2}{\pi}$.

39.69. (a) $\psi(x) = \int_0^\infty B(k) \cos kx dk = \int_0^{k_0} \left(\frac{1}{k_0}\right) \cos kx dk = \frac{\sin kx}{k_0 x} \Big|_0^{k_0} = \frac{\sin k_0 x}{k_0 x}$

(b) $\psi(x)$ has a maximum value at the origin $x = 0$. $\psi(x_0) = 0$ when $k_0 x_0 = \pi$ so $x_0 = \frac{\pi}{k_0}$. Thus the width of this

function $w_x = 2x_0 = \frac{2\pi}{k_0}$. If $k_0 = \frac{2\pi}{L}$, $w_x = L$. $B(k)$ versus k is graphed in Figure 39.69a. The graph of $\psi(x)$ versus x is in Figure 39.69b.

(c) If $k_0 = \frac{\pi}{L}$, $w_x = 2L$.

(d) $w_p w_x = \left(\frac{hw_k}{2\pi}\right) \left(\frac{2\pi}{k_0}\right) = \frac{hw_k}{k_0} = \frac{hk_0}{k_0} = h$. The uncertainty principle states that $w_p w_x \geq \frac{h}{2\pi}$. For us, no matter what k_0 is, $w_p w_x = h$, which is greater than $\frac{h}{2\pi}$.

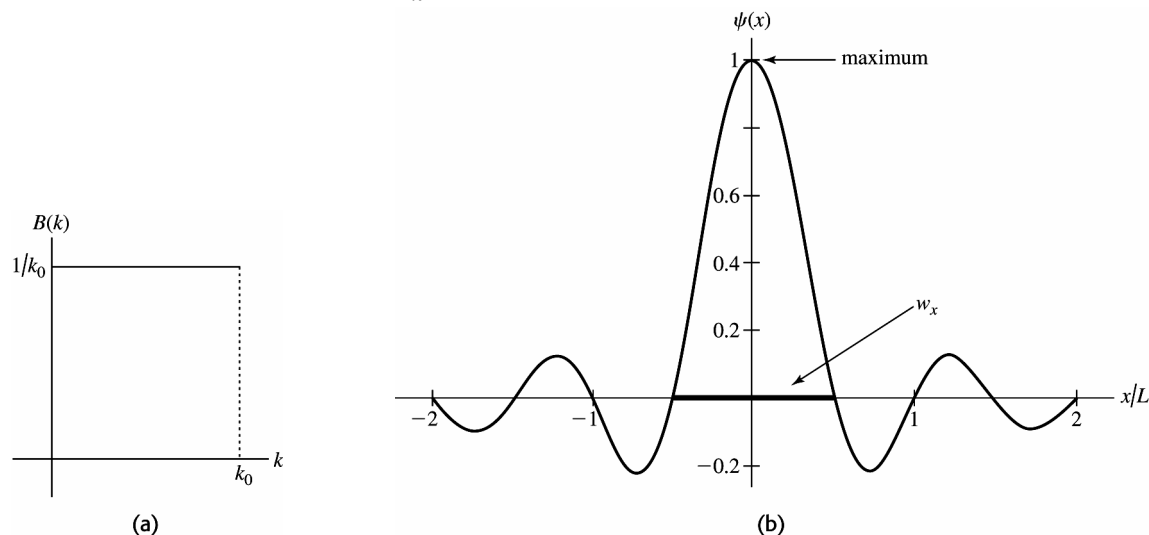


Figure 39.69

39.70. (a) For a standing wave, $n\lambda = 2L$, and $E_n = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{n^2 h^2}{8mL^2}$.

(b) With $L = a_0 = 0.5292 \times 10^{-10}$ m, $E_1 = 2.15 \times 10^{-17}$ J = 134 eV.

39.71. Time of flight of the marble, from a free-fall kinematic equation is just $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(25.0 \text{ m})}{9.81 \text{ m/s}^2}} = 2.26 \text{ s}$.

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\Delta p_x}{m}\right)t = \frac{ht}{2\pi\Delta x_i m} + \Delta x_i$$

$$\text{To minimize } \Delta x_f \text{ with respect to } \Delta x_i, \frac{d(\Delta x_f)}{d(\Delta x_i)} = 0 = \frac{-ht}{2\pi m(\Delta x_i)^2} + 1$$

$$\Rightarrow \Delta x_i(\text{min}) = \sqrt{\left(\frac{ht}{2\pi m}\right)}$$

$$\Rightarrow \Delta x_f(\text{min}) = \sqrt{\frac{ht}{2\pi m}} + \sqrt{\frac{ht}{2\pi m}} = \sqrt{\frac{2ht}{\pi m}} = \sqrt{\frac{2(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(2.26 \text{ s})}{\pi(0.0200 \text{ kg})}} = 2.18 \times 10^{-16} \text{ m} = 2.18 \times 10^{-7} \text{ nm}.$$

QUANTUM MECHANICS

40.1. IDENTIFY and SET UP: The energy levels for a particle in a box are given by $E_n = \frac{n^2 h^2}{8mL^2}$.

EXECUTE: (a) The lowest level is for $n=1$, and $E_1 = \frac{(1)(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(0.20 \text{ kg})(1.5 \text{ m})^2} = 1.2 \times 10^{-67} \text{ J}$.

(b) $E = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(1.2 \times 10^{-67} \text{ J})}{0.20 \text{ kg}}} = 1.1 \times 10^{-33} \text{ m/s}$. If the ball has this speed the time it would take it

to travel from one side of the table to the other is $t = \frac{1.5 \text{ m}}{1.1 \times 10^{-33} \text{ m/s}} = 1.4 \times 10^{33} \text{ s}$.

(c) $E_1 = \frac{h^2}{8mL^2}$, $E_2 = 4E_1$, so $\Delta E = E_2 - E_1 = 3E_1 = 3(1.2 \times 10^{-67} \text{ J}) = 3.6 \times 10^{-67} \text{ J}$

(d) **EVALUATE:** No, quantum mechanical effects are not important for the game of billiards. The discrete, quantized nature of the energy levels is completely unobservable.

40.2. $L = \frac{h}{\sqrt{8mE_1}}$

$$L = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{8(1.673 \times 10^{-27} \text{ kg})(5.0 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}} = 6.4 \times 10^{-15} \text{ m}.$$

40.3. IDENTIFY: An electron in the lowest energy state in this box must have the same energy as it would in the ground state of hydrogen.

SET UP: The energy of the n^{th} level of an electron in a box is $E_n = \frac{nh^2}{8mL^2}$.

EXECUTE: An electron in the ground state of hydrogen has an energy of -13.6 eV , so find the width corresponding to an energy of $E_1 = 13.6 \text{ eV}$. Solving for L gives

$$L = \frac{h}{\sqrt{8mE_1}} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{8(9.11 \times 10^{-31} \text{ kg})(13.6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}} = 1.66 \times 10^{-10} \text{ m}.$$

EVALUATE: This width is of the same order of magnitude as the diameter of a Bohr atom with the electron in the K shell.

40.4. (a) The energy of the given photon is

$$E = hf = h \frac{c}{\lambda} = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \frac{(3.00 \times 10^3 \text{ m/s})}{(122 \times 10^{-9} \text{ m})} = 1.63 \times 10^{-18} \text{ J}.$$

The energy levels of a particle in a box are given by Eq.40.9

$$\Delta E = \frac{h^2}{8mL^2}(n^2 - n_2). \quad L = \sqrt{\frac{h^2(n_1^2 - n_2^2)}{8m\Delta E}} = \sqrt{\frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2(2^2 - 1^2)}{8(9.11 \times 10^{-31} \text{ kg})(1.63 \times 10^{-20} \text{ J})}} = 3.33 \times 10^{-10} \text{ m}.$$

(b) The ground state energy for an electron in a box of the calculated dimensions is

$$E = \frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(3.33 \times 10^{-10} \text{ m})^2} = 5.43 \times 10^{-19} \text{ J} = 3.40 \text{ eV (one-third of the original photon energy),}$$

which does not correspond to the -13.6 eV ground state energy of the hydrogen atom. Note that the energy levels for a particle in a box are proportional to n^2 , whereas the energy levels for the hydrogen atom are proportional to $-\frac{1}{n^2}$.

- 40.5. IDENTIFY and SET UP:** Eq.(40.9) gives the energy levels. Use this to obtain an expression for $E_2 - E_1$ and use the value given for this energy difference to solve for L .

EXECUTE: Ground state energy is $E_1 = \frac{h^2}{8mL^2}$; first excited state energy is $E_2 = \frac{4h^2}{8mL^2}$. The energy separation

between these two levels is $\Delta E = E_2 - E_1 = \frac{3h^2}{8mL^2}$. This gives $L = h \sqrt{\frac{3}{8m\Delta E}} =$

$$L = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \sqrt{\frac{3}{8(9.109 \times 10^{-31} \text{ kg})(3.0 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}} = 6.1 \times 10^{-10} \text{ m} = 0.61 \text{ nm}.$$

EVALUATE: This energy difference is typical for an atom and L is comparable to the size of an atom.

- 40.6. (a)** The wave function for $n=1$ vanishes only at $x=0$ and $x=L$ in the range $0 \leq x \leq L$.
(b) In the range for x , the sine term is a maximum only at the middle of the box, $x=L/2$.
(c) The answers to parts (a) and (b) are consistent with the figure.
- 40.7. IDENTIFY and SET UP:** For the $n=2$ first excited state the normalized wave function is given by Eq.(40.13).

$\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$. $|\psi_2(x)|^2 dx = \frac{2}{L} \sin^2\left(\frac{2\pi x}{L}\right) dx$. Examine $|\psi_2(x)|^2 dx$ and find where it is zero and where it is maximum.

EXECUTE: (a) $|\psi_2|^2 dx = 0$ implies $\sin\left(\frac{2\pi x}{L}\right) = 0$

$$\frac{2\pi x}{L} = m\pi, \quad m = 0, 1, 2, \dots; \quad x = m(L/2)$$

For $m=0$, $x=0$; for $m=1$, $x=L/2$; for $m=2$, $x=L$

The probability of finding the particle is zero at $x=0$, $L/2$, and L .

(b) $|\psi_2|^2 dx$ is maximum when $\sin\left(\frac{2\pi x}{L}\right) = \pm 1$

$$\frac{2\pi x}{L} = m(\pi/2), \quad m = 1, 3, 5, \dots; \quad x = m(L/4)$$

For $m=1$, $x=L/4$; for $m=3$, $x=3L/4$

The probability of finding the particle is largest at $x=L/4$ and $3L/4$.

(c) EVALUATE: The answers to part (a) correspond to the zeros of $|\psi|^2$ shown in Fig.40.5 in the textbook and the answers to part (b) correspond to the two values of x where $|\psi|^2$ in the figure is maximum.

- 40.8. $\frac{d^2\psi}{dx^2} = -k^2\psi$** , and for ψ to be a solution of Eq.(40.3), $k^2 = E \frac{8\pi^2 m}{h^2} = E \frac{2m}{\hbar^2}$.

(b) The wave function must vanish at the rigid walls; the given function will vanish at $x=0$ for any k , but to vanish at $x=L$, $kL = n\pi$ for integer n .

- 40.9. (a) IDENTIFY and SET UP:** $\psi = A \cos kx$. Calculate $d^2\psi/dx^2$ and substitute into Eq.(40.3) to see if this equation is satisfied.

EXECUTE: Eq.(40.3): $-\frac{h^2}{8\pi^2 m} \frac{d^2\psi}{dx^2} = E\psi$

$$\frac{d\psi}{dx} = A(-k \sin kx) = -Ak \sin kx$$

$$\frac{d^2\psi}{dx^2} = -Ak(k \cos kx) = -Ak^2 \cos kx$$

Thus Eq.(40.3) requires $-\frac{h^2}{8\pi^2 m} (-Ak^2 \cos kx) = E(A \cos kx)$.

$$\text{This says } -\frac{h^2 k^2}{8\pi^2 m} = E; \quad k = \frac{\sqrt{2mE}}{\hbar/2\pi} = \frac{\sqrt{2mE}}{\hbar}$$

$\psi = A \cos kx$ is a solution to Eq.(40.3) if $k = \frac{\sqrt{2mE}}{\hbar}$.

(b) EVALUATE: The wave function for a particle in a box with rigid walls at $x=0$ and $x=L$ must satisfy the boundary conditions $\psi=0$ at $x=0$ and $\psi=0$ at $x=L$. $\psi(0) = A \cos 0 = A$, since $\cos 0 = 1$. Thus ψ is not 0 at $x=0$ and this wave function isn't acceptable because it doesn't satisfy the required boundary condition, even though it is a solution to the Schrödinger equation.

- 40.10.** (a) The third excited state is $n = 4$, so

$$\Delta E = (4^2 - 1) \frac{h^2}{8mL^2} = \frac{15(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(0.125 \times 10^{-9} \text{ m})^2} = 5.78 \times 10^{-17} \text{ J} = 361 \text{ eV}.$$

$$(b) \lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{5.78 \times 10^{-17} \text{ J}} = 3.44 \text{ nm}$$

- 40.11.** Recall $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$.

$$(a) E_1 = \frac{h^2}{8mL^2} \Rightarrow \lambda_1 = \frac{h}{\sqrt{2mE_1}} = 2L = 2(3.0 \times 10^{-10} \text{ m}) = 6.0 \times 10^{-10} \text{ m}. \text{ The wavelength is twice the width of}$$

$$\text{the box. } p_1 = \frac{h}{\lambda_1} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{6.0 \times 10^{-10} \text{ m}} = 1.1 \times 10^{-24} \text{ kg} \cdot \text{m/s}$$

$$(b) E_2 = \frac{4h^2}{8mL^2} \Rightarrow \lambda_2 = L = 3.0 \times 10^{-10} \text{ m}. \text{ The wavelength is the same as the width of the box.}$$

$$p_2 = \frac{h}{\lambda_2} = 2p_1 = 2.2 \times 10^{-24} \text{ kg} \cdot \text{m/s}.$$

$$(c) E_3 = \frac{9h^2}{8mL^2} \Rightarrow \lambda_3 = \frac{2}{3}L = 2.0 \times 10^{-10} \text{ m}. \text{ The wavelength is two-thirds the width of the box.}$$

$$p_3 = 3p_1 = 3.3 \times 10^{-24} \text{ kg} \cdot \text{m/s}.$$

- 40.12. IDENTIFY:** If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: We must substitute the equation $\Psi(x, t) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) e^{-iE_n t/\hbar}$ into the one-dimensional Schrödinger

$$\text{equation } -\frac{\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x) = E\Psi(x).$$

$$\text{EXECUTE: Taking the second derivative of } \Psi(x, t) \text{ with respect to } x \text{ gives } \frac{d^2\Psi(x, t)}{dx^2} = -\left(\frac{n\pi}{L}\right)^2 \Psi(x, t)$$

$$\text{Substituting this result into } -\frac{\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x) = E\Psi(x), \text{ we get } \frac{\hbar^2}{2m} \left(\frac{n\pi}{L}\right)^2 \Psi(x, t) = E\Psi(x, t) \text{ which}$$

$$\text{gives } E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{L}\right)^2, \text{ the energies of a particle in a box.}$$

EVALUATE: Since this process gives us the energies of a particle in a box, the given wave function is a solution to the Schrödinger equation.

- 40.13.** (a) Eq.(40.1): $-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U_0\psi = E\psi.$

$$\text{Left-hand side: } \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} (A \sin kx) + U_0 A \sin kx = \frac{\hbar^2 k^2}{2m} A \sin kx + U_0 A \sin kx = \left(\frac{\hbar^2 k^2}{2m} + U_0 \right) \psi.$$

$$\text{But } \frac{\hbar^2 k^2}{2m} + U_0 > U_0 > E \text{ for constant } k. \text{ But } \frac{\hbar^2 k^2}{2m} + U_0 \text{ should equal } E \Rightarrow \text{no solution.}$$

$$(b) \text{ If } E > U_0, \text{ then } \frac{\hbar^2 k^2}{2m} + U_0 = E \text{ is consistent and so } \psi = A \sin kx \text{ is a solution of Eq.(40.1) for this case.}$$

- 40.14.** According to Eq.(40.17), the wavelength of the electron inside of the square well is given by

$$k = \frac{\sqrt{2mE}}{\hbar} \Rightarrow \lambda_{\text{in}} = \frac{h}{\sqrt{2m(3U_0)}}. \text{ By an analysis similar to that used to derive Eq.40.17, we can show that outside}$$

the box

$$\lambda_{\text{out}} = \frac{h}{\sqrt{2m(E - U_0)}} = \frac{h}{\sqrt{2m(2U_0)}}.$$

$$\text{Thus, the ratio of the wavelengths is } \frac{\lambda_{\text{out}}}{\lambda_{\text{in}}} = \frac{\sqrt{2m(3U_0)}}{\sqrt{2m(2U_0)}} = \sqrt{\frac{3}{2}}.$$

40.15. $E_1 = 0.625E_\infty = 0.625 \frac{\pi^2 \hbar^2}{2mL^2}$; $E_1 = 2.00 \text{ eV} = 3.20 \times 10^{-19} \text{ J}$

$$L = \pi \hbar \left(\frac{0.625}{2(9.109 \times 10^{-31} \text{ kg})(3.20 \times 10^{-19} \text{ J})} \right)^{1/2} = 3.43 \times 10^{-10} \text{ m}$$

40.16. Since $U_0 = 6E_\infty$ we can use the result $E_1 = 0.625E_\infty$ from Section 40.3, so $U_0 - E_1 = 5.375E_\infty$ and the maximum wavelength of the photon would be

$$\lambda = \frac{hc}{U_0 - E_1} = \frac{hc}{(5.375)(h^2/8mL^2)} = \frac{8mL^2c}{(5.375)h}$$

$$\lambda = \frac{8(9.11 \times 10^{-31} \text{ kg})(1.50 \times 10^{-9} \text{ m})^2(3.00 \times 10^8 \text{ m/s})}{(5.375)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})} = 1.38 \times 10^{-6} \text{ m}.$$

40.17. Eq.(40.16): $\psi = A \sin \frac{\sqrt{2mE}}{\hbar} x + B \cos \frac{\sqrt{2mE}}{\hbar} x$

$$\frac{d^2\psi}{dx^2} = -A \left(\frac{2mE}{\hbar^2} \right) \sin \frac{\sqrt{2mE}}{\hbar} x - B \left(\frac{2mE}{\hbar^2} \right) \cos \frac{\sqrt{2mE}}{\hbar} x = \frac{-2mE}{\hbar^2} (\psi) = \text{Eq. (40.15)}.$$

40.18. $\frac{d\psi}{dx} = \kappa(Ce^{\kappa x} - De^{-\kappa x})$, $\frac{d^2\psi}{dx^2} = \kappa^2(Ce^{\kappa x} + De^{-\kappa x}) = \kappa^2\psi$ for all constants C and D . Hence ψ is a solution to

Eq.(40.1) for $-\frac{\hbar^2}{2m}\kappa^2 + U_0 = E$, or $\kappa = [2m(U_0 - E)]^{1/2}/\hbar$, and κ is real for $E < U_0$.

40.19. IDENTIFY: Find the transition energy ΔE and set it equal to the energy of the absorbed photon. Use $E = hc/\lambda$ to find the wavelength of the photon.

SET UP: $U_0 = 6E_\infty$, as in Fig.40.8 in the textbook, so $E_1 = 0.625E_\infty$ and $E_3 = 5.09E_\infty$ with $E_\infty = \frac{\pi^2 \hbar^2}{2mL^2}$. In this

problem the particle bound in the well is a proton, so $m = 1.673 \times 10^{-27} \text{ kg}$.

EXECUTE: $E_\infty = \frac{\pi^2 \hbar^2}{2mL^2} = \frac{\pi^2 (1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(1.673 \times 10^{-27} \text{ kg})(4.0 \times 10^{-15} \text{ m})^2} = 2.052 \times 10^{-12} \text{ J}$. The transition energy is

$$\Delta E = E_3 - E_1 = (5.09 - 0.625)E_\infty = 4.465E_\infty. \quad \Delta E = 4.465(2.052 \times 10^{-12} \text{ J}) = 9.162 \times 10^{-12} \text{ J}$$

The wavelength of the photon that is absorbed is related to the transition energy by $\Delta E = hc/\lambda$, so

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{9.162 \times 10^{-12} \text{ J}} = 2.2 \times 10^{-14} \text{ m} = 22 \text{ fm}.$$

EVALUATE: The wavelength of the photon is comparable to the size of the box.

40.20. IDENTIFY: The longest wavelength corresponds to the smallest energy change.

SET UP: The ground level energy level of the infinite well is $E_\infty = \frac{h^2}{8mL^2}$, and the energy of the photon must be equal to the energy difference between the two shells.

EXECUTE: The 400.0 nm photon must correspond to the $n=1$ to $n=2$ transition. Since $U_0 = 6E_\infty$, we have $E_2 = 2.43E_\infty$ and $E_1 = 0.625E_\infty$. The energy of the photon is equal to the energy difference between the two levels,

and $E_\infty = \frac{h^2}{8mL^2}$, which gives $E_\gamma = E_2 - E_1 \Rightarrow \frac{hc}{\lambda} = (2.43 - 0.625)E_\infty = \frac{1.805}{8mL^2} h^2$

$$\text{Solving for } L \text{ gives } L = \sqrt{\frac{(1.805)h\lambda}{8mc}} = \sqrt{\frac{(1.805)(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(4.00 \times 10^{-7} \text{ m})}{8(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})}} = 4.68 \times 10^{-10} \text{ m} = 0.468 \text{ nm}.$$

EVALUATE: This width is approximately half that of a Bohr hydrogen atom.

40.21. $T = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0} \right) e^{-2L\sqrt{2m(U_0 - E)}/\hbar}$. $\frac{E}{U_0} = \frac{6.0 \text{ eV}}{11.0 \text{ eV}}$ and $E - U_0 = 5 \text{ eV} = 8.0 \times 10^{-19} \text{ J}$.

(a) $L = 0.80 \times 10^{-9} \text{ m}$: $T = 16 \left(\frac{6.0 \text{ eV}}{11.0 \text{ eV}} \right) \left(1 - \frac{6.0 \text{ eV}}{11.0 \text{ eV}} \right) e^{-2(0.80 \times 10^{-9} \text{ m})\sqrt{2(9.11 \times 10^{-31} \text{ kg})(8.0 \times 10^{-19} \text{ J})}/1.055 \times 10^{-34} \text{ J} \cdot \text{s}} = 4.4 \times 10^{-8}$

(b) $L = 0.40 \times 10^{-9} \text{ m}$: $T = 4.2 \times 10^{-4}$.

40.22. The transmission coefficient is $T = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right) e^{-2\sqrt{2m(U_0-E)}L/\hbar}$, with $E = 5.0 \text{ eV}$, $L = 0.60 \times 10^{-9} \text{ m}$, and

$$m = 9.11 \times 10^{-31} \text{ kg}$$

(a) $U_0 = 7.0 \text{ eV} \Rightarrow T = 5.5 \times 10^{-4}$.

(b) $U_0 = 9.0 \text{ eV} \Rightarrow T = 1.8 \times 10^{-5}$

(c) $U_0 = 13.0 \text{ eV} \Rightarrow T = 1.1 \times 10^{-7}$.

40.23. IDENTIFY and SET UP: Use Eq.(39.1), where $K = p^2/2m$ and $E = K + U$.

EXECUTE: $\lambda = h/p = h/\sqrt{2mK}$, so $\lambda\sqrt{K}$ is constant

$$\lambda_1\sqrt{K_1} = \lambda_2\sqrt{K_2}; \lambda_1 \text{ and } K_1 \text{ are for } x > L \text{ where } K_1 = 2U_0 \text{ and } \lambda_2 \text{ and } K_2 \text{ are for } 0 < x < L \text{ where}$$

$$K_2 = E - U_0 = U_0$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{U_0}{2U_0}} = \frac{1}{\sqrt{2}}$$

EVALUATE: When the particle is passing over the barrier its kinetic energy is less and its wavelength is larger.

40.24. IDENTIFY: The probability of tunneling depends on the energy of the particle and the width of the barrier.

SET UP: The probability of tunneling is approximately $T = Ge^{-2\kappa L}$, where $G = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right)$ and

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}.$$

EXECUTE: $G = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right) = 16 \frac{50.0 \text{ eV}}{70.0 \text{ eV}} \left(1 - \frac{50.0 \text{ eV}}{70.0 \text{ eV}}\right) = 3.27$.

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar} = \frac{\sqrt{2(1.67 \times 10^{-27} \text{ kg})(70.0 \text{ eV} - 50.0 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})/2\pi} = 9.8 \times 10^{11} \text{ m}^{-1}$$

Solving $T = Ge^{-2\kappa L}$ for L gives $L = \frac{1}{2\kappa} \ln(G/T) = \frac{1}{2(9.8 \times 10^{11} \text{ m}^{-1})} \ln\left(\frac{3.27}{0.0030}\right) = 3.6 \times 10^{-12} \text{ m} = 3.6 \text{ pm}$

If the proton were replaced with an electron, the electron's mass is much smaller so L would be larger.

EVALUATE: An electron can tunnel through a much wider barrier than a proton of the same energy.

40.25. IDENTIFY and SET UP: The probability is $T = Ae^{-2\kappa L}$, with $A = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right)$ and $\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$.

$$E = 32 \text{ eV}, U_0 = 41 \text{ eV}, L = 0.25 \times 10^{-9} \text{ m. Calculate } T.$$

EXECUTE: (a) $A = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right) = 16 \frac{32}{41} \left(1 - \frac{32}{41}\right) = 2.741$.

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

$$\kappa = \frac{\sqrt{2(9.109 \times 10^{-31} \text{ kg})(41 \text{ eV} - 32 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}}{1.055 \times 10^{-34} \text{ J} \cdot \text{s}} = 1.536 \times 10^{10} \text{ m}^{-1}$$

$$T = Ae^{-2\kappa L} = (2.741)e^{-2(1.536 \times 10^{10} \text{ m}^{-1})(0.25 \times 10^{-9} \text{ m})} = 2.741e^{-7.68} = 0.0013$$

(b) The only change in the mass m , which appears in κ .

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

$$\kappa = \frac{\sqrt{2(1.673 \times 10^{-27} \text{ kg})(41 \text{ eV} - 32 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}}{1.055 \times 10^{-34} \text{ J} \cdot \text{s}} = 6.584 \times 10^{11} \text{ m}^{-1}$$

$$\text{Then } T = Ae^{-2\kappa L} = (2.741)e^{-2(6.584 \times 10^{11} \text{ m}^{-1})(0.25 \times 10^{-9} \text{ m})} = 2.741e^{-392.2} = 10^{-143}$$

EVALUATE: The more massive proton has a much smaller probability of tunneling than the electron does.

40.26. $T = Ge^{-2\kappa L}$ with $G = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right)$ and $\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$, so $T = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right) e^{\frac{-2\sqrt{2m(U_0 - E)}}{\hbar} L}$.

(a) If $U_0 = 30.0 \times 10^6$ eV, $L = 2.0 \times 10^{-15}$ m, $m = 6.64 \times 10^{-27}$ kg and

$$U_0 - E = 1.0 \times 10^6 \text{ eV } (E = 29.0 \times 10^6 \text{ eV}), T = 0.090.$$

(b) If $U_0 - E = 10.0 \times 10^6$ eV ($E = 20.0 \times 10^6$ eV), $T = 0.014$.

40.27. IDENTIFY and SET UP: The energy levels are given by Eq.(40.26), where $\omega = \sqrt{\frac{k'}{m}}$.

EXECUTE: $\omega = \sqrt{\frac{k'}{m}} = \sqrt{\frac{110 \text{ N/m}}{0.250 \text{ kg}}} = 21.0 \text{ rad/s}$

The ground state energy is given by Eq.(40.26):

$$E_0 = \frac{1}{2} \hbar \omega = \frac{1}{2} (1.055 \times 10^{-34} \text{ J} \cdot \text{s}) (21.0 \text{ rad/s}) = 1.11 \times 10^{-33} \text{ J} (1 \text{ eV} / 1.602 \times 10^{-19} \text{ J}) = 6.93 \times 10^{-15} \text{ eV}$$

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega, \quad E_{(n+1)} = \left(n + 1 + \frac{1}{2}\right) \hbar \omega$$

The energy separation between these adjacent levels is

$$\Delta E = E_{n+1} - E_n = \hbar \omega = 2E_0 = 2(1.11 \times 10^{-33} \text{ J}) = 2.22 \times 10^{-33} \text{ J} = 1.39 \times 10^{-14} \text{ eV}$$

EVALUATE: These energies are extremely small; quantum effects are not important for this oscillator.

40.28. Let $\sqrt{mk'}/2\hbar = \delta$, and so $\frac{d\psi}{dx} = -2x\delta\psi$ and $\frac{d^2\psi}{dx^2} = (4x^2\delta^2 - 2\delta)\psi$, and ψ is a solution of Eq.(40.21) if

$$E = \frac{\hbar^2}{m} \delta^2 = \frac{1}{2} \hbar \sqrt{k'/m} = \frac{1}{2} \hbar \omega.$$

40.29. IDENTIFY: We can model the molecule as a harmonic oscillator. The energy of the photon is equal to the energy difference between the two levels of the oscillator.

SET UP: The energy of a photon is $E_\gamma = hf = hc/\lambda$, and the energy levels of a harmonic oscillator are given by

$$E_n = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k'}{m}} = \left(n + \frac{1}{2}\right) \hbar \omega.$$

EXECUTE: (a) The photon's energy is $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{5.8 \times 10^{-6} \text{ m}} = 0.21 \text{ eV}$

(b) The transition energy is $\Delta E = E_{n+1} - E_n = \hbar \omega = \hbar \sqrt{\frac{k'}{m}}$, which gives $\frac{2\pi\hbar c}{\lambda} = \hbar \sqrt{\frac{k'}{m}}$. Solving for k' , we get

$$k' = \frac{4\pi^2 c^2 m}{\lambda^2} = \frac{4\pi^2 (3.00 \times 10^8 \text{ m/s})^2 (5.6 \times 10^{-26} \text{ kg})}{(5.8 \times 10^{-6} \text{ m})^2} = 5,900 \text{ N/m}.$$

EVALUATE: This would be a rather strong spring in the physics lab.

40.30. According to Eq.(40.26), the energy released during the transition between two adjacent levels is twice the ground state energy $E_3 - E_2 = \hbar \omega = 2E_0 = 11.2 \text{ eV}$.

For a photon of energy E

$$E = hf \Rightarrow \lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(11.2 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 111 \text{ nm}.$$

40.31. IDENTIFY and SET UP: Use the energies given in Eq.(40.26) to solve for the amplitude A and maximum speed $v_{\max}$ of the oscillator. Use these to estimate Δx and Δp_x and compute the uncertainty product $\Delta x \Delta p_x$.

EXECUTE: The total energy of a Newtonian oscillator is given by $E = \frac{1}{2} k' A^2$ where k' is the force constant and A is the amplitude of the oscillator. Set this equal to the energy $E = (n + \frac{1}{2}) \hbar \omega$ of an excited level that has quantum

number n , where $\omega = \sqrt{\frac{k'}{m}}$, and solve for A : $\frac{1}{2} k' A^2 = (n + \frac{1}{2}) \hbar \omega$

$$A = \sqrt{\frac{(2n+1)\hbar\omega}{k'}}$$

The total energy of the Newtonian oscillator can also be written as $E = \frac{1}{2} m v_{\max}^2$. Set this equal to $E = (n + \frac{1}{2}) \hbar \omega$ and solve for $v_{\max}$: $\frac{1}{2} m v_{\max}^2 = (n + \frac{1}{2}) \hbar \omega$

$$v_{\max} = \sqrt{\frac{(2n+1)\hbar\omega}{m}}$$

Thus the maximum linear momentum of the oscillator is $p_{\max} = mv_{\max} = \sqrt{(2n+1)\hbar m\omega}$. Assume that A represents the uncertainty Δx in position and that $p_{\max}$ is the corresponding uncertainty Δp_x in momentum. Then the uncertainty product is $\Delta x \Delta p_x = \sqrt{\frac{(2n+1)\hbar\omega}{k'}} \sqrt{(2n+1)\hbar m\omega} = (2n+1)\hbar\omega \sqrt{\frac{m}{k'}} = (2n+1)\hbar\omega \left(\frac{1}{\omega}\right) = (2n+1)\hbar$.

EVALUATE: For $n=1$ this gives $\Delta x \Delta p_x = 3\hbar$, in agreement with the result derived in Section 40.4. The uncertainty product $\Delta x \Delta p_x$ increases with n .

40.32. (a) $\frac{|\psi(A)|^2}{|\psi(0)|^2} = \exp\left(-\frac{\sqrt{mk'} }{\hbar} A^2\right) = \exp\left(-\sqrt{mk'} \frac{\omega}{k'}\right) = e^{-1} = 0.368$.

This is consistent with what is shown in Figure 40.20 in the textbook.

(b) $\frac{|\psi(2A)|^2}{|\psi(0)|^2} = \exp\left(-\frac{\sqrt{mk'} }{\hbar} (2A)^2\right) = \exp\left(-\sqrt{mk'} 4 \frac{\omega}{k'}\right) = e^{-4} = 1.83 \times 10^{-2}$.

This figure cannot be read this precisely, but the qualitative decrease in amplitude with distance is clear.

40.33. IDENTIFY: We model the atomic vibration in the crystal as a harmonic oscillator.

SET UP: The energy levels of a harmonic oscillator are given by $E_n = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k'}{m}} = \left(n + \frac{1}{2}\right) \hbar \omega$.

EXECUTE: (a) The ground state energy of a simple harmonic oscillator is

$$E_0 = \frac{1}{2} \hbar \omega = \frac{1}{2} \hbar \sqrt{\frac{k'}{m}} = \frac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})}{2} \sqrt{\frac{12.2 \text{ N/m}}{3.82 \times 10^{-26} \text{ kg}}} = 9.43 \times 10^{-22} \text{ J} = 5.89 \times 10^{-3} \text{ eV}$$

(b) $E_4 - E_3 = \hbar \omega = 2E_0 = 0.0118 \text{ eV}$, so $\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{1.88 \times 10^{-21} \text{ J}} = 106 \mu\text{m}$

(c) $E_{n+1} - E_n = \hbar \omega = 2E_0 = 0.0118 \text{ eV}$

EVALUATE: These energy differences are much smaller than those due to electron transitions in the hydrogen atom.

40.34. IDENTIFY: If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given function is $\psi(x) = Ae^{ikx}$, and the one-dimensional Schrödinger equation is

$$-\frac{\hbar}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x).$$

EXECUTE: Start with the given function and take the indicated derivatives: $\psi(x) = Ae^{ikx}$, $\frac{d\psi(x)}{dx} = Aike^{ikx}$.

$$\frac{d^2\psi(x)}{dx^2} = Aik^2e^{ikx} = -Ak^2e^{ikx}, \quad \frac{d^2\psi(x)}{dx^2} = -k^2\psi(x), \quad -\frac{\hbar}{2m} \frac{d^2\psi(x)}{dx^2} = \frac{\hbar^2}{2m} k^2\psi(x).$$

Substituting these results into the one-dimensional Schrödinger equation gives $\frac{\hbar^2 k^2}{2m} \psi(x) + U_0 \psi(x) = E \psi(x)$.

EVALUATE: $\psi(x) = A e^{ikx}$ is a solution to the one-dimensional Schrödinger equation if $E - U_0 = \frac{\hbar^2 k^2}{2m}$ or

$k = \sqrt{\frac{2m(E - U_0)}{\hbar^2}}$. (Since $U_0 < E$ was given, k is the square root of a positive quantity.) In terms of the particle's momentum p : $k = p/\hbar$, and in terms of the particle's de Broglie wavelength λ : $k = 2\pi/\lambda$.

40.35. IDENTIFY: Let I refer to the region $x < 0$ and let II refer to the region $x > 0$, so $\psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$ and

$\psi_{II}(x) = Ce^{ik_2x}$. Set $\psi_I(0) = \psi_{II}(0)$ and $\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$ at $x = 0$.

SET UP: $\frac{d}{dx}(e^{ikx}) = ike^{ikx}$.

EXECUTE: $\psi_I(0) = \psi_{II}(0)$ gives $A + B = C$. $\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$ at $x = 0$ gives $ik_1A - ik_1B = ik_2C$. Solving this pair of

equations for B and C gives $B = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)A$ and $C = \left(\frac{2k_2}{k_1 + k_2}\right)A$.

EVALUATE: The probability of reflection is $R = \frac{B^2}{A^2} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$. The probability of transmission is

$$T = \frac{C^2}{A^2} = \frac{4k_1^2}{(k_1 + k_2)^2}. \text{ Note that } R + T = 1.$$

- 40.36.** (a) $R_n = \frac{(n+1)^2 - n^2}{n^2} = \frac{2n+1}{n^2} = \frac{2}{n} + \frac{1}{n^2}$. This is never larger than it is for $n=1$, and $R_1 = 3$.

(b) R approaches zero; in the classical limit, there is no quantization, and the spacing of successive levels is vanishingly small compared to the energy levels.

- 40.37. IDENTIFY and SET UP:** The energy levels are given by Eq.(40.9): $E_n = \frac{n^2 h^2}{8mL^2}$. Calculate ΔE for the transition and set $\Delta E = hc/\lambda$, the energy of the photon.

EXECUTE: (a) Ground level, $n=1$, $E_1 = \frac{h^2}{8mL^2}$

First excited level, $n=2$, $E_2 = \frac{4h^2}{8mL^2}$

The transition energy is $\Delta E = E_2 - E_1 = \frac{3h^2}{8mL^2}$. Set the transition energy equal to the energy hc/λ of the emitted

photon. This gives $\frac{hc}{\lambda} = \frac{3h^2}{8mL^2}$.

$$\lambda = \frac{8mL^2}{3h} = \frac{8(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})(4.18 \times 10^{-9} \text{ m})^2}{3(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}$$

$$\lambda = 1.92 \times 10^{-5} \text{ m} = 19.2 \text{ } \mu\text{m}.$$

(b) Second excited level has $n=3$ and $E_3 = \frac{9h^2}{8mL^2}$. The transition energy is $\Delta E = E_3 - E_2 = \frac{9h^2}{8mL^2} - \frac{4h^2}{8mL^2} = \frac{5h^2}{8mL^2}$.

$$\frac{hc}{\lambda} = \frac{5h^2}{8mL^2} \text{ so } \lambda = \frac{8mL^2}{5h} = \frac{3}{5}(19.2 \text{ } \mu\text{m}) = 11.5 \text{ } \mu\text{m}.$$

EVALUATE: The energy spacing between adjacent levels increases with n , and this corresponds to a shorter wavelength and more energetic photon in part (b) than in part (a).

- 40.38.** (a) $\frac{2}{L} \int_0^{L/4} \sin^2 \frac{\pi x}{L} dx = \frac{2}{L} \int_0^{L/4} \frac{1}{2} \left(1 - \cos \frac{2\pi x}{L} \right) dx = \frac{1}{L} \left(x - \frac{L}{2\pi} \sin \frac{2\pi x}{L} \right)_0^{L/4} = \frac{1}{4} - \frac{1}{2\pi}$, which is about 0.0908.

(b) Repeating with limits of $L/4$ and $L/2$ gives $\frac{1}{L} \left(x - \frac{L}{2\pi} \sin \frac{2\pi x}{L} \right)_{L/4}^{L/2} = \frac{1}{4} + \frac{1}{2\pi}$,

about 0.409.

(c) The particle is much likely to be nearer the middle of the box than the edge.

(d) The results sum to exactly $1/2$, which means that the particle is as likely to be between $x=0$ and $L/2$ as it is to be between $x=L/2$ and $x=L$.

(e) These results are represented in Figure 40.5b in the textbook.

- 40.39. IDENTIFY:** The probability of the particle being between x_1 and x_2 is $\int_{x_1}^{x_2} |\psi|^2 dx$, where ψ is the normalized wave function for the particle.

(a) **SET UP:** The normalized wave function for the ground state is $\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$.

EXECUTE: The probability P of the particle being between $x=L/4$ and $x=3L/4$ is

$$P = \int_{L/4}^{3L/4} |\psi_1|^2 dx = \frac{2}{L} \int_{L/4}^{3L/4} \sin^2 \left(\frac{\pi x}{L} \right) dx. \text{ Let } y = \pi x/L; dx = (L/\pi) dy \text{ and the integration limits become } \pi/4 \text{ and } 3\pi/4.$$

$$P = \frac{2}{L} \left(\frac{L}{\pi} \right) \int_{\pi/4}^{3\pi/4} \sin^2 y dy = \frac{2}{\pi} \left[\frac{1}{2} y - \frac{1}{4} \sin 2y \right]_{\pi/4}^{3\pi/4}$$

$$P = \frac{2}{\pi} \left[\frac{3\pi}{8} - \frac{\pi}{8} - \frac{1}{4} \sin \left(\frac{3\pi}{2} \right) + \frac{1}{4} \sin \left(\frac{\pi}{2} \right) \right]$$

$$P = \frac{2}{\pi} \left(\frac{\pi}{4} - \frac{1}{4}(-1) + \frac{1}{4}(1) \right) = \frac{1}{2} + \frac{1}{\pi} = 0.818. \text{ (Note: The integral formula } \int \sin^2 y dy = \frac{1}{2} y - \frac{1}{4} \sin 2y \text{ was used.)}$$

(b) SET UP: The normalized wave function for the first excited state is $\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$

EXECUTE: $P = \int_{L/4}^{3L/4} |\psi_2|^2 dx = \frac{2}{L} \int_{L/4}^{3L/4} \sin^2\left(\frac{2\pi x}{L}\right) dx$. Let $y = 2\pi x/L$; $dx = (L/2\pi) dy$ and the integration limits become $\pi/2$ and $3\pi/2$.

$$P = \frac{2}{L} \left(\frac{L}{2\pi} \right) \int_{\pi/2}^{3\pi/2} \sin^2 y dy = \frac{1}{\pi} \left[\frac{1}{2} y - \frac{1}{4} \sin 2y \right]_{\pi/2}^{3\pi/2} = \frac{1}{\pi} \left(\frac{3\pi}{4} - \frac{\pi}{4} \right) = 0.500$$

(c) EVALUATE: These results are consistent with Fig.40.4b in the textbook. That figure shows that $|\psi|^2$ is more concentrated near the center of the box for the ground state than for the first excited state; this is consistent with the answer to part (a) being larger than the answer to part (b). Also, this figure shows that for the first excited state half the area under $|\psi|^2$ curve lies between $L/4$ and $3L/4$, consistent with our answer to part (b).

40.40. Using the normalized wave function $\psi_1 = \sqrt{2/L} \sin(\pi x/L)$, the probabilities $|\psi|^2 dx$ are

(a) $(2/L) \sin^2(\pi/4) dx = dx/L$

(b) $(2/L) \sin^2(\pi/2) dx = 2dx/L$

(c) $(2/L) \sin^2(3\pi/4) dx = dx/L$.

40.41. IDENTIFY and SET UP: The normalized wave function for the $n=2$ first excited level is $\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$.

$P = |\psi(x)|^2 dx$ is the probability that the particle will be found in the interval x to $x + dx$.

EXECUTE: **(a)** $x = L/4$

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{L}{4}\right)\right) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{2}\right) = \sqrt{\frac{2}{L}}.$$

$$P = (2/L) dx$$

(b) $x = L/2$

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{L}{2}\right)\right) = \sqrt{\frac{2}{L}} \sin(\pi) = 0$$

$$P = 0$$

(c) $x = 3L/4$

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{3L}{4}\right)\right) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{2}\right) = -\sqrt{\frac{2}{L}}.$$

$$P = (2/L) dx$$

EVALUATE: Our results are consistent with the $n=2$ part of Fig.40.5 in the textbook. $|\psi|^2$ is zero at the center of the box and is symmetric about this point.

40.42. $\Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$. $|\vec{p}| = \hbar k = \frac{\hbar n \pi}{L} = \frac{\hbar n}{2L}$. At $x=0$ the initial momentum at the wall is $\vec{p}_{\text{initial}} = -\frac{\hbar n}{2L} \hat{i}$ and the final

momentum, after turning around, is $\vec{p}_{\text{final}} = +\frac{\hbar n}{2L} \hat{i}$. So, $\Delta \vec{p} = +\frac{\hbar n}{2L} \hat{i} - \left(-\frac{\hbar n}{2L} \hat{i}\right) = +\frac{\hbar n}{L} \hat{i}$. At $x=L$ the initial

momentum is $\vec{p}_{\text{initial}} = +\frac{\hbar n}{2L} \hat{i}$ and the final momentum, after turning around, is $\vec{p}_{\text{final}} = -\frac{\hbar n}{2L} \hat{i}$. So,

$$\Delta \vec{p} = -\frac{\hbar n}{2L} \hat{i} - \frac{\hbar n}{2L} \hat{i} = -\frac{\hbar n}{L} \hat{i}$$

40.43. (a) For a free particle, $U(x) = 0$ so Schrödinger's equation becomes $\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2} E \psi(x)$. The graph is given in Figure 40.43.

(b) For $x < 0$: $\psi(x) = e^{+\kappa x}$. $\frac{d\psi(x)}{dx} = \kappa e^{+\kappa x}$. $\frac{d^2\psi(x)}{dx^2} = \kappa^2 e^{+\kappa x}$. So $\kappa^2 = -\frac{2m}{\hbar^2} E \Rightarrow E = -\frac{\hbar^2 \kappa^2}{2m}$.

(c) For $x > 0$: $\psi(x) = e^{-\kappa x}$. $\frac{d\psi(x)}{dx} = -\kappa e^{-\kappa x}$. $\frac{d^2\psi(x)}{dx^2} = \kappa^2 e^{-\kappa x}$

So again $\kappa^2 = -\frac{2m}{\hbar^2} E \Rightarrow E = \frac{-\hbar^2 \kappa^2}{2m}$. Parts (c) and (d) show $\psi(x)$ satisfies the Schrödinger's equation, provided

$$E = \frac{-\hbar^2 \kappa^2}{2m}.$$

(d) Note $\frac{d\psi(x)}{dx}$ is discontinuous at $x = 0$. (That is, negative for $x > 0$ and positive for $x < 0$.)

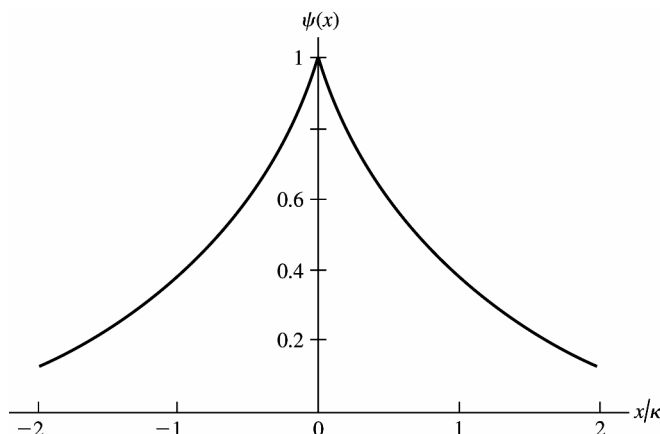


Figure 40.43

40.44. IDENTIFY: We start with the penetration distance formula given in the problem.

SET UP: The given formula is $\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$.

EXECUTE: (a) Substitute the given numbers into the formula:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(20 \text{ eV} - 13 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}} = 7.4 \times 10^{-11} \text{ m}$$

$$(b) \eta = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(1.67 \times 10^{-27} \text{ kg})(30 \text{ MeV} - 20 \text{ MeV})(1.602 \times 10^{-13} \text{ J/MeV})}} = 1.44 \times 10^{-15} \text{ m}$$

EVALUATE: The penetration depth varies widely depending on the mass and energy of the particle.

40.45. (a) We set the solutions for inside and outside the well equal to each other at the well boundaries, $x = 0$ and L .

$x = 0$: $A \sin(0) + B = C \Rightarrow B = C$, since we must have $D = 0$ for $x < 0$.

$x = L$: $A \sin \frac{\sqrt{2mEL}}{\hbar} + B \cos \frac{\sqrt{2mEL}}{\hbar} = +De^{-\kappa L}$ since $C = 0$ for $x > L$.

This gives $A \sin kL + B \cos kL = De^{-\kappa L}$, where $k = \frac{\sqrt{2mE}}{\hbar}$.

(b) Requiring continuous derivatives at the boundaries yields

$x = 0$: $\frac{d\psi}{dx} = kA \cos(k \cdot 0) - kB \sin(k \cdot 0) = kA = \kappa C e^{k \cdot 0} \Rightarrow kA = \kappa C$

$x = L$: $kA \cos kL - kB \sin kL = -\kappa D e^{-\kappa L}$.

$$\mathbf{40.46.} \quad T = Ge^{-2\kappa L} \text{ with } G = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0}\right) \text{ and } \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar} \Rightarrow L = -\frac{1}{2\kappa} \ln \left(\frac{T}{G}\right).$$

If $E = 5.5 \text{ eV}$, $U_0 = 10.0 \text{ eV}$, $m = 9.11 \times 10^{-31} \text{ kg}$, and $T = 0.0010$. Then

$$\kappa = \frac{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(4.5 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}}{(1.054 \times 10^{-34} \text{ J} \cdot \text{s})} = 1.09 \times 10^{10} \text{ m}^{-1} \text{ and } G = 16 \frac{5.5 \text{ eV}}{10.0 \text{ eV}} \left(1 - \frac{5.5 \text{ eV}}{10.0 \text{ eV}}\right) = 3.96$$

$$\text{so } L = -\frac{1}{2(1.09 \times 10^{10} \text{ m}^{-1})} \ln \left(\frac{0.0010}{3.96}\right) = 3.8 \times 10^{-10} \text{ m} = 0.38 \text{ nm}.$$

40.47. IDENTIFY and SET UP: When κL is large, then $e^{\kappa L}$ is large and $e^{-\kappa L}$ is small. When κL is small, $\sinh \kappa L \rightarrow \kappa L$. Consider both κL large and κL small limits.

$$\mathbf{EXECUTE:} \quad (a) \quad T = \left[1 + \frac{(U_0 \sinh \kappa L)^2}{4E(U_0 - E)}\right]^{-1}$$

$$\sinh \kappa L = \frac{e^{\kappa L} - e^{-\kappa L}}{2}$$

$$\text{For } \kappa L \gg 1, \sinh \kappa L \rightarrow \frac{e^{\kappa L}}{2} \text{ and } T \rightarrow \left[1 + \frac{U_0^2 e^{2\kappa L}}{16E(U_0 - E)} \right]^{-1} = \frac{16E(U_0 - E)}{16E(U_0 - E) + U_0^2 e^{2\kappa L}}$$

$$\text{For } \kappa L \gg 1, 16E(U_0 - E) + U_0^2 e^{2\kappa L} \rightarrow U_0^2 e^{2\kappa L}$$

$$T \rightarrow \frac{16E(U_0 - E)}{U_0^2 e^{2\kappa L}} = 16 \left(\frac{E}{U_0} \right) \left(1 - \frac{E}{U_0} \right) e^{-2\kappa L}, \text{ which is Eq.(40.21).}$$

$$(b) \kappa L = \frac{L\sqrt{2m(U_0 - E)}}{\hbar}. \text{ So } \kappa L \gg 1 \text{ when } L \text{ is large (barrier is wide) or } U_0 - E \text{ is large. (} E \text{ is small compared to } U_0 \text{.)}$$

$$(c) \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}; \kappa \text{ becomes small as } E \text{ approaches } U_0. \text{ For } \kappa \text{ small, } \sinh \kappa L \rightarrow \kappa L \text{ and}$$

$$T \rightarrow \left[1 + \frac{U_0^2 \kappa^2 L^2}{4E(U_0 - E)} \right]^{-1} = \left[1 + \frac{U_0^2 2m(U_0 - E)L^2}{\hbar^2 4E(U_0 - E)} \right]^{-1} \text{ (using the definition of } \kappa \text{)}$$

$$\text{Thus } T \rightarrow \left[1 + \frac{2U_0^2 L^2 m}{4E\hbar^2} \right]^{-1}$$

$$U_0 \rightarrow E \text{ so } \frac{U_0^2}{E} \rightarrow E \text{ and } T \rightarrow \left[1 + \frac{2EL^2 m}{4\hbar^2} \right]^{-1}$$

$$\text{But } k^2 = \frac{2mE}{\hbar^2}, \text{ so } T \rightarrow \left[1 + \left(\frac{kL}{2} \right)^2 \right]^{-1}, \text{ as was to be shown.}$$

EVALUATE: When κL is large Eq.(40.20) applies and T is small. When $E \rightarrow U_0$, T does not approach unity.

40.48. (a) $E = \frac{1}{2}mv^2 = (n + (1/2))\hbar\omega = (n + (1/2))hf$, and solving for n ,

$$n = \frac{\frac{1}{2}mv^2}{hf} - \frac{1}{2} = \frac{(1/2)(0.020 \text{ kg})(0.360 \text{ m/s})^2}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(1.50 \text{ Hz})} - \frac{1}{2} = 1.3 \times 10^{30}.$$

(b) The difference between energies is $\hbar\omega = hf = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(1.50 \text{ Hz}) = 9.95 \times 10^{-34} \text{ J}$. This energy is too small to be detected with current technology

40.49. IDENTIFY and SET UP: Calculate the angular frequency ω of the pendulum and apply Eq.(40.26) for the energy levels.

EXECUTE: $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.500 \text{ s}} = 4\pi \text{ s}^{-1}$

The ground-state energy is $E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.055 \times 10^{-34} \text{ J} \cdot \text{s})(4\pi \text{ s}^{-1}) = 6.63 \times 10^{-34} \text{ J}$.

$$E_0 = 6.63 \times 10^{-34} \text{ J} (1 \text{ eV} / 1.602 \times 10^{-19} \text{ J}) = 4.14 \times 10^{-15} \text{ eV}$$

$$E_n = \left(n + \frac{1}{2} \right) \hbar\omega$$

$$E_{n+1} = \left(n + 1 + \frac{1}{2} \right) \hbar\omega$$

The energy difference between the adjacent energy levels is

$$\Delta E = E_{n+1} - E_n = \hbar\omega = 2E_0 = 1.33 \times 10^{-33} \text{ J} = 8.30 \times 10^{-15} \text{ eV}$$

EVALUATE: These energies are much too small to detect. Quantum effects are not important for ordinary size objects.

40.50. IDENTIFY: We model the electrons in the lattice as a particle in a box. The energy of the photon is equal to the energy difference between the two energy states in the box.

SET UP: The energy of an electron in the n^{th} level is $E_n = \frac{n^2 h^2}{8mL^2}$. We do not know the initial or final levels, but we do know they differ by 1. The energy of the photon, hc/λ , is equal to the energy difference between the two states.

EXECUTE: The energy difference between the levels is $\Delta E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{1.649 \times 10^{-7} \text{ m}} =$

$1.206 \times 10^{-18} \text{ J}$. Using the formula for the energy levels in a box, this energy difference is equal to

$$\Delta E = \left[n^2 - (n-1)^2 \right] \frac{h^2}{8mL^2} = (2n-1) \frac{h^2}{8mL^2}.$$

Solving for n gives $n = \left(\frac{\Delta E 8mL^2}{h^2} + 1 \right) = \frac{1}{2} \left(\frac{(1.206 \times 10^{-18} \text{ J}) 8(9.11 \times 10^{-31} \text{ kg})(0.500 \times 10^{-9} \text{ m})^2}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2} + 1 \right) = 3$.

The transition is from $n = 3$ to $n = 2$.

EVALUATE: We know the transition is not from the $n = 4$ to the $n = 3$ state because we let n be the higher state and $n - 1$ the lower state.

40.51. IDENTIFY: If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given wave function is $\psi_0(x) = A_0 e^{-\alpha^2 x^2/2}$ and the Schrödinger equation is

$$-\frac{\hbar}{2m} \frac{d^2 \psi(x)}{dx^2} + \frac{k'x^2}{2} \psi(x) = E \psi(x).$$

EXECUTE: (a) Start by taking the derivatives: $\psi_0(x) = A_0 e^{-\alpha^2 x^2/2}$. $\frac{d\psi_0(x)}{dx} = -\alpha^2 x A_0 e^{-\alpha^2 x^2/2}$.

$$\frac{d^2 \psi_0(x)}{dx^2} = -A_0 \alpha^2 e^{-\alpha^2 x^2/2} + (\alpha^2)^2 x^2 A_0 e^{-\alpha^2 x^2/2}. \quad \frac{d^2 \psi_0(x)}{dx^2} = [-\alpha^2 + (\alpha^2)^2 x^2] \psi_0(x).$$

$$-\frac{\hbar}{2m} \frac{d^2 \psi_0(x)}{dx^2} = -\frac{\hbar^2}{2m} [-\alpha^2 + (\alpha^2)^2 x^2] \psi_0(x). \text{ Equation (40.22) is } -\frac{\hbar}{2m} \frac{d^2 \psi(x)}{dx^2} + \frac{k'x^2}{2} \psi(x) = E \psi(x). \text{ Substituting}$$

the above result into that equation gives $-\frac{\hbar^2}{2m} [-\alpha^2 + (\alpha^2)^2 x^2] \psi_0(x) + \frac{k'x^2}{2} \psi_0(x) = E \psi_0(x)$. Since $\alpha^2 = \frac{m\omega}{\hbar}$ and

$$\omega = \sqrt{\frac{k'}{m}}, \text{ the coefficient of } x^2 \text{ is } -\frac{\hbar^2}{2m} (\alpha^2)^2 + \frac{k'}{2} = -\frac{\hbar^2}{2m} \left(\frac{m\omega}{\hbar} \right)^2 + \frac{m\omega^2}{2} = 0.$$

(b) $A_0 = \left(\frac{m\omega}{\hbar\pi} \right)^{1/4}$

(c) The classical turning points are at $A = \pm \sqrt{\frac{\hbar}{\omega m}}$. The probability density function $|\psi|^2$ is

$$|\psi_0(x)|^2 = A_0^2 e^{-\alpha^2 x^2} = \left(\frac{m\omega}{\hbar\pi} \right)^{1/2} e^{-m\omega x^2/\hbar}. \text{ At } x = 0, |\psi_0|^2 = \left(\frac{m\omega}{\hbar\pi} \right)^{1/2}.$$

$$\frac{d|\psi_0(x)|^2}{dx} = \left(\frac{m\omega}{\hbar\pi} \right)^{1/2} (-\alpha^2 2x) e^{-\alpha^2 x^2} = -2 \frac{m\omega}{\hbar} \left(\frac{m\omega}{\hbar\pi} \right)^{1/2} x e^{-\alpha^2 x^2}. \text{ At } x = 0, \frac{d|\psi_0(x)|^2}{dx} = 0.$$

$$\frac{d^2 |\psi_0(x)|^2}{dx^2} = -2 \frac{m\omega}{\hbar} \left(\frac{m\omega}{\hbar\pi} \right)^{1/2} [1 - 2\alpha^2 x^2] e^{-\alpha^2 x^2}. \text{ At } x = 0, \frac{d^2 |\psi_0(x)|^2}{dx^2} < 0. \text{ Therefore, at } x = 0, \text{ the first derivative is}$$

zero and the second derivative is negative. Therefore, the probability density function has a maximum at $x = 0$.

EVALUATE: $\psi_0(x) = A_0 e^{-\alpha^2 x^2/2}$ is a solution to equation (40.22) if $-\frac{\hbar^2}{2m} (-\alpha^2) \psi_0(x) = E \psi_0(x)$ or

$$E = \frac{\hbar^2 \alpha^2}{2m} = \frac{\hbar\omega}{2}. \quad E_0 = \frac{\hbar\omega}{2} \text{ corresponds to } n = 0 \text{ in Equation (40.26).}$$

40.52. IDENTIFY: If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given wave function is $\psi_1(x) = A_1 2xe^{-\alpha^2 x^2/2}$ and the Schrödinger equation is

$$-\frac{\hbar}{2m} \frac{d^2 \psi(x)}{dx^2} + \frac{k'x^2}{2} \psi(x) = E \psi(x).$$

EXECUTE: (a) Start by taking the indicated derivatives: $\psi_1(x) = A_1 2xe^{-\alpha^2 x^2/2}$.

$$\frac{d\psi_1(x)}{dx} = -2\alpha^2 x^2 A_1 e^{-\alpha^2 x^2/2} + 2A_1 e^{-\alpha^2 x^2/2}. \quad \frac{d^2 \psi_1(x)}{dx^2} = -2A_1 \alpha^2 2xe^{-\alpha^2 x^2/2} - 2A_1 \alpha^2 x^2 (-\alpha^2 x) e^{-\alpha^2 x^2/2} + 2A_1 (-\alpha^2 x) e^{-\alpha^2 x^2/2}.$$

$$\frac{d^2 \psi_1(x)}{dx^2} = [-2\alpha^2 + (\alpha^2)^2 x^2 - \alpha^2] \psi_1(x) = [-3\alpha^2 + (\alpha^2)^2 x^2] \psi_1(x)$$

$$-\frac{\hbar}{2m} \frac{d^2 \psi_1(x)}{dx^2} = -\frac{\hbar^2}{2m} [-3\alpha^2 + (\alpha^2)^2 x^2] \psi_1(x)$$

Equation (40.22) is $-\frac{\hbar}{2m} \frac{d^2\psi(x)}{dx^2} + \frac{k'x^2}{2}\psi(x) = E\psi(x)$. Substituting the above result into that equation gives

$-\frac{\hbar^2}{2m}[-3\alpha^2 + (\alpha^2)^2 x^2]\psi_1(x) + \frac{k'x^2}{2}\psi_1(x) = E\psi_1(x)$. Since $\alpha^2 = \frac{m\omega}{\hbar}$ and $\omega = \sqrt{\frac{k'}{m}}$, the coefficient of x^2 is

$$-\frac{\hbar^2}{2m}(\alpha^2)^2 + \frac{k'}{2} = -\frac{\hbar^2}{2m}\left(\frac{m\omega}{\hbar}\right)^2 + \frac{m\omega^2}{2} = 0$$

$$(b) A_1 = \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\hbar\pi} \right)^{1/4}$$

(c) The probability density function $|\psi|^2$ is $|\psi_1(x)|^2 = A_1^2 4x^2 e^{-\alpha^2 x^2} = \frac{1}{2} \left(\frac{m\omega}{\hbar\pi} \right)^{1/2} 4x^2 e^{-\frac{m\omega x^2}{\hbar}}$

$$\text{At } x=0, \left| \frac{d|\psi_1(x)|^2}{dx} \right| = A_1^2 8xe^{-\alpha^2 x^2} + A_1^2 4x^2(-\alpha^2 2x)e^{-\alpha^2 x^2} = A_1^2 8xe^{-\alpha^2 x^2} - A_1^2 8x^3 \alpha^2 e^{-\alpha^2 x^2}$$

$$\text{At } x=0, \frac{d|\psi_1(x)|^2}{dx} = 0. \text{ At } x = \pm \frac{1}{\alpha}, \frac{d|\psi_1(x)|^2}{dx} = 0.$$

$$\frac{d^2|\psi_1(x)|^2}{dx^2} = A_1^2 8e^{-\alpha^2 x^2} + A_1^2 8x(-\alpha^2 2x)e^{-\alpha^2 x^2} - A_1^2 8(3x^2)\alpha^2 e^{-\alpha^2 x^2} - A_1^2 8x^3 \alpha^2(-\alpha^2 2x)e^{-\alpha^2 x^2}.$$

$$\frac{d^2|\psi_1(x)|^2}{dx^2} = A_1^2 8e^{-\alpha^2 x^2} - A_1^2 16x^2 \alpha^2 e^{-\alpha^2 x^2} - A_1^2 24x^2 \alpha^2 e^{-\alpha^2 x^2} + A_1^2 16x^4 (\alpha^2)^2 e^{-\alpha^2 x^2}. \text{ At } x=0, \frac{d^2|\psi_1(x)|^2}{dx^2} > 0. \text{ So at}$$

$x=0$, the first derivative is zero and the second derivative is positive. Therefore, the probability density function has a minimum at $x=0$. At $x = \pm \frac{1}{\alpha}$, $\frac{d^2|\psi_1(x)|^2}{dx^2} < 0$. So at $x = \pm \frac{1}{\alpha}$, the first derivative is zero and the second derivative is negative. Therefore, the probability density function has maxima at $x = \pm \frac{1}{\alpha}$, corresponding to the classical turning points for $n=0$ as found in the previous question.

EVALUATE: $\psi_1(x) = A_1 2xe^{-\alpha^2 x^2/2}$ is a solution to equation (40.22) if $-\frac{\hbar^2}{2m}(-3\alpha^2)\psi_1(x) = E\psi_1(x)$ or

$$E = \frac{3\hbar^2 \alpha^2}{2m} = \frac{3\hbar\omega}{2}. E_1 = \frac{3\hbar\omega}{2} \text{ corresponds to } n=1 \text{ in Equation (40.26).}$$

40.53. IDENTIFY and SET UP: Evaluate $\partial^2\psi/\partial x^2$, $\partial^2\psi/\partial y^2$, and $\partial^2\psi/\partial z^2$ for the proposed ψ and put Eq.(40.29). Use that ψ_{n_x} , ψ_{n_y} , and ψ_{n_z} are each solutions to Eq.(40.22).

$$\text{EXECUTE: (a) } -\frac{\hbar^2}{2m} \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \right) + U\psi = E\psi$$

$$\psi_{n_x}, \psi_{n_y}, \psi_{n_z} \text{ are each solutions of Eq.(40.22), so } -\frac{\hbar^2}{2m} \frac{d^2\psi_{n_x}}{dx^2} + \frac{1}{2}k'x^2\psi_{n_x} = E_{n_x}\psi_{n_x}$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_{n_y}}{dy^2} + \frac{1}{2}k'y^2\psi_{n_y} = E_{n_y}\psi_{n_y}$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_{n_z}}{dz^2} + \frac{1}{2}k'z^2\psi_{n_z} = E_{n_z}\psi_{n_z}$$

$$\psi = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z), U = \frac{1}{2}k'x^2 + \frac{1}{2}k'y^2 + \frac{1}{2}k'z^2$$

$$\frac{\partial^2\psi}{\partial x^2} = \left(\frac{d^2\psi_{n_x}}{dx^2} \right) \psi_{n_y}\psi_{n_z}, \frac{\partial^2\psi}{\partial y^2} = \left(\frac{d^2\psi_{n_y}}{dy^2} \right) \psi_{n_x}\psi_{n_z}, \frac{\partial^2\psi}{\partial z^2} = \left(\frac{d^2\psi_{n_z}}{dz^2} \right) \psi_{n_x}\psi_{n_y}.$$

$$\text{So } -\frac{\hbar^2}{2m} \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \right) + U\psi = \left(-\frac{\hbar^2}{2m} \frac{d^2\psi_{n_x}}{dx^2} + \frac{1}{2}k'x^2\psi_{n_x} \right) \psi_{n_y}\psi_{n_z}$$

$$+ \left(-\frac{\hbar^2}{2m} \frac{d^2\psi_{n_y}}{dy^2} + \frac{1}{2}k'y^2\psi_{n_y} \right) \psi_{n_x}\psi_{n_z} + \left(-\frac{\hbar^2}{2m} \frac{d^2\psi_{n_z}}{dz^2} + \frac{1}{2}k'z^2\psi_{n_z} \right) \psi_{n_x}\psi_{n_y}$$

$$- \frac{\hbar^2}{2m} \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \right) + U\psi = (E_{n_x} + E_{n_y} + E_{n_z})\psi$$

Therefore, we have shown that this ψ is a solution to Eq.(40.29), with energy

$$E_{n_x n_y n_z} = E_{n_x} + E_{n_y} + E_{n_z} = \left(n_x + n_y + n_z + \frac{3}{2} \right) \hbar \omega$$

(b) and (c) The ground state has $n_x = n_y = n_z = 0$, so the energy is $E_{000} = \frac{3}{2} \hbar \omega$. There is only one set of n_x, n_y and n_z that give this energy.

First-excited state: $n_x = 1, n_y = n_z = 0$ or $n_y = 1, n_x = n_z = 0$ or $n_z = 1, n_x = n_y = 0$ and $E_{100} = E_{010} = E_{001} = \frac{5}{2} \hbar \omega$

There are three different sets of n_x, n_y, n_z quantum numbers that give this energy, so there are three different quantum states that have this same energy.

EVALUATE: For the three-dimensional isotropic harmonic oscillator, the wave function is a product of one-dimensional harmonic oscillator wavefunctions for each dimension. The energy is a sum of energies for three one-dimensional oscillators. All the excited states are degenerate, with more than one state having the same energy.

- 40.54.** $\omega_1 = \sqrt{k'_1/m}, \omega_2 = \sqrt{k'_2/m}$. Let $\psi_{n_x}(x)$ be a solution of Eq.(40.22) with $E_{n_x} = \left(n_x + \frac{1}{2} \right) \hbar \omega_1$, $\psi_{n_y}(y)$ be a similar solution, $\psi_{n_z}(z)$ be a solution of Eq.(40.22) but with z as the independent variable instead of x , and energy $E_{n_z} = \left(n_z + \frac{1}{2} \right) \hbar \omega_2$.

(a) As in Problem 40.53, look for a solution of the form $\psi(x, y, z) = \psi_{n_x}(x) \psi_{n_y}(y) \psi_{n_z}(z)$. Then,

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} &= \left(E_{n_x} - \frac{1}{2} k'_1 x^2 \right) \psi \text{ with similar relations for } \frac{\partial^2 \psi}{\partial y^2} \text{ and } \frac{\partial^2 \psi}{\partial z^2}. \text{ Adding,} \\ -\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) &= \left(E_{n_x} + E_{n_y} + E_{n_z} - \frac{1}{2} k'_1 x^2 - \frac{1}{2} k'_1 y^2 - \frac{1}{2} k'_2 z^2 \right) \psi \\ &= (E_{n_x} + E_{n_y} + E_{n_z} - U) \psi = (E - U) \psi \end{aligned}$$

where the energy E is $E = E_{n_x} + E_{n_y} + E_{n_z} = \hbar \left[\left(n_x + n_y + 1 \right) \omega_1^2 + \left(n_z + \frac{1}{2} \right) \omega_2^2 \right]$, with n_x, n_y and n_z all nonnegative integers.

(b) The ground level corresponds to $n_x = n_y = n_z = 0$, and $E = \hbar \left(\omega_1^2 + \frac{1}{2} \omega_2^2 \right)$. The first excited level corresponds to $n_x = n_y = 0$ and $n_z = 1$, since $\omega_1^2 > \omega_2^2$, and $E = \hbar \omega \left(\omega_1^2 + \frac{3}{2} \omega_2^2 \right)$. There is only one set of quantum numbers for both the ground state and the first excited state.

- 40.55.** (a) $\psi(x) = A \sin kx$ and $\psi(-L/2) = 0 = \psi(+L/2)$

$$\Rightarrow 0 = A \sin \left(\frac{kL}{2} \right) \Rightarrow \frac{kL}{2} = n\pi \Rightarrow k = \frac{2n\pi}{L} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{L}{n} \Rightarrow p_n = \frac{h}{\lambda n} = \frac{nh}{L} \Rightarrow E_n = \frac{p_n^2}{2m} = \frac{n^2 h^2}{2mL^2} = \frac{(2n)^2 h^2}{8mL^2}, \text{ where } n = 1, 2, \dots$$

(b) $\psi(x) = A \cos kx$ and $\psi(-L/2) = 0 = \psi(+L/2)$

$$\Rightarrow 0 = A \cos \left(\frac{kL}{2} \right) \Rightarrow \frac{kL}{2} = (2n+1) \frac{\pi}{2} \Rightarrow k = \frac{(2n+1)\pi}{L} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2L}{(2n+1)} \Rightarrow p_n = \frac{(2n+1)h}{2L}$$

$$\Rightarrow E_n = \frac{(2n+1)^2 h^2}{8mL^2} \quad n = 0, 1, 2, \dots$$

(c) The combination of all the energies in parts (a) and (b) is the same energy levels as given in Eq.(40.9), where

$$E_n = \frac{n^2 h^2}{8mL^2}.$$

(d) Part (a)'s wave functions are odd, and part (b)'s are even.

- 40.56.** (a) As with the particle in a box, $\psi(x) = A \sin kx$, where A is a constant and $k^2 = 2mE/\hbar^2$. Unlike the particle in a box, however, k and hence E do not have simple forms.

(b) For $x > L$, the wave function must have the form of Eq.(40.18). For the wave function to remain finite as $x \rightarrow \infty$, $C = 0$. The constant $\kappa^2 = 2m(U_0 - E)/\hbar^2$, as in Eq.(14.17) and Eq.(40.18).

(c) At $x = L$, $A \sin kL = De^{-\kappa L}$ and $kA \cos kL = -\kappa De^{-\kappa L}$. Dividing the second of these by the first gives $k \cot kL = -\kappa$, a transcendental equation that must be solved numerically for different values of the length L and the ratio E/U_0 .

40.57. (a) $E = K + U(x) = \frac{p^2}{2m} + U(x) \Rightarrow p = \sqrt{2m(E - U(x))}$. $\lambda = \frac{h}{p} \Rightarrow \lambda(x) = \frac{h}{\sqrt{2m(E - U(x))}}$.

(b) As $U(x)$ gets larger (i.e., $U(x)$ approaches E from below—recall $k \geq 0$), $E - U(x)$ gets smaller, so $\lambda(x)$ gets larger.

(c) When $E = U(x)$, $E - U(x) = 0$, so $\lambda(x) \rightarrow \infty$.

(d) $\int_a^b \frac{dx}{\lambda(x)} = \int_a^b \frac{dx}{h/\sqrt{2m(E - U(x))}} = \frac{1}{h} \int_a^b \sqrt{2m(E - U(x))} dx = \frac{n}{2} \Rightarrow \int_a^b \sqrt{2m(E - U(x))} dx = \frac{hn}{2}$.

(e) $U(x) = 0$ for $0 < x < L$ with classical turning points at $x = 0$ and $x = L$. So,

$$\int_a^b \sqrt{2m(E - U(x))} dx = \int_0^L \sqrt{2mE} dx = \sqrt{2mE} \int_0^L dx = \sqrt{2mE} L. \text{ So, from part (d),}$$

$$\sqrt{2mE} L = \frac{hn}{2} \Rightarrow E = \frac{1}{2m} \left(\frac{hn}{2L} \right)^2 = \frac{h^2 n^2}{8mL^2}.$$

(f) Since $U(x) = 0$ in the region between the turning points at $x = 0$ and $x = L$, the results is the *same* as part (e). The height U_0 never enters the calculation. WKB is best used with *smoothly* varying potentials $U(x)$.

40.58. (a) At the turning points $E = \frac{1}{2} k' x_{\text{TP}}^2 \Rightarrow x_{\text{TP}} = \pm \sqrt{\frac{2E}{k'}}$.

(b) $\int_{-\sqrt{2E/k'}}^{+\sqrt{2E/k'}} \sqrt{2m \left(E - \frac{1}{2} k' x^2 \right)} dx = \frac{nh}{2}$. To evaluate the integral, we want to get it into a form that matches the

standard integral given. $\sqrt{2m \left(E - \frac{1}{2} k' x^2 \right)} = \sqrt{2mE - mk' x^2} = \sqrt{mk'} \sqrt{\frac{2mE}{mk'} - x^2} = \sqrt{mk'} \sqrt{\frac{2E}{k'} - x^2}$.

Letting $A^2 = \frac{2E}{k'}$, $a = -\sqrt{\frac{2E}{k'}}$, and $b = +\sqrt{\frac{2E}{k'}}$

$$\Rightarrow \sqrt{mk'} \int_a^b \sqrt{A^2 - x^2} dx = 2 \frac{\sqrt{mk'}}{2} \left[x \sqrt{A^2 - x^2} + A^2 \arcsin \left(\frac{x}{A} \right) \right]_0^b$$

$$= \sqrt{mk'} \left[\sqrt{\frac{2E}{k'}} \sqrt{\frac{2E}{k'} - \frac{2E}{k'}} + \frac{2E}{k'} \arcsin \left(\frac{\sqrt{2E/k'}}{\sqrt{2E/k'}} \right) \right] = \sqrt{mk'} \frac{2E}{k'} \arcsin(1) = 2E \sqrt{\frac{m}{k'}} \left(\frac{\pi}{2} \right).$$

Using WKB, this is equal to $\frac{hn}{2}$, so $E \sqrt{\frac{m}{k'}} \pi = \frac{hn}{2}$. Recall $\omega = \sqrt{\frac{k'}{m}}$, so $E = \frac{h}{2\pi} \omega n = \hbar \omega n$.

(c) We are missing the zero-point-energy offset of $\frac{\hbar \omega}{2}$ (recall $E = \hbar \omega \left(n + \frac{1}{2} \right)$). However, our approximation isn't bad at all!

40.59. (a) At the turning points $E = A|x_{\text{TP}}| \Rightarrow x_{\text{TP}} = \pm \frac{E}{A}$.

(b) $\int_{-E/A}^{+E/A} \sqrt{2m(E - A|x|)} dx = 2 \int_0^{E/A} \sqrt{2m(E - Ax)} dx$. Let $y = 2m(E - Ax) \Rightarrow$

$$dy = -2mA dx \text{ when } x = \frac{E}{A}, y = 0, \text{ and when } x = 0, y = 2mE. \text{ So}$$

$$2 \int_0^{E/A} \sqrt{2m(E - Ax)} dx = -\frac{1}{mA} \int_{2mE}^0 y^{1/2} dy = -\frac{2}{3mA} y^{3/2} \Big|_{2mE}^0 = \frac{2}{3mA} (2mE)^{3/2}. \text{ Using WKB, this is equal to } \frac{hn}{2}. \text{ So,}$$

$$\frac{2}{3mA} (2mE)^{3/2} = \frac{hn}{2} \Rightarrow E = \frac{1}{2m} \left(\frac{3mA\hbar}{4} \right)^{2/3} n^{2/3}.$$

(c) The difference in energy decreases between successive levels. For example:

$$1^{2/3} - 0^{2/3} = 1, 2^{2/3} - 1^{2/3} = 0.59, 3^{2/3} - 2^{2/3} = 0.49, \dots$$

- A sharp ∞ step gave ever-increasing level differences ($\sim n^2$).
- A parabola ($\sim x^2$) gave evenly spaced levels ($\sim n$).
- Now, a linear potential ($\sim x$) gives ever-decreasing level differences ($\sim n^{2/3}$).

Roughly speaking, if the curvature of the potential ($\sim$ second derivative) is bigger than that of a parabola, then the level differences will increase. If the curvature is less than a parabola, the differences will decrease.

ATOMIC STRUCTURE

- 41.1. IDENTIFY and SET UP:** $L = \sqrt{l(l+1)}\hbar$. $L_z = m_l\hbar$. $l = 0, 1, 2, \dots, n-1$. $m_l = 0, \pm 1, \pm 2, \dots, \pm l$. $\cos\theta = L_z/L$.
- EXECUTE:** (a) $l=0$: $L=0$, $L_z=0$. $l=1$: $L=\sqrt{2}\hbar$, $L_z=\hbar, 0, -\hbar$. $l=2$: $L=\sqrt{6}\hbar$, $L_z=2\hbar, \hbar, 0, -\hbar, -2\hbar$.
- (b) In each case $\cos\theta = L_z/L$. $L=0$: θ not defined. $L=\sqrt{2}\hbar$: $45.0^\circ, 90.0^\circ, 135.0^\circ$. $L=\sqrt{6}\hbar$: $35.3^\circ, 65.9^\circ, 90.0^\circ, 114.1^\circ, 144.7^\circ$.
- EVALUATE:** There is no state where $\vec{L}$ is totally aligned along the z axis.
- 41.2. IDENTIFY and SET UP:** $L = \sqrt{l(l+1)}\hbar$. $L_z = m_l\hbar$. $l = 0, 1, 2, \dots, n-1$. $m_l = 0, \pm 1, \pm 2, \dots, \pm l$. $\cos\theta = L_z/L$.
- EXECUTE:** (a) $l=0$: $L=0$, $L_z=0$. $l=1$: $L=\sqrt{2}\hbar$, $L_z=\hbar, 0, -\hbar$. $l=2$: $L=\sqrt{6}\hbar$, $L_z=2\hbar, \hbar, 0, -\hbar, -2\hbar$. $l=3$: $L=2\sqrt{3}\hbar$, $L_z=3\hbar, 2\hbar, \hbar, 0, -\hbar, -2\hbar, -3\hbar$. $l=4$: $L=2\sqrt{5}\hbar$, $L_z=4\hbar, 3\hbar, 2\hbar, \hbar, 0, -\hbar, -2\hbar, -3\hbar, -4\hbar$.
- (b) $L=0$: θ not defined. $L=\sqrt{2}\hbar$: $45.0^\circ, 90.0^\circ, 135.0^\circ$. $L=\sqrt{6}\hbar$: $35.3^\circ, 65.9^\circ, 90.0^\circ, 114.1^\circ, 144.7^\circ$. $L=2\sqrt{3}\hbar$: $54.7^\circ, 73.2^\circ, 90.0^\circ, 106.8^\circ, 125.3^\circ, 150.0^\circ$. $L=2\sqrt{5}\hbar$: $26.6^\circ, 47.9^\circ, 63.4^\circ, 77.1^\circ, 90.0^\circ, 102.9^\circ, 116.6^\circ, 132.1^\circ, 153.4^\circ$.
- (c) The minimum angle is 26.6° and occurs for $l=4$, $m_l=+4$. The maximum angle is 153.4° and occurs for $l=4$, $m_l=-4$.
- 41.3. IDENTIFY and SET UP:** The magnitude of the orbital angular momentum L is related to the quantum number l by Eq.(41.4): $L = \sqrt{l(l+1)}\hbar$, $l = 0, 1, 2, \dots$
- EXECUTE:** $l(l+1) = \left(\frac{L}{\hbar}\right)^2 = \left(\frac{4.716 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s}}{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}\right)^2 = 20$
- And then $l(l+1) = 20$ gives that $l = 4$.
- EVALUATE:** l must be integer.
- 41.4. (a)** $(m_l)_{\max} = 2$, so $(L_z)_{\max} = 2\hbar$.
- (b)** $\sqrt{l(l+1)}\hbar = \sqrt{6}\hbar = 2.45\hbar$.
- (c)** The angle is $\arccos\left(\frac{L_z}{L}\right) = \arccos\left(\frac{m_l}{\sqrt{6}}\right)$, and the angles are, for $m_l = -2$ to $m_l = 2$, 144.7° , 114.1° , 90.0° , 65.9° , 35.3° . The angle corresponding to $m_l = l$ will always be larger for larger l .
- 41.5. IDENTIFY and SET UP:** The angular momentum L is related to the quantum number l by Eq.(41.4), $L = \sqrt{l(l+1)}\hbar$. The maximum l , $l_{\max}$, for a given n is $l_{\max} = n-1$.
- EXECUTE:** For $n=2$, $l_{\max}=1$ and $L=\sqrt{2}\hbar=1.414\hbar$.
- For $n=20$, $l_{\max}=19$ and $L=\sqrt{(19)(20)}\hbar=19.49\hbar$.
- For $n=200$, $l_{\max}=199$ and $L=\sqrt{(199)(200)}\hbar=199.5\hbar$.
- EVALUATE:** As n increases, the maximum L gets closer to the value $n\hbar$ postulated in the Bohr model.
- 41.6.** The (l, m_l) combinations are $(0, 0)$, $(1, 0)$, $(1, \pm 1)$, $(2, 0)$, $(2, \pm 1)$, $(2, \pm 2)$, $(3, 0)$, $(3, \pm 1)$, $(3, \pm 2)$, $(3, \pm 3)$, $(4, 0)$, $(4, \pm 1)$, $(4, \pm 2)$, $(4, \pm 3)$, and $(4, \pm 4)$, a total of 25.
- (b)** Each state has the same energy (n is the same), $-\frac{13.60 \text{ eV}}{25} = -0.544 \text{ eV}$.
- 41.7.** $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = \frac{-1}{4\pi\epsilon_0} \frac{(1.60 \times 10^{-19} \text{ C})^2}{1.0 \times 10^{-10} \text{ m}} = -2.3 \times 10^{-18} \text{ J}$
- $$U = \frac{-2.3 \times 10^{-18} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = -14.4 \text{ eV}.$$

- 41.8. (a) As in Example 41.3, the probability is

$$P = \int_0^{a/2} |\psi_{1s}|^2 4\pi r^2 dr = \frac{4}{a^3} \left[\left(-\frac{ar^2}{2} - \frac{a^2 r}{2} - \frac{a^3}{4} \right) e^{-2r/a} \right]_0^{a/2} = 1 - \frac{5e^{-1}}{2} = 0.0803.$$

(b) The difference in the probabilities is $(1 - 5e^{-2}) - (1 - (5/2)e^{-1}) = (5/2)(e^{-1} - 2e^{-2}) = 0.243$.

- 41.9. (a) $|\psi|^2 = \psi^* \psi = |R(r)|^2 |\Theta(\theta)|^2 (Ae^{-im_l \phi})(Ae^{+im_l \phi}) = A^2 |R(r)|^2 |\Theta(\theta)|^2$, which is independent of ϕ .

(b) $\int_0^{2\pi} |\Phi(\phi)|^2 d\phi = A^2 \int_0^{2\pi} d\phi = 2\pi A^2 = 1 \Rightarrow A = \frac{1}{\sqrt{2\pi}}.$

41.10. $E_n = -\frac{1}{(4\pi\epsilon_0)^2} \frac{m_r e^4}{2n^2 \hbar^2} \Delta E_{12} = E_2 - E_1 = \frac{E_1}{2^2} - E_1 = -(0.75)E_1.$

(a) If $m_r = m = 9.11 \times 10^{-31} \text{ kg}$

$$\frac{m_r e^4}{(4\pi\epsilon_0)^2 \hbar^2} = \frac{(9.109 \times 10^{-31} \text{ kg})(1.602 \times 10^{-19} \text{ C})^4}{2(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2} (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)^2 = 2.177 \times 10^{-18} \text{ J} = 13.59 \text{ eV}$$

For $2 \rightarrow 1$ transition, the coefficient is $(0.75)(13.59 \text{ eV}) = 10.19 \text{ eV}$.

(b) If $m_r = \frac{m}{2}$, using the result from part (a),

$$\frac{m_r e^4}{(4\pi\epsilon_0)^2 \hbar^2} = (13.59 \text{ eV}) \left(\frac{m/2}{m} \right) = \left(\frac{13.59 \text{ eV}}{2} \right) = 6.795 \text{ eV}.$$

Similarly, the $2 \rightarrow 1$ transition, $\Rightarrow \left(\frac{10.19 \text{ eV}}{2} \right) = 5.095 \text{ eV}$.

(c) If $m_r = 185.8m$, using the result from part (a),

$$\frac{m_r e^4}{(4\pi\epsilon_0)^2 \hbar^2} = (13.59 \text{ eV}) \left(\frac{185.8m}{m} \right) = 2525 \text{ eV},$$

and the $2 \rightarrow 1$ transition gives $\Rightarrow (10.19 \text{ eV})(185.8) = 1893 \text{ eV}$.

- 41.11. **IDENTIFY and SET UP:** Eq.(41.8) gives $a = \frac{4\pi\epsilon_0 \hbar^2}{m_r e^2} = \frac{\epsilon_0 \hbar^2}{\pi m_r e^2}.$

EXECUTE: (a) $m_r = m$

$$a = \frac{\epsilon_0 \hbar^2}{\pi m_r e^2} = \frac{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{\pi(9.109 \times 10^{-31} \text{ kg})(1.602 \times 10^{-19} \text{ C})^2} = 0.5293 \times 10^{-10} \text{ m}$$

(b) $m_r = m/2$

$$a = 2 \left(\frac{\epsilon_0 \hbar^2}{\pi m_r e^2} \right) = 1.059 \times 10^{-10} \text{ m}$$

(c) $m_r = 185.8m$

$$a = \frac{1}{185.8} \left(\frac{\epsilon_0 \hbar^2}{\pi m_r e^2} \right) = 2.849 \times 10^{-13} \text{ m}$$

EVALUATE: a is the radius for the $n=1$ level in the Bohr model. When the reduced mass m_r increases, a decreases. For positronium and muonium the reduced mass effect is large.

- 41.12. $e^{im_l \phi} = \cos(m_l \phi) + i \sin(m_l \phi)$, and to be periodic with period 2π , $m_l 2\pi$ must be an integer multiple of 2π , so m_l must be an integer.

41.13. $P(a) = \int_0^a |\psi_{1s}|^2 2V = \int_0^a \frac{1}{\pi a^3} e^{-2r/a} (4\pi r^2 dr).$

$$P(a) = \frac{4}{a^3} \int_0^a r^2 e^{-2r/a} dr = \frac{4}{a^3} \left[\left(-\frac{ar^2}{2} - \frac{a^2 r}{2} - \frac{a^3}{4} \right) e^{-2r/a} \right]_0^a = \frac{4}{a^3} \left[\left(-\frac{a^3}{2} - \frac{a^3}{2} - \frac{a^3}{4} \right) e^{-2} + \frac{a^3}{4} e^0 \right]$$

$$\Rightarrow P(a) = 1 - 5e^{-2}.$$

- 41.14. (a) $\Delta E = \mu_B B = (5.79 \times 10^{-5} \text{ eV/T})(0.400 \text{ T}) = 2.32 \times 10^{-5} \text{ eV}$
 (b) $m_l = -2$ the lowest possible value of m_l .

(c) The energy level diagram is sketched in Figure 41.14.

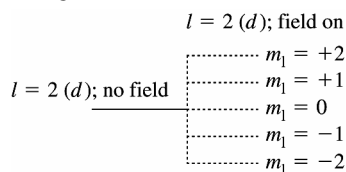


Figure 41.14

41.15. IDENTIFY and SET UP: The interaction energy between an external magnetic field and the orbital angular momentum of the atom is given by Eq.(41.18). The energy depends on m_l with the most negative m_l value having the lowest energy.

EXECUTE: (a) For the 5g level, $l = 4$ and there are $2l + 1 = 9$ different m_l states. The 5g level is split into 9 levels by the magnetic field.

(b) Each m_l level is shifted in energy an amount given by $U = m_l \mu_B B$. Adjacent levels differ in m_l by one, so $\Delta U = \mu_B B$.

$$\mu_B = \frac{e\hbar}{2m} = \frac{(1.602 \times 10^{-19} \text{ C})(1.055 \times 10^{-34} \text{ J} \cdot \text{s})}{2(9.109 \times 10^{-31} \text{ kg})} = 9.277 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

$$\Delta U = \mu_B B = (9.277 \times 10^{-24} \text{ A} \cdot \text{m}^2)(0.600 \text{ T}) = 5.566 \times 10^{-24} \text{ J} (1 \text{ eV} / 1.602 \times 10^{-19} \text{ J}) = 3.47 \times 10^{-5} \text{ eV}$$

(c) The level of highest energy is for the largest m_l , which is $m_l = l = 4$; $U_4 = 4\mu_B B$. The level of lowest energy is for the smallest m_l , which is $m_l = -l = -4$; $U_{-4} = -4\mu_B B$. The separation between these two levels is

$$U_4 - U_{-4} = 8\mu_B B = 8(3.47 \times 10^{-5} \text{ eV}) = 2.78 \times 10^{-4} \text{ eV}.$$

EVALUATE: The energy separations are proportional to the magnetic field. The energy of the $n = 5$ level in the absence of the external magnetic field is $(-13.6 \text{ eV})/5^2 = -0.544 \text{ eV}$, so the interaction energy with the magnetic field is much less than the binding energy of the state.

41.16. (a) According to Figure 41.11 in the textbook there are three different transitions that are consistent with the selection rules. The initial m_l values are 0, ± 1 ; and the final m_l value is 0.

(b) The transition from $m_l = 0$ to $m_l = 0$ produces the same wavelength (122 nm) that was seen without the magnetic field.

(c) The larger wavelength (smaller energy) is produced from the $m_l = -1$ to $m_l = 0$ transition.

(d) The shorter wavelength (greater energy) is produced from the $m_l = +1$ to $m_l = 0$ transition.

41.17. $3p \Rightarrow n = 3, l = 1, \Delta U = \mu_B B \Rightarrow B = \frac{U}{\mu_B} = \frac{(2.71 \times 10^{-5} \text{ eV})}{(5.79 \times 10^{-5} \text{ eV/T})} = 0.468 \text{ T}$

(b) Three: $m_l = 0, \pm 1$.

41.18. (a) $U = +(2.00232) \left(\frac{e}{2m} \right) \left(\frac{-\hbar}{2} \right) B = -\frac{(2.00232)}{2} \mu_B B$

$$U = -\frac{(2.00232)}{2} (5.788 \times 10^{-5} \text{ eV/T})(0.480 \text{ T}) = -2.78 \times 10^{-5} \text{ eV}.$$

(b) Since $n = 1, l = 0$ so there is no orbital magnetic dipole interaction. But if $n \neq 0$ there could be since $l < n$ allows for $l \neq 0$.

41.19. IDENTIFY and SET UP: The interaction energy is $U = -\vec{\mu} \cdot \vec{B}$, with μ_z given by Eq.(41.22).

EXECUTE: $U = -\vec{\mu} \cdot \vec{B} = +\mu_z B$, since the magnetic field is in the negative z -direction.

$$\mu_z = -(2.00232) \left(\frac{e}{2m} \right) S_z, \text{ so } U = -(2.00232) \left(\frac{e}{2m} \right) S_z B$$

$$S_z = m_s \hbar, \text{ so } U = -2.00232 \left(\frac{e\hbar}{2m} \right) m_s B$$

$$\frac{e\hbar}{2m} = \mu_B = 5.788 \times 10^{-5} \text{ eV/T}$$

$$U = -2.00232 \mu_B m_s B$$

The $m_s = +\frac{1}{2}$ level has lower energy.

$$\Delta U = U \left(m_s = -\frac{1}{2} \right) - U \left(m_s = +\frac{1}{2} \right) = -2.00232 \mu_B B \left(-\frac{1}{2} - \left(+\frac{1}{2} \right) \right) = +2.00232 \mu_B B$$

$$\Delta U = +2.00232 (5.788 \times 10^{-5} \text{ eV/T})(1.45 \text{ T}) = 1.68 \times 10^{-4} \text{ eV}$$

EVALUATE: The interaction energy with the electron spin is the same order of magnitude as the interaction energy with the orbital angular momentum for states with $m_l \neq 0$. But a $1s$ state has $l = 0$ and $m_l = 0$, so there is no orbital magnetic interaction.

41.20. The allowed (l, j) combinations are $\left(0, \frac{1}{2}\right), \left(1, \frac{1}{2}\right), \left(1, \frac{3}{2}\right), \left(2, \frac{3}{2}\right)$ and $\left(2, \frac{5}{2}\right)$.

41.21. IDENTIFY and SET UP: j can have the values $l + 1/2$ and $l - 1/2$.

EXECUTE: If j takes the values $7/2$ and $9/2$ it must be that $l - 1/2 = 7/2$ and $l = 8/2 = 4$. The letter that labels this l is g .

EVALUATE: l must be an integer.

41.22. (a) $\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(300 \times 10^8 \text{ m/s})}{(5.9 \times 10^{-6} \text{ eV})} = 21 \text{ cm}$, $f = \frac{c}{\lambda} = \frac{(3.00 \times 10^8 \text{ m/s})}{0.21 \text{ m}} = 1.4 \times 10^9 \text{ Hz}$, a short radio wave.

(b) As in Example 41.6, the effective field is $B \equiv \Delta E / 2\mu_B = 5.1 \times 10^{-2} \text{ T}$, for smaller than that found in the example.

41.23. IDENTIFY and SET UP: For a classical particle $L = I\omega$. For a uniform sphere with mass m and radius R ,

$I = \frac{2}{5}mR^2$, so $L = \left(\frac{2}{5}mR^2\right)\omega$. Solve for ω and then use $v = r\omega$ to solve for v .

EXECUTE: (a) $L = \sqrt{\frac{3}{4}}\hbar$ so $\frac{2}{5}mR^2\omega = \sqrt{\frac{3}{4}}\hbar$

$$\omega = \frac{5\sqrt{3/4}\hbar}{2mR^2} = \frac{5\sqrt{3/4}(1.055 \times 10^{-34} \text{ J} \cdot \text{s})}{2(9.109 \times 10^{-31} \text{ kg})(1.0 \times 10^{-17} \text{ m})^2} = 2.5 \times 10^{30} \text{ rad/s}$$

(b) $v = r\omega = (1.0 \times 10^{-17} \text{ m})(2.5 \times 10^{30} \text{ rad/s}) = 2.5 \times 10^{13} \text{ m/s}$.

EVALUATE: This is much greater than the speed of light c , so the model cannot be valid.

41.24. However the number of electrons is obtained, the results must be consistent with Table (41.3); adding two more electrons to the zinc configuration gives $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^2$.

41.25. The ten lowest energy levels for electrons are in the $n = 1$ and $n = 2$ shells.

$$n = 1, l = 0, m_l = 0, \quad m_s = \pm \frac{1}{2} : 2 \text{ states.}$$

$$n = 2, l = 0, m_l = 0, \quad m_s = \pm \frac{1}{2} : 2 \text{ states.}$$

$$n = 2, l = 1, m_l = 0, \pm 1, \quad m_s = \pm \frac{1}{2} : 6 \text{ states.}$$

41.26. For the outer electrons, there are more inner electrons to screen the nucleus.

41.27. IDENTIFY and SET UP: The energy of an atomic level is given in terms of n and Z_{eff} by Eq.(41.27),

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV}). \text{ The ionization energy for a level with energy } -E_n \text{ is } +E_n.$$

$$\text{EXECUTE: } n = 5 \text{ and } Z_{\text{eff}} = 2.771 \text{ gives } E_5 = -\frac{(2.771)^2}{5^2}(13.6 \text{ eV}) = -4.18 \text{ eV}$$

The ionization energy is 4.18 eV.

EVALUATE: The energy of an atomic state is proportional to Z_{eff}^2 .

41.28. For the $4s$ state, $E = -4.339 \text{ eV}$ and $Z_{\text{eff}} = 4\sqrt{(-4.339)/(-13.6)} = 2.26$. Similarly, $Z_{\text{eff}} = 1.79$ for the $4p$ state and 1.05 for the $4d$ state. The electrons in the states with higher l tend to be further away from the filled subshells and the screening is more complete.

41.29. IDENTIFY and SET UP: Use the exclusion principle to determine the ground-state electron configuration, as in Table 41.3. Estimate the energy by estimating Z_{eff} , taking into account the electron screening of the nucleus.

EXECUTE: (a) $Z = 7$ for nitrogen so a nitrogen atom has 7 electrons. N^{2+} has 5 electrons: $1s^2 2s^2 2p$.

(b) $Z_{\text{eff}} = 7 - 4 = 3$ for the $2p$ level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV}) = -\frac{3^2}{2^2}(13.6 \text{ eV}) = -30.6 \text{ eV}$$

(c) $Z = 15$ for phosphorus so a phosphorus atom has 15 electrons.

P^{2+} has 13 electrons: $1s^2 2s^2 2p^6 3s^2 3p$

(d) $Z_{\text{eff}} = 15 - 12 = 3$ for the $3p$ level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV}) = -\frac{3^2}{3^2}(13.6 \text{ eV}) = -13.6 \text{ eV}$$

EVALUATE: In these ions there is one electron outside filled subshells, so it is a reasonable approximation to assume full screening by these inner-subshell electrons.

41.30. (a) $E_2 = -\frac{13.6 \text{ eV}}{4} Z_{\text{eff}}^2$, so $Z_{\text{eff}} = 1.26$.

(b) Similarly, $Z_{\text{eff}} = 2.26$.

(c) Z_{eff} becomes larger going down the columns in the periodic table.

41.31. **IDENTIFY and SET UP:** Estimate Z_{eff} by considering electron screening and use Eq.(41.27) to calculate the energy. Z_{eff} is calculated as in Example 41.8.

EXECUTE: (a) The element Be has nuclear charge $Z = 4$. The ion Be^+ has 3 electrons. The outermost electron sees the nuclear charge screened by the other two electrons so $Z_{\text{eff}} = 4 - 2 = 2$.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6 \text{ eV}) \text{ so } E_2 = -\frac{2^2}{2^2}(13.6 \text{ eV}) = -13.6 \text{ eV}$$

(b) The outermost electron in Ca^+ sees a $Z_{\text{eff}} = 2$. $E_4 = -\frac{2^2}{4^2}(13.6 \text{ eV}) = -3.4 \text{ eV}$

EVALUATE: For the electron in the highest l -state it is reasonable to assume full screening by the other electrons, as in Example 41.8. The highest l -states of Be^+ , Mg^+ , Ca^+ , etc. all have a $Z_{\text{eff}} = 2$. But the energies are different because for each ion the outermost sublevel has a different n quantum number.

41.32. $E_{kx} \cong (Z-1)^2(10.2 \text{ eV})$. $Z \approx 1 + \sqrt{\frac{7.46 \times 10^3 \text{ eV}}{10.2 \text{ eV}}} = 28.0$, which corresponds to the element Nickel (Ni).

41.33. (a) $Z = 20$: $f = (2.48 \times 10^{15} \text{ Hz})(20-1)^2 = 8.95 \times 10^{17} \text{ Hz}$.

$$E = hf = (4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(8.95 \times 10^{17} \text{ Hz}) = 3.71 \text{ keV}. \quad \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{8.95 \times 10^{17} \text{ Hz}} = 3.35 \times 10^{-10} \text{ m}.$$

(b) $Z = 27$: $f = 1.68 \times 10^{18} \text{ Hz}$. $E = 6.96 \text{ keV}$. $\lambda = 1.79 \times 10^{-10} \text{ m}$.

(c) $Z = 48$: $f = 5.48 \times 10^{18} \text{ Hz}$, $E = 22.7 \text{ keV}$, $\lambda = 5.47 \times 10^{-11} \text{ m}$.

41.34. **IDENTIFY:** The orbital angular momentum is limited by the shell the electron is in.

SET UP: For an electron in the n shell, its orbital angular momentum quantum number l is limited by $0 \leq l < n$, and its orbital angular momentum is given by $L = \sqrt{l(l+1)}\hbar$. The z -component of its angular momentum is

$L_z = m_l\hbar$, where $m_l = 0, \pm 1, \dots, \pm l$, and its spin angular momentum is $S = \sqrt{3/4}\hbar$ for all electrons. Its energy in the n^{th} shell is $E_n = -(13.6 \text{ eV})/n^2$.

EXECUTE: (a) $L = \sqrt{l(l+1)}\hbar = 12\hbar \Rightarrow l = 3$. Therefore the smallest that n can be is 4, so $E_n = -(13.6 \text{ eV})/n^2 = -(13.6 \text{ eV})/4^2 = -0.8500 \text{ eV}$.

(b) For $l = 3$, $m_l = \pm 3, \pm 2, \pm 1, 0$. Since $L_z = m_l\hbar$, the largest L_z can be is $3\hbar$ and the smallest it can be is $-3\hbar$.

(c) $S = \sqrt{3/4}\hbar$ for all electrons.

(d) In this case, $n = 3$, so $l = 2, 1, 0$. Therefore the maximum that L can be is $L_{\text{max}} = \sqrt{2(2+1)}\hbar = \sqrt{6}\hbar$. The minimum L can be is zero when $l = 0$.

EVALUATE: At the quantum level, electrons in atoms can have only certain allowed values of their angular momentum.

41.35. **IDENTIFY:** The total energy determines what shell the electron is in, which limits its angular momentum.

SET UP: The electron's orbital angular momentum is given by $L = \sqrt{l(l+1)}\hbar$, and its total energy in the n^{th} shell is $E_n = -(13.6 \text{ eV})/n^2$.

EXECUTE: (a) First find n : $E_n = -(13.6 \text{ eV})/n^2 = -0.5440 \text{ eV}$ which gives $n = 5$, so $l = 4, 3, 2, 1, 0$. Therefore the possible values of L are given by $L = \sqrt{l(l+1)}\hbar$, giving $L = 0, \sqrt{2}\hbar, \sqrt{6}\hbar, \sqrt{12}\hbar, \sqrt{20}\hbar$.

(b) $E_6 = -(13.6 \text{ eV})/6^2 = -0.3778 \text{ eV}$. $\Delta E = E_6 - E_5 = -0.3778 \text{ eV} - (-0.5440 \text{ eV}) = +0.1662 \text{ eV}$

This must be the energy of the photon, so $\Delta E = hc/\lambda$, which gives

$$\lambda = hc/\Delta E = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(0.1662 \text{ eV}) = 7.47 \times 10^{-6} \text{ m} = 7470 \text{ nm}, \text{ which is in the infrared and hence not visible.}$$

EVALUATE: The electron can have any of the five possible values for its angular momentum, but it cannot have any others.

41.36. IDENTIFY: For the N shell, $n = 4$, which limits the values of the other quantum numbers.

SET UP: In the n^{th} shell, $0 \leq l < n$, $m_l = 0, \pm 1, \dots, \pm l$, and $m_s = \pm 1/2$. The orbital angular momentum of the electron is $L = \sqrt{l(l+1)}\hbar$ and its spin angular momentum is $S = \sqrt{3/4}\hbar$.

EXECUTE: (a) For $l = 3$ we can have $m_l = \pm 3, \pm 2, \pm 1, 0$ and $m_s = \pm 1/2$; for $l = 2$ we can have $m_l = \pm 2, \pm 1, 0$ and $m_s = \pm 1/2$; for $l = 1$, we can have $m_l = \pm 1, 0$ and $m_s = \pm 1/2$; for $l = 0$, we can have $m_l = 0$ and $m_s = \pm 1/2$.

(b) For the N shell, $n = 4$, and for an f -electron, $l = 3$, giving $L = \sqrt{l(l+1)}\hbar = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar$. $L_z =$

$m_l\hbar = \pm 3\hbar, \pm 2\hbar, \pm \hbar, 0$, so the maximum value is $3\hbar$. $S = \sqrt{3/4}\hbar$ for all electrons.

(c) For a d -state electron, $l = 2$, giving $L = \sqrt{2(2+1)}\hbar = \sqrt{6}\hbar$. $L_z = m_l\hbar$, and the maximum value of m_l is 2, so the maximum value of L_z is $2\hbar$. The smallest angle occurs when L_z is most closely aligned along the angular

momentum vector, which is when L_z is greatest. Therefore $\cos\theta_{\min} = \frac{L_z}{L} = \frac{2\hbar}{\sqrt{6}\hbar} = \frac{2}{\sqrt{6}}$ and $\theta_{\min} = 35.3^\circ$. The largest angle occurs when L_z is as far as possible from the L -vector, which is when L_z is most negative. Therefore

$$\cos\theta_{\max} = \frac{-2\hbar}{\sqrt{6}\hbar} = -\frac{2}{\sqrt{6}} \quad \text{and} \quad \theta_{\max} = 144.7^\circ.$$

(d) This is not possible since $l = 3$ for an f -electron, but in the M shell the maximum value of l is 2.

EVALUATE: The fact that the angle in part (c) cannot be zero tells us that the orbital angular momentum of the electron cannot be totally aligned along any specified direction.

41.37. IDENTIFY: The inner electrons shield part of the nuclear charge from the outer electron.

SET UP: The electron's energy in the n^{th} shell, due to shielding, is $E_n = -\frac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$, where Z_{eff} is the effective charge that the electron "sees" for the nucleus.

EXECUTE: (a) $E_n = -\frac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$ and $n = 4$ for the $4s$ state. Solving for Z_{eff} gives $Z_{\text{eff}} = \sqrt{-\frac{(4^2)(-13.6\text{ eV})}{13.6\text{ eV}}}$

$= 1.51$. The nucleus contains a charge of $+11e$, so the average number of electrons that screen this nucleus must be $11 - 1.51 = 9.49$ electrons

(b) (i) The charge of the nucleus is $+19e$, but $17.2e$ is screened by the electrons, so the outer electron "sees" $19e - 17.2e = 1.8e$ and $Z_{\text{eff}} = 1.8$.

$$(ii) E_n = -\frac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV}) = -\frac{(1.8)^2}{4^2}(13.6\text{ eV}) = -2.75\text{ eV}$$

EVALUATE: Sodium has 11 protons, so the inner 10 electrons shield a large portion of this charge from the outer electron. But they don't shield 10 of the protons, since the inner electrons are not totally equivalent to a uniform spherical shell. (They are lumpy.)

41.38. See Example 41.3; $r^2|\psi|^2 = Cr^2e^{-2r/a}$, $\frac{d(r^2|\psi|^2)}{dr} = Ce^{-2r/a}(2r - (2r^2/a))$, and for a maximum, $r = a$, the distance of the electron from the nucleus in the Bohr model.

41.39. (a) IDENTIFY and SET UP: The energy is given by Eq.(38.18), which is identical to Eq.(41.3). The potential energy is given by Eq.(23.9), with $q = +Ze$ and $q_0 = -e$.

$$\text{EXECUTE: } E_{1s} = -\frac{1}{(4\pi\epsilon_0)^2} \frac{me^4}{2\hbar^2}; U(r) = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

$$E_{1s} = U(r) \text{ gives } -\frac{1}{(4\pi\epsilon_0)^2} \frac{me^4}{2\hbar^2} = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

$$r = \frac{(4\pi\epsilon_0)2\hbar^2}{me^2} = 2a$$

EVALUATE: The turning point is twice the Bohr radius.

(b) **IDENTIFY and SET UP:** For the $1s$ state the probability that the electron is in the classically forbidden region is $P(r > 2a) = \int_{2a}^{\infty} |\psi_{1s}|^2 dV = 4\pi \int_{2a}^{\infty} |\psi_{1s}|^2 r^2 dr$. The normalized wave function of the $1s$ state of hydrogen is given in

Example 41.3: $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$. Evaluate the integral; the integrand is the same as in Example 41.3.

$$\text{EXECUTE: } P(r > 2a) = 4\pi \left(\frac{1}{\pi a^3} \right) \int_{2a}^{\infty} r^2 e^{-2r/a} dr$$

Use the integral formula $\int r^2 e^{-\alpha r} dr = -e^{-\alpha r} \left(\frac{r^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3} \right)$, with $\alpha = 2/a$.

$$P(r > 2a) = -\frac{4}{a^3} \left[e^{-2r/a} \left(\frac{ar^2}{2} + \frac{a^2 r}{2} + \frac{a^3}{4} \right) \right]_{2a}^{\infty} = +\frac{4}{a^3} e^{-4} (2a^3 + a^3 + a^3/4)$$

$$P(r > 2a) = 4e^{-4}(13/4) = 13e^{-4} = 0.238.$$

EVALUATE: These is a 23.8% probability of the electron being found in the classically forbidden region, where classically its kinetic energy would be negative.

- 41.40.** (a) For large values of n , the inner electrons will completely shield the nucleus, so $Z_{\text{eff}} = 1$ and the ionization energy would be $\frac{13.60 \text{ eV}}{n^2}$.

(b) $\frac{13.60 \text{ eV}}{350^2} = 1.11 \times 10^{-4} \text{ eV}$, $r_{350} = (350)^2 a_0 = (350)^2 (0.529 \times 10^{-10} \text{ m}) = 6.48 \times 10^{-6} \text{ m}$.

(c) Similarly for $n = 650$, $\frac{13.60 \text{ eV}}{(650)^2} = 3.22 \times 10^{-5} \text{ eV}$, $r_{650} = (650)^2 (0.529 \times 10^{-10} \text{ m}) = 2.24 \times 10^{-5} \text{ m}$.

41.41. $\psi_{2s}(r) = \frac{1}{\sqrt{32\pi a^3}} \left(2 - \frac{r}{a} \right) e^{-r/2a}$

(a) **IDENTIFY and SET UP:** Let $I = \int_0^{\infty} |\psi_{2s}|^2 dV = 4\pi \int_0^{\infty} |\psi_{2s}|^2 r^2 dr$. If ψ_{2s} is normalized then we will find that $I = 1$.

EXECUTE: $I = 4\pi \left(\frac{1}{32\pi a^3} \right) \int_0^{\infty} \left(2 - \frac{r}{a} \right)^2 e^{-r/a} r^2 dr = \frac{1}{8a^3} \int_0^{\infty} \left(4r^2 - \frac{4r^3}{a} + \frac{r^4}{a^2} \right) e^{-r/a} dr$

Use the integral formula $\int_0^{\infty} x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$, with $\alpha = 1/a$

$$I = \frac{1}{8a^3} \left(4(2!)(a^3) - \frac{4}{a}(3!)(a^4) + \frac{1}{a^2}(4!)(a^5) \right) = \frac{1}{8}(8 - 24 + 24) = 1; \text{ this } \psi_{2s} \text{ is normalized.}$$

(b) **SET UP:** For a spherically symmetric state such as the $2s$, the probability that the electron will be found at $r < 4a$ is $P(r < 4a) = \int_0^{4a} |\psi_{2s}|^2 dV = 4\pi \int_0^{4a} |\psi_{2s}|^2 r^2 dr$.

EXECUTE: $P(r < 4a) = \frac{1}{8a^3} \int_0^{4a} \left(4r^2 - \frac{4r^3}{a} + \frac{r^4}{a^2} \right) e^{-r/a} dr$

Let $P(r < 4a) = \frac{1}{8a^3} (I_1 + I_2 + I_3)$.

$$I_1 = 4 \int_0^{4a} r^2 e^{-r/a} dr$$

Use the integral formula $\int r^2 e^{-\alpha r} dr = -e^{-\alpha r} \left(\frac{r^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3} \right)$ with $\alpha = 1/a$.

$$I_1 = -4[e^{-r/a}(r^2 a + 2ra^2 + 2a^3)]_0^{4a} = (-104e^{-4} + 8)a^3.$$

$$I_2 = -\frac{4}{a} \int_0^{4a} r^3 e^{-r/a} dr$$

Use the integral formula $\int r^3 e^{-\alpha r} dr = -e^{-\alpha r} \left(\frac{r^3}{\alpha} + \frac{3r^2}{\alpha^2} + \frac{6r}{\alpha^3} + \frac{6}{\alpha^4} \right)$ with $\alpha = 1/a$.

$$I_2 = \frac{4}{a} [e^{-r/a}(r^3 a + 3r^2 a^2 + 6ra^3 + 6a^4)]_0^{4a} = (568e^{-4} - 24)a^3.$$

$$I_3 = \frac{1}{a^2} \int_0^{4a} r^4 e^{-r/a} dr$$

Use the integral formula $\int r^4 e^{-\alpha r} dr = -e^{-\alpha r} \left(\frac{r^4}{\alpha} + \frac{4r^3}{\alpha^2} + \frac{12r^2}{\alpha^3} + \frac{24r}{\alpha^4} + \frac{24}{\alpha^5} \right)$ with $\alpha = 1/a$.

$$I_3 = -\frac{1}{a^2} [e^{-r/a}(r^4 a + 4r^3 a^2 + 12r^2 a^3 + 24ra^4 + 24a^5)]_0^{4a} = (-824e^{-4} + 24)a^3.$$

$$\text{Thus } P(r < 4a) = \frac{1}{8a^3}(I_1 + I_2 + I_3) = \frac{1}{8a^3}a^3([8 - 24 + 24] + e^{-4}[-104 + 568 - 824])$$

$$P(r < 4a) = \frac{1}{8}(8 - 360e^{-4}) = 1 - 45e^{-4} = 0.176.$$

EVALUATE: There is an 82.4% probability that the electron will be found at $r > 4a$. In the Bohr model the electron is for certain at $r = 4a$; this is a poor description of the radial probability distribution for this state.

- 41.42. (a)** Since the given $\psi(r)$ is real, $r^2 |\psi|^2 = r^2 \psi^2$. The probability density will be an extreme when

$$\frac{d}{dr}(r^2 \psi^2) = 2 \left(r\psi^2 + r^2 \psi \frac{d\psi}{dr} \right) = 2r\psi \left(\psi + r \frac{d\psi}{dr} \right) = 0. \text{ This occurs at } r = 0, \text{ a minimum, and when } \psi = 0, \text{ also a}$$

minimum. A maximum must correspond to $\psi + r \frac{d\psi}{dr} = 0$. Within a multiplicative constant, $\psi(r) = (2 - r/a)e^{-r/2a}$,

$$\frac{d\psi}{dr} = -\frac{1}{a}(2 - r/2a)e^{-r/2a}, \text{ and the condition for a maximum is } (2 - r/a) = (r/a)(2 - r/2a), \text{ or } r^2 - 6ra + 4a^2 = 0.$$

The solutions to the quadratic are $r = a(3 \pm \sqrt{5})$. The ratio of the probability densities at these radii is 3.68, with the larger density at $r = a(3 + \sqrt{5})$.

(b) $\psi = 0$ at $r = 2a$

Parts (a) and (b) are consistent with Figure 41.5 in the textbook; note the two relative maxima, one on each side of the minimum of zero at $r = 2a$.

- 41.43. IDENTIFY:** Use Figure 41.2 in the textbook to relate θ_L to L_z and L : $\cos \theta_L = \frac{L_z}{L}$ so $\theta_L = \arccos\left(\frac{L_z}{L}\right)$

(a) SET UP: The smallest angle $(\theta_L)_{\min}$ is for the state with the largest L and the largest L_z . This is the state with $l = n - 1$ and $m_l = l = n - 1$.

EXECUTE: $L_z = m_l \hbar = (n - 1)\hbar$

$$L = \sqrt{l(l+1)}\hbar = \sqrt{(n-1)n}\hbar$$

$$(\theta_L)_{\min} = \arccos\left(\frac{(n-1)\hbar}{\sqrt{(n-1)n}\hbar}\right) = \arccos\left(\frac{(n-1)}{\sqrt{(n-1)n}}\right) = \arccos\left(\sqrt{\frac{n-1}{n}}\right) = \arccos(\sqrt{1-1/n}).$$

EVALUATE: Note that $(\theta_L)_{\min}$ approaches 0° as $n \rightarrow \infty$.

(b) SET UP: The largest angle $(\theta_L)_{\max}$ is for $l = n - 1$ and $m_l = -l = -(n - 1)$.

EXECUTE: A similar calculation to part (a) yields $(\theta_L)_{\max} = \arccos(-\sqrt{1-1/n})$

EVALUATE: Note that $(\theta_L)_{\max}$ approaches 180° as $n \rightarrow \infty$.

- 41.44. (a)** $L_x^2 + L_y^2 = L^2 - L_z^2 = l(l+1)\hbar^2 - m_l^2\hbar^2$ so $\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\hbar$.

(b) This is the magnitude of the component of angular momentum perpendicular to the z -axis.

(c) The maximum value is $\sqrt{l(l+1)}\hbar = L$, when $m_l = 0$. That is, if the electron is known to have no z -component of angular momentum, the angular momentum must be perpendicular to the z -axis. The minimum is $\sqrt{l}\hbar$ when $m_l = \pm l$.

- 41.45.** $P(r) = \left(\frac{1}{24a^5}\right)r^4 e^{-r/2a}$. $\frac{dP}{dr} = \left(\frac{1}{24a^5}\right)\left(4r^3 - \frac{r^4}{a}\right)e^{-r/2a}$. $\frac{dP}{dr} = 0$ when $4r^3 - \frac{r^4}{a} = 0$; $r = 4a$. In the Bohr model, $r_n = n^2 a$ so $r_2 = 4a$, which agrees.

- 41.46.** The time required to transit the horizontal 50 cm region is $t = \frac{\Delta x}{v_x} = \frac{0.500 \text{ m}}{525 \text{ m/s}} = 0.952 \text{ ms}$. The force required to

$$\text{deflect each spin component by } 0.50 \text{ mm is } F_z = ma_z = \pm m \frac{2\Delta z}{t^2} = \pm \left(\frac{0.1079 \text{ kg/mol}}{6.022 \times 10^{23} \text{ atoms/mol}}\right) \frac{2(0.50 \times 10^{-3} \text{ m})}{(0.952 \times 10^{-3} \text{ s})^2} =$$

$\pm 1.98 \times 10^{-22} \text{ N}$. According to Eq.(41.22), the value of μ_z is $|\mu_z| = 9.28 \times 10^{-24} \text{ A} \cdot \text{m}^2$. Thus, the required

$$\text{magnetic-field gradient is } \left|\frac{dB_z}{dz}\right| = \left|\frac{F_z}{\mu_z}\right| = \frac{1.98 \times 10^{-22} \text{ N}}{9.28 \times 10^{-24} \text{ J/T}} = 21.3 \text{ T/m}.$$

- 41.47.** Decay from a $3d$ to $2p$ state in hydrogen means that $n = 3 \rightarrow n = 2$ and $m_l = \pm 2, \pm 1, 0 \rightarrow m_l = \pm 1, 0$. However selection rules limit the possibilities for decay. The emitted photon carries off one unit of angular momentum so l must change by 1 and hence m_l must change by 0 or ± 1 . The shift in the transition energy from the zero field value is just $U = (m_{l_3} - m_{l_2})\mu_B B = \frac{e\hbar B}{2m}(m_{l_3} - m_{l_2})$, where m_{l_3} is the $3d$ m_l value and m_{l_2} is the $2p$ m_l value. Thus there are only three different energy shifts. They and the transitions that have them, labeled by the m_l names, are:

$$\begin{aligned} \frac{e\hbar B}{2m} : 2 \rightarrow 1, \quad 1 \rightarrow 0, \quad 0 \rightarrow -1 \\ 0 : 1 \rightarrow 1, \quad 0 \rightarrow 0, \quad -1 \rightarrow -1 \\ -\frac{e\hbar B}{2m} : 0 \rightarrow 1, \quad -1 \rightarrow 0, \quad -2 \rightarrow -1 \end{aligned}$$

- 41.48. IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l .

SET UP: The selection rules tell us that for allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1 .

EXECUTE: (a) $E = hc/\lambda = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(475.082 \text{ nm}) = 2.612 \text{ eV}$.

(b) For allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1 . For the $3d$ state, $n = 3$, $l = 2$, and m_l can have the values 2, 1, 0, -1, -2. In the $2p$ state, $n = 2$, $l = 1$, and m_l can be 1, 0, -1. Therefore the 9 allowed transitions from the $3d$ state in the presence of a magnetic field are:

$$\begin{aligned} l = 2, m_l = 2 &\rightarrow l = 1, m_l = 1; \\ l = 2, m_l = 1 &\rightarrow l = 1, m_l = 0 \\ l = 2, m_l = 1 &\rightarrow l = 1, m_l = 1 \\ l = 2, m_l = 0 &\rightarrow l = 1, m_l = 0 \\ l = 2, m_l = 0 &\rightarrow l = 1, m_l = 1 \\ l = 2, m_l = 0 &\rightarrow l = 1, m_l = -1 \\ l = 2, m_l = -1 &\rightarrow l = 1, m_l = 0 \\ l = 2, m_l = -1 &\rightarrow l = 1, m_l = -1 \\ l = 2, m_l = -2 &\rightarrow l = 1, m_l = -1 \end{aligned}$$

(c) $\Delta E = \mu_B B = (5.788 \times 10^{-5} \text{ eV/T})(3.500 \text{ T}) = 0.000203 \text{ eV}$

So the energies of the new states are $-8.50000 \text{ eV} + 0$ and $-8.50000 \text{ eV} \pm 0.000203 \text{ eV}$, giving energies of: -8.50020 eV , -8.50000 eV , and -8.49980 eV

(d) The energy differences of the allowed transitions are equal to the energy differences if no magnetic field were present (2.61176 eV, from part (a)), and that value $\pm \Delta E$ (0.000203 eV, from part (c)). Therefore we get the following.

For $E = 2.61176 \text{ eV}$: $\lambda = 475.082 \text{ nm}$ (which was given)

For $E = 2.61176 \text{ eV} + 0.000203 \text{ eV} = 2.611963 \text{ eV}$:

$$\lambda = hc/E = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(2.611963 \text{ eV}) = 475.045 \text{ nm}$$

For $E = 2.61176 \text{ eV} - 0.000203 \text{ eV} = 2.61156 \text{ eV}$:

$$\lambda = hc/E = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(2.61156 \text{ eV}) = 475.119 \text{ nm}$$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

- 41.49. IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l .

SET UP: The energy difference due to the magnetic field is $\Delta E = \mu_B B$ and the energy of a photon is $E = hc/\lambda$.

EXECUTE: For the p state, $m_l = 0$ or ± 1 , and for the s state $m_l = 0$. Between any two adjacent lines, $\Delta E = \mu_B B$. Since the change in the wavelength ($\Delta\lambda$) is very small, the energy change (ΔE) is also very small, so we can use

differentials. $E = hc/\lambda$. $|dE| = \frac{hc}{\lambda^2} d\lambda$ and $\Delta E = \frac{hc\Delta\lambda}{\lambda^2}$. Since $\Delta E = \mu_B B$, we get $\mu_B B = \frac{hc\Delta\lambda}{\lambda^2}$ and $B = \frac{hc\Delta\lambda}{\mu_B \lambda^2}$.

$$B = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})(0.0462 \text{ nm})/(5.788 \times 10^{-5} \text{ eV/T})(575.050 \text{ nm})^2 = 3.00 \text{ T}$$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

- 41.50. (a)** The energy shift from zero field is $\Delta U_0 = m_l \mu_B B$.

$$\text{For } m_l = 2, \Delta U_0 = (2)(5.79 \times 10^{-5} \text{ eV/T})(1.40 \text{ T}) = 1.62 \times 10^{-4} \text{ eV}.$$

$$\text{For } m_l = 1, \Delta U_0 = (1)(5.79 \times 10^{-5} \text{ eV/T})(1.40 \text{ T}) = 8.11 \times 10^{-5} \text{ eV}.$$

(b) $|\Delta\lambda| = \lambda_0 \frac{|\Delta E|}{E_0}$, where $E_0 = (13.6 \text{ eV})((1/4) - (1/9))$, $\lambda_0 = \left(\frac{36}{5}\right)\frac{1}{R} = 6.563 \times 10^{-7} \text{ m}$

and $\Delta E = 1.62 \times 10^{-4} \text{ eV} - 8.11 \times 10^{-5} \text{ eV} = 8.09 \times 10^{-5} \text{ eV}$ from part (a). Then, $|\Delta \lambda| = 2.81 \times 10^{-11} \text{ m} = 0.0281 \text{ nm}$. The wavelength corresponds to a larger energy change, and so the wavelength is smaller.

- 41.51. IDENTIFY:** The ratio according to the Boltzmann distribution is given by Eq.(38.21): $\frac{n_1}{n_0} = e^{-(E_1 - E_0)/kT}$, where 1 is the higher energy state and 0 is the lower energy state.

SET UP: The interaction energy with the magnetic field is $U = -\mu_z B = 2.00232 \left(\frac{e\hbar}{2m} \right) m_s B$ (Example 41.5.). The

energy of the $m_s = +\frac{1}{2}$ level is increased and the energy of the $m_s = -\frac{1}{2}$ level is decreased.

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-(U_{1/2} - U_{-1/2})/kT}$$

EXECUTE: $U_{1/2} - U_{-1/2} = 2.00232 \left(\frac{e\hbar}{2m} \right) B \left(\frac{1}{2} - \left(-\frac{1}{2} \right) \right) = 2.00232 \left(\frac{e\hbar}{2m} \right) B = 2.00232 \mu_B B$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-(2.00232) \mu_B B / kT}$$

(a) $B = 5.00 \times 10^{-5} \text{ T}$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-2.00232 (9.274 \times 10^{-24} \text{ A}\cdot\text{m}^2) (5.00 \times 10^{-5} \text{ T}) / ([1.381 \times 10^{-23} \text{ J/K}] [300 \text{ K}])}$$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24 \times 10^{-7}} = 0.99999978 = 1 - 2.2 \times 10^{-7}$$

(b) $B = 5.00 \times 10^{-5} \text{ T}$, $\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24 \times 10^{-3}} = 0.9978$

(c) $B = 5.00 \times 10^{-5} \text{ T}$, $\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24 \times 10^{-2}} = 0.978$

EVALUATE: For small fields the energy separation between the two spin states is much less than kT for $T = 300 \text{ K}$ and the states are equally populated. For $B = 5.00 \text{ T}$ the energy spacing is large enough for there to be a small excess of atoms in the lower state.

- 41.52.** Using Eq.(41.4), $L = mvr = \sqrt{l(l+1)}\hbar$, and the Bohr radius from Eq.(38.15), we obtain the following value for v :

$$v = \frac{\sqrt{l(l+1)}\hbar}{m(n^2 a_0)} = \frac{\sqrt{2}(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{2\pi(9.11 \times 10^{-31} \text{ kg})(4)(5.29 \times 10^{-11} \text{ m})} = 7.74 \times 10^5 \text{ m/s.}$$

The magnetic field generated by the “moving” proton at the electrons position can be calculated from Eq.(28.1):

$$B = \frac{\mu_0}{4\pi} \frac{q \mid v \sin \phi}{r^2} = (10^{-7} \text{ T}\cdot\text{m/A}) \frac{(1.60 \times 10^{-19} \text{ C})(7.74 \times 10^5 \text{ m/s}) \sin(90^\circ)}{(4)^2 (5.29 \times 10^{-11} \text{ m})^2} = 0.277 \text{ T.}$$

- 41.53. IDENTIFY and SET UP:** m_s can take on 4 different values: $m_s = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$. Each nlm_l state can have 4 electrons, each with one of the four different m_s values. Apply the exclusion principle to determine the electron configurations.

EXECUTE: (a) For a filled $n = 1$ shell, the electron configuration would be $1s^4$; four electrons and $Z = 4$. For a filled $n = 2$ shell, the electron configuration would be $1s^4 2s^4 2p^{12}$; twenty electrons and $Z = 20$.

(b) Sodium has $Z = 11$; 11 electrons. The ground-state electron configuration would be $1s^4 2s^4 2p^3$.

EVALUATE: The chemical properties of each element would be very different.

- 41.54.** (a) $Z^2 (-13.6 \text{ eV}) = (7)^2 (-13.6 \text{ eV}) = -666 \text{ eV}$.

(b) The negative of the result of part (a), 666 eV.

(c) The radius of the ground state orbit is inversely proportional to the nuclear charge, and

$$\frac{a}{Z} = (0.529 \times 10^{-10} \text{ m}) / 7 = 7.56 \times 10^{-12} \text{ m.}$$

(d) $\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_0 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)}$, where E_0 is the energy found in part (b), and $\lambda = 2.49 \text{ nm}$.

- 41.55. (a) IDENTIFY and SET UP:** The energy of the photon equals the transition energy of the atom: $\Delta E = hc/\lambda$. The energies of the states are given by Eq.(41.3).

EXECUTE: $E_n = -\frac{13.60 \text{ eV}}{n^2}$ so $E_2 = -\frac{13.60 \text{ eV}}{4}$ and $E_1 = -\frac{13.60 \text{ eV}}{1}$

$$\Delta E = E_2 - E_1 = 13.60 \text{ eV} \left(-\frac{1}{4} + 1 \right) = \frac{3}{4} (13.60 \text{ eV}) = 10.20 \text{ eV} = (10.20 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV}) = 1.634 \times 10^{-18} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.634 \times 10^{-18} \text{ J}} = 1.22 \times 10^{-7} \text{ m} = 122 \text{ nm}$$

(b) IDENTIFY and SET UP: Calculate the change in ΔE due to the orbital magnetic interaction energy, Eq.(41.17), and relate this to the shift $\Delta\lambda$ in the photon wavelength.

EXECUTE: The shift of a level due to the energy of interaction with the magnetic field in the z -direction is $U = m_l \mu_B B$. The ground state has $m_l = 0$ so is unaffected by the magnetic field. The $n = 2$ initial state has

$$m_l = -1 \text{ so its energy is shifted downward an amount } U = m_l \mu_B B = (-1)(9.274 \times 10^{-24} \text{ A/m}^2)(2.20 \text{ T}) =$$

$$(-2.040 \times 10^{-23} \text{ J})(1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 1.273 \times 10^{-4} \text{ eV}$$

Note that the shift in energy due to the magnetic field is a very small fraction of the 10.2 eV transition energy.

Problem 39.56c shows that in this situation $|\Delta\lambda/\lambda| = |\Delta E/E|$. This gives

$$|\Delta\lambda| = \lambda |\Delta E/E| = 122 \text{ nm} \left(\frac{1.273 \times 10^{-4} \text{ eV}}{10.2 \text{ eV}} \right) = 1.52 \times 10^{-3} \text{ nm} = 1.52 \text{ pm}.$$

EVALUATE: The upper level in the transition is lowered in energy so the transition energy is decreased. A smaller ΔE means a larger λ ; the magnetic field increases the wavelength. The fractional shift in wavelength, $\Delta\lambda/\lambda$ is small, only 1.2×10^{-5} .

- 41.56.** The effective field is that which gives rise to the observed difference in the energy level transition,

$$B = \frac{\Delta E}{\mu_B} = \frac{hc}{\mu_B} \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right) = \frac{2\pi mc}{e} \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right). \text{ Substitution of numerical values gives } B = 3.64 \times 10^{-3} \text{ T, much smaller}$$

than that for sodium.

- 41.57. IDENTIFY:** Estimate the atomic transition energy and use Eq.(38.6) to relate this to the photon wavelength.

(a) SET UP: vanadium, $Z = 23$

minimum wavelength; corresponds to largest transition energy

EXECUTE: The highest occupied shell is the N shell ($n = 4$). The highest energy transition is $N \rightarrow K$, with transition energy $\Delta E = E_N - E_K$. Since the shell energies scale like $1/n^2$ neglect E_N relative to E_K , so

$$\Delta E = E_K = (Z - 1)^2 (13.6 \text{ eV}) = (23 - 1)^2 (13.6 \text{ eV}) = 6.582 \times 10^3 \text{ eV} = 1.055 \times 10^{-15} \text{ J. The energy of the emitted photon equals this transition energy, so the photon's wavelength is given by } \Delta E = hc/\lambda \text{ so } \lambda = hc/\Delta E.$$

$$\lambda = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.055 \times 10^{-15} \text{ J}} = 1.88 \times 10^{-10} \text{ m} = 0.188 \text{ nm}.$$

SET UP: maximum wavelength; corresponds to smallest transition energy, so for the K_α transition

EXECUTE: The frequency of the photon emitted in this transition is given by Moseley's law (Eq.41.29):

$$f = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2 = (2.48 \times 10^{15} \text{ Hz})(23 - 1)^2 = 1.200 \times 10^{18} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{1.200 \times 10^{18} \text{ Hz}} = 2.50 \times 10^{-10} \text{ m} = 0.250 \text{ nm}$$

(b) rhenium, $Z = 45$

Apply the analysis of part (a), just with this different value of Z .

minimum wavelength

$$\Delta E = E_K = (Z - 1)^2 (13.6 \text{ eV}) = (45 - 1)^2 (13.6 \text{ eV}) = 2.633 \times 10^4 \text{ eV} = 4.218 \times 10^{-15} \text{ J.}$$

$$\lambda = hc/\Delta E = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{4.218 \times 10^{-15} \text{ J}} = 4.71 \times 10^{-11} \text{ m} = 0.0471 \text{ nm}.$$

maximum wavelength

$$f = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2 = (2.48 \times 10^{15} \text{ Hz})(45 - 1)^2 = 4.801 \times 10^{18} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{4.801 \times 10^{18} \text{ Hz}} = 6.24 \times 10^{-11} \text{ m} = 0.0624 \text{ nm}$$

EVALUATE: Our calculated wavelengths have values corresponding to x rays. The transition energies increase when Z increases and the photon wavelengths decrease.

41.58. (a) $\Delta E = (2.00232) \frac{e}{2m} B \Delta S_z \approx \frac{e\hbar}{m} B = \frac{hc}{\lambda} \Rightarrow B = \frac{2\pi mc}{\lambda e}.$

(b) $B = \frac{2\pi(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})}{(0.0350 \text{ m})(1.60 \times 10^{-19} \text{ C})} = 0.307 \text{ T}.$

- 41.59. (a) To calculate the total number of states for the n^{th} principal quantum number shell we must multiply all the possibilities. The spin states multiply everything by 2. The maximum l value is $(n-1)$, and each l value has $(2l+1)m_l$ values. So the total number of states is

$$N = 2 \sum_{l=0}^{n-1} (2l+1) = 2 \sum_{l=0}^{n-1} 1 + 4 \sum_{l=0}^{n-1} l = 2n + \frac{4(n-1)(n)}{2} = 2n + 2n^2 - 2n = 2n^2.$$

(b) The $n = 5$ shell (O -shell) has 50 states.

- 41.60. IDENTIFY: We treat the Earth as an electron.

SET UP: The intrinsic spin angular momentum of an electron is $S = \sqrt{\frac{3}{4}} \hbar$, and the angular momentum of the spinning Earth is $S = I\omega$, where $I = \frac{2}{5} mR^2$.

EXECUTE: (a) Using $S = I\omega = \sqrt{\frac{3}{4}} \hbar$ and solving for ω gives

$$\omega = \frac{\sqrt{\frac{3}{4}} \hbar}{\frac{2}{5} mR^2} = \frac{\sqrt{\frac{3}{4}} (1.055 \times 10^{-34} \text{ J}\cdot\text{s})}{\frac{2}{5} (5.97 \times 10^{24} \text{ kg})(6.38 \times 10^6 \text{ m})^2} = 9.40 \times 10^{-73} \text{ rad/s}$$

(b) We could not use this approach on the electron because in quantum physics we do not view it in the classical sense as a spinning ball.

EVALUATE: The angular velocity we have just calculated for the Earth would certainly be masked by its present angular spin of one revolution per day.

- 41.61. The potential $U(x) = \frac{1}{2} k' x^2$ is that of a simple harmonic oscillator. Treated quantum mechanically (see Section 40.4) each energy state has energy $E_n = \hbar\omega (n + \frac{1}{2})$. Since electrons obey the exclusion principle, this allows us to put *two* electrons (one for each $m_s = \pm \frac{1}{2}$) for every value of n —each quantum state is then defined by the ordered pair of quantum numbers (n, m_s) . By placing two electrons in each energy level the lowest energy is then

$$2 \left(\sum_{n=0}^{N-1} E_n \right) = 2 \left(\sum_{n=0}^{N-1} \hbar\omega \left(n + \frac{1}{2} \right) \right) = 2\hbar\omega \left[\sum_{n=0}^{N-1} n + \sum_{n=0}^{N-1} \frac{1}{2} \right] = 2\hbar\omega \left[\frac{(N-1)(N)}{2} + \frac{N}{2} \right] =$$

$$\hbar\omega [N^2 - N + N] = \hbar\omega N^2 = \hbar N^2 \sqrt{\frac{k'}{m}}.$$

Here we used the hint from Problem 41.59 to do the first sum, realizing that the first value of n is zero and the last value of n is $N-1$, giving us a total of N energy levels filled.

- 41.62. (a) Apply Coulomb's law to the orbiting electron and set it equal to the centripetal force. There is an attractive force with charge $+2e$ a distance r away and a repulsive force a distance $2r$ away. So, $\frac{(+2e)(-e)}{4\pi\epsilon_0 r^2} + \frac{(-e)(-e)}{4\pi\epsilon_0 (2r)^2} = \frac{-mv^2}{r}$. But, from the quantization of angular momentum in the first Bohr orbit, $L = mvr = \hbar \Rightarrow v = \frac{\hbar}{mr}$.

$$\text{So } \frac{-2e^2}{4\pi\epsilon_0 r^2} + \frac{e^2}{4\pi\epsilon_0 (4r)^2} = \frac{-mv^2}{r} = \frac{-m \left(\frac{\hbar}{mr} \right)^2}{r} = \frac{-\hbar^2}{mr^3} \Rightarrow \frac{-7e^2}{4r^2} = \frac{-4\pi\epsilon_0 \hbar^2}{mr^3}.$$

$$r = \frac{4 \left(\frac{4\pi\epsilon_0 \hbar^2}{me^2} \right)}{7} = \frac{4}{7} a_0 = \frac{4}{7} (0.529 \times 10^{-10} \text{ m}) = 3.02 \times 10^{-11} \text{ m}.$$

$$\text{And } v = \frac{\hbar}{mr} = \frac{\hbar}{4ma_0} = \frac{1.054 \times 10^{-34} \text{ J}\cdot\text{s}}{4(9.11 \times 10^{-31} \text{ kg})(0.529 \times 10^{-10} \text{ m})} = 3.83 \times 10^6 \text{ m/s}.$$

(b) $K = 2 \left(\frac{1}{2} mv^2 \right) = 9.11 \times 10^{-31} \text{ kg} (3.83 \times 10^6 \text{ m/s})^2 = 1.34 \times 10^{-17} \text{ J} = 83.5 \text{ eV}.$

$$(c) U = 2 \left(\frac{-2e^2}{4\pi\epsilon_0 r} \right) + \frac{e^2}{4\pi\epsilon_0(2r)} = \frac{-4e^2}{4\pi\epsilon_0 r} + \frac{e^2}{4\pi\epsilon_0(2r)} = \frac{-7}{2} \left(\frac{e^2}{4\pi\epsilon_0 r} \right) = -2.67 \times 10^{-17} \text{ J} = -166.9 \text{ eV}$$

(d) $E_\infty = -[-166.9 \text{ eV} + 83.5 \text{ eV}] = 83.4 \text{ eV}$, which is only off by about 5% from the real value of 79.0 eV.

41.63. (a) The radius is inversely proportional to Z , so the classical turning radius is $2a/Z$.

(b) The normalized wave function is $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3/Z^3}} e^{-Zr/a}$ and the probability of the electron being found

outside the classical turning point is $P = \int_{2a/Z}^{\infty} |\psi_{1s}|^2 4\pi r^2 dr = \frac{4}{a^3/Z^3} \int_{2a/Z}^{\infty} e^{-2Zr/a} r^2 dr$. Making the change of variable

$u = Zr/a$, $dr = (a/Z)du$ changes the integral to $P = 4 \int_2^{\infty} e^{-2u} u^2 du$, which is independent of Z . The probability is that found in Problem 41.39, 0.238, independent of Z .

MOLECULES AND CONDENSED MATTER

42.1. (a) $K = \frac{3}{2}kT \Rightarrow T = \frac{2K}{3k} = \frac{2(7.9 \times 10^{-4} \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{3(1.38 \times 10^{-23} \text{ J/K})} = 6.1 \text{ K}$

(b) $T = \frac{2(4.48 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{3(1.38 \times 10^{-23} \text{ J/K})} = 34,600 \text{ K}.$

(c) The thermal energy associated with room temperature (300 K) is much greater than the bond energy of He_2 (calculated in part (a)), so the typical collision at room temperature will be more than enough to break up He_2 . However, the thermal energy at 300 K is much less than the bond energy of H_2 , so we would expect it to remain intact at room temperature.

42.2. (a) $U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -5.0 \text{ eV}.$

(b) $-5.0 \text{ eV} + (4.3 \text{ eV} - 3.5 \text{ eV}) = -4.2 \text{ eV}.$

42.3. IDENTIFY: The energy given to the photon comes from a transition between rotational states.

SET UP: The rotational energy of a molecule is $E = l(l+1)\frac{\hbar^2}{2I}$ and the energy of the photon is $E = hc/\lambda$.

EXECUTE: Use the energy formula, the energy difference between the $l = 3$ and $l = 1$ rotational levels of the molecule is $\Delta E = \frac{\hbar^2}{2I}[3(3+1) - 1(1+1)] = \frac{5\hbar^2}{I}$. Since $\Delta E = hc/\lambda$, we get $hc/\lambda = 5\hbar^2/I$. Solving for I gives

$$I = \frac{5\hbar\lambda}{2\pi c} = \frac{5(1.055 \times 10^{-34} \text{ J}\cdot\text{s})(1.780 \text{ nm})}{2\pi(3.00 \times 10^8 \text{ m/s})} = 4.981 \times 10^{-52} \text{ kg}\cdot\text{m}^2.$$

Using $I = m_r r_0^2$, we can solve for r_0 : $r_0 = \sqrt{\frac{I(m_N + m_H)}{m_N m_H}} = \sqrt{\frac{(4.981 \times 10^{-52} \text{ kg}\cdot\text{m}^2)(2.33 \times 10^{-26} \text{ kg} + 1.67 \times 10^{-27} \text{ kg})}{(2.33 \times 10^{-26} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}}$

$$r_0 = 5.65 \times 10^{-13} \text{ m}$$

EVALUATE: This separation is much smaller than the diameter of a typical atom and is not very realistic. But we are treating a *hypothetical* NH molecule.

42.4. The energy of the emitted photon is $1.01 \times 10^{-5} \text{ eV}$, and so its frequency and wavelength are

$$f = \frac{E}{h} = \frac{(1.01 \times 10^{-5} \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})} = 2.44 \text{ GHz} \text{ and } \lambda = \frac{c}{f} = \frac{(3.00 \times 10^8 \text{ m/s})}{(2.44 \times 10^9 \text{ Hz})} = 0.123 \text{ m}.$$
 This frequency

corresponds to that given for a microwave oven.

42.5. Let 1 refer to C and 2 to O. $m_1 = 1.993 \times 10^{-26} \text{ kg}$, $m_2 = 2.656 \times 10^{-26} \text{ kg}$, $r_0 = 0.1128 \text{ nm}$.

$$r_1 = \left(\frac{m_2}{m_1 + m_2}\right)r_0 = 0.0644 \text{ nm (carbon)}; \quad r_2 = \left(\frac{m_1}{m_1 + m_2}\right)r_0 = 0.0484 \text{ nm (oxygen)}$$

(b) $I = m_1 r_1^2 + m_2 r_2^2 = 1.45 \times 10^{-46} \text{ kg}\cdot\text{m}^2$; yes, this agrees with Example 42.2.

42.6. Each atom has a mass m and is at a distance $L/2$ from the center, so the moment of inertia is

$$2(m)(L/2)^2 = mL^2/2 = 2.21 \times 10^{-44} \text{ kg}\cdot\text{m}^2.$$

42.7. IDENTIFY and SET UP: Set $K = E_1$ from Example 42.2. Use $K = \frac{1}{2}I\omega^2$ to solve for ω and $v = r\omega$ to solve for v .

EXECUTE: (a) From Example 42.2, $E_1 = 0.479 \text{ meV} = 7.674 \times 10^{-23} \text{ J}$ and $I = 1.449 \times 10^{-46} \text{ kg}\cdot\text{m}^2$

$$K = \frac{1}{2}I\omega^2 \text{ and } K = E \text{ gives } \omega = \sqrt{2E_1/I} = 1.03 \times 10^{12} \text{ rad/s}$$

(b) $v_1 = r_1 \omega_1 = (0.0644 \times 10^{-9} \text{ m})(1.03 \times 10^{12} \text{ rad/s}) = 66.3 \text{ m/s (carbon)}$

$v_2 = r_2 \omega_2 = (0.0484 \times 10^{-9} \text{ m})(1.03 \times 10^{12} \text{ rad/s}) = 49.8 \text{ m/s (oxygen)}$

(c) $T = 2\pi / \omega = 6.10 \times 10^{-12} \text{ s}$

EVALUATE: From the information in Example 42.3 we can calculate the vibrational period to be $T = 2\pi / \omega = 2\pi \sqrt{m_r / k'} = 1.5 \times 10^{-14} \text{ s}$. The rotational motion is over an order of magnitude slower than the vibrational motion.

42.8. $\Delta E = \frac{hc}{\lambda} = \hbar \sqrt{k' / m_r}$, and solving for k' , $k' = \left(\frac{2\pi c}{\lambda} \right)^2 m_r = 205 \text{ N/m}$.

42.9. IDENTIFY and SET UP: The energy of a rotational level with quantum number l is $E_l = l(l+1)\hbar^2 / 2I$ (Eq.(42.3)). $I = m_r r^2$, with the reduced mass m_r given by Eq.(42.4). Calculate I and ΔE and then use $\Delta E = hc / \lambda$ to find λ .

EXECUTE: (a) $m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_{\text{Li}} m_{\text{H}}}{m_{\text{Li}} + m_{\text{H}}} = \frac{(1.17 \times 10^{-26} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{1.17 \times 10^{-26} \text{ kg} + 1.67 \times 10^{-27} \text{ kg}} = 1.461 \times 10^{-27} \text{ kg}$

$I = m_r r^2 = (1.461 \times 10^{-27} \text{ kg})(0.159 \times 10^{-9} \text{ m})^2 = 3.694 \times 10^{-47} \text{ kg} \cdot \text{m}^2$

$l = 3: E = 3(4) \left(\frac{\hbar^2}{2I} \right) = 6 \left(\frac{\hbar^2}{I} \right)$

$l = 4: E = 4(5) \left(\frac{\hbar^2}{2I} \right) = 10 \left(\frac{\hbar^2}{I} \right)$

$\Delta E = E_4 - E_3 = 4 \left(\frac{\hbar^2}{I} \right) = 4 \left(\frac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{3.694 \times 10^{-47} \text{ kg} \cdot \text{m}^2} \right) = 1.20 \times 10^{-21} \text{ J} = 7.49 \times 10^{-3} \text{ eV}$

(b) $\Delta E = hc / \lambda$ so $\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV})(2.998 \times 10^8 \text{ m/s})}{7.49 \times 10^{-3} \text{ eV}} = 166 \mu\text{m}$

EVALUATE: LiH has a smaller reduced mass than CO and λ is somewhat smaller here than the λ calculated for CO in Example 42.2

42.10. IDENTIFY: The vibrational energy of the molecule is related to its force constant and reduced mass, while the rotational energy depends on its moment of inertia, which in turn depends on the reduced mass.

SET UP: The vibrational energy is $E_n = \left(n + \frac{1}{2} \right) \hbar \omega = \left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k'}{m_r}}$ and the rotational energy is $E_l = l(l+1) \frac{\hbar^2}{2I}$.

EXECUTE: For a vibrational transition, we have $\Delta E_v = \hbar \sqrt{\frac{k'}{m_r}}$, so we first need to find m_r . The energy for a

rotational transition is $\Delta E_r = \frac{\hbar^2}{2I} [2(2+1) - 1(1+1)] = \frac{2\hbar^2}{I}$. Solving for I and using the fact that $I = m_r r_0^2$, we have

$m_r r_0^2 = \frac{2\hbar^2}{\Delta E_r}$, which gives

$$m_r = \frac{2\hbar^2}{r_0^2 \Delta E_r} = \frac{2(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2 (6.583 \times 10^{-16} \text{ eV} \cdot \text{s})}{(0.8860 \times 10^{-9} \text{ m})^2 (8.841 \times 10^{-4} \text{ eV})} = 2.0014 \times 10^{-28} \text{ kg}$$

Now look at the vibrational transition to find the force constant.

$$\Delta E_v = \hbar \sqrt{\frac{k'}{m_r}} \Rightarrow k' = \frac{m_r (\Delta E_v)^2}{\hbar^2} = \frac{(2.0014 \times 10^{-28} \text{ kg})(0.2560 \text{ eV})^2}{(6.583 \times 10^{-16} \text{ eV} \cdot \text{s})^2} = 30.27 \text{ N/m}$$

EVALUATE: This would be a rather weak spring in the laboratory.

42.11. (a) $E_l = \frac{l(l+1)\hbar^2}{2I}$, $E_{l-1} = \frac{l(l-1)\hbar^2}{2I} \Rightarrow \Delta E = \frac{\hbar^2}{2I} (l^2 + l - l^2 + l) = \frac{l\hbar^2}{I}$

(b) $f = \frac{\Delta E}{h} = \frac{\Delta E}{2\pi\hbar} = \frac{l\hbar}{2\pi I}$.

42.12. IDENTIFY: Find ΔE for the transition and compute λ from $\Delta E = hc/\lambda$.

SET UP: From Example 42.2, $E_l = l(l+1)\frac{\hbar^2}{2I}$, with $\frac{\hbar^2}{2I} = 0.2395 \times 10^{-3}$ eV. From Example 42.3, $\Delta E = 0.2690$ eV is the spacing between vibrational levels. Thus $E_n = (n + \frac{1}{2})\hbar\omega$, with $\hbar\omega = 0.2690$ eV. By Eq.(42.9),

$$E = E_n + E_l = (n + \frac{1}{2})\hbar\omega + l(l+1)\frac{\hbar^2}{2I}.$$

EXECUTE: (a) $n=0 \rightarrow n=1$ and $l=1 \rightarrow l=2$

$$\text{For } n=0, l=1, E_i = \frac{1}{2}\hbar\omega + 2\left(\frac{\hbar^2}{2I}\right).$$

$$\text{For } n=1, l=2, E_f = \frac{3}{2}\hbar\omega + 6\left(\frac{\hbar^2}{2I}\right).$$

$$\Delta E = E_f - E_i = \hbar\omega + 4\left(\frac{\hbar^2}{2I}\right) = 0.2690 \text{ eV} + 4(0.2395 \times 10^{-3} \text{ eV}) = 0.2700 \text{ eV}$$

$$\frac{hc}{\lambda} = \Delta E \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2700 \text{ eV}} = 4.592 \times 10^{-6} \text{ m} = 4.592 \mu\text{m}$$

(b) $n=0 \rightarrow n=1$ and $l=2 \rightarrow l=1$

$$\text{For } n=0, l=2, E_i = \frac{1}{2}\hbar\omega + 6\left(\frac{\hbar^2}{2I}\right).$$

$$\text{For } n=1, l=1, E_f = \frac{3}{2}\hbar\omega + 2\left(\frac{\hbar^2}{2I}\right).$$

$$\Delta E = E_f - E_i = \hbar\omega - 4\left(\frac{\hbar^2}{2I}\right) = 0.2690 \text{ eV} - 4(0.2395 \times 10^{-3} \text{ eV}) = 0.2680 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2680 \text{ eV}} = 4.627 \times 10^{-6} \text{ m} = 4.627 \mu\text{m}$$

(c) $n=0 \rightarrow n=1$ and $l=3 \rightarrow l=2$

$$\text{For } n=0, l=3, E_i = \frac{1}{2}\hbar\omega + 12\left(\frac{\hbar^2}{2I}\right).$$

$$\text{For } n=1, l=2, E_f = \frac{3}{2}\hbar\omega + 6\left(\frac{\hbar^2}{2I}\right).$$

$$\Delta E = E_f - E_i = \hbar\omega - 6\left(\frac{\hbar^2}{2I}\right) = 0.2690 \text{ eV} - 6(0.2395 \times 10^{-3} \text{ eV}) = 0.2676 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2676 \text{ eV}} = 4.634 \times 10^{-6} \text{ m} = 4.634 \mu\text{m}$$

EVALUATE: All three transitions are for $n=0 \rightarrow n=1$. The spacing between vibrational levels is larger than the spacing between rotational levels, so the difference in λ for the various rotational transitions is small. When the transition is to a larger l , $\Delta E > \hbar\omega$ and when the transition is to a smaller l , $\Delta E < \hbar\omega$.

42.13. (a) IDENTIFY and SET UP: Use $\omega = \sqrt{k'/m_r}$ and $\omega = 2\pi f$ to calculate k' . The atomic masses are used in Eq.(42.4) to calculate m_r .

$$\text{EXECUTE: } f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k'}{m_r}}, \text{ so } k' = m_r (2\pi f)^2$$

$$m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_H m_F}{m_H + m_F} = \frac{(1.67 \times 10^{-27} \text{ kg})(3.15 \times 10^{-26} \text{ kg})}{1.67 \times 10^{-27} \text{ kg} + 3.15 \times 10^{-26} \text{ kg}} = 1.586 \times 10^{-27} \text{ kg}$$

$$k' = m_r (2\pi f)^2 = (1.586 \times 10^{-27} \text{ kg})(2\pi[1.24 \times 10^{14} \text{ Hz}])^2 = 963 \text{ N/m}$$

(b) **IDENTIFY and SET UP:** The energy levels are given by Eq.(42.7). $E_n = (n + \frac{1}{2})\hbar\omega = (n + \frac{1}{2})hf$, since $\hbar\omega = (h/2\pi)\omega$ and $(\omega/2\pi) = f$. The energy spacing between adjacent levels is $\Delta E = E_{n+1} - E_n = (n+1 + \frac{1}{2} - n - \frac{1}{2})hf = hf$, independent of n .

EXECUTE: $\Delta E = hf = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(1.24 \times 10^{14} \text{ Hz}) = 8.22 \times 10^{-20} \text{ J} = 0.513 \text{ eV}$

(c) IDENTIFY and SET UP: The photon energy equals the transition energy so $\Delta E = hc/\lambda$.

EXECUTE: $hf = hc/\lambda$ so $\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{1.24 \times 10^{14} \text{ Hz}} = 2.42 \times 10^{-6} \text{ m} = 2.42 \mu\text{m}$

EVALUATE: This photon is infrared, which is typical for vibrational transitions.

42.14. For an average spacing a , the density is $\rho = m/a^3$, where m is the average of the ionic masses, and so

$$a^3 = \frac{m}{\rho} = \frac{(6.49 \times 10^{-26} \text{ kg} + 1.33 \times 10^{-25} \text{ kg})/2}{(2.75 \times 10^3 \text{ kg/m}^3)} = 3.60 \times 10^{-29} \text{ m}^3,$$

and $a = 3.30 \times 10^{-10} \text{ m} = 0.330 \text{ nm}$.

(b) The larger (higher atomic number) atoms have the larger spacing.

42.15. IDENTIFY and SET UP: Find the volume occupied by each atom. The density is the average mass of Na and Cl divided by this volume.

EXECUTE: Each atom occupies a cube with side length 0.282 nm . Therefore, the volume occupied by each atom is $V = (0.282 \times 10^{-9} \text{ m})^3 = 2.24 \times 10^{-29} \text{ m}^3$. In NaCl there are equal numbers of Na and Cl atoms, so the average mass of the atoms in the crystal is $m = \frac{1}{2}(m_{\text{Na}} + m_{\text{Cl}}) = \frac{1}{2}(3.82 \times 10^{-26} \text{ kg} + 5.89 \times 10^{-26} \text{ kg}) = 4.855 \times 10^{-26} \text{ kg}$

The density then is $\rho = \frac{m}{V} = \frac{4.855 \times 10^{-26} \text{ kg}}{2.24 \times 10^{-29} \text{ m}^3} = 2.17 \times 10^3 \text{ kg/m}^3$.

EVALUATE: The density of water is $1.00 \times 10^3 \text{ kg/m}^3$, so our result is reasonable.

42.16. (a) As a photon, $\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(6.20 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 0.200 \text{ nm}$.

(b) As a matter wave,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(37.6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}} = 0.200 \text{ nm}$$

(c) As a matter wave,

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\sqrt{2(1.67 \times 10^{-27} \text{ kg})(0.0205 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}} = 0.200 \text{ nm}.$$

42.17. IDENTIFY: The energy gap is the energy of the maximum-wavelength photon.

SET UP: The energy difference is equal to the energy of the photon, so $\Delta E = hc/\lambda$.

EXECUTE: **(a)** Using the photon wavelength to find the energy difference gives

$$\Delta E = hc/\lambda = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(1.11 \times 10^{-6} \text{ m}) = 1.12 \text{ eV}$$

(b) A wavelength of $1.11 \mu\text{m} = 1110 \text{ nm}$ is in the infrared, shorter than that of visible light.

EVALUATE: Since visible photons have more than enough energy to excite electrons from the valence to the conduction band, visible light will be absorbed, which makes silicon opaque.

42.18. (a) $\frac{hc}{\Delta E} = 2.27 \times 10^{-7} \text{ m} = 227 \text{ nm}$, in the ultraviolet.

(b) Visible light lacks enough energy to excite the electrons into the conduction band, so visible light passes through the diamond unabsorbed.

(c) Impurities can lower the gap energy making it easier for the material to absorb shorter wavelength visible light. This allows longer wavelength visible light to pass through, giving the diamond color.

42.19. $\Delta E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{9.31 \times 10^{-13} \text{ m}} = 2.14 \times 10^{-13} \text{ J} = 1.34 \times 10^6 \text{ eV}$. So the number of electrons that can be

excited to the conduction band is $n = \frac{1.34 \times 10^6 \text{ eV}}{1.12 \text{ eV}} = 1.20 \times 10^6$ electrons

42.20. $1 = \int |\psi|^2 dV$

$$= A^2 \left(\int_0^L \sin^2 \left(\frac{n_x \pi x}{L} \right) dx \right) \left(\int_0^L \sin^2 \left(\frac{n_y \pi y}{L} \right) dy \right) \left(\int_0^L \sin^2 \left(\frac{n_z \pi z}{L} \right) dz \right) = A^2 \left(\frac{L}{2} \right)^3$$

so $A = (2/L)^{3/2}$ (assuming A to be real positive).

42.21. Density of states:

$$g(E) = \frac{(2m)^{3/2} V}{2\pi^2 \hbar^3} E^{1/2} = \frac{(2(9.11 \times 10^{-31} \text{ kg}))^{3/2} (1.0 \times 10^{-6} \text{ m}^3) (5.0 \text{ eV})^{1/2} (1.60 \times 10^{-19} \text{ J/eV})^{1/2}}{2\pi^2 (1.054 \times 10^{-34} \text{ J} \cdot \text{s})^3}$$

$$g(E) = (9.5 \times 10^{40} \text{ states/J}) (1.60 \times 10^{-19} \text{ J/eV}) = 1.5 \times 10^{22} \text{ states/eV}.$$

42.22. $v_{\text{rms}} = \sqrt{3kT/m} = 1.17 \times 10^5 \text{ m/s}$, as found in Example 42.9. The equipartition theorem does not hold for the electrons at the Fermi energy. Although these electrons are very energetic, they cannot lose energy, unlike electrons in a free electron gas.

42.23. (a) **IDENTIFY and SET UP:** The three-dimensional Schrödinger equation is $-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + U\psi = E\psi$

(Eq.40.29). For free electrons, $U = 0$. Evaluate $\partial^2 \psi / \partial x^2$, $\partial^2 \psi / \partial y^2$, and $\partial^2 \psi / \partial z^2$ for ψ as given by Eq.(42.10). Put the results into Eq.(40.20) and see if the equation is satisfied.

EXECUTE: $\frac{\partial \psi}{\partial x} = \frac{n_x \pi}{L} A \cos\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right)$

$$\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{n_x \pi}{L}\right)^2 A \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right) = -\left(\frac{n_x \pi}{L}\right)^2 \psi$$

Similarly $\frac{\partial^2 \psi}{\partial y^2} = -\left(\frac{n_y \pi}{L}\right)^2 \psi$ and $\frac{\partial^2 \psi}{\partial z^2} = -\left(\frac{n_z \pi}{L}\right)^2 \psi$.

Therefore, $-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = \frac{\hbar^2}{2m} \left(\frac{\pi^2}{L^2} \right) (n_x^2 + n_y^2 + n_z^2) \psi = \frac{(n_x^2 + n_y^2 + n_z^2) \pi^2 \hbar^2}{2mL^2} \psi$

This equals $E\psi$, with $E = \frac{(n_x^2 + n_y^2 + n_z^2) \pi^2 \hbar^2}{2mL^2}$, which is Eq.(42.11).

EVALUATE: ψ given by Eq.(42.10) is a solution to Eq.(40.29), with E as given by Eq.(42.11).

(b) **IDENTIFY and SET UP:** Find the set of quantum numbers n_x , n_y , and n_z that give the lowest three values of E . The degeneracy is the number of sets n_x , n_y , n_z and m_s that give the same E .

EXECUTE: Ground level: lowest E so $n_x = n_y = n_z = 1$ and $E = \frac{3\pi^2 \hbar^2}{2mL^2}$. No other combination of n_x , n_y , and n_z gives this same E , so the only degeneracy is the degeneracy of two due to spin.

First excited level: next lower E so one n equals 2 and the others equal 1. $E = (2^2 + 1^2 + 1^2) \frac{\pi^2 \hbar^2}{2mL^2} = \frac{6\pi^2 \hbar^2}{2mL^2}$

There are three different sets of n_x , n_y , n_z values that give this E :

$$n_x = 2, n_y = 1, n_z = 1; n_x = 1, n_y = 2, n_z = 1; n_x = 1, n_y = 1, n_z = 2$$

This gives a degeneracy of 3 so the total degeneracy, with the factor of 2 from spin, is 6.

Second excited level: next lower E so two of n_x , n_y , n_z equal 2 and the other equals 1.

$$E = (2^2 + 2^2 + 1^2) \frac{\pi^2 \hbar^2}{2mL^2} = \frac{9\pi^2 \hbar^2}{2mL^2}$$

There are different sets of n_x , n_y , n_z values that give this E :

$$n_x = 2, n_y = 2, n_z = 1; n_x = 2, n_y = 1, n_z = 2; n_x = 1, n_y = 2, n_z = 2.$$

Thus, as for the first excited level, the total degeneracy, including spin, is 6.

EVALUATE: The wavefunction for the 3-dimensional box is a product of the wavefunctions for a 1-dimensional box in the x , y , and z coordinates and the energy is the sum of energies for three 1-dimensional boxes. All levels except for the ground level have a degeneracy greater than two. Compare to the 3-dimensional isotropic harmonic oscillator treated in Problem 40.53.

42.24. Eq.(42.13) may be solved for $n_{\text{fs}} = (2mE)^{1/2} (L/\hbar\pi)$, and substituting this into Eq. (42.12), using $L^3 = V$, gives Eq.(42.14).

42.25. (a) **IDENTIFY and SET UP:** The electron contribution to the molar heat capacity at constant volume of a metal is

$$C_V = \left(\frac{\pi^2 K T}{2E_F} \right) R.$$

EXECUTE: $C_V = \frac{\pi^2 (1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}{2(5.48 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})} R = 0.0233R.$

(b) EVALUATE: The electron contribution found in part (a) is $0.0233R = 0.194 \text{ J/mol} \cdot \text{K}$. This is $0.194/25.3 = 7.67 \times 10^{-3} = 0.767\%$ of the total C_V .

(c) Only a small fraction of C_V is due to the electrons. Most of C_V is due to the vibrational motion of the ions.

42.26. (a) From Eq. (42.22), $E_{\text{av}} = \frac{3}{5}E_F = 1.94 \text{ eV}$.

$$\text{(b) } \sqrt{2E/m} = \sqrt{\frac{2(1.94 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}} = 8.25 \times 10^5 \text{ m/s}.$$

$$\text{(c) } \frac{E_F}{k} = \frac{(3.23 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{(1.38 \times 10^{-23} \text{ J/K})} = 3.74 \times 10^4 \text{ K}.$$

42.27. IDENTIFY: The probability is given by the Fermi-Dirac distribution.

SET UP: The Fermi-Dirac distribution is $f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$.

EXECUTE: We calculate the value of $f(E)$, where $E = 8.520 \text{ eV}$, $E_F = 8.500 \text{ eV}$, $k = 1.38 \times 10^{-23} \text{ J/K} = 8.625 \times 10^{-5} \text{ eV/K}$, and $T = 20^\circ\text{C} = 293 \text{ K}$. The result is $f(E) = 0.312 = 31.2\%$.

EVALUATE: Since the energy is close to the Fermi energy, the probability is quite high that the state is occupied by an electron.

42.28. (a) See Example 42.10: The probabilities are 1.78×10^{-7} , 2.37×10^{-6} , and 1.51×10^{-5} .

(b) The Fermi distribution, Eq.(42.17), has the property that $f(E_F - E) = 1 - f(E)$ (see Problem (42.48)), and so the probability that a state at the top of the valence band is occupied is the same as the probability that a state of the bottom of the conduction band is filled (this result depends on having the Fermi energy in the middle of the gap).

42.29. IDENTIFY: Use Eq.(42.17), $f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$. Solve for $E - E_F$.

$$\text{SET UP: } e^{(E-E_F)/kT} = \frac{1}{f(E)} - 1$$

The problem states that $f(E) = 4.4 \times 10^{-4}$ for E at the bottom of the conduction band.

$$\text{EXECUTE: } e^{(E-E_F)/kT} = \frac{1}{4.4 \times 10^{-4}} - 1 = 2.272 \times 10^3.$$

$$E - E_F = kT \ln(2.272 \times 10^3) = (1.3807 \times 10^{-23} \text{ J/K})(300 \text{ K}) \ln(2.272 \times 10^3) = 3.201 \times 10^{-20} \text{ J} = 0.20 \text{ eV}$$

$E_F = E - 0.20 \text{ eV}$; the Fermi level is 0.20 eV below the bottom of the conduction band.

EVALUATE: The energy gap between the Fermi level and bottom of the conduction band is large compared to kT at $T = 300 \text{ K}$ and as a result $f(E)$ is small.

42.30. IDENTIFY: The current depends on the voltage across the diode and its temperature, so the resistance also depends on these quantities.

SET UP: The current is $I = I_s(e^{eV/kT} - 1)$ and the resistance is $R = V/I$.

$$\text{EXECUTE: (a) The resistance is } R = \frac{V}{I} = \frac{V}{I_s(e^{eV/kT} - 1)}. \text{ The exponent is } \frac{eV}{kT} = \frac{e(0.0850 \text{ V})}{(8.625 \times 10^{-5} \text{ eV/K})(293 \text{ K})} =$$

$$3.3635, \text{ giving } R = \frac{85.0 \text{ mV}}{(0.750 \text{ mA})(e^{3.3635} - 1)} = 4.06 \Omega.$$

$$\text{(b) In this case, the exponent is } \frac{eV}{kT} = \frac{e(-0.050 \text{ V})}{(8.625 \times 10^{-5} \text{ eV/K})(293 \text{ K})} = -1.979$$

$$\text{which gives } R = \frac{-50.0 \text{ mV}}{(0.750 \text{ mA})(e^{-1.979} - 1)} = 77.4 \Omega$$

EVALUATE: Reversing the voltage can make a considerable change in the resistance of a diode.

42.31. IDENTIFY and SET UP: The voltage-current relation is given by Eq.(42.23): $I = I_s(e^{eV/kT} - 1)$. Use the current for $V = +15.0 \text{ mV}$ to solve for the constant I_s .

EXECUTE: (a) Find I_s : $V = +15.0 \times 10^{-3} \text{ V}$ gives $I = 9.25 \times 10^{-3} \text{ A}$

$$\frac{eV}{kT} = \frac{(1.602 \times 10^{-19} \text{ C})(15.0 \times 10^{-3} \text{ V})}{(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 0.5800$$

$$I_s = \frac{I}{e^{eV/kT} - 1} = \frac{9.25 \times 10^{-3} \text{ A}}{e^{0.5800} - 1} = 1.177 \times 10^{-2} = 11.77 \text{ mA}$$

Then can calculate I for $V = 10.0$ mV: $\frac{eV}{kT} = \frac{(1.602 \times 10^{-19} \text{ C})(10.0 \times 10^{-3} \text{ V})}{(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 0.3867$

$$I = I_s (e^{eV/kT} - 1) = (11.77 \text{ mA})(e^{0.3867} - 1) = 5.56 \text{ mA}$$

(b) $\frac{eV}{kT}$ has the same magnitude as in part (a) but not V is negative so $\frac{eV}{kT}$ is negative.

$$V = -15.0 \text{ mV}: \frac{eV}{kT} = -0.5800 \text{ and } I = I_s (e^{eV/kT} - 1) = (11.77 \text{ mA})(e^{-0.5800} - 1) = -5.18 \text{ mA}$$

$$V = -10.0 \text{ mV}: \frac{eV}{kT} = -0.3867 \text{ and } I = I_s (e^{eV/kT} - 1) = (11.77 \text{ mA})(e^{-0.3867} - 1) = -3.77 \text{ mA}$$

EVALUATE: There is a directional asymmetry in the current, with a forward-bias voltage producing more current than a reverse-bias voltage of the same magnitude, but the voltage is small enough for the asymmetry not be pronounced. Compare to Example 42.11, where more extreme voltages are considered.

42.32. (a) Solving Eq.(42.23) for the voltage as a function of current,

$$V = \frac{kT}{e} \ln \left(\frac{I}{I_s} + 1 \right) = \frac{kT}{e} \ln \left(\frac{40.0 \text{ mA}}{3.60 \text{ mA}} + 1 \right) = 0.0645 \text{ V}.$$

(b) From part (a), the quantity $e^{eV/kT} = 12.11$, so far a reverse-bias voltage of the same magnitude,

$$I = I_s (e^{-eV/kT} - 1) = I_s \left(\frac{1}{12.11} - 1 \right) = -3.30 \text{ mA}.$$

42.33. IDENTIFY: During the transition, the molecule emits a photon of light having energy equal to the energy difference between the two vibrational states of the molecule.

SET UP: The vibrational energy is $E_n = \left(n + \frac{1}{2} \right) \hbar \omega = \left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k'}{m_t}}$.

EXECUTE: (a) The energy difference between two adjacent energy states is $\Delta E = \hbar \sqrt{\frac{k'}{m_t}}$, and this is the energy of

the photon, so $\Delta E = hc/\lambda$. Equating these two expressions for ΔE and solving for k' , we have $k' = m_t \left(\frac{\Delta E}{\hbar} \right)^2 =$

$\frac{m_H m_O}{m_H + m_O} \left(\frac{\Delta E}{\hbar} \right)^2$, and using $\frac{\Delta E}{\hbar} = \frac{hc/\lambda}{\hbar} = \frac{2\pi c}{\lambda}$ with the appropriate numbers gives us

$$k' = \frac{(1.67 \times 10^{-27} \text{ kg})(2.656 \times 10^{-26} \text{ kg})}{1.67 \times 10^{-27} \text{ kg} + 2.656 \times 10^{-26} \text{ kg}} \left[\frac{2\pi(3.00 \times 10^8 \text{ m/s})}{2.39 \times 10^{-6} \text{ m}} \right]^2 = 977 \text{ N/m}$$

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k'}{m_t}} = \frac{1}{2\pi} \sqrt{\frac{m_H m_O}{m_H + m_O} \frac{k'}{m_t}}$. Substituting the appropriate numbers gives us

$$f = \frac{1}{2\pi} \sqrt{\frac{(1.67 \times 10^{-27} \text{ kg})(2.656 \times 10^{-26} \text{ kg})}{1.67 \times 10^{-27} \text{ kg} + 2.656 \times 10^{-26} \text{ kg}} \frac{977 \text{ N/m}}{m_t}} = 1.25 \times 10^{14} \text{ Hz}$$

EVALUATE: The frequency is close to, but not quite in, the visible range.

42.34. $I = \frac{2\hbar^2}{\Delta E} = \frac{h\lambda}{2\pi^2 c} = 7.14 \times 10^{-48} \text{ kg} \cdot \text{m}^2$.

42.35. IDENTIFY and SET UP: Eq.(21.14) gives the electric dipole moment as $p = qd$, where the dipole consists of charges $\pm q$ separated by distance d .

EXECUTE: (a) Point charges $+e$ and $-e$ separated by distance d , so

$$p = ed = (1.602 \times 10^{-19} \text{ C})(0.24 \times 10^{-9} \text{ m}) = 3.8 \times 10^{-29} \text{ C} \cdot \text{m}$$

(b) $p = qd$ so $q = \frac{p}{d} = \frac{3.0 \times 10^{-29} \text{ C} \cdot \text{m}}{0.24 \times 10^{-9} \text{ m}} = 1.3 \times 10^{-19} \text{ C}$

(c) $\frac{q}{e} = \frac{1.3 \times 10^{-19} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 0.81$

$$(d) \quad q = \frac{p}{d} = \frac{1.5 \times 10^{-30} \text{ C} \cdot \text{m}}{0.16 \times 10^{-9} \text{ m}} = 9.37 \times 10^{-21} \text{ C}$$

$$\frac{q}{e} = \frac{9.37 \times 10^{-21} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 0.058$$

EVALUATE: The fractional ionic character for the bond in HI is much less than the fractional ionic character for the bond in NaCl. The bond in HI is mostly covalent and not very ionic.

42.36. The electrical potential energy is $U = -5.13 \text{ eV}$, and $r = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{U} = 2.8 \times 10^{-10} \text{ m}$.

42.37. (a) IDENTIFY: $E(\text{Na}) + E(\text{Cl}) = E(\text{Na}^+) + E(\text{Cl}^-) + U(r)$. Solving for $U(r)$ gives

$$U(r) = -[E(\text{Na}^+) - E(\text{Na})] + [E(\text{Cl}) - E(\text{Cl}^-)].$$

SET UP: $[E(\text{Na}^+) - E(\text{Na})]$ is the ionization energy of Na, the energy required to remove one electron, and is equal to 5.1 eV. $[E(\text{Cl}) - E(\text{Cl}^-)]$ is the electron affinity of Cl, the magnitude of the decrease in energy when an electron is attached to a neutral Cl atom, and is equal to 3.6 eV.

EXECUTE: $U = -5.1 \text{ eV} + 3.6 \text{ eV} = -1.5 \text{ eV} = -2.4 \times 10^{-19} \text{ J}$, and $-\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -2.4 \times 10^{-19} \text{ J}$

$$r = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{e^2}{2.4 \times 10^{-19} \text{ J}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.602 \times 10^{-19} \text{ C})^2}{2.4 \times 10^{-19} \text{ J}}$$

$$r = 9.6 \times 10^{-10} \text{ m} = 0.96 \text{ nm}$$

(b) ionization energy of K = 4.3 eV; electron affinity of Br = 3.5 eV

Thus $U = -4.3 \text{ eV} + 3.5 \text{ eV} = -0.8 \text{ eV} = -1.28 \times 10^{-19} \text{ J}$, and $-\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -1.28 \times 10^{-19} \text{ J}$

$$r = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{e^2}{1.28 \times 10^{-19} \text{ J}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.602 \times 10^{-19} \text{ C})^2}{1.28 \times 10^{-19} \text{ J}}$$

$$r = 1.8 \times 10^{-9} \text{ m} = 1.8 \text{ nm}$$

EVALUATE: K has a smaller ionization energy than Na and the electron affinities of Cl and Br are very similar, so it takes less energy to make $\text{K}^+ + \text{Br}^-$ from $\text{K} + \text{Br}$ than to make $\text{Na}^+ + \text{Cl}^-$ from $\text{Na} + \text{Cl}$. Thus, the stabilization distance is larger for KBr than for NaCl.

42.38. The energies corresponding to the observed wavelengths are $3.29 \times 10^{-21} \text{ J}$, $2.87 \times 10^{-21} \text{ J}$, $2.47 \times 10^{-21} \text{ J}$, $2.06 \times 10^{-21} \text{ J}$ and $1.65 \times 10^{-21} \text{ J}$. The average spacing of these energies is $0.410 \times 10^{-21} \text{ J}$ and these are seen to

correspond to transition from levels 8, 7, 6, 5 and 4 to the respective next lower levels. Then, $\frac{\hbar^2}{I} = 0.410 \times 10^{-21} \text{ J}$,

from which $I = 2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2$.

42.39. (a) IDENTIFY: The rotational energies of a molecule depend on its moment of inertia, which in turn depends on the separation between the atoms in the molecule.

SET UP: Problem 42.38 gives $I = 2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2$. $I = m_r r^2$. Calculate m_r and solve for r .

EXECUTE: $m_r = \frac{m_{\text{H}} m_{\text{Cl}}}{m_{\text{H}} + m_{\text{Cl}}} = \frac{(1.67 \times 10^{-27} \text{ kg})(5.81 \times 10^{-26} \text{ kg})}{1.67 \times 10^{-27} \text{ kg} + 5.81 \times 10^{-26} \text{ kg}} = 1.623 \times 10^{-27} \text{ kg}$

$$r = \sqrt{\frac{I}{m_r}} = \sqrt{\frac{2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2}{1.623 \times 10^{-27} \text{ kg}}} = 1.29 \times 10^{-10} \text{ m} = 0.129 \text{ nm}$$

EVALUATE: This is a typical atomic separation for a diatomic molecule; see Example 42.2 for the corresponding distance for CO.

(b) IDENTIFY: Each transition is from the level l to the level $l-1$. The rotational energies are given by Eq.(42.3). The transition energy is related to the photon wavelength by $\Delta E = hc/\lambda$.

SET UP: $E_l = l(l+1)\hbar^2/2I$, so $\Delta E = E_l - E_{l-1} = [l(l+1) - l(l-1)]\left(\frac{\hbar^2}{2I}\right) = l\left(\frac{\hbar^2}{I}\right)$.

EXECUTE: $l\left(\frac{\hbar^2}{I}\right) = \frac{hc}{\lambda}$

$$l = \frac{2\pi c I}{\hbar \lambda} = \frac{2\pi(2.998 \times 10^8 \text{ m/s})(2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2)}{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})\lambda} = \frac{4.843 \times 10^{-4} \text{ m}}{\lambda}$$

$$\text{For } \lambda = 60.4 \mu\text{m}, l = \frac{4.843 \times 10^{-4} \text{ m}}{60.4 \times 10^{-6} \text{ m}} = 8.$$

$$\text{For } \lambda = 69.0 \mu\text{m}, l = \frac{4.843 \times 10^{-4} \text{ m}}{69.0 \times 10^{-6} \text{ m}} = 7.$$

$$\text{For } \lambda = 80.4 \mu\text{m}, l = \frac{4.843 \times 10^{-4} \text{ m}}{80.4 \times 10^{-6} \text{ m}} = 6.$$

$$\text{For } \lambda = 96.4 \mu\text{m}, l = \frac{4.843 \times 10^{-4} \text{ m}}{96.4 \times 10^{-6} \text{ m}} = 5.$$

$$\text{For } \lambda = 120.4 \mu\text{m}, l = \frac{4.843 \times 10^{-4} \text{ m}}{120.4 \times 10^{-6} \text{ m}} = 4.$$

EVALUATE: In each case l is an integer, as it must be.

(c) IDENTIFY and SET UP: Longest λ implies smallest ΔE , and this is for the transition from $l=1$ to $l=0$.

$$\text{EXECUTE: } \Delta E = l \left(\frac{\hbar^2}{I} \right) = (1) \frac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2} = 4.099 \times 10^{-22} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{4.099 \times 10^{-22} \text{ J}} = 4.85 \times 10^{-4} \text{ m} = 485 \mu\text{m}.$$

EVALUATE: This is longer than any wavelengths in part (b).

(d) IDENTIFY: What changes is m_r , the reduced mass of the molecule.

SET UP: The transition energy is $\Delta E = l \left(\frac{\hbar^2}{I} \right)$ and $\Delta E = \frac{hc}{\lambda}$, so $\lambda = \frac{2\pi cl}{I\hbar}$ (part (b)). $I = m_r r^2$, so λ is directly

$$\text{proportional to } m_r. \quad \frac{\lambda(\text{HCl})}{m_r(\text{HCl})} = \frac{\lambda(\text{DCl})}{m_r(\text{DCl})} \quad \text{so} \quad \lambda(\text{DCl}) = \lambda(\text{HCl}) \frac{m_r(\text{DCl})}{m_r(\text{HCl})}$$

EXECUTE: The mass of a deuterium atom is approximately twice the mass of a hydrogen atom, so $m_D = 3.34 \times 10^{-27} \text{ kg}$.

$$m_r(\text{DCl}) = \frac{m_D m_{\text{Cl}}}{m_D + m_{\text{Cl}}} = \frac{(3.34 \times 10^{-27} \text{ kg})(5.81 \times 10^{-27} \text{ kg})}{3.34 \times 10^{-27} \text{ kg} + 5.81 \times 10^{-26} \text{ kg}} = 3.158 \times 10^{-27} \text{ kg}$$

$$\lambda(\text{DCl}) = \lambda(\text{HCl}) \left(\frac{3.158 \times 10^{-27} \text{ kg}}{1.623 \times 10^{-27} \text{ kg}} \right) = (1.946) \lambda(\text{HCl})$$

$$l=8 \rightarrow l=7; \lambda = (60.4 \mu\text{m})(1.946) = 118 \mu\text{m}$$

$$l=7 \rightarrow l=6; \lambda = (69.0 \mu\text{m})(1.946) = 134 \mu\text{m}$$

$$l=6 \rightarrow l=5; \lambda = (80.4 \mu\text{m})(1.946) = 156 \mu\text{m}$$

$$l=5 \rightarrow l=4; \lambda = (96.4 \mu\text{m})(1.946) = 188 \mu\text{m}$$

$$l=4 \rightarrow l=3; \lambda = (120.4 \mu\text{m})(1.946) = 234 \mu\text{m}$$

EVALUATE: The moment of inertia increases when H is replaced by D, so the transition energies decrease and the wavelengths increase. The larger the rotational inertia the smaller the rotational energy for a given l (Eq.42.3).

42.40. From the result of Problem 42.11, the moment inertia of the molecule is $I = \frac{\hbar^2 l}{\Delta E} = \frac{hl\lambda}{4\pi^2 c} = 6.43 \times 10^{-46} \text{ kg} \cdot \text{m}^2$ and

$$\text{from Eq.(42.6) the separation is } r_0 = \sqrt{\frac{I}{m_r}} = 0.193 \text{ nm}.$$

42.41. (a) $E_{\text{ex}} = \frac{L^2}{2I} = \frac{\hbar^2 l(l+1)}{2I}$. $E_g = 0$ ($l=0$), and there is an additional multiplicative factor of $2l+1$ because for each l

state there are really $(2l+1)$ m_l -states with the same energy. So $\frac{n_l}{n_0} = (2l+1)e^{-\hbar^2 l(l+1)/(2lRT)}$.

(b) $T = 300 \text{ K}, I = 1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2$.

$$(i) E_{l=1} = \frac{\hbar^2 (1)(1+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)} = 7.67 \times 10^{-23} \text{ J}. \quad \frac{E_{l=1}}{kT} = \frac{7.67 \times 10^{-23} \text{ J}}{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 0.0185.$$

$$(2l+1) = 3, \text{ so } \frac{n_{l=1}}{n_0} = (3)e^{-0.0185} = 2.95.$$

$$(ii) \frac{E_{l=2}}{kT} = \frac{\hbar^2(2)(2+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 0.0556.$$

$$(2l+1) = 5, \text{ so } \frac{n_{l=1}}{n_0} = (5)(e^{-0.0556}) = 4.73.$$

$$(iii) \frac{E_{l=10}}{kT} = \frac{\hbar^2(10)(10+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 1.02.$$

$$(2l+1) = 21, \text{ so } \frac{n_{l=10}}{n_0} = (21)(e^{-1.02}) = 7.57.$$

$$(iv) \frac{E_{l=20}}{kT} = \frac{\hbar^2(20)(20+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 3.89.$$

$$(2l+1) = 41, \text{ so } \frac{n_{l=20}}{n_0} = (41)e^{-3.89} = 0.838.$$

$$(v) \frac{E_{l=50}}{kT} = \frac{\hbar^2(50)(50+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 23.6.$$

$$(2l+1) = 101, \text{ so } \frac{n_{l=50}}{n_0} = (101)e^{-23.6} = 5.69 \times 10^{-9}.$$

(c) There is a competing effect between the $(2l+1)$ term and the decaying exponential. The $2l+1$ term dominates for small l , while the exponential term dominates for large l .

42.42. (a) $I_{\text{CO}} = 1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2.$

$$E_{l=1} = \frac{\hbar^2 l(l+1)}{2I} = \frac{(1.054 \times 10^{-34} \text{ J} \cdot \text{s})^2 (1)(1+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)} = 7.67 \times 10^{-23} \text{ J}. \quad E_{l=0} = 0.$$

$$\Delta E = 7.67 \times 10^{-23} \text{ J} = 4.79 \times 10^{-4} \text{ eV}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(7.67 \times 10^{-23} \text{ J})} = 2.59 \times 10^{-3} \text{ m} = 2.59 \text{ mm}.$$

(b) Let's compare the value of kT when $T = 20 \text{ K}$ to that of ΔE for the $l=1 \rightarrow l=0$ rotational transition:

$$kT = (1.38 \times 10^{-23} \text{ J/K})(20 \text{ K}) = 2.76 \times 10^{-22} \text{ J}.$$

$$\Delta E = 7.67 \times 10^{-23} \text{ J (from part (a)). So } \frac{kT}{\Delta E} = 3.60.$$

Therefore, although T is quite small, there is still plenty of energy to excite CO molecules into the first rotational level. This allows astronomers to detect the 2.59 mm wavelength radiation from such molecular clouds.

42.43. IDENTIFY and SET UP: $E_l = l(l+1)\hbar^2/2I$, so E_l and the transition energy ΔE depend on l . Different isotopic molecules have different I .

EXECUTE: (a) Calculate I for Na^{35}Cl : $m_r = \frac{m_{\text{Na}} m_{\text{Cl}}}{m_{\text{Na}} + m_{\text{Cl}}} = \frac{(3.8176 \times 10^{-26} \text{ kg})(5.8068 \times 10^{-26} \text{ kg})}{3.8176 \times 10^{-26} \text{ kg} + 5.8068 \times 10^{-26} \text{ kg}} = 2.303 \times 10^{-26} \text{ kg}$

$$I = m_r r^2 = (2.303 \times 10^{-26} \text{ kg})(0.2361 \times 10^{-9} \text{ m})^2 = 1.284 \times 10^{-45} \text{ kg} \cdot \text{m}^2$$

$l=2 \rightarrow l=1$ transition

$$\Delta E = E_2 - E_1 = (6-2) \left(\frac{\hbar^2}{2I} \right) = \frac{2\hbar^2}{I} = \frac{2(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{1.284 \times 10^{-45} \text{ kg} \cdot \text{m}^2} = 1.734 \times 10^{-23} \text{ J}$$

$$\Delta E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.734 \times 10^{-23} \text{ J}} = 1.146 \times 10^{-2} \text{ m} = 1.146 \text{ cm}$$

$l=1 \rightarrow l=0$ transition

$$\Delta E = E_1 - E_0 = (2-0) \left(\frac{\hbar^2}{2I} \right) = \frac{\hbar^2}{I} = \frac{1}{2}(1.734 \times 10^{-23} \text{ J}) = 8.67 \times 10^{-24} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{8.67 \times 10^{-24} \text{ J}} = 2.291 \text{ cm}$$

(b) Calculate I for Na^{37}Cl : $m_r = \frac{m_{\text{Na}} m_{\text{Cl}}}{m_{\text{Na}} + m_{\text{Cl}}} = \frac{(3.8176 \times 10^{-26} \text{ kg})(6.1384 \times 10^{-26} \text{ kg})}{3.8176 \times 10^{-26} \text{ kg} + 6.1384 \times 10^{-26} \text{ kg}} = 2.354 \times 10^{-26} \text{ kg}$

$$I = m_r r^2 = (2.354 \times 10^{-26} \text{ kg})(0.2361 \times 10^{-9} \text{ m})^2 = 1.312 \times 10^{-45} \text{ kg} \cdot \text{m}^2$$

$l = 2 \rightarrow l = 1$ transition

$$\Delta E = \frac{2\hbar^2}{I} = \frac{2(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{1.312 \times 10^{-45} \text{ kg} \cdot \text{m}^2} = 1.697 \times 10^{-23} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.697 \times 10^{-23} \text{ J}} = 1.171 \times 10^{-2} \text{ m} = 1.171 \text{ cm}$$

$l = 1 \rightarrow l = 0$ transition

$$\Delta E = \frac{\hbar^2}{I} = \frac{1}{2}(1.697 \times 10^{-23} \text{ J}) = 8.485 \times 10^{-24} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{8.485 \times 10^{-24} \text{ J}} = 2.341 \text{ cm}$$

The differences in the wavelengths for the two isotopes are:

$$l = 2 \rightarrow l = 1 \text{ transition: } 1.171 \text{ cm} - 1.146 \text{ cm} = 0.025 \text{ cm}$$

$$l = 1 \rightarrow l = 0 \text{ transition: } 2.341 \text{ cm} - 2.291 \text{ cm} = 0.050 \text{ cm}$$

EVALUATE: Replacing ^{35}Cl by ^{37}Cl increases I , decreases ΔE and increases λ . The effect on λ is small but measurable.

- 42.44.** The vibration frequency is, from Eq.(42.8), $f = \frac{\Delta E}{h} = 1.12 \times 10^{14} \text{ Hz}$. The force constant is

$$k' = (2\pi f)^2 m_r = 777 \text{ N/m}.$$

- 42.45.** $E_n = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k'}{m_r}} \Rightarrow E_0 = \frac{1}{2} \hbar \sqrt{\frac{2k'}{m_H}}$

$$\Rightarrow E_0 = \frac{1}{2} (1.054 \times 10^{-34} \text{ J} \cdot \text{s}) \sqrt{\frac{2(576 \text{ N/m})}{1.67 \times 10^{-27} \text{ kg}}} = 4.38 \times 10^{-20} \text{ J} = 0.274 \text{ eV}.$$

This is much less than the H_2 bond energy.

- 42.46.** (a) The frequency is proportional to the reciprocal of the square root of the reduced mass, and in terms of the atomic masses, the frequency of the isotope with the deuterium atom is

$$f = f_0 \left(\frac{m_F m_H / (m_H + m_F)}{m_F m_D / (m_D + m_F)} \right)^{1/2} = f_0 \left(\frac{1 + (m_F / m_D)}{1 + (m_F / m_H)} \right)^{1/2}.$$

Using f_0 from Exercise 42.13 and the given masses, $f = 8.99 \times 10^{13} \text{ Hz}$.

- 42.47.** **IDENTIFY and SET UP:** Use Eq.(42.6) to calculate I . The energy levels are given by Eq.(42.9). The transition energy ΔE is related to the photon wavelength by $\Delta E = hc / \lambda$.

EXECUTE: (a) $m_r = \frac{m_H m_I}{m_H + m_I} = \frac{(1.67 \times 10^{-27} \text{ kg})(2.11 \times 10^{-25} \text{ kg})}{1.67 \times 10^{-27} \text{ kg} + 2.11 \times 10^{-25} \text{ kg}} = 1.657 \times 10^{-27} \text{ kg}$

$$I = m_r r^2 = (1.657 \times 10^{-27} \text{ kg})(0.160 \times 10^{-9} \text{ m})^2 = 4.24 \times 10^{-47} \text{ kg} \cdot \text{m}^2$$

(b) The energy levels are $E_{nl} = l(l+1) \left(\frac{\hbar^2}{2I} \right) + (n + \frac{1}{2}) \hbar \sqrt{\frac{k'}{m_r}}$ (Eq.(42.9))

$$\sqrt{\frac{k'}{m}} = \omega = 2\pi f \text{ so } E_{nl} = l(l+1) \left(\frac{\hbar^2}{2I} \right) + (n + \frac{1}{2}) \hbar f$$

(i) transition $n = 1 \rightarrow n = 0$, $l = 1 \rightarrow l = 0$

$$\Delta E = (2-0) \left(\frac{\hbar^2}{2I} \right) + (1 + \frac{1}{2} - \frac{1}{2}) \hbar f = \frac{\hbar^2}{I} + \hbar f$$

$$\Delta E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{hc}{(\hbar^2 / I) + \hbar f} = \frac{c}{(\hbar / 2\pi I) + f}$$

$$\frac{\hbar}{2\pi I} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(4.24 \times 10^{-47} \text{ kg} \cdot \text{m}^2)} = 3.960 \times 10^{11} \text{ Hz}$$

$$\lambda = \frac{c}{(\hbar / 2\pi I) + f} = \frac{2.998 \times 10^8 \text{ m/s}}{3.960 \times 10^{11} \text{ Hz} + 6.93 \times 10^{13} \text{ Hz}} = 4.30 \text{ } \mu\text{m}$$

(ii) transition $n = 1 \rightarrow n = 0, l = 2 \rightarrow l = 1$

$$\Delta E = (6 - 2) \left(\frac{\hbar^2}{2I} \right) + hf = \frac{2\hbar^2}{I} + hf$$

$$\lambda = \frac{c}{2(\hbar/2\pi I) + f} = \frac{2.998 \times 10^8 \text{ m/s}}{2(3.960 \times 10^{11} \text{ Hz}) + 6.93 \times 10^{13} \text{ Hz}} = 4.28 \text{ } \mu\text{m}$$

(iii) transition $n = 2 \rightarrow n = 1, l = 2 \rightarrow l = 3$

$$\Delta E = (6 - 12) \left(\frac{\hbar^2}{2I} \right) + hf = -\frac{3\hbar^2}{I} + hf$$

$$\lambda = \frac{c}{-3(\hbar/2\pi I) + f} = \frac{2.998 \times 10^8 \text{ m/s}}{-3(3.960 \times 10^{11} \text{ Hz}) + 6.93 \times 10^{13} \text{ Hz}} = 4.40 \text{ } \mu\text{m}$$

EVALUATE: The vibrational energy change for the $n = 1 \rightarrow n = 0$ transition is the same as for the $n = 2 \rightarrow n = 1$ transition. The rotational energies are much smaller than the vibrational energies, so the wavelengths for all three transitions don't differ much.

42.48. The sum of the probabilities is $f(E_F + \Delta E) + f(E_F - \Delta E) = \frac{1}{e^{-\Delta E/kT} + 1} + \frac{1}{e^{\Delta E/kT} + 1} = \frac{1}{e^{-\Delta E/kT} + 1} + \frac{e^{-\Delta E/kT}}{1 + e^{-\Delta E/kT}} = 1$.

42.49. Since potassium is a metal we approximate $E_F = E_{F0}$. $\Rightarrow E_F = \frac{3^{2/3} \pi^{4/3} \hbar^2 n^{2/3}}{2m}$.

But the electron concentration $n = \frac{\rho}{m} \Rightarrow n = \frac{851 \text{ kg/m}^3}{6.49 \times 10^{-26} \text{ kg}} = 1.31 \times 10^{28} \text{ electron/m}^3$

$$\Rightarrow E_F = \frac{3^{2/3} \pi^{4/3} (1.054 \times 10^{-34} \text{ J} \cdot \text{s})^2 (1.31 \times 10^{28} / \text{m}^3)^{2/3}}{2(9.11 \times 10^{-31} \text{ kg})} = 3.24 \times 10^{-19} \text{ J} = 2.03 \text{ eV}.$$

42.50. IDENTIFY: The only difference between the two isotopes is their mass, which will affect their reduced mass and hence their moment of inertia.

SET UP: The rotational energy states are given by $E = l(l+1) \frac{\hbar^2}{2I}$ and the reduced mass is given by $m_r = m_1 m_2 / (m_1 + m_2)$.

EXECUTE: (a) If we call m the mass of the H-atom, the mass of the deuterium atom is $2m$ and the reduced masses of the molecules are

$$\text{H}_2 \text{ (hydrogen): } m_r(\text{H}) = mm/(m + m) = m/2$$

$$\text{D}_2 \text{ (deuterium): } m_r(\text{D}) = (2m)(2m)/(2m + 2m) = m$$

Using $I = m_r r_0^2$, the moments of inertia are $I_H = m r_0^2/2$ and $I_D = m r_0^2$. The ratio of the rotational energies is then

$$\frac{E_H}{E_D} = \frac{l(l+1) \left(\frac{\hbar^2}{2I_H} \right)}{l(l+1) \left(\frac{\hbar^2}{2I_D} \right)} = \frac{I_D}{I_H} = \frac{m r_0^2}{\frac{m}{2} r_0^2} = 2.$$

(b) The ratio of the vibrational energies is $\frac{E_H}{E_D} = \frac{\left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k'}{m_r(\text{H})}}}{\left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k'}{m_r(\text{D})}}} = \sqrt{\frac{m_r(\text{D})}{m_r(\text{H})}} = \sqrt{\frac{m}{m/2}} = \sqrt{2}.$

EVALUATE: The electrical force is the same for both molecules since both H and D have the same charge, so it is reasonable that the force constant would be the same for both of them.

41.51. IDENTIFY and SET UP: Use the description of the bcc lattice in Fig. 42.12c in the textbook to calculate the number of atoms per unit cell and then the number of atoms per unit volume.

EXECUTE: (a) Each unit cell has one atom at its center and 8 atoms at its corners that are each shared by 8 other unit cells. So there are $1 + 8/8 = 2$ atoms per unit cell.

$$\frac{n}{V} = \frac{2}{(0.35 \times 10^{-9} \text{ m})^3} = 4.66 \times 10^{28} \text{ atoms/m}^3$$

(b) $E_{F0} = \frac{3^{2/3} \pi^{4/3} \hbar^2 \left(\frac{N}{V} \right)^{2/3}}{2m}$

In this equation N/V is the number of free electrons per m^3 . But the problem says to assume one free electron per atom, so this is the same as n/V calculated in part (a).

$$m = 9.109 \times 10^{-31} \text{ kg (the electron mass), so } E_{F0} = 7.563 \times 10^{-19} \text{ J} = 4.7 \text{ eV}$$

EVALUATE: Our result for metallic lithium is similar to that calculated for copper in Example 42.8.

42.52. (a) $\frac{d}{dr}U_{\text{tot}} = \frac{ae^2}{4\pi\epsilon_0 r^2} - 8A \frac{1}{r^9}$. Setting this equal to zero when $r = r_0$ gives $r_0^7 = \frac{8A4\pi\epsilon_0}{ae^2}$

and so $U_{\text{tot}} = \frac{ae^2}{4\pi\epsilon_0} \left(-\frac{1}{r} + \frac{r_0^7}{8r^8} \right)$. At $r = r_0$, $U_{\text{tot}} = -\frac{7ae^2}{32\pi\epsilon_0 r_0} = -1.26 \times 10^{-18} \text{ J} = -7.85 \text{ eV}$.

(b) To remove a Na^+Cl^- ion pair from the crystal requires 7.85 eV. When neutral Na and Cl atoms are formed from the Na^+ and Cl^- atoms there is a net release of energy $-5.14 \text{ eV} + 3.61 \text{ eV} = -1.53 \text{ eV}$, so the net energy required to remove a neutral Na, Cl pair from the crystal is $7.85 \text{ eV} - 1.53 \text{ eV} = 6.32 \text{ eV}$.

42.53. (a) **IDENTIFY and SET UP:** $p = -\frac{dE_{\text{tot}}}{dV}$. Relate E_{tot} to E_{F0} and evaluate the derivative.

EXECUTE: $E_{\text{tot}} = NE_{\text{av}} = \frac{3N}{5} E_{\text{F0}} = \frac{3}{5} \left(\frac{3^{2/3} \pi^{4/3} \hbar^2}{2m} \right) N^{5/3} V^{-2/3}$

$\frac{dE_{\text{tot}}}{dV} = \frac{3}{5} \left(\frac{3^{2/3} \pi^{4/3} \hbar^2}{2m} \right) N^{5/3} \left(-\frac{2}{3} V^{-5/3} \right)$ so $p = \left(\frac{3^{2/3} \pi^{4/3} \hbar^2}{5m} \right) \left(\frac{N}{V} \right)^{5/3}$, as was to be shown.

(b) $N/V = 8.45 \times 10^{28} \text{ m}^{-3}$

$p = \left(\frac{3^{2/3} \pi^{4/3} (1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{5(9.109 \times 10^{-31} \text{ kg})} \right) (8.45 \times 10^{28} \text{ m}^{-3})^{5/3} = 3.81 \times 10^{10} \text{ Pa} = 3.76 \times 10^5 \text{ atm}$.

(c) **EVALUATE:** Normal atmospheric pressure is about 10^5 Pa , so these pressures are extremely large. The electrons are held in the metal by the attractive force exerted on them by the copper ions.

42.54. (a) From Problem 42.53, $p = \frac{3^{2/3} \pi^{4/3} \hbar^2}{5m} \left(\frac{N}{V} \right)^{5/3}$. $B = -V \frac{dp}{dV} = -V \left[\frac{5}{3} \cdot \frac{3^{2/3} \pi^{4/3} \hbar^2}{5m} \cdot \left(\frac{N}{V} \right)^{2/3} \left(\frac{-N}{V^2} \right) \right] = \frac{5}{3} p$.

(b) $\frac{N}{V} = 8.45 \times 10^{28} \text{ m}^{-3}$. $B = \frac{5}{3} \cdot \frac{3^{2/3} \pi^{4/3} \hbar^2}{5m} (8.45 \times 10^{28} \text{ m}^{-3})^{5/3} = 6.33 \times 10^{10} \text{ Pa}$.

(c) $\frac{6.33 \times 10^{10} \text{ Pa}}{1.4 \times 10^{11} \text{ Pa}} = 0.45$. The copper ions themselves make up the remaining fraction.

42.55. (a) $E_{\text{F0}} = \frac{3^{2/3} \pi^{4/3} \hbar^2}{2m} \left(\frac{N}{V} \right)^{2/3}$. Let $E_{\text{F0}} = \frac{1}{100} mc^2$.

$\left(\frac{N}{V} \right) = \left[\frac{2m^2 c^2}{(100) 3^{2/3} \pi^{4/3} \hbar^2} \right]^{3/2} = \frac{2^{3/2} m^3 c^3}{100^{3/2} 3 \pi^2 \hbar^3} = \frac{2^{3/2} m^3 c^3}{3000 \pi^2 \hbar^3} = 1.67 \times 10^{33} \text{ m}^{-3}$.

(b) $\frac{8.45 \times 10^{28} \text{ m}^{-3}}{1.67 \times 10^{33} \text{ m}^{-3}} = 5.06 \times 10^{-5}$. Since the real concentration of electrons in copper is less than one part in 10^{-4} of the concentration where relativistic effects are important, it is safe to ignore relativistic effects for most applications.

(c) The number of electrons is $N_e = \frac{6(2 \times 10^{30} \text{ kg})}{1.99 \times 10^{-26} \text{ kg}} = 6.03 \times 10^{56}$. The concentration is

$\frac{N_e}{V} = \frac{6.03 \times 10^{56}}{\frac{4}{3} \pi (6.00 \times 10^6 \text{ m})^3} = 6.66 \times 10^{35} \text{ m}^{-3}$.

(d) Comparing this to the result from part (a) $\frac{6.66 \times 10^{35} \text{ m}^{-3}}{1.67 \times 10^{33} \text{ m}^{-3}} \cong 400$ so relativistic effects will be very important.

42.56. **IDENTIFY:** The current through the diode is related to the voltage across it.

SET UP: The current through the diode is given by $I = I_S (e^{eV/kT} - 1)$.

EXECUTE: (a) The current through the resistor is $(35.0 \text{ V})/(125 \Omega) = 0.280 \text{ A} = 280 \text{ mA}$, which is also the current through the diode. This current is given by $I = I_S (e^{eV/kT} - 1)$, giving $280 \text{ mA} = 0.625 \text{ mA} (e^{eV/kT} - 1)$ and $1 +$

$(280/0.625) = 449 = e^{eV/kT}$. Solving for V at $T = 293 \text{ K}$ gives $V = \frac{kT \ln 449}{e} = \frac{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) \ln 449}{1.60 \times 10^{-19} \text{ C}} =$

0.154 V

(b) $R = V/I = (0.154 \text{ V})/(0.280 \text{ A}) = 0.551 \Omega$

EVALUATE: At a different voltage, the diode would have different resistance.

$$42.57. \quad (a) \quad U = \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}} = \frac{q^2}{4\pi\epsilon_0} \left(\frac{-1}{d} + \frac{1}{r} - \frac{1}{r+d} - \frac{1}{r-d} + \frac{1}{r} - \frac{1}{d} \right) = \frac{q^2}{4\pi\epsilon_0} \left(\frac{2}{r} - \frac{2}{d} - \frac{1}{r+d} - \frac{1}{r-d} \right).$$

$$\text{But } \frac{1}{r+d} + \frac{1}{r-d} = \frac{1}{r} \left(\frac{1}{1+\frac{d}{r}} + \frac{1}{1-\frac{d}{r}} \right) \approx \frac{1}{r} \left(1 - \frac{d}{r} + \frac{d^2}{r^2} + \cdots + 1 + \frac{d}{r} + \frac{d^2}{r^2} \right) \approx \frac{2}{r} + \frac{2d^2}{r^3}$$

$$\Rightarrow U = \frac{-2q^2}{4\pi\epsilon_0} \left(\frac{1}{d} + \frac{d^2}{r^3} \right) = \frac{-2p^2}{4\pi\epsilon_0 r^3} - \frac{2p^2}{4\pi\epsilon_0 d^3}.$$

$$(b) \quad U = \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}} = \frac{q^2}{4\pi\epsilon_0} \left(\frac{-1}{d} - \frac{1}{r} + \frac{1}{r+d} + \frac{1}{r-d} - \frac{1}{r} - \frac{1}{d} \right) = \frac{q^2}{4\pi\epsilon_0} \left(\frac{-2}{d} - \frac{2}{r} + \frac{2}{r} + \frac{2d^2}{r^3} \right) =$$

$$\frac{-2q^2}{4\pi\epsilon_0} \left(\frac{1}{d} - \frac{d^2}{r^3} \right) \Rightarrow U = \frac{-2p^2}{4\pi\epsilon_0 d^3} + \frac{2p^2}{4\pi\epsilon_0 r^3}.$$

If we ignore the potential energy involved in forming each individual molecule, which just involves a different choice for the zero of potential energy, then the answers are:

$$(a) \quad U = \frac{-2p^2}{4\pi\epsilon_0 r^3}. \quad \text{The interaction is attractive.}$$

$$(b) \quad U = \frac{+2p^2}{4\pi\epsilon_0 r^3}. \quad \text{The interaction is repulsive.}$$

$$42.58. \quad (a) \quad \text{Following the hint, } k' dr = -d \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \right)_{r=r_0} = \frac{1}{2\pi\epsilon_0} \frac{e^2}{r_0^3} dr \quad \text{and} \quad \hbar\omega = \hbar\sqrt{2k'/m} = \hbar\sqrt{\frac{1}{\pi\epsilon_0} \frac{e^2}{mr_0^3}} =$$

$1.23 \times 10^{-19} \text{ J} = 0.77 \text{ eV}$, where $(m/2)$ has been used for the reduced mass.

(b) The reduced mass is doubled, and the energy is reduced by a factor of $\sqrt{2}$ to 0.54 eV .

NUCLEAR PHYSICS

- 43.1.** (a) ${}^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.
 (b) ${}^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.
 (c) ${}^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.
- 43.2.** (a) Using $R = (1.2 \text{ fm})A^{1/3}$, the radii are roughly 3.6 fm, 5.3 fm, and 7.1 fm.
 (b) Using $4\pi R^2$ for each of the radii in part (a), the areas are 163 fm^2 , 353 fm^2 and 633 fm^2 .
 (c) $\frac{4}{3}\pi R^3$ gives 195 fm^3 , 624 fm^3 and 1499 fm^3 .
 (d) The density is the same, since the volume and the mass are both proportional to A : $2.3 \times 10^{17} \text{ kg/m}^3$ (see Example 43.1).
 (e) Dividing the result of part (d) by the mass of a nucleon, the number density is $0.14/\text{fm}^3 = 1.40 \times 10^{44}/\text{m}^3$.
- 43.3. IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the $1s$ level. Calculate the energy splitting between these states and relate this to the frequency of the photons.
SET UP: When the spin component is parallel to the field the interaction energy is $U = -\mu_z B$. When the spin component is antiparallel to the field the interaction energy is $U = +\mu_z B$. The transition energy for a transition between these two states is $\Delta E = 2\mu_z B$, where $\mu_z = 2.7928\mu_n$. The transition energy is related to the photon frequency by $\Delta E = hf$, so $2\mu_z B = hf$.
EXECUTE: $B = \frac{hf}{2\mu_z} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(22.7 \times 10^6 \text{ Hz})}{2(2.7928)(5.051 \times 10^{-27} \text{ J/T})} = 0.533 \text{ T}$
EVALUATE: This magnetic field is easily achievable. Photons of this frequency have wavelength $\lambda = c/f = 13.2 \text{ m}$. These are radio waves.
- 43.4.** (a) As in Example 43.2, $\Delta E = 2(1.9130)(3.15245 \times 10^{-8} \text{ eV/T})(2.30 \text{ T}) = 2.77 \times 10^{-7} \text{ eV}$. Since $\vec{\mu}$ and $\vec{S}$ are in opposite directions for a neutron, the antiparallel configuration is lower energy. This result is smaller than but comparable to that found in the example for protons.
 (b) $f = \frac{\Delta E}{h} = 66.9 \text{ MHz}$, $\lambda = \frac{c}{f} = 4.48 \text{ m}$.
- 43.5. IDENTIFY:** Calculate the spin magnetic energy shift for each spin component. Calculate the energy splitting between these states and relate this to the frequency of the photons.
(a) SET UP: From Example 43.2, when the z -component of $\vec{S}$ (and $\vec{\mu}$) is parallel to $\vec{B}$, $U = -|\mu_z|B = -2.7928\mu_n B$. When the z -component of $\vec{S}$ (and $\vec{\mu}$) is antiparallel to $\vec{B}$, $U = -|\mu_z|B = +2.7928\mu_n B$. The state with the proton spin component parallel to the field lies lower in energy. The energy difference between these two states is $\Delta E = 2(2.7928\mu_n B)$.
EXECUTE: $\Delta E = hf$ so $f = \frac{\Delta E}{h} = \frac{2(2.7928\mu_n)B}{h} = \frac{2(2.7928)(5.051 \times 10^{-27} \text{ J/T})(1.65 \text{ T})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}$
 $f = 7.03 \times 10^7 \text{ Hz} = 7.03 \text{ MHz}$
 And then $\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{7.03 \times 10^7 \text{ Hz}} = 4.26 \text{ m}$
EVALUATE: From Figure 32.4 in the textbook, these are radio waves.

(b) SET UP: From Eqs. (27.27) and (41.22) and Fig. 41.14 in the textbook, the state with the z -component of $\vec{\mu}$ parallel to $\vec{B}$ has lower energy. But, since the charge of the electron is negative, this is the state with the electron spin component antiparallel to $\vec{B}$. That is, for the $m_s = -\frac{1}{2}$ state lies lower in energy.

EXECUTE: For the $m_s = +\frac{1}{2}$ state, $U = +(2.00232)\left(\frac{e}{2m}\right)\left(+\frac{\hbar}{2}\right)B = +\frac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right)B = +\frac{1}{2}(2.00232)\mu_B B$.

For the $m_s = -\frac{1}{2}$ state, $U = -\frac{1}{2}(2.00232)\mu_B B$. The energy difference between these two states is $\Delta E = (2.00232)\mu_B B$.

$$\Delta E = hf \text{ so } f = \frac{\Delta E}{h} = \frac{2.00232\mu_B B}{h} = \frac{(2.00232)(9.274 \times 10^{-24} \text{ J/T})(1.65 \text{ T})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = 4.62 \times 10^{10} \text{ Hz} = 46.2 \text{ GHz. And}$$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{4.62 \times 10^{10} \text{ Hz}} = 6.49 \times 10^{-3} \text{ m} = 6.49 \text{ mm.}$$

EVALUATE: From Figure 32.4 in the textbook, these are microwaves. The interaction energy with the magnetic field is inversely proportional to the mass of the particle, so it is less for the proton than for the electron. The smaller transition energy for the proton produces a larger wavelength.

43.6. (a) $146m_n + 92m_H - m_U = 1.93 \text{ u}$

(b) $1.80 \times 10^3 \text{ MeV}$

(c) 7.56 MeV per nucleon (using 931.5 MeV/u and 238 nucleons).

43.7. IDENTIFY and SET UP: The text calculates that the binding energy of the deuteron is 2.224 MeV. A photon that breaks the deuteron up into a proton and a neutron must have at least this much energy.

$$E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E}$$

$$\text{EXECUTE: } \lambda = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{2.224 \times 10^6 \text{ eV}} = 5.575 \times 10^{-13} \text{ m} = 0.5575 \text{ pm.}$$

EVALUATE: This photon has gamma-ray wavelength.

43.8. IDENTIFY: The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

SET UP: In terms of the masses of the particles involved, the binding energy is

$$E_B = (6m_H + 6m_n - m_{C-12})c^2.$$

EXECUTE: (a) Using the values from Table 43.2, we get

$$E_B = [6(1.007825 \text{ u}) + 6(1.008665 \text{ u}) - 12.000000 \text{ u}](931.5 \text{ MeV/u}) = 92.16 \text{ MeV}$$

(b) The binding energy per nucleon is $(92.16 \text{ MeV})/(12 \text{ nucleons}) = 7.680 \text{ MeV/nucleon}$

(c) The energy of the C-12 nucleus is $(12.0000 \text{ u})(931.5 \text{ MeV/u}) = 11178 \text{ MeV}$. Therefore the percent of the mass

$$\text{that is binding energy is } \frac{92.16 \text{ MeV}}{11178 \text{ MeV}} = 0.8245\%.$$

EVALUATE: The binding energy of 92.16 MeV binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

43.9. IDENTIFY: Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

SET UP: The binding energy of a deuteron is 2.224 MeV and the energy of the photon is $E = hc/\lambda$. Kinetic energy is $K = \frac{1}{2}mv^2$.

EXECUTE: (a) The energy of the photon is

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{3.50 \times 10^{-13} \text{ m}} = 5.68 \times 10^{-13} \text{ J.}$$

The binding of the deuteron is $E_B = 2.224 \text{ MeV} = 3.56 \times 10^{-13} \text{ J}$. Therefore the kinetic energy

$$\text{is } K = (5.68 - 3.56) \times 10^{-13} \text{ J} = 2.12 \times 10^{-13} \text{ J} = 1.32 \text{ MeV.}$$

(b) The particles share the energy equally, so each gets half. Solving the kinetic energy for v gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06 \times 10^{-13} \text{ J})}{1.6605 \times 10^{-27} \text{ kg}}} = 1.13 \times 10^7 \text{ m/s}$$

EVALUATE: Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

43.10. (a) $7(m_n + m_H) - m_N = 0.112 \text{ u}$, which is 105 MeV, or 7.48 MeV per nucleon.

(b) Similarly, $2(m_H + m_n) - m_{\text{He}} = 0.03038 \text{ u} = 28.3 \text{ MeV}$, or 7.07 MeV per nucleon, slightly lower (compare to Figure 43.2 in the textbook).

- 43.11. (a) IDENTIFY:** Find the energy equivalent of the mass defect.

SET UP: A $^{11}_5\text{B}$ atom has 5 protons, $11 - 5 = 6$ neutrons, and 5 electrons. The mass defect therefore is

$$\Delta M = 5m_p + 6m_n + 5m_e - M(^{11}_5\text{B}).$$

EXECUTE: $\Delta M = 5(1.0072765 \text{ u}) + 6(1.0086649 \text{ u}) + 5(0.0005485799 \text{ u}) - 11.009305 \text{ u} = 0.08181 \text{ u}$. The energy equivalent is $E_B = (0.08181 \text{ u})(931.5 \text{ MeV/u}) = 76.21 \text{ MeV}$.

(b) IDENTIFY and SET UP: Eq.(43.11): $E_B = C_1A - C_2A^{2/3} - C_3Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$

The fifth term is zero since Z is odd but N is even. $A = 11$ and $Z = 5$.

EXECUTE: $E_B = (15.75 \text{ MeV})(11) - (17.80 \text{ MeV})(11)^{2/3} - (0.7100 \text{ MeV})5(4)/11^{1/3} - (23.69 \text{ MeV})(11-10)^2/11$.

$$E_B = +173.25 \text{ MeV} - 88.04 \text{ MeV} - 6.38 \text{ MeV} - 2.15 \text{ MeV} = 76.68 \text{ MeV}$$

The percentage difference between the calculated and measured E_B is $\frac{76.68 \text{ MeV} - 76.21 \text{ MeV}}{76.21 \text{ MeV}} = 0.6\%$

EVALUATE: Eq.(43.11) has a greater percentage accuracy for ^{62}Ni . The semi-empirical mass formula is more accurate for heavier nuclei.

- 43.12. (a)** $34m_n + 29m_H - m_{\text{Cu}} = 34(1.008665) \text{ u} + 29(1.007825) \text{ u} - 62.929601 \text{ u} = 0.592 \text{ u}$, which is 551 MeV, or 8.75 MeV per nucleon (using 931.5 MeV/u and 63 nucleons).

(b) In Eq.(43.11), $Z = 29$ and $N = 34$, so the fifth term is zero. The predicted binding energy is

$$E_B = (15.75 \text{ MeV})(63) - (17.80 \text{ MeV})(63)^{2/3} - (0.7100 \text{ MeV})\frac{(29)(28)}{(63)^{1/3}} - (23.69 \text{ MeV})\frac{(5)^2}{(63)}.$$

$E_B = 556 \text{ MeV}$. The fifth term is zero since the number of neutrons is even while the number of protons is odd, making the pairing term zero. This result differs from the binding energy found from the mass deficit by 0.86%, a very good agreement comparable to that found in Example 43.4.

- 43.13. IDENTIFY** In each case determine how the decay changes A and Z of the nucleus. The β^+ and β^- particles have charge but their nucleon number is $A = 0$.

(a) SET UP: α -decay: Z increases by 2, $A = N + Z$ decreases by 4 (an α particle is a ^4_2He nucleus)

EXECUTE: $^{239}_{94}\text{Pu} \rightarrow ^4_2\text{He} + ^{235}_{92}\text{U}$

(b) SET UP: β^- decay: Z increases by 1, $A = N + Z$ remains the same (a β^- particle is an electron, $^0_{-1}\text{e}$)

EXECUTE: $^{24}_{11}\text{Na} \rightarrow ^0_{-1}\text{e} + ^{24}_{12}\text{Mg}$

(c) SET UP β^+ decay: Z decreases by 1, $A = N + Z$ remains the same (a β^+ particle is a positron, $^0_{+1}\text{e}$)

EXECUTE: $^{15}_8\text{O} \rightarrow ^0_{+1}\text{e} + ^{15}_7\text{N}$

EVALUATE: In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.

- 43.14. (a)** The energy released is the energy equivalent of $m_n - m_p - m_e = 8.40 \times 10^{-4} \text{ u}$, or 783 keV.

(b) $m_n > m_p$, and the decay is not possible.

- 43.15. IDENTIFY:** The energy of the photon must be equal to the difference in energy of the two nuclear energy levels.

SET UP: The energy difference is $\Delta E = hc/\lambda$.

$$\text{EXECUTE: } \Delta E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{0.0248 \times 10^{-9} \text{ J}} = 8.015 \times 10^{-15} \text{ J} = 0.0501 \text{ MeV}$$

EVALUATE: Since the wavelength of this photon is *much* shorter than the wavelengths of visible light, its energy is much greater than visible-light photons which are frequently emitted during electron transitions in atoms. This tells us that the energy difference between the nuclear shells is *much* greater than the energy difference between electron shells in atoms, meaning that nuclear energies are *much* greater than the energies of orbiting electrons.

- 43.16. IDENTIFY:** The energy released is equal to the mass defect of the initial and final nuclei.

SET UP: The mass defect is equal to the difference between the initial and final masses of the constituent particles.

EXECUTE: (a) The mass defect is $238.050788 \text{ u} - 234.043601 \text{ u} - 4.002603 \text{ u} = 0.004584 \text{ u}$. The energy released is $(0.004584 \text{ u})(931.5 \text{ MeV/u}) = 4.270 \text{ MeV}$.

(b) Take the ratio of the two kinetic energies, using the fact that $K = p^2/2m$:

$$\frac{K_{\text{Th}}}{K_{\alpha}} = \frac{\frac{p_{\text{Th}}^2}{2m_{\text{Th}}}}{\frac{p_{\alpha}^2}{2m_{\alpha}}} = \frac{m_{\alpha}}{m_{\text{Th}}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{\text{Th}} = \frac{4}{234+4} K_{\text{Total}} = \frac{4}{238} (4.270 \text{ MeV}) = 0.07176 \text{ MeV} = 1.148 \times 10^{-14} \text{ J}$$

Solving for v in the kinetic energy gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.148 \times 10^{-14} \text{ J})}{(234.043601)(1.6605 \times 10^{-27} \text{ kg})}} = 2.431 \times 10^5 \text{ m/s}$$

EVALUATE: As we can see by the ratio of kinetic energies in part (b), the alpha particle will have a much higher kinetic energy than the thorium.

- 43.17.** If β^- decay of ^{14}C is possible, then we are considering the decay $^{14}_6\text{C} \rightarrow ^{14}_7\text{N} + \beta^-$.

$$\Delta m = M(^{14}_6\text{C}) - M(^{14}_7\text{N}) - m_e$$

$$\Delta m = (14.003242 \text{ u} - 6(0.000549 \text{ u})) - (14.003074 \text{ u} - 7(0.000549 \text{ u})) - 0.0005491 \text{ u}$$

$$\Delta m = +1.68 \times 10^{-4} \text{ u. So } E = (1.68 \times 10^{-4} \text{ u})(931.5 \text{ MeV/u}) = 0.156 \text{ MeV} = 156 \text{ keV}$$

- 43.18.** (a) A proton changes to a neutron, so the emitted particle is a positron (β^+).

(b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

(c) A neutron changes to a proton, so the emitted particle is an electron (β^-).

- 43.19.** (a) As in the example, $(0.000898 \text{ u})(931.5 \text{ MeV/u}) = 0.836 \text{ MeV}$.

$$(b) 0.836 \text{ MeV} - 0.122 \text{ MeV} - 0.014 \text{ MeV} = 0.700 \text{ MeV}.$$

- 43.20.** (a) $^{90}_{39}\text{Sr} \rightarrow \beta^- + ^{90}_{39}\text{X}$. X has 39 protons and 90 protons plus neutrons, so it must be ^{90}Y .

(b) Use base 2 because we know the half life. $A = A_0 2^{-t/T_{1/2}}$ and $0.01A_0 = A_0 2^{-t/T_{1/2}}$.

$$t = -\frac{T_{1/2} \log 0.01}{\log 2} = -\frac{(28 \text{ yr}) \log 0.01}{\log 2} = 190 \text{ yr}.$$

- 43.21. IDENTIFY and SET UP:** $T_{1/2} = \frac{\ln 2}{\lambda}$ The mass of a single nucleus is $124m_p = 2.07 \times 10^{-25} \text{ kg}$.

$$|\Delta N / \Delta t| = 0.350 \text{ Ci} = 1.30 \times 10^{10} \text{ Bq}; |\Delta N / \Delta t| = \lambda N$$

$$\text{EXECUTE: } N = \frac{6.13 \times 10^{-3} \text{ kg}}{2.07 \times 10^{-25} \text{ kg}} = 2.96 \times 10^{22}; \lambda = \frac{\Delta N / \Delta t}{N} = \frac{1.30 \times 10^{10} \text{ Bq}}{2.96 \times 10^{22}} = 4.39 \times 10^{-13} \text{ s}^{-1}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = 1.58 \times 10^{12} \text{ s} = 5.01 \times 10^4 \text{ yr}$$

- 43.22.** Note that Eq.(43.17) can be written as follows: $N = N_0 2^{-t/T_{1/2}}$. The amount of elapsed time since the source was created is roughly 2.5 years. Thus, we expect the current activity to be $N = (5000 \text{ Ci})2^{-(2.5 \text{ yr})/(5.271 \text{ yr})} = 3600 \text{ Ci}$. The

source is barely usable. Alternatively, we could calculate $\lambda = \frac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$ and use the Eq. 43.17 directly

to obtain the same answer.

- 43.23. IDENTIFY and SET UP:** As discussed in Section 43.4, the activity $A = |dN/dt|$ obeys the same decay equation as

Eq. (43.17): $A = A_0 e^{-\lambda t}$. For ^{14}C , $T_{1/2} = 5730 \text{ y}$ and $\lambda = \ln 2 / T_{1/2}$ so $A = A_0 e^{-(\ln 2)t / T_{1/2}}$; Calculate A at each t ;

$$A_0 = 180.0 \text{ decays/min.}$$

EXECUTE: (a) $t = 1000 \text{ y}$, $A = 159 \text{ decays/min}$

(b) $t = 50,000 \text{ y}$, $A = 0.43 \text{ decays/min}$

EVALUATE: The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor of $(\frac{1}{2})^{8.73}$.

- 43.24. IDENTIFY and SET UP:** The decay rate decreases by a factor of 2 in a time of one half-life.

EXECUTE: (a) 24 d is $3T_{1/2}$ so the activity is $(375 \text{ Bq})/(2^3) = 46.9 \text{ Bq}$

(b) The activity is proportional to the number of radioactive nuclei, so the percent is $\frac{17.0 \text{ Bq}}{46.9 \text{ Bq}} = 36.2\%$

(c) $^{131}_{53}\text{I} \rightarrow ^0_{-1}\text{e} + ^{131}_{54}\text{Xe}$ The nucleus $^{131}_{54}\text{Xe}$ is produced.

EVALUATE: Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

43.25. (a) ${}^3_1\text{H} \rightarrow {}^0_{-1}\text{e} + {}^3_2\text{He}$

(b) $N = N_0 e^{-\lambda t}$, $N = 0.100 N_0$ and $\lambda = (\ln 2)/T_{1/2}$

$$0.100 = e^{-t(\ln 2)/T_{1/2}}; \quad -t(\ln 2)/T_{1/2} = \ln(0.100); \quad t = \frac{-\ln(0.100)T_{1/2}}{\ln 2} = 40.9 \text{ y}$$

43.26. (a) $\frac{dN}{dt} = 500 \mu\text{Ci} = (500 \times 10^{-6})(3.70 \times 10^{10} \text{ s}^{-1}) = 1.85 \times 10^7 \text{ decays/s}$

$$T_{1/2} = \frac{\ln 2}{\lambda} \rightarrow \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12 \text{ d}(86,400 \text{ s/d})} = 6.69 \times 10^{-7} \text{ s}^{-1}$$

$$\frac{dN}{dt} = \lambda N \Rightarrow N = \frac{dN/dt}{\lambda} = \frac{1.85 \times 10^7 \text{ decays/s}}{6.69 \times 10^{-7} \text{ s}^{-1}} = 2.77 \times 10^{13} \text{ nuclei. The mass of this many } {}^{131}\text{Ba} \text{ nuclei is}$$

$$m = 2.77 \times 10^{13} \text{ nuclei} \times (131 \times 1.66 \times 10^{-27} \text{ kg/nucleus}) = 6.0 \times 10^{-12} \text{ kg} = 6.0 \times 10^{-9} \text{ g} = 6.0 \text{ ng}$$

(b) $A = A_0 e^{-\lambda t}$. $1 \mu\text{Ci} = (500 \mu\text{Ci}) e^{-\lambda t}$. $\ln(1/500) = -\lambda t$.

$$t = -\frac{\ln(1/500)}{\lambda} = -\frac{\ln(1/500)}{6.69 \times 10^{-7} \text{ s}^{-1}} = 9.29 \times 10^6 \text{ s} \left(\frac{1 \text{ d}}{86,400 \text{ s}} \right) = 108 \text{ days}$$

43.27. $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$. $-\frac{(\ln 2)t}{T_{1/2}} = \ln(A/A_0)$.

$$T_{1/2} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00 \text{ days})}{\ln(3091/8318)} = 2.80 \text{ days}$$

43.28. $\frac{dN}{dt} = \lambda N$. $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1620 \text{ yr}(3.15 \times 10^7 \text{ s/yr})} = 1.36 \times 10^{-11} \text{ s}^{-1}$.

$$N = 1 \text{ g} \left(\frac{6.022 \times 10^{23} \text{ atoms}}{226 \text{ g}} \right) = 2.665 \times 10^{25} \text{ atoms.}$$

$$\frac{dN}{dt} = \lambda N = (2.665 \times 10^{25})(1.36 \times 10^{-11} \text{ s}^{-1}) = 3.62 \times 10^{10} \text{ decays/s} = 3.62 \times 10^{10} \text{ Bq}$$

Convert to Ci: $3.62 \times 10^{10} \text{ Bq} \left(\frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ Bq}} \right) = 0.98 \text{ Ci}$

43.29. **IDENTIFY and SET UP:** Calculate the number N of ${}^{14}\text{C}$ atoms in the sample and then use Eq. (43.17) to find the decay constant λ . Eq. (43.18) then gives $T_{1/2}$.

EXECUTE: Find the total number of carbon atoms in the sample.

$$n = m/M;$$

$$N_{\text{tot}} = nN_A = mN_A/M = (12.0 \times 10^{-3} \text{ kg})(6.022 \times 10^{23} \text{ atoms/mol})/(12.011 \times 10^{-3} \text{ kg/mol})$$

$$N_{\text{tot}} = 6.016 \times 10^{23} \text{ atoms, so } (1.3 \times 10^{-12})(6.016 \times 10^{23}) = 7.82 \times 10^{11} \text{ carbon-14 atoms}$$

$$\Delta N/\Delta t = -180 \text{ decays/min} = -3.00 \text{ decays/s}$$

$$\Delta N/\Delta t = -\lambda N; \quad \lambda = \frac{-\Delta N/\Delta t}{N} = 3.836 \times 10^{-12} \text{ s}^{-1}$$

$$T_{1/2} = (\ln 2)/\lambda = 1.807 \times 10^{11} \text{ s} = 5730 \text{ y}$$

EVALUATE: The value we calculated agrees with the value given in Section 43.4.

43.30. $\frac{360 \times 10^6 \text{ decays}}{86,400 \text{ s}} = 4.17 \times 10^3 \text{ Bq} = 1.13 \times 10^{-7} \text{ Ci} = 0.113 \mu\text{Ci}$.

43.31. (a) $\left| \frac{dN}{dt} \right| = 7.56 \times 10^{11} \text{ Bq} = 7.56 \times 10^{11} \text{ decays/s}$. $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(30.8 \text{ min})(60 \text{ s/min})} = 3.75 \times 10^{-4} \text{ s}^{-1}$.

$$N_0 = \frac{1}{\lambda} \left| \frac{dN}{dt} \right| = \frac{7.56 \times 10^{11} \text{ decays/s}}{3.75 \times 10^{-4} \text{ s}^{-1}} = 2.02 \times 10^{15} \text{ nuclei.}$$

(b) The number of nuclei left after one half-life is $\frac{N_0}{2} = 1.01 \times 10^{15}$ nuclei, and the activity is half:

$$\left| \frac{dN}{dt} \right| = 3.78 \times 10^{11} \text{ decays/s.}$$

(c) After three half lives (92.4 minutes) there is an eighth of the original amount, so $N = 2.53 \times 10^{14}$ nuclei, and an eighth of the activity: $\left(\frac{dN}{dt} \right) = 9.45 \times 10^{10} \text{ decays/s.}$

43.32. The activity of the sample is $\frac{3070 \text{ decays/min}}{(60 \text{ sec/min})(0.500 \text{ kg})} = 102 \text{ Bq/kg}$, while the activity of atmospheric carbon is

$$255 \text{ Bq/kg (see Example 43.9). The age of the sample is then } t = -\frac{\ln(102/255)}{\lambda} = -\frac{\ln(102/255)}{1.21 \times 10^{-4} \text{ /y}} = 7573 \text{ y.}$$

43.33. IDENTIFY and SET UP: Find λ from the half-life and the number N of nuclei from the mass of one nucleus and the mass of the sample. Then use Eq.(43.16) to calculate $|dN/dt|$, the number of decays per second.

EXECUTE: (a) $|dN/dt| = \lambda N$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1.28 \times 10^9 \text{ y})(3.156 \times 10^7 \text{ s/y})} = 1.715 \times 10^{-17} \text{ s}^{-1}$$

The mass of ^{40}K atom is approximately 40 u, so the number of ^{40}K nuclei in the sample is

$$N = \frac{1.63 \times 10^{-9} \text{ kg}}{40 \text{ u}} = \frac{1.63 \times 10^{-9} \text{ kg}}{40(1.66054 \times 10^{-27} \text{ kg})} = 2.454 \times 10^{16}.$$

$$\text{Then } |dN/dt| = \lambda N = (1.715 \times 10^{-17} \text{ s}^{-1})(2.454 \times 10^{16}) = 0.421 \text{ decays/s}$$

$$\text{(b) } |dN/dt| = (0.421 \text{ decays/s})(1 \text{ Ci}/(3.70 \times 10^{10} \text{ decays/s})) = 1.14 \times 10^{-11} \text{ Ci}$$

EVALUATE: The very small sample still contains a very large number of nuclei. But the half life is very large, so the decay rate is small.

43.34. (a) $\text{rem} = \text{rad} \times \text{RBE}$. $200 = x(10)$ and $x = 20 \text{ rad}$.

(b) 1 rad deposits 0.010 J/kg, so 20 rad deposit 0.20 J/kg. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is $(0.025 \text{ kg})(0.20 \text{ J/kg}) = 5.0 \times 10^{-3} \text{ J} = 5.0 \text{ mJ}$

(c) Since $\text{RBE} = 1$ for β -rays, so $\text{rem} = \text{rad}$. Therefore $20 \text{ rad} = 20 \text{ rem}$.

43.35. 1 rad = 10^{-2} Gy, so 1 Gy = 100 rad and the dose was 500 rad.

$$\text{rem} = (\text{rad})(\text{RBE}) = (500 \text{ rad})(4.0) = 2000 \text{ rem. } 1 \text{ Gy} = 1 \text{ J/kg, so } 5.0 \text{ J/kg.}$$

43.36. IDENTIFY and SET UP: For x rays $\text{RBE} = 1$ so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

EXECUTE: One whole-body scan delivers $(75 \text{ kg})(12 \times 10^{-3} \text{ J/kg}) = 0.90 \text{ J}$. One chest x ray delivers

$$(5.0 \text{ kg})(0.20 \times 10^{-3} \text{ J/kg}) = 1.0 \times 10^{-3} \text{ J. It takes } \frac{0.90 \text{ J}}{1.0 \times 10^{-3} \text{ J}} = 900 \text{ chest x rays to deliver the same total energy.}$$

43.37. IDENTIFY and SET UP: For x rays $\text{RBE} = 1$ and the equivalent dose equals the absorbed dose.

EXECUTE: (a) $175 \text{ krad} = 175 \text{ krem} = 1.75 \text{ kGy} = 1.75 \text{ kSv}$

$$(1.75 \times 10^3 \text{ J/kg})(0.150 \text{ kg}) = 2.62 \times 10^2 \text{ J}$$

$$\text{(b) } 175 \text{ krad} = 1.75 \text{ kGy; } (1.50)(175 \text{ krad}) = 262 \text{ krem} = 2.62 \text{ kSv}$$

The energy deposited would be $2.62 \times 10^2 \text{ J}$, the same as in (a).

EVALUATE: The energy required to raise the temperature of 0.150 kg of water 1°C is 628 J, and $2.62 \times 10^2 \text{ J}$ is less than this. The energy deposited corresponds to a very small amount of heating.

43.38. (a) $5.4 \text{ Sv} (100 \text{ rem/Sv}) = 540 \text{ rem}$.

(b) The RBE of 1 gives an absorbed dose of 540 rad.

(c) The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4 \text{ Gy})(65 \text{ kg}) = 351 \text{ J}$. The energy required to raise the temperature of 65 kg by 0.010°C is $(65 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(0.01^\circ\text{C}) = 3 \text{ kJ}$.

43.39. (a) We need to know how many decays per second occur.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(12.3 \text{ y})(3.156 \times 10^7 \text{ s/y})} = 1.79 \times 10^{-9} \text{ s}^{-1}.$$

$$\text{The number of tritium atoms is } N_0 = \frac{1}{\lambda} \left| \frac{dN}{dt} \right| = \frac{(0.35 \text{ Ci})(3.70 \times 10^{10} \text{ Bq/Ci})}{1.79 \times 10^{-9} \text{ s}^{-1}} = 7.2540 \times 10^{18} \text{ nuclei.}$$

The number of remaining nuclei after one week is

$$N = N_0 e^{-\lambda t} = (7.25 \times 10^{18}) e^{-(1.79 \times 10^{-9} \text{ s}^{-1})(7)(24)(3600 \text{ s})} = 7.2462 \times 10^{18} \text{ nuclei. } \Delta N = N_0 - N = 7.8 \times 10^{15} \text{ decays.}$$

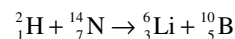
So the energy absorbed is $E_{\text{total}} = \Delta N E_\gamma = (7.8 \times 10^{15})(5000 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 6.24 \text{ J}$. The absorbed dose is $\frac{(6.24 \text{ J})}{(50 \text{ kg})} = 0.125 \text{ J/kg} = 12.5 \text{ rad}$. Since RBE = 1, then the equivalent dose is 12.5 rem.

(b) In the decay, antineutrinos are also emitted. These are not absorbed by the body, and so some of the energy of the decay is lost (about 12 keV).

- 43.40. $(0.72 \times 10^{-6} \text{ Ci})(3.7 \times 10^{10} \text{ Bq/Ci})(3.156 \times 10^7 \text{ s}) = 8.41 \times 10^{11} \alpha$ particles. The absorbed dose is $\frac{(8.41 \times 10^{11})(4.0 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(0.50 \text{ kg})} = 1.08 \text{ Gy} = 108 \text{ rad}$. The equivalent dose is (20)(108 rad) = 2160 rem.

- 43.41. (a) **IDENTIFY and SET UP:** Determine X by balancing the charge and nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = 2 + 14 - 10 = 6$ and $Z = 1 + 7 - 5 = 3$. Thus X is ${}^6_3\text{Li}$ and the reaction is



(b) **IDENTIFY and SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: The neutral atoms on each side of the reaction equation have a total of 8 electrons, so the electron masses cancel when neutral atom masses are used. The neutral atom masses are found in Table 43.2.

$$\text{mass of } {}^2_1\text{H} + {}^{14}_7\text{N} \text{ is } 2.014102 \text{ u} + 14.003074 \text{ u} = 16.017176 \text{ u}$$

$$\text{mass of } {}^6_3\text{Li} + {}^{10}_5\text{B} \text{ is } 6.015121 \text{ u} + 10.012937 \text{ u} = 16.028058 \text{ u}$$

The mass increases, so energy is absorbed by the reaction. The Q value is $(16.017176 \text{ u} - 16.028058 \text{ u})(931.5 \text{ MeV/u}) = -10.14 \text{ MeV}$

(c) **IDENTIFY and SET UP:** The available energy in the collision, the kinetic energy K_{cm} in the center of mass reference frame, is related to the kinetic energy K of the bombarding particle by Eq. (43.24).

EXECUTE: The kinetic energy that must be available to cause the reaction is 10.14 MeV. Thus

$K_{\text{cm}} = 10.14 \text{ MeV}$. The mass M of the stationary target (${}^{14}_7\text{N}$) is $M = 14 \text{ u}$. The mass m of the colliding particle (${}^2_1\text{H}$) is 2 u. Then by Eq. (43.24) the minimum kinetic energy K that the ${}^2_1\text{H}$ must have is

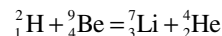
$$K = \left(\frac{M+m}{M} \right) K_{\text{cm}} = \left(\frac{14 \text{ u} + 2 \text{ u}}{14 \text{ u}} \right) (10.14 \text{ MeV}) = 11.59 \text{ MeV}$$

EVALUATE: The projectile (${}^2_1\text{H}$) is much lighter than the target (${}^{14}_7\text{N}$) so K is not much larger than K_{cm} . The K we have calculated is what is required to allow the mass increase. We would also need to check to see if at this energy the projectile can overcome the Coulomb repulsion to get sufficiently close to the target nucleus for the reaction to occur.

- 43.42. $m_{{}^3_2\text{He}} + m_{{}^2_1\text{H}} - m_{{}^4_2\text{He}} - m_{{}^1_1\text{H}} = 1.97 \times 10^{-2} \text{ u}$, so the energy released is 18.4 MeV.

- 43.43. **IDENTIFY and SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = +2 + 9 - 4 = 7$ and $Z = +1 + 4 - 2 = 3$. Thus X is ${}^7_3\text{Li}$ and the reaction is



(b) **IDENTIFY and SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2.

$${}^2_1\text{H} + {}^9_4\text{Be} \text{ has mass } 2.014102 \text{ u} + 9.012182 \text{ u} = 11.026284 \text{ u}$$

$${}^7_3\text{Li} + {}^4_2\text{He} \text{ has mass } 7.016003 \text{ u} + 4.002603 \text{ u} = 11.018606 \text{ u}$$

$$\text{The mass decrease is } 11.026284 \text{ u} - 11.018606 \text{ u} = 0.007678 \text{ u}.$$

$$\text{This corresponds to an energy release of } 0.007678 \text{ u}(931.5 \text{ MeV/u}) = 7.152 \text{ MeV}.$$

(c) **IDENTIFY and SET UP:** Estimate the threshold energy by calculating the Coulomb potential energy when the ${}^2_1\text{H}$ and ${}^9_4\text{Be}$ nuclei just touch. Obtain the nuclear radii from Eq. (43.1).

$$\text{EXECUTE: The radius } R_{\text{Be}} \text{ of the } {}^9_4\text{Be} \text{ nucleus is } R_{\text{Be}} = (1.2 \times 10^{-15} \text{ m})(9)^{1/3} = 2.5 \times 10^{-15} \text{ m}.$$

$$\text{The radius } R_{\text{H}} \text{ of the } {}^2_1\text{H} \text{ nucleus is } R_{\text{H}} = (1.2 \times 10^{-15} \text{ m})(2)^{1/3} = 1.5 \times 10^{-15} \text{ m}.$$

The nuclei touch when their center-to-center separation is

$$R = R_{\text{Be}} + R_{\text{H}} = 4.0 \times 10^{-15} \text{ m}.$$

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e(4e)}{r}$$

$$U = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{4(1.602 \times 10^{-19} \text{ C})^2}{(4.0 \times 10^{-15} \text{ m})(1.602 \times 10^{-19} \text{ J/eV})} = 1.4 \text{ MeV}$$

This is an estimate of the threshold energy for this reaction.

EVALUATE: The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

- 43.44. IDENTIFY and SET UP:** 0.7% of naturally occurring uranium is the isotope ^{235}U . The mass of one ^{235}U nucleus is about $235m_p$.

EXECUTE: (a) The number of fissions needed is $\frac{1.0 \times 10^{19} \text{ J}}{(200 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 3.13 \times 10^{29}$. The mass of

^{235}U required is $(3.13 \times 10^{29})(235m_p) = 1.23 \times 10^5 \text{ kg}$.

(b) $\frac{1.23 \times 10^5 \text{ kg}}{0.7 \times 10^{-2}} = 1.76 \times 10^7 \text{ kg}$

EVALUATE: The calculation assumes 100% conversion of fission energy to electrical energy.

- 43.45. IDENTIFY and SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to 931.5 MeV. The mass of one ^{235}U nucleus is $235m_p$.

EXECUTE: (a) $^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^1_0\text{n}$

We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is

$$\Delta M = m(^{235}_{92}\text{U}) + m({}^1_0\text{n}) - [m(^{144}_{56}\text{Ba}) + m(^{89}_{36}\text{Kr}) + 3m({}^1_0\text{n})]$$

$$\Delta M = 235.043930 \text{ u} + 1.0086649 \text{ u} - 143.922953 \text{ u} - 88.917630 \text{ u} - 3(1.0086649 \text{ u})$$

$$\Delta M = 0.1860 \text{ u}. \text{ The energy released is } (0.1860 \text{ u})(931.5 \text{ MeV/u}) = 173.3 \text{ MeV}.$$

(b) The number of ^{235}U nuclei in 1.00 g is $\frac{1.00 \times 10^{-3} \text{ kg}}{235m_p} = 2.55 \times 10^{21}$. The energy released per gram is

$$(173.3 \text{ MeV/nucleus})(2.55 \times 10^{21} \text{ nuclei/g}) = 4.42 \times 10^{23} \text{ MeV/g}.$$

- 43.46. (a)** $^{28}_{14}\text{Si} + \gamma \rightarrow ^{24}_{12}\text{Mg} + {}^4_Z\text{X}$. $A + 24 = 28$ so $A = 4$. $Z + 12 = 14$ so $Z = 2$. X is an α particle.

(b) $E_\gamma = -\Delta mc^2 = (23.985042 \text{ u} + 4.002603 \text{ u} - 27.976927 \text{ u})(931.5 \text{ MeV/u}) = 9.984 \text{ MeV}$

- 43.47.** The energy liberated will be

$$M({}^3_2\text{He}) + M({}^4_2\text{He}) - M({}^7_4\text{Be}) = (3.016029 \text{ u} + 4.002603 \text{ u} - 7.016929 \text{ u})(931.5 \text{ MeV/u}) = 1.586 \text{ MeV}.$$

- 43.48. (a)** $Z = 3 + 2 - 0 = 5$ and $A = 4 + 7 - 1 = 10$.

(b) The nuclide is a boron nucleus, and $m_{\text{He}} + m_{\text{Li}} - m_{\text{n}} - m_{\text{B}} = -3.00 \times 10^{-3} \text{ u}$, and so 2.79 MeV of energy is absorbed.

- 43.49.** Nuclei: ${}^A_Z\text{X}^{Z+} \rightarrow {}^{A-4}_{Z-2}\text{Y}^{(Z-2)+} + {}^4_2\text{He}^{2+}$. Add the mass of Z electrons to each side and we find:

$\Delta m = M({}^A_Z\text{X}) - M({}^{A-4}_{Z-2}\text{Y}) - M({}^4_2\text{He})$, where now we have the mass of the neutral atoms. So as long as the mass of the original neutral atom is greater than the sum of the neutral products masses, the decay can happen.

- 43.50.** Denote the reaction as ${}^A_Z\text{X} \rightarrow {}^A_{Z+1}\text{Y} + e^-$. The mass defect is related to the change in the neutral atomic masses by

$$[m_X - Zm_e] - [m_Y - (Z+1)m_e] - m_e = (m_X - m_Y),$$

where m_X and m_Y are the masses as tabulated in, for instance, Table (43.2).

- 43.51.** ${}^A_Z\text{X}^{Z+} \rightarrow {}^A_{Z-1}\text{Y}^{(Z-1)+} + \beta^+$. Adding $(Z-1)$ electrons to both sides yields ${}^A_Z\text{X}^+ \rightarrow {}^A_{Z-1}\text{Y} + \beta^+$. So in terms of masses:

$\Delta m = M({}^A_Z\text{X}^+) - M({}^A_{Z-1}\text{Y}) - m_e = (M({}^A_Z\text{X}) - m_e) - M({}^A_{Z-1}\text{Y}) - m_e = M({}^A_Z\text{X}) - M({}^A_{Z-1}\text{Y}) - 2m_e$. So the decay will occur as long as the original neutral mass is greater than the sum of the neutral product mass and two electron masses.

- 43.52. IDENTIFY and SET UP:** $m = \rho V$. 1 gal = 3.788 L = $3.788 \times 10^{-3} \text{ m}^3$. The mass of a ^{235}U nucleus is $235m_p$.

$$1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$$

EXECUTE: (a) For 1 gallon, $m = \rho V = (737 \text{ kg/m}^3)(3.788 \times 10^{-3} \text{ m}^3) = 2.79 \text{ kg} = 2.79 \times 10^3 \text{ g}$

$$\frac{1.3 \times 10^8 \text{ J/gal}}{2.79 \times 10^3 \text{ g/gal}} = 4.7 \times 10^4 \text{ J/g}$$

(b) 1 g contains $\frac{1.00 \times 10^{-3} \text{ kg}}{235m_p} = 2.55 \times 10^{21}$ nuclei

$(200 \text{ MeV/nucleus})(1.60 \times 10^{-13} \text{ J/MeV})(2.55 \times 10^{21} \text{ nuclei}) = 8.2 \times 10^{10} \text{ J/g}$

(c) A mass of $6m_p$ produces 26.7 MeV.

$$\frac{(26.7 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{6m_p} = 4.26 \times 10^{14} \text{ J/kg} = 4.26 \times 10^{11} \text{ J/g}$$

(d) The total energy available would be $(1.99 \times 10^{30} \text{ kg})(4.7 \times 10^7 \text{ J/kg}) = 9.4 \times 10^{37} \text{ J}$

$$\text{power} = \frac{\text{energy}}{t} \text{ so } t = \frac{\text{energy}}{\text{power}} = \frac{9.4 \times 10^{37} \text{ J}}{3.86 \times 10^{26} \text{ W}} = 2.4 \times 10^{11} \text{ s} = 7600 \text{ yr}$$

EVALUATE: If the mass of the sun were all proton fuel, it would contain enough fuel to last

$$(7600 \text{ yr}) \left(\frac{4.3 \times 10^{11} \text{ J/g}}{4.7 \times 10^4 \text{ J/g}} \right) = 7.0 \times 10^{10} \text{ yr}.$$

43.53. Using Eq: (43.12): ${}_Z^A M = ZM_H + Nm_n - E_B/c^2 \Rightarrow M({}_{11}^{24}\text{Na}) = 11M_H + 13m_n - E_B/c^2$.

$$\text{But } E_B = (15.75 \text{ MeV})(24) - (17.80 \text{ MeV})(24)^{2/3} - (0.7100 \text{ MeV}) \frac{(11)(10)}{(24)^{1/3}} -$$

$$(23.69 \text{ MeV}) \frac{(24 - 2(11))^2}{24} - (39 \text{ MeV})(24)^{-4/3} = 198.31 \text{ MeV}.$$

$$\Rightarrow M({}_{11}^{24}\text{Na}) = 11(1.007825 \text{ u}) + 13(1.008665 \text{ u}) - \frac{(198.31 \text{ MeV})}{931.5 \text{ MeV/u}} = 23.9858 \text{ u}$$

$$\% \text{ error} = \frac{23.990963 - 23.9858}{23.990963} \times 100 = 0.022\%.$$

If the binding energy term is neglected, $M({}_{11}^{24}\text{Na}) = 24.1987 \text{ u}$ and the percentage error would be

$$\frac{24.1987 - 23.990963}{23.990963} \times 100 = 0.87\%.$$

43.54. The α -particle will have $\frac{226}{230}$ of the released energy (see Example 43.5). $\frac{226}{230}(m_{\text{Th}} - m_{\text{Ra}} - m_{\alpha}) = 5.032 \times 10^{-3} \text{ u}$ or 4.69 MeV.

43.55. (a) IDENTIFY and SET UP: The heavier nucleus will decay into the lighter one.

EXECUTE: ${}_{13}^{25}\text{Al}$ will decay into ${}_{12}^{25}\text{Mg}$.

(b) IDENTIFY and SET UP: Determine the emitted particle by balancing A and Z in the decay reaction.

EXECUTE: This gives ${}_{13}^{25}\text{Al} \rightarrow {}_{12}^{25}\text{Mg} + {}_{+1}^0\text{e}$. The emitted particle must have charge $+e$ and its nucleon number must be zero. Therefore, it is a β^+ particle, a positron.

(c) IDENTIFY and SET UP: Calculate the energy defect ΔM for the reaction and find the energy equivalent of ΔM . Use the nuclear masses for ${}_{13}^{25}\text{Al}$ and ${}_{12}^{25}\text{Mg}$, to avoid confusion in including the correct number of electrons if neutral atom masses are used.

EXECUTE: The nuclear mass for ${}_{13}^{25}\text{Al}$ is $M_{\text{nuc}}({}_{13}^{25}\text{Al}) = 24.990429 \text{ u} - 13(0.000548580 \text{ u}) = 24.983297 \text{ u}$.

The nuclear mass for ${}_{12}^{25}\text{Mg}$ is $M_{\text{nuc}}({}_{12}^{25}\text{Mg}) = 24.985837 \text{ u} - 12(0.000548580 \text{ u}) = 24.979254 \text{ u}$.

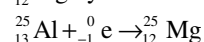
The mass defect for the reaction is

$$\Delta M = M_{\text{nuc}}({}_{13}^{25}\text{Al}) - M_{\text{nuc}}({}_{12}^{25}\text{Mg}) - M({}_{+1}^0\text{e}) = 24.983297 \text{ u} - 24.979254 \text{ u} - 0.00054858 \text{ u} = 0.003494 \text{ u}$$

$$Q = (\Delta M)c^2 = 0.003494 \text{ u}(931.5 \text{ MeV/1 u}) = 3.255 \text{ MeV}$$

EVALUATE: The mass decreases in the decay and energy is released. Note: ${}_{13}^{25}\text{Al}$ can also decay into

${}_{12}^{25}\text{Mg}$ by the electron capture.



The ${}_{-1}^0\text{e}$ electron in the reaction is an orbital electron in the neutral ${}_{13}^{25}\text{Al}$ atom. The mass defect can be calculated using the nuclear masses:

$$\Delta M = M_{\text{nuc}}({}_{13}^{25}\text{Al}) + M({}_{-1}^0\text{e}) - M_{\text{nuc}}({}_{12}^{25}\text{Mg}) = 24.983287 \text{ u} + 0.00054858 \text{ u} - 24.979254 \text{ u} = 0.004592 \text{ u}.$$

$$Q = (\Delta M)c^2 = (0.004592 \text{ u})(931.5 \text{ MeV/1 u}) = 4.277 \text{ MeV}$$

The mass decreases in the decay and energy is released.

43.56. (a) $m_{84}^{210}\text{Po} - m_{82}^{206}\text{Pb} - m_{2}^{4}\text{He} = 5.81 \times 10^{-3} \text{ u}$, or $Q = 5.41 \text{ MeV}$. The energy of the alpha particle is $(206/210)$ times this, or 5.30 MeV (see Example 43.5).

(b) $m_{84}^{210}\text{Po} - m_{83}^{209}\text{Bi} - m_{1}^{1}\text{H} = -5.35 \times 10^{-3} \text{ u} < 0$, so the decay is not possible.

(c) $m_{84}^{210}\text{Po} - m_{84}^{209}\text{Po} - m_{0}^{0} = -8.22 \times 10^{-3} \text{ u} < 0$, so the decay is not possible.

(d) $m_{85}^{210}\text{At} > m_{84}^{210}\text{Po}$, so the decay is not possible (see Problem (43.50)).

(e) $m_{83}^{210}\text{Bi} + 2m_e > m_{84}^{210}\text{Po}$, so the decay is not possible (see Problem (43.51)).

43.57. IDENTIFY and SET UP: The amount of kinetic energy released is the energy equivalent of the mass change in the decay. $m_e = 0.0005486 \text{ u}$ and the atomic mass of ${}^{14}_7\text{N}$ is 14.003074 u . The energy equivalent of 1 u is 931.5 MeV . ${}^{14}_6\text{C}$ has a half-life of $T_{1/2} = 5730 \text{ yr} = 1.81 \times 10^{11} \text{ s}$. The RBE for an electron is 1.0 .

EXECUTE: (a) ${}^{14}_6\text{C} \rightarrow e^- + {}^{14}_7\text{N} + \bar{\nu}_e$

(b) The mass decrease is $\Delta M = m({}^{14}_6\text{C}) - [m_e + m({}^{14}_7\text{N})]$. Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of ${}^{14}_6\text{C}$ is $14.003242 \text{ u} - 6m_e = 13.999950 \text{ u}$.

The nuclear mass of ${}^{14}_7\text{N}$ is $14.003074 \text{ u} - 7m_e = 13.999234 \text{ u}$.

$\Delta M = 13.999950 \text{ u} - 13.999234 \text{ u} - 0.000549 \text{ u} = 1.67 \times 10^{-4} \text{ u}$. The energy equivalent of ΔM is 0.156 MeV .

(c) The mass of carbon is $(0.18)(75 \text{ kg}) = 13.5 \text{ kg}$. From Example 43.9, the activity due to 1 g of carbon in a living organism is 0.255 Bq . The number of decay/s due to 13.5 kg of carbon is $(13.5 \times 10^3)(0.255 \text{ Bq/g}) = 3.4 \times 10^3 \text{ decays/s}$.

(d) Each decay releases 0.156 MeV so $3.4 \times 10^3 \text{ decays/s}$ releases $530 \text{ MeV/s} = 8.5 \times 10^{-11} \text{ J/s}$.

(e) The total energy absorbed in 1 yr is $(8.5 \times 10^{-11} \text{ J/s})(3.156 \times 10^7 \text{ s}) = 2.7 \times 10^{-3} \text{ J}$. The absorbed dose is $\frac{2.7 \times 10^{-3} \text{ J}}{75 \text{ kg}} = 3.6 \times 10^{-5} \text{ J/kg} = 36 \mu\text{Gy} = 3.6 \text{ mrad}$. With $\text{RBE} = 1.0$, the equivalent dose is $36 \mu\text{Sv} = 3.6 \text{ mrem}$.

43.58. IDENTIFY and SET UP: $m_\pi = 264m_e = 2.40 \times 10^{-28} \text{ kg}$. The total energy of the two photons equals the rest mass energy $m_\pi c^2$ of the pion.

EXECUTE: (a) $E_{\text{ph}} = \frac{1}{2}m_\pi c^2 = \frac{1}{2}(2.40 \times 10^{-28} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 1.08 \times 10^{-11} \text{ J} = 67.5 \text{ MeV}$

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E_{\text{ph}}} = \frac{1.24 \times 10^{-6} \text{ eV} \cdot \text{m}}{67.5 \times 10^6 \text{ eV}} = 1.84 \times 10^{-14} \text{ m} = 18.4 \text{ fm}$$

These are gamma ray photons, so they have $\text{RBE} = 1.0$.

(b) Each pion delivers $2(1.08 \times 10^{-11} \text{ J}) = 2.16 \times 10^{-11} \text{ J}$.

The absorbed dose is $200 \text{ rad} = 2.00 \text{ Gy} = 2.00 \text{ J/kg}$.

The energy deposited is $(25 \times 10^{-3} \text{ kg})(2.00 \text{ J/kg}) = 0.050 \text{ J}$.

The number of π^0 mesons needed is $\frac{0.050 \text{ J}}{2.16 \times 10^{-11} \text{ J/meson}} = 2.3 \times 10^9 \text{ mesons}$.

EVALUATE: Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same λ and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and $E = pc$.

43.59. IDENTIFY and SET UP: Find the energy equivalent of the mass decrease. Part of the released energy appears as the emitted photon and the rest as kinetic energy of the electron.

EXECUTE: ${}^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^0_{-1}\text{e}$

The mass change is $197.968225 \text{ u} - 197.966752 \text{ u} = 1.473 \times 10^{-3} \text{ u}$

(The neutral atom masses include 79 electrons before the decay and 80 electrons after the decay. This one additional electron in the products accounts correctly for the electron emitted by the nucleus.) The total energy released in the decay is $(1.473 \times 10^{-3} \text{ u})(931.5 \text{ MeV/u}) = 1.372 \text{ MeV}$. This energy is divided between the energy of the emitted photon and the kinetic energy of the β^- particle. Thus the β^- particle has kinetic energy equal to $1.372 \text{ MeV} - 0.412 \text{ MeV} = 0.960 \text{ MeV}$.

EVALUATE: The emitted electron is much lighter than the ${}^{198}_{80}\text{Hg}$ nucleus, so the electron has almost all the final kinetic energy. The final kinetic energy of the ${}^{198}_{80}\text{Hg}$ nucleus is very small.

43.60. (See Problem (43.51)) $m_{^{11}\text{C}} - m_{^{11}\text{B}} - 2m_e = 1.03 \times 10^{-3} \text{ u}$. Decay is energetically possible.

43.61. IDENTIFY and SET UP: The decay is energetically possible if the total mass decreases. Determine the nucleus produced by the decay by balancing A and Z on both sides of the equation. $^{13}_7\text{N} \rightarrow ^0_{+1}\text{e} + ^{13}_6\text{C}$. To avoid confusion in including the correct number of electrons with neutral atom masses, use nuclear masses, obtained by subtracting the mass of the atomic electrons from the neutral atom masses.

EXECUTE: The nuclear mass for $^{13}_7\text{N}$ is $M_{\text{nuc}}(^{13}_7\text{N}) = 13.005739 \text{ u} - 7(0.00054858 \text{ u}) = 13.001899 \text{ u}$.

The nuclear mass for $^{13}_6\text{C}$ is $M_{\text{nuc}}(^{13}_6\text{C}) = 13.003355 \text{ u} - 6(0.00054858 \text{ u}) = 13.000064 \text{ u}$.

The mass defect for the reaction is

$$\Delta M = M_{\text{nuc}}(^{13}_7\text{N}) - M_{\text{nuc}}(^{13}_6\text{C}) - M(^0_{+1}\text{e}). \Delta M = 13.001899 \text{ u} - 13.000064 \text{ u} - 0.00054858 \text{ u} = 0.001286 \text{ u}.$$

EVALUATE: The mass decreases in the decay, so energy is released. This decay is energetically possible.

43.62. (a) A least-squares fit to log of the activity vs. time gives a slope of $\lambda = 0.5995 \text{ h}^{-1}$, for a half-life of $\frac{\ln 2}{\lambda} = 1.16 \text{ h}$.

(b) The initial activity is $N_0\lambda$, and this gives $N_0 = \frac{(2.00 \times 10^4 \text{ Bq})}{(0.5995 \text{ hr}^{-1})(1 \text{ hr}/3600 \text{ s})} = 1.20 \times 10^8$.

(c) $N_0 e^{-\lambda t} = 1.81 \times 10^6$.

43.63. The activity $A(t) \equiv \frac{dN(t)}{dt}$ but $\frac{dN(t)}{dt} = -\lambda N(t)$ so $-\lambda N_0 = A_0$. Taking the derivative of

$$N(t) = N_0 e^{-\lambda t} \Rightarrow \frac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}, \text{ or } A(t) = A_0 e^{-\lambda t}.$$

43.64. From Eq.43.17 $N(t) = N_0 e^{-\lambda t}$ but $N_0 e^{-\lambda t} = N_0 e^{-(\ln 2)(t/T_{1/2})}$

$$= N_0 \left[e^{-(\ln 2)} \right]^{(t/T_{1/2})} = N_0 \left[e^{\ln(1/2)} \right]^{(t/T_{1/2})}. \text{ So } N(t) = N_0 \left(\frac{1}{2} \right)^n \text{ where } n = \frac{t}{T_{1/2}}.$$

(We have used that $a \ln x = \ln(x^a)$, $e^{ax} = (e^x)^a$, and $e^{\ln x} = x$.)

43.65. IDENTIFY and SET UP: One-half of the sample decays in a time of $T_{1/2}$.

EXECUTE: (a) $\frac{10 \times 10^9 \text{ yr}}{200,000 \text{ yr}} = 5.0 \times 10^4$

(b) $(\frac{1}{2})^{5.0 \times 10^4}$. This exponent is too large for most hand-held calculators. But $(\frac{1}{2}) = 10^{-0.301}$ so

$$(\frac{1}{2})^{5.0 \times 10^4} = (10^{-0.301})^{5.0 \times 10^4} = 10^{-15,000}$$

43.66. IDENTIFY and SET UP: $T_{1/2} = \frac{\ln 2}{\lambda}$. The mass of a single nucleus is $149m_p = 2.49 \times 10^{-25} \text{ kg}$. $\Delta N / \Delta t = -\lambda N$.

EXECUTE: $N = \frac{12.0 \times 10^{-3} \text{ kg}}{2.49 \times 10^{-25} \text{ kg}} = 4.82 \times 10^{22}$. $\Delta N / \Delta t = -2.65 \text{ decays/s}$

$$\lambda = -\frac{\Delta N / \Delta t}{N} = \frac{2.65 \text{ decays/s}}{4.82 \times 10^{22}} = 5.50 \times 10^{-23} \text{ s}^{-1}; \quad T_{1/2} = \frac{\ln 2}{\lambda} = 1.26 \times 10^{22} \text{ s} = 3.99 \times 10^{14} \text{ yr}$$

43.67. IDENTIFY: Use Eq. (43.17) to relate the initial number of radioactive nuclei, N_0 , to the number, N , left after time t .

SET UP: We have to be careful; after ^{87}Rb has undergone radioactive decay it is no longer a rubidium atom. Let N_{85} be the number of ^{85}Rb atoms; this number doesn't change. Let N_0 be the number of ^{87}Rb atoms on earth when the solar system was formed. Let N be the present number of ^{87}Rb atoms.

EXECUTE: The present measurements say that $0.2783 = N / (N + N_{85})$.

$(N + N_{85})(0.2783) = N$, so $N = 0.3856 N_{85}$. The percentage we are asked to calculate is $N_0 / (N_0 + N_{85})$.

N and N_0 are related by $N = N_0 e^{-\lambda t}$ so $N_0 = e^{+\lambda t} N$.

$$\text{Thus } \frac{N_0}{N_0 + N_{85}} = \frac{N e^{\lambda t}}{N e^{\lambda t} + N_{85}} = \frac{(0.3856 e^{\lambda t}) N_{85}}{(0.3856 e^{\lambda t}) N_{85} + N_{85}} = \frac{0.3856 e^{\lambda t}}{0.3856 e^{\lambda t} + 1}.$$

$$t = 4.6 \times 10^9 \text{ y}; \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4.75 \times 10^{10} \text{ y}} = 1.459 \times 10^{-11} \text{ y}^{-1}$$

$$e^{\lambda t} = e^{(1.459 \times 10^{-11} \text{ y}^{-1})(4.6 \times 10^9 \text{ y})} = e^{0.16711} = 1.0694$$

$$\text{Thus } \frac{N_0}{N_0 + N_{85}} = \frac{(0.3856)(1.0694)}{(0.3856)(1.0694) + 1} = 29.2\%.$$

EVALUATE: The half-life for ^{87}Rb is a factor of 10 larger than the age of the solar system, so only a small fraction of the ^{87}Rb nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

43.68. (a) $(6.25 \times 10^{12})(4.77 \times 10^6 \text{ MeV})(1.602 \times 10^{-19} \text{ J/eV}) / (70.0 \text{ kg}) = 0.0682 \text{ Gy} = 0.682 \text{ rad}$

(b) $(20)(6.82 \text{ rad}) = 136 \text{ rem}$

(c) $N\lambda = \frac{m \ln(2)}{Am_p T_{1/2}} = 1.17 \times 10^9 \text{ Bq} = 31.6 \text{ mCi}.$

(d) $\frac{6.25 \times 10^{12}}{1.17 \times 10^9 \text{ Bq}} = 5.34 \times 10^3 \text{ s}$, about an hour and a half. Note that this time is so small in comparison with the

half-life that the decrease in activity of the source may be neglected.

43.69. IDENTIFY and SET UP: Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

EXECUTE: (a) First find the number of decays each second:

$$2.6 \times 10^{-4} \text{ Ci} \left(\frac{3.70 \times 10^{10} \text{ decays/s}}{1 \text{ Ci}} \right) = 9.6 \times 10^6 \text{ decays/s}$$

The average energy per decay is 1.25 MeV, and one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is

$$\frac{1}{2}(9.6 \times 10^6 \text{ decays/s})(1.25 \times 10^6 \text{ eV/decay})(1.602 \times 10^{-19} \text{ J/eV}) = 9.6 \times 10^{-7} \text{ J/s}.$$

(b) The absorbed dose is the energy absorbed divided by the mass of the tissue:

$$\frac{9.6 \times 10^{-7} \text{ J/s}}{0.500 \text{ kg}} = (1.9 \times 10^{-6} \text{ J/kg} \cdot \text{s})(1 \text{ rad}/(0.01 \text{ J/kg})) = 1.9 \times 10^{-4} \text{ rad/s}$$

(c) equivalent dose (REM) = RBE $\times$ absorbed dose (rad)

In one second the equivalent dose is $0.70(1.9 \times 10^{-4} \text{ rad}) = 1.3 \times 10^{-4} \text{ rem}$.

(d) $(200 \text{ rem}/1.3 \times 10^{-4} \text{ rem/s}) = 1/5 \times 10^6 \text{ s} (1 \text{ h}/3600 \text{ s}) = 420 \text{ h} = 17 \text{ days}$.

EVALUATE: The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200 rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

43.70. (a) After 4.0 min = 240 s, the ratio of the number of nuclei is $\frac{2^{-240/122.2}}{2^{-240/26.9}} = 2^{(240)\left(\frac{1}{26.9} - \frac{1}{122.2}\right)} = 124$.

(b) After 15.0 min = 900 s, the ratio is 7.15×10^7 .

43.71. IDENTIFY and SET UP: The number of radioactive nuclei left after time t is given by $N = N_0 e^{-\lambda t}$. The problem says $N/N_0 = 0.21$; solve for t .

EXECUTE: $0.21 = e^{-\lambda t}$ so $\ln(0.21) = -\lambda t$ and $t = -\ln(0.21)/\lambda$

Example 43.9 gives $\lambda = 1.209 \times 10^{-4} \text{ y}^{-1}$ for ^{14}C . Thus $t = \frac{-\ln(0.21)}{1.209 \times 10^{-4} \text{ y}} = 1.3 \times 10^4 \text{ y}$.

EVALUATE: The half-life of ^{14}C is 5730 y, so our calculated t is more than two half-lives, so the fraction

remaining is less than $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

43.72. IDENTIFY: The tritium (H-3) decays to He-3. The ratio of the number of He-3 atoms to H-3 atoms allows us to calculate the time since the decay began, which is when the H-3 was formed by the nuclear explosion. The H-3 decay is exponential.

SET UP: The number of tritium (H-3) nuclei decreases exponentially as $N_{\text{H}} = N_{0,\text{H}} e^{-\lambda t}$, with a half-life of 12.3 years. The amount of He-3 present after a time t is equal to the original amount of tritium minus the number of tritium nuclei that are still undecayed after time t .

EXECUTE: The number of He-3 nuclei after time t is

$$N_{\text{He}} = N_{0,\text{H}} - N_{\text{H}} = N_{0,\text{H}} - N_{0,\text{H}} e^{-\lambda t} = N_{0,\text{H}} (1 - e^{-\lambda t}).$$

Taking the ratio of the number of He-3 atoms to the number of tritium (H-3) atoms gives

$$\frac{N_{\text{He}}}{N_{\text{H}}} = \frac{N_{0,\text{H}} (1 - e^{-\lambda t})}{N_{0,\text{H}} e^{-\lambda t}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

Solving for t gives $t = \frac{\ln(1 + N_{\text{He}}/N_{\text{H}})}{\lambda}$. Using the given numbers and $T_{1/2} = \frac{\ln 2}{\lambda}$, we have

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12.3 \text{ y}} = 0.0563/\text{y} \text{ and } t = \frac{\ln(1 + 4.3)}{0.0563/\text{y}} = 30 \text{ years.}$$

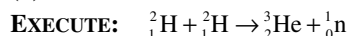
EVALUATE: One limitation on this method would be that after many years the ratio of H to He would be too small to measure accurately.

- 43.73. (a) IDENTIFY and SET UP:** Use Eq.(43.1) to calculate the radius R of a ${}^2_1\text{H}$ nucleus. Calculate the Coulomb potential energy (Eq.23.9) of the two nuclei when they just touch.

EXECUTE: The radius of ${}^2_1\text{H}$ is $R = (1.2 \times 10^{-15} \text{ m})(2)^{1/3} = 1.51 \times 10^{-15} \text{ m}$. The barrier energy is the Coulomb potential energy of two ${}^2_1\text{H}$ nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.602 \times 10^{-19} \text{ C})^2}{2(1.51 \times 10^{-15} \text{ m})} = 7.64 \times 10^{-14} \text{ J} = 0.48 \text{ MeV}$$

(b) IDENTIFY and SET UP: Find the energy equivalent of the mass decrease.



If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

$${}^2_1\text{H} + {}^2_1\text{H} \text{ has mass } 2(2.014102 \text{ u}) = 4.028204 \text{ u}$$

$${}^3_2\text{He} + {}^1_0\text{n} \text{ has mass } 3.016029 \text{ u} + 1.008665 \text{ u} = 4.024694 \text{ u}$$

The mass decrease is $4.028204 \text{ u} - 4.024694 \text{ u} = 3.510 \times 10^{-3} \text{ u}$. This corresponds to a liberated energy of $(3.510 \times 10^{-3} \text{ u})(931.5 \text{ MeV/u}) = 3.270 \text{ MeV}$, or $(3.270 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV}) = 5.239 \times 10^{-13} \text{ J}$.

(c) IDENTIFY and SET UP: We know the energy released when two ${}^2_1\text{H}$ nuclei fuse. Find the number of reactions obtained with one mole of ${}^2_1\text{H}$.

EXECUTE: Each reaction takes two ${}^2_1\text{H}$ nuclei. Each mole of D_2 has 6.022×10^{23} molecules, so 6.022×10^{23} pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is $(6.022 \times 10^{23})(5.239 \times 10^{-13} \text{ J}) = 3.155 \times 10^{11} \text{ J/mol}$.

EVALUATE: The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

- 43.74.** In terms of the number N of cesium atoms that decay in one week and the mass $m = 1.0 \text{ kg}$, the equivalent dose is

$$3.5 \text{ Sv} = \frac{N}{m} ((\text{RBE})_\gamma E_\gamma + (\text{RBE})_e E_e) = \frac{N}{m} ((1)(0.66 \text{ MeV}) + (1.5)(0.51 \text{ MeV})) = \frac{N}{m} (2.283 \times 10^{-13} \text{ J}), \text{ so}$$

$$N = \frac{(1.0 \text{ kg})(3.5 \text{ Sv})}{(2.283 \times 10^{-13} \text{ J})} = 1.535 \times 10^{13}. \text{ The number } N_0 \text{ of atoms present is related to}$$

$$N \text{ by } N_0 = Ne^{\lambda t}. \lambda = \frac{\ln 2}{T_{y_2}} = \frac{0.693}{(30.07 \text{ yr})(3.156 \times 10^7 \text{ sec/yr})} = 7.30 \times 10^{-10} \text{ sec}^{-1}.$$

$$\text{Then } N_0 = Ne^{\lambda t} = (1.535 \times 10^{13})e^{(7.30 \times 10^{-10} \text{ s}^{-1})(7 \text{ days})(8.64 \times 10^4 \text{ s/day})} = 1.536 \times 10^{13}.$$

- 43.75. (a)** $v_{\text{cm}} = v \frac{m}{m+M}$. $v'_m = v - v \frac{m}{m+M} = \left(\frac{M}{m+M} \right) v$. $v'_M = \frac{vm}{m+M}$.

$$K' = \frac{1}{2} m v_m'^2 + \frac{1}{2} M v_M'^2 = \frac{1}{2} \frac{mM^2}{(m+M)^2} v^2 + \frac{1}{2} \frac{Mm^2}{(m+M)^2} v^2 = \frac{1}{2} \frac{M}{(m+M)} \left(\frac{mM}{m+M} + \frac{m^2}{m+M} \right) v^2.$$

$$K' = \frac{M}{m+M} \left(\frac{1}{2} m v^2 \right) \Rightarrow K' = \frac{M}{m+M} K \equiv K_{\text{cm}}.$$

(b) For an endoergic reaction $K_{\text{cm}} = -Q (Q < 0)$ at threshold. Putting this into part (a) gives

$$-Q = \frac{M}{M+m} K_{\text{th}} \Rightarrow K_{\text{th}} = \frac{-(M+m)}{M} Q$$

- 43.76. $K = \frac{M_\alpha}{M_\alpha + m} K_\infty$, where K_∞ is the energy that the α -particle would have if the nucleus were infinitely massive.

$$\text{Then, } M = M_{\text{Os}} - M_\alpha - K_\infty = M_{\text{Os}} - M_\alpha - \frac{186}{182} (2.76 \text{ MeV}/c^2) = 181.94821 \text{ u.}$$

- 43.77. $\Delta m = M({}^{235}_{92}\text{U}) - M({}^{140}_{54}\text{Xe}) - M({}^{94}_{38}\text{Sr}) - m_n$
 $\Delta m = 235.043923 \text{ u} - 139.921636 \text{ u} - 93.915360 \text{ u} - 1.008665 \text{ u} = 0.1983 \text{ u}$
 $\Rightarrow E = (\Delta m)c^2 = (0.1983 \text{ u})(931.5 \text{ MeV/u}) = 185 \text{ MeV.}$

- 43.78. (a) A least-squares fit of the log of the activity vs. time for the times later than 4.0 h gives a fit with correlation $-(1 - 2 \times 10^{-6})$ and decay constant of 0.361 h^{-1} , corresponding to a half-life of 1.92 h. Extrapolating this back to time 0 gives a contribution to the rate of about 2500/s for this longer-lived species. A least-squares fit of the log of the activity vs. time for times earlier than 2.0 h gives a fit with correlation = 0.994, indicating the presence of only two species.
 (b) By trial and error, the data is fit by a decay rate modeled by $R = (5000 \text{ Bq})e^{-t(1.733/\text{h})} + (2500 \text{ Bq})e^{-t(0.361/\text{h})}$. This would correspond to half-lives of 0.400 h and 1.92 h.
 (c) In this model, there are 1.04×10^7 of the shorter-lived species and 2.49×10^7 of the longer-lived species.
 (d) After 5.0 h, there would be 1.80×10^3 of the shorter-lived species and 4.10×10^6 of the longer-lived species.

- 43.79. (a) There are two processes occurring: the creation of ${}^{128}\text{I}$ by the neutron irradiation, and the decay of the newly produced ${}^{128}\text{I}$. So $\frac{dN}{dt} = K - \lambda N$ where K is the rate of production by the neutron irradiation. Then

$$\int_0^N \frac{dN'}{K - \lambda N'} = \int_0^t dt. \quad [\ln(K - \lambda N')]_0^N = -\lambda t. \quad \ln(K - \lambda N) = \ln K - \lambda t. \quad N(t) = \frac{K(1 - e^{-\lambda t})}{\lambda}. \quad \text{The graph is given in Figure 43.79.}$$

- (b) The activity of the sample is $\lambda N(t) = K(1 - e^{-\lambda t}) = (1.5 \times 10^6 \text{ decays/s}) \times \left(1 - e^{-\left(\frac{0.693}{25 \text{ min}}\right)t}\right)$. So the activity is

$$(1.5 \times 10^6 \text{ decays/s})(1 - e^{-0.02772t}), \text{ with } t \text{ in minutes. So the activity } \left(\frac{-dN'}{dt}\right) \text{ at various times is:}$$

$$\begin{aligned} \frac{-dN'}{dt}(t = 1 \text{ min}) &= 4.1 \times 10^4 \text{ Bq}; & \frac{-dN'}{dt}(t = 10 \text{ min}) &= 3.6 \times 10^5 \text{ Bq}; \\ \frac{-dN'}{dt}(t = 25 \text{ min}) &= 7.5 \times 10^5 \text{ Bq}; & \frac{-dN'}{dt}(t = 50 \text{ min}) &= 1.1 \times 10^6 \text{ Bq}; \\ \frac{-dN'}{dt}(t = 75 \text{ min}) &= 1.3 \times 10^6 \text{ Bq}; & \frac{-dN'}{dt}(t = 180 \text{ min}) &= 1.5 \times 10^6 \text{ Bq}; \end{aligned}$$

$$(c) N_{\text{max}} = \frac{K}{\lambda} = \frac{(1.5 \times 10^6)(60)}{(0.02772)} = 3.2 \times 10^9 \text{ atoms.}$$

- (d) The maximum activity is at saturation, when the rate being produced equals that decaying and so it equals $1.5 \times 10^6 \text{ decays/s}$.

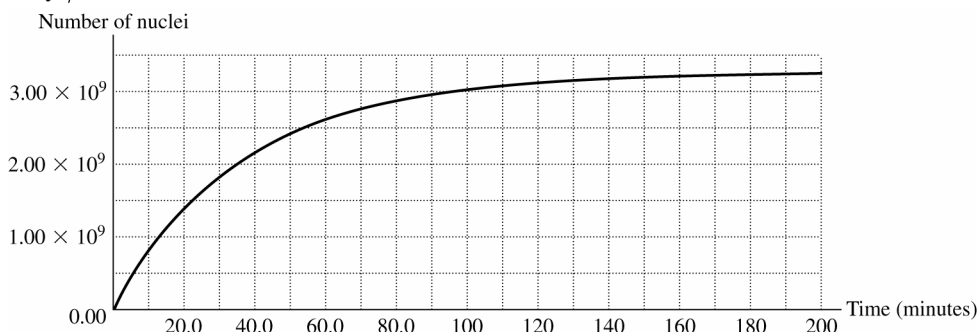


Figure 43.79

- 43.80. The activity of the original iron, after 1000 hours of operation, would be $(9.4 \times 10^{-6} \text{ Ci})(3.7 \times 10^{10} \text{ Bq/Ci})2^{-(1000 \text{ h})/(45 \text{ d} \times 24 \text{ h/d})} = 1.8306 \times 10^5 \text{ Bq}$. The activity of the oil is 84 Bq, or 4.5886×10^{-4} of the total iron activity, and this must be the fraction of the mass worn, or mass of $4.59 \times 10^{-2} \text{ g}$. The rate at which the piston rings lost their mass is then $4.59 \times 10^{-5} \text{ g/h}$.

PARTICLE PHYSICS AND COSMOLOGY

- 44.1. (a) IDENTIFY and SET UP:** Use Eq.(37.36) to calculate the kinetic energy K .

EXECUTE: $K = mc^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) = 0.1547mc^2$

$m = 9.109 \times 10^{-31} \text{ kg}$, so $K = 1.27 \times 10^{-14} \text{ J}$

- (b) IDENTIFY and SET UP:** The total energy of the particles equals the sum of the energies of the two photons. Linear momentum must also be conserved.

EXECUTE: The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 = 9.46 \times 10^{-13} \text{ J}$. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy $9.46 \times 10^{-14} \text{ J}$.

- (c) IDENTIFY and SET UP:** Use Eq. (38.2) to relate the photon energy to the photon wavelength.

EXECUTE: $E = hc/\lambda$ so $\lambda = hc/E = hc/(9.46 \times 10^{-14} \text{ J}) = 2.10 \text{ pm}$

EVALUATE: The wavelength calculated in Example 44.1 is 2.43 pm. When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

- 44.2.** The total energy of the positron is

$$E = K + mc^2 = 5.00 \text{ MeV} + 0.511 \text{ MeV} = 5.51 \text{ MeV}.$$

We can calculate the speed of the positron from Eq.(37.38):

$$E = \frac{mc^2}{\sqrt{1-v^2/c^2}} \Rightarrow \frac{v}{c} = \sqrt{1 - \left(\frac{mc^2}{E} \right)^2} = \sqrt{1 - \left(\frac{0.511 \text{ MeV}}{5.51 \text{ MeV}} \right)^2} = 0.996.$$

- 44.3. IDENTIFY and SET UP:** By momentum conservation the two photons must have equal and opposite momenta. Then $E = pc$ says the photons must have equal energies. Their total energy must equal the rest mass energy

$E = mc^2$ of the pion. Once we have found the photon energy we can use $E = hf$ to calculate the photon frequency and use $\lambda = c/f$ to calculate the wavelength.

EXECUTE: The mass of the pion is $270m_e$, so the rest energy of the pion is $270(0.511 \text{ MeV}) = 138 \text{ MeV}$. Each

photon has half this energy, or 69 MeV. $E = hf$ so $f = \frac{E}{h} = \frac{(69 \times 10^6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = 1.7 \times 10^{22} \text{ Hz}$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{1.7 \times 10^{22} \text{ Hz}} = 1.8 \times 10^{-14} \text{ m} = 18 \text{ fm}.$$

EVALUATE: These photons are in the gamma ray part of the electromagnetic spectrum.

- 44.4. (a)** The energy will be the proton rest energy, 938.3 MeV, corresponding to a frequency of $2.27 \times 10^{23} \text{ Hz}$ and a wavelength of $1.32 \times 10^{-15} \text{ m}$.

(b) The energy of each photon will be $938.3 \text{ MeV} + 830 \text{ MeV} = 1768 \text{ MeV}$, with frequency $42.8 \times 10^{22} \text{ Hz}$ and wavelength $7.02 \times 10^{-16} \text{ m}$.

- 44.5. (a)** $\Delta m = m_{\pi^+} - m_{\mu^+} = 270 m_e - 207 m_e = 63 m_e \Rightarrow E = 63(0.511 \text{ MeV}) = 32 \text{ MeV}$.

(b) A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

$$44.6. \quad (a) \quad \lambda = \frac{hc}{E} = \frac{hc}{m_{\mu}c^2} = \frac{h}{m_{\mu}c} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{(207)(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} = 1.17 \times 10^{-14} \text{ m} = 0.0117 \text{ pm}$$

In this case, the muons are created at rest (no kinetic energy).

(b) Shorter wavelengths would mean higher photon energy, and the muons would be created with non-zero kinetic energy.

44.7. **IDENTIFY:** The energy released comes from the mass difference.

SET UP: The mass difference is the initial mass minus the final mass.

$$\Delta m = m_{\mu^-} - m_{e^-} - m_{e^+}$$

EXECUTE: Using the masses from Table 44.2, we have

$$\Delta m = m_{\mu^-} - m_{e^-} - m_{e^+} = (105.7 \text{ MeV}/c^2) - (0.511 \text{ MeV}/c^2) - (0.511 \text{ MeV}/c^2) = 105 \text{ MeV}/c^2$$

Multiplying these masses by c^2 gives $E = 105 \text{ MeV}$.

EVALUATE: This energy is observed as kinetic energy of the electron and positron.

44.8. **IDENTIFY and SET UP:** Calculate the mass change in each reaction, using the atomic masses in Table 44.2. A mass change of 1 u is equivalent to an energy of 931.5 MeV.

EXECUTE: (a) and (b) Eq.(44.1): ${}^4_2\text{He} + {}^9_4\text{Be} \rightarrow {}^{12}_6\text{C} + {}^1_0\text{n}$

$$\Delta M = m({}^4_2\text{He}) + m({}^9_4\text{Be}) - [m({}^{12}_6\text{C}) + m({}^1_0\text{n})]$$

$$\Delta M = 4.00260 \text{ u} + 9.01218 \text{ u} - 12.00000 \text{ u} - 1.00866 \text{ u} = 0.00612 \text{ u}$$

The mass decreases and the energy liberated is 5.70 MeV. The reaction is exoergic.

Eq.(44.2): ${}^1_0\text{n} + {}^{10}_5\text{B} \rightarrow {}^7_3\text{Li} + {}^4_2\text{He}$

$$\Delta M = m({}^1_0\text{n}) + m({}^{10}_5\text{B}) - [m({}^7_3\text{Li}) + m({}^4_2\text{He})]$$

$$\Delta M = 1.00866 \text{ u} + 10.01294 \text{ u} - 7.01600 \text{ u} - 4.00260 \text{ u} = 0.00300 \text{ u}$$

The mass decreases and the energy liberated is 2.79 MeV. The reaction is exoergic.

(c) The reactants in the reactions of Eq.(44.1) have positive nuclear charges and a threshold kinetic energy is required for the reactants to overcome their Coulomb repulsion and get close enough for the reaction to occur. The neutron in Eq.(44.2) is neutral so there is no Coulomb repulsion and no threshold energy for this reaction.

44.9. **IDENTIFY:** The antimatter annihilates with an equal amount of matter.

SET UP: The energy of the matter is $E = (\Delta m)c^2$.

EXECUTE: Putting in the numbers gives

$$E = (\Delta m)c^2 = (400 \text{ kg} + 400 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 7.2 \times 10^{19} \text{ J}.$$

This is about 70% of the annual energy use in the U.S.

EVALUATE: If this huge amount of energy were released suddenly, it would blow up the *Enterprise*! Getting useable energy from matter-antimatter annihilation is not so easy to do!

44.10. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2(E_m + mc^2)$ for two particles of equal mass when one is initially stationary. In this case, the initial kinetic energy ($20.0 \text{ GeV} = 20,000 \text{ MeV}$) is much more than the rest energy of the electron (0.511 MeV), so the formula for available energy reduces to $E_a = \sqrt{2mc^2E_m}$.

EXECUTE: (a) Using the formula for available energy gives

$$E_a = \sqrt{2mc^2E_m} = \sqrt{2(0.511 \text{ MeV})(20.0 \text{ GeV})} = 143 \text{ MeV}$$

(b) For colliding beams of equal mass, each particle has half the available energy, so each has 71.5 MeV. The *total* energy is twice this, or 143 MeV.

EVALUATE: Colliding beams provide considerably more available energy to do experiments than do beams hitting a stationary target. With a stationary electron target in part (a), we had to give the moving electron 20,000 MeV of energy to get the same available energy that we got with only 143 MeV of energy with the colliding beams.

44.11. (a) **IDENTIFY and SET UP:** Eq. (44.7) says $\omega = |q|B/m$ so $B = m\omega/|q|$. And since $\omega = 2\pi f$, this becomes

$$B = 2\pi mf / |q|.$$

EXECUTE: A deuteron is a deuterium nucleus (${}^2_1\text{H}$). Its charge is $q = +e$. Its mass is the mass of the neutral ${}^2_1\text{H}$ atom (Table 43.2) minus the mass of the one atomic electron:

$$m = 2.014102 \text{ u} - 0.0005486 \text{ u} = 2.013553 \text{ u} (1.66054 \times 10^{-27} \text{ kg/1 u}) = 3.344 \times 10^{-27} \text{ kg}$$

$$B = \frac{2\pi mf}{|q|} = \frac{2\pi(3.344 \times 10^{-27} \text{ kg})(9.00 \times 10^6 \text{ Hz})}{1.602 \times 10^{-19} \text{ C}} = 1.18 \text{ T}$$

$$(b) \text{ Eq.(44.8): } K = \frac{q^2 B^2 R^2}{2m} = \frac{[(1.602 \times 10^{-19} \text{ C})(1.18 \text{ T})(0.320 \text{ m})]^2}{2(3.344 \times 10^{-27} \text{ kg})}.$$

$$K = 5.471 \times 10^{-13} \text{ J} = (5.471 \times 10^{-13} \text{ J})(1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 3.42 \text{ MeV}$$

$$K = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.471 \times 10^{-13} \text{ J})}{3.344 \times 10^{-27} \text{ kg}}} = 1.81 \times 10^7 \text{ m/s}$$

EVALUATE: $v/c = 0.06$, so it is ok to use the nonrelativistic expression for kinetic energy.

$$44.12. (a) 2f = \frac{\omega}{\pi} = \frac{eB}{m\pi} = 3.97 \times 10^7 /s.$$

$$(b) \omega R = \frac{eBR}{m} = 3.12 \times 10^7 \text{ m/s}$$

(c) For three-figure precision, the relativistic form of the kinetic energy must be used,

$$eV = (\gamma - 1)mc^2, \text{ so } eV = (\gamma - 1)mc^2, \text{ so } V = \frac{(\gamma - 1)mc^2}{e} = 5.11 \times 10^6 \text{ V}.$$

44.13. (a) **IDENTIFY and SET UP:** The masses of the target and projectile particles are equal, so Eq. (44.10) can be used. $E_a^2 = 2mc^2(E_m + mc^2)$. E_a is specified; solve for the energy E_m of the beam particles.

$$\text{EXECUTE: } E_m = \frac{E_a^2}{2mc^2} - mc^2$$

The mass for the alpha particle can be calculated by subtracting two electron masses from the ${}^4\text{He}$ atomic mass:

$$m = m_\alpha = 4.002603 \text{ u} - 2(0.0005486 \text{ u}) = 4.001506 \text{ u}$$

$$\text{Then } mc^2 = (4.001506 \text{ u})(931.5 \text{ MeV/u}) = 3.727 \text{ GeV}.$$

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(16.0 \text{ GeV})^2}{2(3.727 \text{ GeV})} - 3.727 \text{ GeV} = 30.6 \text{ GeV}.$$

(b) Each beam must have $\frac{1}{2}E_a = 8.0 \text{ GeV}$.

EVALUATE: For a stationary target the beam energy is nearly twice the available energy. In a colliding beam experiment all the energy is available and each beam needs to have just half the required available energy.

$$44.14. (a) \gamma = \frac{1000 \times 10^3 \text{ MeV}}{938.3 \text{ MeV}} = 1065.8, \text{ so } v = 0.999999559c.$$

$$(b) \text{ Nonrelativistic: } \omega = \frac{eB}{m} = 3.83 \times 10^8 \text{ rad/s}.$$

$$\text{Relativistic: } \omega = \frac{eB}{m} \frac{1}{\gamma} = 3.59 \times 10^5 \text{ rad/s}.$$

44.15. (a) **IDENTIFY and SET UP:** For a proton beam on a stationary proton target and since E_a is much larger than the proton rest energy we can use Eq.(44.11): $E_a^2 = 2mc^2 E_m$.

$$\text{EXECUTE: } E_m = \frac{E_a^2}{2mc^2} = \frac{(77.4 \text{ GeV})^2}{2(0.938 \text{ GeV})} = 3200 \text{ GeV}$$

(b) **IDENTIFY and SET UP:** For colliding beams the total momentum is zero and the available energy E_a is the total energy for the two colliding particles.

EXECUTE: For proton-proton collisions the colliding beams each have the same energy, so the total energy of each beam is $\frac{1}{2}E_a = 38.7 \text{ GeV}$.

EVALUATE: For a stationary target less than 3% of the beam energy is available for conversion into mass. The beam energy for a colliding beam experiment is a factor of (1/83) times smaller than the required energy for a stationary target experiment.

44.16. **IDENTIFY:** Only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy.

SET UP: To create the η^0 , the minimum available energy must be equal to the rest mass energy of the products, which in this case is the η^0 plus two protons. In a collider, all of the initial energy is available, so the beam energy is the available energy.

EXECUTE: The minimum amount of available energy must be rest mass energy

$$E_a = 2m_p + m_{\eta} = 2(938.3 \text{ MeV}) + 547.3 \text{ MeV} = 2420 \text{ MeV}$$

Each incident proton has half of the rest mass energy, or $1210 \text{ MeV} = 1.21 \text{ GeV}$.

EVALUATE: As we saw in problem 44.10, we would need much more initial energy if one of the initial protons were stationary. The result here (1.21 GeV) is the *minimum* amount of energy needed; the original protons could have more energy and still trigger this reaction.

- 44.17. Section 44.3 says $m(Z^0) = 91.2 \text{ GeV}/c^2$.

$$E = 91.2 \times 10^9 \text{ eV} = 1.461 \times 10^{-8} \text{ J}; m = E/c^2 = 1.63 \times 10^{-25} \text{ kg}; m(Z^0)/m(p) = 97.2$$

- 44.18. (a) We shall assume that the kinetic energy of the Λ^0 is negligible. In that case we can set the value of the photon's energy equal to Q :

$$Q = (1193 - 1116) \text{ MeV} = 77 \text{ MeV} = E_{\text{photon}}.$$

(b) The momentum of this photon is

$$p = \frac{E_{\text{photon}}}{c} = \frac{(77 \times 10^6 \text{ eV})(1.60 \times 10^{-18} \text{ J/eV})}{(3.00 \times 10^8 \text{ m/s})} = 4.1 \times 10^{-20} \text{ kg} \cdot \text{m/s}$$

To justify our original assumption, we can calculate the kinetic energy of a Λ^0 that has this value of momentum

$$K_{\Lambda^0} = \frac{p^2}{2m} = \frac{E^2}{2mc^2} = \frac{(77 \text{ MeV})^2}{2(1116 \text{ MeV})} = 2.7 \text{ MeV} \ll Q = 77 \text{ MeV}.$$

Thus, we can ignore the momentum of the Λ^0 without introducing a large error.

- 44.19. **IDENTIFY and SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: The mass decrease is $m(\Sigma^+) - m(p) - m(\pi^0)$ and the energy released is

$$mc^2(\Sigma^+) - mc^2(p) - mc^2(\pi^0) = 1189 \text{ MeV} - 938.3 \text{ MeV} - 135.0 \text{ MeV} = 116 \text{ MeV. (The } mc^2 \text{ values for each particle were taken from Table 44.3.)}$$

EVALUATE: The mass of the decay products is less than the mass of the original particle, so the decay is energetically allowed and energy is released.

- 44.20. **IDENTIFY:** If the initial and final rest mass energies were equal, there would be no left over energy for kinetic energy. Therefore the kinetic energy of the products is the difference between the mass energy of the initial particles and the final particles.

SET UP: The difference in mass is $\Delta m = M_{\Omega^-} - m_{\Lambda^0} - m_{K^-}$.

EXECUTE: Using Table 44.3, the energy difference is

$$E = (\Delta m)c^2 = 1672 \text{ MeV} - 1116 \text{ MeV} - 494 \text{ MeV} = 62 \text{ MeV}$$

EVALUATE: There is less rest mass energy after the reaction than before because 62 MeV of the initial energy was converted to kinetic energy of the products.

- 44.21. Conservation of lepton number.

(a) $\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu \Rightarrow L_\mu : +1 \neq -1, L_e : 0 \neq +1 + 1$, so lepton numbers are not conserved.

(b) $\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau \Rightarrow L_e : 0 = +1 - 1; L_\tau : +1 = +1$, so lepton numbers are conserved.

(c) $\pi^+ \rightarrow e^+ + \gamma$. Lepton numbers are not conserved since just one lepton is produced from zero original leptons.

(d) $n \rightarrow p + e^- + \bar{\nu}_e \Rightarrow L_e : 0 = +1 - 1$, so the lepton numbers are conserved.

- 44.22. **IDENTIFY and SET UP:** p and n have baryon number +1 and $\bar{p}$ has baryon number -1. e^+ , e^- , $\bar{\nu}_e$ and γ all have baryon number zero. Baryon number is conserved if the total baryon number of the products equals the total baryon number of the reactants.

EXECUTE: (a) reactants: $B = 1 + 1 = 2$. Products: $B = 1 + 0 = 1$. Not conserved.

(b) reactants: $B = 1 + 1 = 2$. Products: $B = 0 + 0 = 0$. Not conserved.

(c) reactants: $B = +1$. Products: $B = 1 + 0 + 0 = +1$. Conserved.

(d) reactants: $B = 1 - 1 = 0$. Products: $B = 0$. Conserved.

- 44.23. **IDENTIFY and SET UP:** Compare the sum of the strangeness quantum numbers for the particles on each side of the decay equation. The strangeness quantum numbers for each particle are given Table 44.3.

EXECUTE: (a) $K^+ \rightarrow \mu^+ + \nu_\mu$; $S_{K^+} = +1$, $S_{\mu^+} = 0$, $S_{\nu_\mu} = 0$

$S = 1$ initially; $S = 0$ for the products; S is not conserved

(b) $n + K^+ \rightarrow p + \pi^0$; $S_n = 0$, $S_{K^+} = +1$, $S_p = 0$, $S_{\pi^0} = 0$

$S = 1$ initially; $S = 0$ for the products; S is not conserved

(c) $K^+ + K^- \rightarrow \pi^0 + \pi^0$; $S_{K^+} = +1$; $S_{K^-} = -1$; $S_{\pi^0} = 0$

$S = +1 - 1 = 0$ initially; $S = 0$ for the products; S is conserved

(d) $p + K^- \rightarrow \Lambda^0 + \pi^0$; $S_p = 0$, $S_{K^-} = -1$, $S_{\Lambda^0} = -1$, $S_{\pi^0} = 0$.

$S = -1$ initially; $S = -1$ for the products; S is conserved

EVALUATE: Strangeness is not a conserved quantity in weak interactions and strangeness non-conserving reactions or decays can occur.

44.24. (a) Using the values of the constants from Appendix F,

$$\frac{e^2}{4\pi\epsilon_0\hbar c} = 7.29660475 \times 10^{-3} = \frac{1}{137.050044}, \text{ or } 1/137 \text{ to three figures.}$$

(b) From Section 38.5, $v_1 = \frac{e^2}{2\epsilon_0\hbar}$. But notice this is just $\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)c$, as claimed.

44.25. $\left[\frac{f^2}{\hbar c}\right] = \frac{(\text{J} \cdot \text{m})}{(\text{J} \cdot \text{s})(\text{m} \cdot \text{s}^{-1})} = 1$ and thus $\frac{f^2}{\hbar c}$ is dimensionless. (Recall f^2 has units of energy times distance.)

44.26. (a) The diagram is given in Figure 44.26. The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a “hole” in an otherwise regular lattice in the $S-Q$ plane. The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this “hole” and mass regularity that led to an accurate prediction of the properties of the Ω^- !

(b) See diagram. Use quark charges $u = +\frac{2}{3}$, $d = -\frac{1}{3}$, and $s = -\frac{1}{3}$ as a guide.

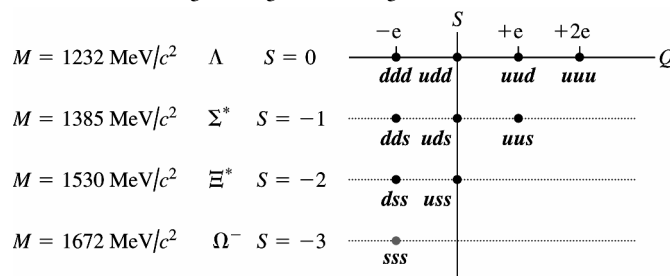


Figure 44.26

44.27. IDENTIFY and SET UP: Each value for the combination is the sum of the values for each quark. Use Table 44.4.

EXECUTE: (a) uds

$$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$$

$$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$S = 0 + 0 - 1 = -1$$

$$C = 0 + 0 + 0 = 0$$

(b) $c\bar{u}$

The values for $\bar{u}$ are the negative for those for u .

$$Q = \frac{2}{3}e - \frac{2}{3}e = 0$$

$$B = \frac{1}{3} - \frac{1}{3} = 0$$

$$S = 0 + 0 = 0$$

$$C = +1 + 0 = +1$$

(c) ddd

$$Q = -\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$$

$$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$$

$$S = 0 + 0 + 0 = 0$$

$$C = 0 + 0 + 0 = 0$$

(d) $d\bar{c}$

$$Q = -\frac{1}{3}e - \frac{2}{3}e = -e$$

$$B = \frac{1}{3} - \frac{1}{3} = 0$$

$$S = 0 + 0 = 0$$

$$C = 0 - 1 = -1$$

EVALUATE: The charge, baryon number, strangeness and charm quantum numbers of a particle are determined by the particle's quark composition.

44.28. $(m_\gamma - 2m_e)c^2 = (9460 \text{ MeV} - 2(1777 \text{ MeV})) = 5906 \text{ MeV}$ (see Sections 44.3 and 44.4 for masses).

44.29. (a) The antiparticle must consist of the antiquarks so $\bar{n} = \bar{u}\bar{d}\bar{d}$.

(b) So $n = uud$ is not its own antiparticle.

(c) $\psi = c\bar{c}$ so $\bar{\psi} = \bar{c}c = \psi$ so the ψ is its own antiparticle.

44.30. (a) $S = 1$ indicates the presence of one $\bar{s}$ antiquark and no s quark. To have baryon number 0 there can be only one other quark, and to have net charge $+e$ that quark must be a u , and the quark content is $u\bar{s}$.

(b) The particle has an $\bar{s}$ antiquark, and for a baryon number of -1 the particle must consist of three antiquarks.

For a net charge of $-e$, the quark content must be $\bar{d}\bar{d}\bar{s}$.

(c) $S = -2$ means that there are two s quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a u quark and the quark content is uss .

44.31. IDENTIFY: A proton is made up of uud quarks and a neutron consists of udd quarks.

SET UP: If a proton decays by β^+ decay, we have $p \rightarrow e^+ + n + \nu_e$ (both charge and lepton number are conserved).

EVALUATE: Since a proton consists of uud quarks and a neutron is udd quarks, it follows that in β^+ decay a u quark changes to a d quark.

44.32. (a) Using the definition of z from Example 44.9 we have that

$$1 + z = 1 + \frac{(\lambda_0 - \lambda_s)}{\lambda_0} = \frac{\lambda_0}{\lambda_s}.$$

$$\text{Now we use Eq.(44.13) to obtain } 1 + z = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+v/c}{1-v/c}} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

$$\text{(b) Solving the above equation for } \beta \text{ we obtain } \beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{1.5^2 - 1}{1.5^2 + 1} = 0.3846.$$

Thus, $v = 0.3846 c = 1.15 \times 10^8 \text{ m/s}$.

(c) We can use Eq.(44.15) to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.15 \times 10^8 \text{ m/s})}{(7.1 \times 10^4 \text{ (m/s)/Mpc})} = 1.6 \times 10^3 \text{ Mpc}$$

44.33. (a) **IDENTIFY and SET UP:** Use Eq.(44.14) to calculate v .

$$\text{EXECUTE: } v = \left[\frac{(\lambda_0 / \lambda_s)^2 - 1}{(\lambda_0 / \lambda_s)^2 + 1} \right] c = \left[\frac{(658.5 \text{ nm}/590 \text{ nm})^2 - 1}{(658.5 \text{ nm}/590 \text{ nm})^2 + 1} \right] c = 0.1094c$$

$$v = (0.1094)(2.998 \times 10^8 \text{ m/s}) = 3.28 \times 10^7 \text{ m/s}$$

(b) **IDENTIFY and SET UP:** Use Eq.(44.15) to calculate r .

$$\text{EXECUTE: } r = \frac{v}{H_0} = \frac{3.28 \times 10^4 \text{ km/s}}{(71 \text{ (km/s)/Mpc})(1 \text{ Mpc}/3.26 \text{ Mly})} = 1510 \text{ Mly}$$

EVALUATE: The red shift $\lambda_0 / \lambda_s - 1$ for this galaxy is 0.116. It is therefore about twice as far from earth as the galaxy in Examples 44.9 and 44.10, that had a red shift of 0.053.

$$\text{44.34. From Eq.(44.15), } r = \frac{c}{H_0} = \frac{3.00 \times 10^8 \text{ m/s}}{20 \text{ (km/s)/Mly}} = 1.5 \times 10^4 \text{ Mly}.$$

(b) This distance represents looking back in time so far that the light has not been able to reach us.

44.35. (a) **IDENTIFY and SET UP:** Hubble's law is Eq.(44.15), with $H_0 = 71 \text{ (km/s)/Mpc}$. $1 \text{ Mpc} = 3.26 \text{ Mly}$.

$$\text{EXECUTE: } r = 5210 \text{ Mly so } v = H_0 r = (71 \text{ km/s/Mpc})(1 \text{ Mpc}/3.26 \text{ Mly})(5210 \text{ Mly}) = 1.1 \times 10^5 \text{ km/s}$$

(b) **IDENTIFY and SET UP:** Use v from part (a) in Eq. (44.13).

$$\text{EXECUTE: } \frac{\lambda_0}{\lambda_s} = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+v/c}{1-v/c}}$$

$$\frac{v}{c} = \frac{1.1 \times 10^5 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}} = 0.367 \text{ so } \frac{\lambda_0}{\lambda_s} = \sqrt{\frac{1+0.367}{1-0.367}} = 1.5$$

EVALUATE: The galaxy in Examples 44.9 and 44.10 is 710 Mly away so has a smaller recession speed and redshift than the galaxy in this problem.

44.36. IDENTIFY and SET UP: $m_H = 1.67 \times 10^{-27} \text{ kg}$. The ideal gas law says $pV = nRT$. Normal pressure is $1.013 \times 10^5 \text{ Pa}$ and normal temperature is about $27^\circ\text{C} = 300 \text{ K}$. 1 mole is 6.02×10^{23} atoms.

EXECUTE: (a) $\frac{6.3 \times 10^{-27} \text{ kg/m}^3}{1.67 \times 10^{-27} \text{ kg/atom}} = 3.8 \text{ atoms/m}^3$

(b) $V = (4 \text{ m})(7 \text{ m})(3 \text{ m}) = 84 \text{ m}^3$ and $(3.8 \text{ atoms/m}^3)(84 \text{ m}^3) = 320 \text{ atoms}$

(c) With $p = 1.013 \times 10^5 \text{ Pa}$, $V = 84 \text{ m}^3$, $T = 300 \text{ K}$ the ideal gas law gives the number of moles to be

$$n = \frac{pV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(84 \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 3.4 \times 10^3 \text{ moles}$$

$$(3.4 \times 10^3 \text{ moles})(6.02 \times 10^{23} \text{ atoms/mol}) = 2.0 \times 10^{27} \text{ atoms}$$

EVALUATE: The average density of the universe is very small. Interstellar space contains a very small number of atoms per cubic meter, compared to the number of atoms per cubic meter in ordinary material on the earth, such as air.

44.37. IDENTIFY and SET UP: Find the energy equivalent of the mass decrease.

EXECUTE: (a) $p + {}^2_1\text{H} \rightarrow {}^3_2\text{He}$ or can write as ${}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He}$

If neutral atom masses are used then the masses of the two atomic electrons on each side of the reaction will cancel.

Taking the atomic masses from Table 43.2, the mass decrease is $m({}^1_1\text{H}) + m({}^2_1\text{H}) - m({}^3_2\text{He}) = 1.007825 \text{ u} + 2.014102 \text{ u} - 3.016029 \text{ u} = 0.005898 \text{ u}$. The energy released is the energy equivalent of this mass decrease: $(0.005898 \text{ u})(931.5 \text{ MeV/u}) = 5.494 \text{ MeV}$

(b) ${}^1_0\text{n} + {}^3_2\text{He} \rightarrow {}^4_2\text{He}$

If neutral helium masses are used then the masses of the two atomic electrons on each side of the reaction equation will cancel. The mass decrease is $m({}^1_0\text{n}) + m({}^3_2\text{He}) - m({}^4_2\text{He}) = 1.008665 \text{ u} + 3.016029 \text{ u} - 4.002603 \text{ u} = 0.022091 \text{ u}$. The energy released is the energy equivalent of this mass decrease: $(0.022091 \text{ u})(931.5 \text{ MeV/u}) = 20.58 \text{ MeV}$

EVALUATE: These are important nucleosynthesis reactions, discussed in Section 44.7.

44.38. $3m({}^4_2\text{He}) - m({}^{12}_6\text{C}) = 7.80 \times 10^{-3} \text{ u}$, or 7.27 MeV .

44.39. $\Delta m = m_e + m_p - m_n - m_{\nu_e}$ so assuming $m_{\nu_e} \approx 0$,

$$\Delta m = 0.0005486 \text{ u} + 1.007276 \text{ u} - 1.008665 \text{ u} = -8.40 \times 10^{-4} \text{ u}$$

$$\Rightarrow E = (\Delta m)c^2 = (-8.40 \times 10^{-4} \text{ u})(931.5 \text{ MeV/u}) = -0.783 \text{ MeV} \text{ and is endoergic.}$$

44.40. $m({}^{12}_6\text{C}) + m({}^{4}_2\text{He}) - m({}^{16}_8\text{O}) = 7.69 \times 10^{-3} \text{ u}$, or 7.16 MeV , an exoergic reaction.

44.41. IDENTIFY and SET UP: The Wien displacement law (Eq.38.30) says $\lambda_m T$ equals a constant. Use this to relate $\lambda_{m,1}$ at T_1 to $\lambda_{m,2}$ at T_2 .

EXECUTE: $\lambda_{m,1} T_1 = \lambda_{m,2} T_2$

$$\lambda_{m,1} = \lambda_{m,2} \left(\frac{T_2}{T_1} \right) = 1.062 \times 10^{-3} \text{ m} \left(\frac{2.728 \text{ K}}{3000 \text{ K}} \right) = 966 \text{ nm}$$

EVALUATE: The peak wavelength was much less when the temperature was much higher.

44.42. (a) The dimensions of $\hbar$ are energy times time, the dimensions of G are energy times time per mass squared, and so the dimensions of $\sqrt{\hbar G/c^3}$ are

$$\left[\frac{(E \cdot T)(E \cdot L/M^2)}{(L/T)^3} \right]^{1/2} = \left[\frac{E}{M} \right] \left[\frac{T^2}{L} \right] = \left[\frac{L}{T} \right] \left[\frac{T^2}{L} \right] = L.$$

(b) $\left(\frac{\hbar G}{c^3} \right)^{1/2} = \left(\frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)}{2\pi(3.00 \times 10^8 \text{ m/s})^3} \right)^{1/2} = 1.616 \times 10^{-35} \text{ m}.$

44.43. IDENTIFY and SET UP: For colliding beams the available energy is twice the beam energy. For a fixed-target experiment only a portion of the beam energy is available energy (Eqs.44.9 and 44.10).

EXECUTE: (a) $E_a = 2(7.0 \text{ TeV}) = 14.0 \text{ TeV}$

(b) Need $E_a = 14.0 \text{ TeV} = 14.0 \times 10^6 \text{ MeV}$. Since the target and projectile particles are both protons Eq. (44.10) can be used: $E_a^2 = 2mc^2(E_m + mc^2)$

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(14.0 \times 10^6 \text{ MeV})^2}{2(938.3 \text{ MeV})} - 938.3 \text{ MeV} = 1.0 \times 10^{11} \text{ MeV} = 1.0 \times 10^5 \text{ TeV}.$$

EVALUATE: This shows the great advantage of colliding beams at relativistic energies.

44.44. $K + m_p c^2 = \frac{hc}{\lambda}$, $K = \frac{hc}{\lambda} - m_p c^2 = 652 \text{ MeV}$.

44.45. IDENTIFY and SET UP: Section 44.3 says the strong interaction is 100 times as strong as the electromagnetic interaction and that the weak interaction is 10^{-9} times as strong as the strong interaction. The Coulomb force is $F_e = \frac{kq_1q_2}{r^2}$ and the gravitational force is $F_g = G \frac{m_1m_2}{r^2}$.

EXECUTE: (a) $F_e = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1 \times 10^{-15} \text{ m})^2} = 200 \text{ N}$

$$F_g = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.67 \times 10^{-27} \text{ kg})^2}{(1 \times 10^{-15} \text{ m})^2} = 2 \times 10^{-34} \text{ N}$$

(b) $F_{\text{str}} \approx 100F_e \approx 2 \times 10^4 \text{ N}$. $F_{\text{weak}} \approx 10^{-9}F_{\text{str}} \approx 2 \times 10^{-5} \text{ N}$

(c) $F_{\text{str}} > F_e > F_{\text{weak}} > F_g$

(d) $F_e \approx 1 \times 10^{36}F_g$. $F_{\text{str}} \approx 100F_e \approx 1 \times 10^{38}F_g$. $F_{\text{weak}} \approx 10^{-9}F_{\text{str}} \approx 1 \times 10^{29}F_g$

EVALUATE: The gravity force is much weaker than any of the other three forces. Gravity is important only when one very massive object is involved.

44.46. In Eq.(44.9), $E_a = (m_{\Sigma^0} + m_{K^0})c^2$, and with $M = m_p$, $m = m_{\pi^-}$ and $E_m = (m_{\pi^-})c^2 + K$,

$$K = \frac{E_a^2 - (m_{\pi^-}c^2)^2 - (m_p c^2)^2}{2m_p c^2} - (m_{\pi^-})c^2$$

$$K = \frac{(1193 \text{ MeV} + 497.7 \text{ MeV})^2 - (139.6 \text{ MeV})^2 - (938.3 \text{ MeV})^2}{2(938.3 \text{ MeV})} - 139.6 \text{ MeV} = 904 \text{ MeV}.$$

44.47. IDENTIFY: With a stationary target, only part of the initial kinetic energy of the moving proton is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2(E_m + mc^2)$ for two particles of equal mass when one is initially stationary. The *minimum* available energy must be equal to the rest mass energies of the products, which in this case is two protons, a K^+ and a K^- . The available energy must be at least the sum of the final rest masses.

EXECUTE: The minimum amount of available energy must be

$$E_a = 2m_p + m_{K^+} + m_{K^-} = 2(938.3 \text{ MeV}) + 493.7 \text{ MeV} + 493.7 \text{ MeV} = 2864 \text{ MeV} = 2.864 \text{ GeV}$$

Solving the available energy formula for E_m gives $E_a^2 = 2mc^2(E_m + mc^2)$ and

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(2864 \text{ MeV})^2}{2(938.3 \text{ MeV})} - 938.3 \text{ MeV} = 3432.6 \text{ MeV}$$

Recalling that E_m is the *total* energy of the proton, including its rest mass energy (RME), we have

$$K = E_m - \text{RME} = 3432.6 \text{ MeV} - 938.3 \text{ MeV} = 2494 \text{ MeV} = 2.494 \text{ GeV}$$

Therefore the threshold kinetic energy is $K = 2494 \text{ MeV} = 2.494 \text{ GeV}$.

EVALUATE: Considerably less energy would be needed if the experiment were done using colliding beams of protons.

44.48. (a) The decay products must be neutral, so the only possible combinations are $\pi^0\pi^0\pi^0$ or $\pi^0\pi^+\pi^-$

(b) $m_{\eta_0} - 3m_{\pi^0} = 142.3 \text{ MeV}/c^2$, so the kinetic energy of the π^0 mesons is 142.3 MeV. For the other reaction,

$$K = (m_{\eta_0} - m_{\pi^0} - m_{\pi^+} - m_{\pi^-})c^2 = 133.1 \text{ MeV}.$$

44.49. IDENTIFY and SET UP: Apply conservation of linear momentum to the collision. A photon has momentum

$$p = h/\lambda, \text{ in the direction it is traveling. The energy of a photon is } E = pc = \frac{hc}{\lambda}. \text{ All the mass of the electron and}$$

positron is converted to the total energy of the two photons, according to $E = mc^2$. The mass of an electron and of a positron is $m_e = 9.11 \times 10^{-31} \text{ kg}$

EXECUTE: (a) In the lab frame the initial momentum of the system is zero, since the electron and positron have equal speeds in opposite directions. According to momentum conservation, the final momentum of the system must also be zero. A photon has momentum, so the momentum of a single photon is not zero.

(b) For the two photons to have zero total momentum they must have the same magnitude of momentum and move in opposite directions. Since $E = pc$, equal p means equal E .

(c) $2E_{\text{ph}} = 2m_e c^2$ so $E_{\text{ph}} = m_e c^2$

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \frac{hc}{\lambda} = m_e c^2 \text{ and } \lambda = \frac{h}{m_e c} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} = 2.43 \text{ pm}$$

These are gamma ray photons.

EVALUATE: The total charge of the electron/positron system is zero and the photons have no charge, so charge is conserved in the particle-antiparticle annihilation.

- 44.50.** (a) If the π^- decays, it must end in an electron and neutrinos. The rest energy of π^- (139.6 MeV) is shared between the electron rest energy (0.511 MeV) and kinetic energy (assuming the neutrino masses are negligible). So the energy released is $139.6 \text{ MeV} - 0.511 \text{ MeV} = 139.1 \text{ MeV}$.

(b) Conservation of momentum leads to the neutrinos carrying away most of the energy.

- 44.51.** (a) The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+ .
 (b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero, and the particle is π^- .
 (c) The baryon number is -1 , the charge is 0, the strangeness is zero, all lepton numbers are 0, and the particle is an antineutron.
 (d) The baryon number is 0, the charge is $+e$, the strangeness is 0, the muonic lepton number is -1 , all other lepton numbers are 0, and the particle is μ^+ .

44.52. $\Delta t = 7.6 \times 10^{-21} \text{ s} \Rightarrow \Delta E = \frac{\hbar}{\Delta t} = \frac{1.054 \times 10^{-34} \text{ J} \cdot \text{s}}{7.6 \times 10^{-21} \text{ s}} = 1.39 \times 10^{-14} \text{ J} = 87 \text{ keV}.$

$$\frac{\Delta E}{m_\nu c^2} = \frac{0.087 \text{ MeV}}{3097 \text{ MeV}} = 2.8 \times 10^{-5}.$$

44.53. $\frac{\hbar}{\Delta E} = \frac{(1.054 \times 10^{-34} \text{ J} \cdot \text{s})}{(4.4 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = 1.5 \times 10^{-22} \text{ s}.$

- 44.54. IDENTIFY and SET UP:** $\phi \rightarrow K^+ + K^-$. The total energy released is the energy equivalent of the mass decrease.

(a) **EXECUTE:** The mass decrease is $m(\phi) - m(K^+) - m(K^-)$. The energy equivalent of the mass decrease is $mc^2(\phi) - mc^2(K^+) - mc^2(K^-)$. The rest mass energy mc^2 for the ϕ meson is given in Problem 44.53, and the values for K^+ and K^- are given in Table 44.3. The energy released then is $1019.4 \text{ MeV} - 2(493.7 \text{ MeV}) = 32.0 \text{ MeV}$. The K^+ gets half this, 16.0 MeV.

EVALUATE: (b) Does the decay $\phi \rightarrow K^+ + K^- + \pi^0$ occur? The energy equivalent of the $K^+ + K^- + \pi^0$ mass is $493.7 \text{ MeV} + 493.7 \text{ MeV} + 135.0 \text{ MeV} = 1122 \text{ MeV}$. This is greater than the energy equivalent of the ϕ mass. The mass of the decay products would be greater than the mass of the parent particle; the decay is energetically forbidden.

(c) Does the decay $\phi \rightarrow K^+ + \pi^-$ occur? The reaction $\phi \rightarrow K^+ + K^-$ is observed. K^+ has strangeness $+1$ and K^- has strangeness -1 , so the total strangeness of the decay products is zero. If strangeness must be conserved we deduce that the ϕ particle has strangeness zero. π^- has strangeness 0, so the product $K^+ + \pi^-$ has strangeness $+1$. The decay $\phi \rightarrow K^+ + \pi^-$ violates conservation of strangeness. Does the decay $\phi \rightarrow K^+ + \mu^-$ occur? μ^- has strangeness 0, so this decay would also violate conservation of strangeness.

- 44.55.** (a) The number of protons in a kilogram is

$$(1.00 \text{ kg}) \left(\frac{6.023 \times 10^{23} \text{ molecules/mol}}{18.0 \times 10^{-3} \text{ kg/mol}} \right) (2 \text{ protons/molecule}) = 6.7 \times 10^{25}.$$

Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_p c^2 = 938.3 \text{ MeV} = 1.503 \times 10^{-10} \text{ J}$, and so the energy deposited in a year, per kilogram, is

$$(6.7 \times 10^{25}) \left(\frac{\ln(2)}{1.0 \times 10^{18} \text{ y}} \right) (1 \text{ y}) (1.50 \times 10^{-10} \text{ J}) = 7.0 \times 10^{-3} \text{ Gy} = 0.70 \text{ rad}$$

(b) For an RBE of unity, the equivalent dose is (1) (0.70 rad) = 0.70 rem.

- 44.56. IDENTIFY and SET UP:** The total released energy is the equivalent of the mass decrease. Use conservation of linear momentum to relate the kinetic energies of the decay particles.

EXECUTE: (a) The energy equivalent of the mass decrease is

$$mc^2(\Xi^-) - mc^2(\Lambda^0) - mc^2(\pi^-) = 1321 \text{ MeV} - 1116 \text{ MeV} - 139.6 \text{ MeV} = 65 \text{ MeV}$$

(b) The Ξ^- is at rest means that the linear momentum is zero. Conservation of linear momentum then says that the Λ^0 and π^- must have equal and opposite momenta:

$$m_{\Lambda^0} v_{\Lambda^0} = m_{\pi^-} v_{\pi^-}$$

$$v_{\pi^-} = \left(\frac{m_{\Lambda^0}}{m_{\pi^-}} \right) v_{\Lambda^0}$$

Also, the sum of the kinetic energies of the Λ^0 and π^- must equal the total kinetic energy $K_{\text{tot}} = 65 \text{ MeV}$ calculated in part (a):

$$K_{\text{tot}} = K_{\Lambda^0} + K_{\pi^-}$$

$$K_{\Lambda^0} + \frac{1}{2} m_{\pi^-} v_{\pi^-}^2 = K_{\text{tot}}$$

Use the momentum conservation result:

$$K_{\Lambda^0} + \frac{1}{2} m_{\pi^-} \left(\frac{m_{\Lambda^0}}{m_{\pi^-}} \right)^2 v_{\Lambda^0}^2 = K_{\text{tot}}$$

$$K_{\Lambda^0} + \left(\frac{m_{\Lambda^0}}{m_{\pi^-}} \right) \left(\frac{1}{2} m_{\Lambda^0} v_{\Lambda^0}^2 \right) = K_{\text{tot}}$$

$$K_{\Lambda^0} \left(1 + \frac{m_{\Lambda^0}}{m_{\pi^-}} \right) = K_{\text{tot}}$$

$$K_{\Lambda^0} = \frac{K_{\text{tot}}}{1 + m_{\Lambda^0}/m_{\pi^-}} = \frac{65 \text{ MeV}}{1 + (1116 \text{ MeV})/(139.6 \text{ MeV})} = 7.2 \text{ MeV}$$

$$K_{\Lambda^0} + K_{\pi^-} = K_{\text{tot}} \text{ so } K_{\pi^-} = K_{\text{tot}} - K_{\Lambda^0} = 65 \text{ MeV} - 7.2 \text{ MeV} = 57.8 \text{ MeV}$$

$$\text{The fraction for the } \Lambda^0 \text{ is } \frac{7.2 \text{ MeV}}{65 \text{ MeV}} = 11\%.$$

$$\text{The fraction for the } \pi^- \text{ is } \frac{57.8 \text{ MeV}}{65 \text{ MeV}} = 89\%.$$

EVALUATE: The lighter particle carries off more of the kinetic energy that is released in the decay than the heavier particle does.

44.57. (a) For this model, $\frac{dR}{dt} = HR$, so $\frac{dR/dt}{R} = \frac{HR}{R} = H$, presumed to be the same for all points on the surface.

(b) For constant θ , $\frac{dr}{dt} = \frac{dR}{dt} \theta = HR\theta = Hr$.

(c) See part (a), $H_0 = \frac{dR/dt}{R}$.

(d) The equation $\frac{dR}{dt} = H_0 R$ is a differential equation, the solution to which, for constant H_0 , is $R(t) =$

$R_0 e^{H_0 t}$, where R_0 is the value of R at $t = 0$. This equation may be solved by separation of variables, as

$$\frac{dR/dt}{R} = \frac{d}{dt} \ln(R) = H_0 \text{ and integrating both sides with respect to time.}$$

(e) A constant H_0 would mean a constant critical density, which is inconsistent with uniform expansion.

44.58. From Problem 44.57, $r = R\theta \Rightarrow R = \frac{r}{\theta}$. So $\frac{dR}{dt} = \frac{1}{\theta} \frac{dr}{dt} - \frac{r}{\theta^2} \frac{d\theta}{dt} = \frac{1}{\theta} \frac{dr}{dt}$ since $\frac{d\theta}{dt} = 0$.

$$\text{So } \frac{1}{R} \frac{dR}{dt} = \frac{1}{R\theta} \frac{dr}{dt} = \frac{1}{r} \frac{dr}{dt} \Rightarrow v = \frac{dr}{dt} = \left(\frac{1}{R} \frac{dR}{dt} \right) r = H_0 r. \text{ Now } \frac{dv}{d\theta} = 0 = \frac{d}{d\theta} \left(\frac{r}{R} \frac{dR}{dt} \right) = \frac{d}{d\theta} \left(\theta \frac{dR}{dt} \right)$$

$$\Rightarrow \theta \frac{dR}{dt} = K \text{ where } K \text{ is a constant. } \Rightarrow \frac{dR}{dt} = \frac{K}{\theta} \Rightarrow R = \left(\frac{K}{\theta} \right) t \text{ since } \frac{d\theta}{dt} = 0 \Rightarrow H_0 = \frac{1}{R} \frac{dR}{dt} = \frac{\theta}{Kt} \frac{K}{\theta} = \frac{1}{t}. \text{ So the}$$

current value of the Hubble constant is $\frac{1}{T}$ where T is the present age of the universe.

44.59. (a) For mass m , in Eq. (37.23) $u = -v_{\text{cm}}$, $v' = v_0$, and so $v_m = \frac{v_0 - v_{\text{cm}}}{1 - v_0 v_{\text{cm}}/c^2}$. For mass

$$M, u = -v_{\text{cm}}, v' = 0, \text{ so } v_M = -v_{\text{cm}}.$$

(b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0$, where γ_m and γ_M correspond to the velocities found in part (a). The algebra reduces to $\beta_m \gamma_m = (\beta_0 - \beta') \gamma_0 \gamma_M$, where $\beta_0 = \frac{v_0}{c}$, $\beta' = \frac{v_{cm}}{c}$, and the condition for no net momentum becomes $m(\beta_0 - \beta') \gamma_0 \gamma_M = M \beta' \gamma_M$, or

$$\beta' = \frac{\beta_0}{1 + \frac{M}{m\gamma_0}} = \beta_0 \frac{m}{m + M\sqrt{1 - \beta_0^2}} \cdot v_{cm} = \frac{mv_0}{m + M\sqrt{1 - (v_0/c)^2}}.$$

(c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms $v_m = v_0 \gamma_0 \frac{M}{m + M\gamma_0}$, $v_M = -v_0 \gamma_0 \frac{m}{m\gamma_0 + M}$. After some more algebra,

$$\gamma_m = \frac{m + M\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}, \gamma_M = \frac{M + m\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}, \text{ from which } m\gamma_m + M\gamma_M = \sqrt{m^2 + M^2 + 2mM\gamma_0}. \text{ This last}$$

expression, multiplied by c^2 , is the available energy E_a in the center of mass frame, so that

$$E_a^2 = (m^2 + M^2 + 2mM\gamma_0)c^4 = (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0 c^2) = (mc^2)^2 + (Mc^2)^2 + 2Mc^2 E_m, \text{ which is Eq.(44.9).}$$

44.60. $\Lambda^0 \rightarrow n + \pi^0$

(a) $E = (\Delta m)c^2 = (m_{\Lambda^0}c^2 - (m_n)c^2 - (m_{\pi^0})c^2) = 1116 \text{ MeV} - 939.6 \text{ MeV} - 135.0 \text{ MeV} = 41.4 \text{ MeV}$

(b) Using conservation of momentum and kinetic energy; we know that the momentum of the neutron and pion must have the same magnitude, $p_n = p_\pi$.

$$K_n = E_n - m_n c^2 = \sqrt{(m_n c^2)^2 + (p_n c)^2} - m_n c^2 = \sqrt{(m_n c^2)^2 + (p_n c)^2} - m_n c^2$$

$$K_n = \sqrt{(m_n c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_n c^2 = K_\pi + K_n = K_\pi + \sqrt{(m_n c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_n c^2 = E.$$

$$(m_n c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi = E^2 + (m_n c^2)^2 + K_\pi^2 + 2Em_\pi c^2 - 2EK_\pi - 2m_n c^2 K_\pi. \text{ Collecting terms we find:}$$

$$K_\pi(2m_\pi c^2 + 2E + 2m_n c^2) = E^2 + 2Em_\pi c^2$$

$$\Rightarrow K_\pi = \frac{(41.4 \text{ MeV})^2 + 2(41.4 \text{ MeV})(939.6 \text{ MeV})}{2(135.0 \text{ MeV}) + 2(41.4 \text{ MeV}) + 2(939.6 \text{ MeV})} = 35.62 \text{ MeV}.$$

So the fractional energy carried by the pion is $\frac{35.62}{41.4} = 0.86$, and that of the neutron is 0.14.